Higgs bosons in the two-doublet model with explicit CP violation

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work with

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- Input Lagrangians:
  general two-doublet Higgs sector,
  Higgs boson-squark interactions
- Effective λ_i parameters at the one-loop in the MSSM
- Mass eigenstates in the local minimum of the two-doublet Higgs potential
- Comparison with CPsuperH and FeynHiggs packages

Hermitian two-Higgs-doublet potential in the $-\mu^2\varphi^2+\lambda\varphi^4$ representation

$$
\begin{aligned}
U(\varphi_1,\varphi_2) = \quad & -\mu_1^2(\varphi_1^+\varphi_1)-\mu_2^2(\varphi_2^+\varphi_2) \\
& -\mu_{12}^2(\varphi_1^+\varphi_2)-\overset{*}{\mu_{12}^2}(\varphi_2^+\varphi_1) \\
+\bar\lambda_1(\varphi_1^+\varphi_1)^2+\bar\lambda_2(\varphi_2^+\varphi_2)^2 & +\bar\lambda_3(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2)+\bar\lambda_4(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1) \\
& +\frac{\bar\lambda_5}{2}(\varphi_1^+\varphi_2)(\varphi_1^+\varphi_2)+\frac{\overset{*}{\bar\lambda_5}}{2}(\varphi_2^+\varphi_1)(\varphi_2^+\varphi_1) \\
& +\bar\lambda_6(\varphi_1^+\varphi_1)(\varphi_1^+\varphi_2)+\overset{*}{\bar\lambda_6}(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_1) \\
& +\bar\lambda_7(\varphi_2^+\varphi_2)(\varphi_1^+\varphi_2)+\overset{*}{\bar\lambda_7}(\varphi_2^+\varphi_1)\varphi_2^+\varphi_2)
\end{aligned}
$$

where λ_5, λ_6 λ_7 are complex variables,

the VEV's

$$
<\varphi_1>=\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad <\varphi_2>=\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\zeta} \end{pmatrix}
$$

Mass eigenstates h_1, h_2 h_3

($s_\alpha = \mathtt{sin}\alpha$, $c_\beta = \mathtt{cos}\beta$ etc.)

$$\begin{aligned}
\varphi_1 &= \begin{pmatrix} -i*(-H^+ s_\beta + G^+ c_\beta) \\ \frac{1}{\sqrt{2}}[v_1 + Hc_\alpha - hs_\alpha + i*(A^0 c_\beta + G^{'} s_\beta)] \end{pmatrix} \\
\varphi_2 &= e^{i\xi} \begin{pmatrix} -i*(H^+ c_\beta + G^+ s_\beta) \\ \frac{1}{\sqrt{2}}[v_2 e^{i\zeta} + Hs_\alpha + hc_\alpha + i*(-A^0 s_\beta + G^{'} c_\beta)] \end{pmatrix}
\end{aligned}$$

h, H CP-even bosons, A CP-odd boson, $H^\pm$ charged boson, G Goldstone modes of the CP-conserving limit: $\mathtt{Im}\mu_{12}^2 = 0$, $\mathtt{Im}\lambda_{5,6,7}$ =0, $\xi = \zeta$ =0. With imaginary parts nonzero

$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

In the CP conserving limit the mixing matrix a_{ij}

not only $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, but also $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \ldots$

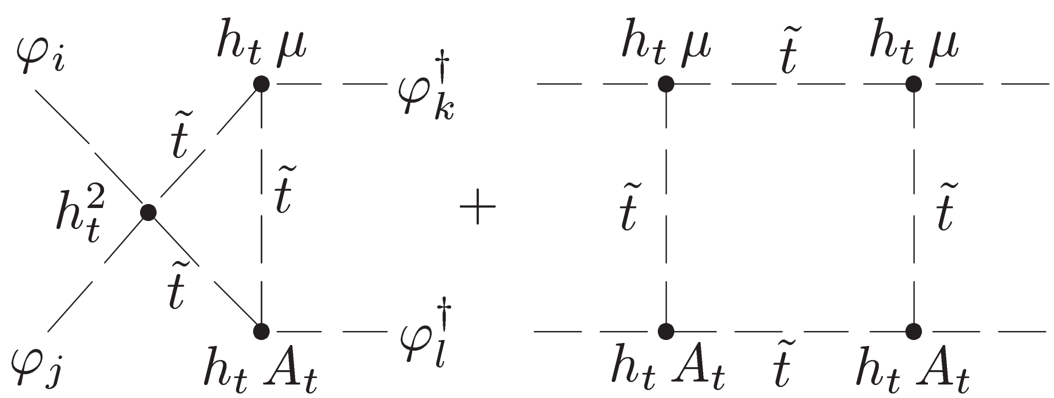
In the MSSM at the scale M_{SUSY}

$$\bar\lambda_1^{SUSY} = \bar\lambda_2^{SUSY} = \frac{g^2+g_1^2}{8},\ \bar\lambda_3^{SUSY} = \frac{g^2-g_1^2}{4},$$
$$\bar\lambda_4^{SUSY} = -\frac{g^2}{2}, \quad \bar\lambda_{5,6,7}^{SUSY} = 0.$$

with radiative corrections $\Delta\lambda$ at the scale m_{top}

$$2(\bar\lambda_{1,2}^{SUSY} - \bar\lambda_{1,2}) = \Delta\bar\lambda_{1,2},\ \ \bar\lambda_{3,4}^{SUSY} - \bar\lambda_{3,4} = \Delta\bar\lambda_{3,4}$$

In the effective two-Higgs-doublet potential $\Delta\lambda_{5,6,7}$ are radiatively induced by the loop contributions which are generally speaking complex.



A. Pilaftsis, C.E.M. Wagner, Nucl.Phys. B553 (1999) 3
M. Carena, J. Ellis, A. Pilaftsis, C.E.M. Wagner, Nucl.Phys. B586 (2000) 92

General potential, including Higgs fields and one generation of scalar quarks

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\widetilde{Q}}\,,$$

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\widetilde{Q}}^2 \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + M_{\widetilde{U}}^2 \widetilde{U}^* \widetilde{U} + M_{\widetilde{D}}^2 \widetilde{D}^* \widetilde{D}$$

$$\begin{aligned}
\mathcal{V}_\Gamma &= \Gamma_i^D \left(\Phi_i^\dagger \widetilde{Q}\right) \widetilde{D} + \Gamma_i^U \left(i\Phi_i^T \sigma_2 \widetilde{Q}\right) \widetilde{U} \\
&\quad + \overset{*}{\Gamma}{}_i^D \left(\widetilde{Q}^\dagger \Phi_i\right) \widetilde{D}^* - \overset{*}{\Gamma}{}_i^U \left(i\widetilde{Q}^\dagger \sigma_2 \Phi_i^*\right) \widetilde{U}^* \\
\mathcal{V}_\Lambda &= \Lambda_{ik}^{jl} \left(\Phi_i^\dagger \Phi_j\right) \left(\Phi_k^\dagger \Phi_l\right) \\
&\quad + \left(\Phi_i^\dagger \Phi_j\right) \left[\Lambda_{ij}^Q \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + \Lambda_{ij}^U \widetilde{U}^* \widetilde{U} + \Lambda_{ij}^D \widetilde{D}^* \widetilde{D}\right] \\
&\quad + \bar{\Lambda}_{ij}^Q \left(\Phi_i^\dagger \widetilde{Q}\right) \left(\widetilde{Q}^\dagger \Phi_j\right) + \frac{1}{2} \left[\Lambda \epsilon_{ij} \left(i\Phi_i^T \sigma_2 \Phi_j\right) \widetilde{D}^* \widetilde{U} + \text{h.c.}\right], \qquad i,j,k,l = 1,2
\end{aligned}$$

where

$\widetilde{Q}^T = (\tilde{t}_L, \tilde{b}_L)$, $\widetilde{U}^* = \tilde{t}_R$, $\widetilde{D}^* = \tilde{b}_R$,

$h_t = \frac{\sqrt{2} m_t}{v \sin\beta}$, $h_b = \frac{\sqrt{2} m_b}{v \cos\beta}$,

$\Gamma^U = h_U(-\overset{*}{\mu}, A_U), \qquad \Gamma^D = h_D(A_D, -\overset{*}{\mu})$

H. Haber, R. Hempfling, Phys.Rev. D48 (1993) 4280

The effective field theory approach:

H. Haber, R. Hempfling, Phys.Rev. D48 (1993) 4280

M. Carena, M. Quiros, C. Wagner, Nucl.Phys. B461 (1996) 407

squark-Higgs boson potential, which explicitly describes sparticle interactions at the scale above M_{SUSY} is matched to an effective THDM potential at the scale below M_{SUSY}, where the sparticles decouple. λ_i parameters are expressed through M_{SUSY}, μ, A_t, A_b,...

We reproduced the result of

A. Pilaftsis, C. Wagner, Nucl.Phys. B553 (1999) 3

for complex parameters $\lambda_{1,...7}$ at the one-loop additionally taking into account the nonleading D-term contributions and the wave-function renormalization, using the effective potential method.

One-loop renormalized effective potential

$$\mathcal{V} = \mathcal{V}^0 + \frac{N_C}{32\pi^2}\mathrm{tr}\mathcal{M}^4\left[\ln\left(\frac{\mathcal{M}^2}{\sigma^2}\right) - \frac{3}{2}\right],$$

σ renormalization scale

Mass matrix of the scalar quarks

$$\mathcal{M}^2 = \mathcal{M}^2_M + \mathcal{M}^2_\Gamma + \mathcal{M}^2_\Lambda, \quad (\mathcal{M}^2_X)_{ab} \equiv \frac{\partial^2\mathcal{V}_X}{\partial\tilde{Q}_a\,\partial\,\tilde{Q}^*_b}$$

$X \equiv \{M,\,\Gamma,\,\Lambda\}$

Decomposition of $\mathcal{V}$ in the inverse powers of M^2_{SUSY}

$$V_{quartic} = \Lambda^{jl}_{ik}(\Phi^\dagger_i\Phi_j)(\Phi^\dagger_k\Phi_l) + \frac{N_C}{32\pi^2}\left\{\ln\left(\frac{M^2_{\mathrm{SUSY}}}{\sigma^2}\right)\mathrm{tr}(\mathcal{M}^2_\Lambda)^2 + \right.$$

$$\left. + \frac{1}{M^2_{\mathrm{SUSY}}}\mathrm{tr}(\mathcal{M}^2_\Gamma)^2\,\mathcal{M}^2_\Lambda - \frac{1}{12M^4_{\mathrm{SUSY}}}\mathrm{tr}(\mathcal{M}^2_\Gamma)^4\right\}.$$

One-loop

$$\lambda_1 = \frac{g_2^2+g_1^2}{8} + \frac{3}{32\pi^2}\left[h_b^4 \frac{|A_b|^2}{M_{\rm SUSY}^2}\left(2 - \frac{|A_b|^2}{6M_{\rm SUSY}^2}\right) - h_t^4 \frac{|\,\mu|^4}{6M_{\rm SUSY}^4} + 2h_b^4 l + \frac{g_2^2+g_1^2}{4\,M_{\rm SUSY}^2}(h_t^2|\,\mu|^2 - h_b^2|A_b|^2)\right] + + \Delta\,\lambda_1^{field} + \frac{1}{768\pi^2}\left(11g_1^4 + 9g_2^4 - 36\,(g_1^2+g_2^2)\,h_b^2\right) l,$$

$$\lambda_2 = \lambda_1\ (t \longleftrightarrow b),$$

$$\lambda_3 = \frac{g_2^2-g_1^2}{4}\left[1 - \frac{3}{16\pi^2}\,(h_t^2+h_b^2)\,l\right] + \frac{3}{8\pi^2}\,h_t^2 h_b^2\left[l + \frac{1}{2}X_{tb}\right] + \frac{3}{96\pi^2}\,\frac{|\mu|^2}{M_{\rm SUSY}^2}\left[h_t^4\left(3 - \frac{|A_t|^2}{M_{\rm SUSY}^2}\right) + h_b^4\left(3 - \frac{|A_b|^2}{M_{\rm SUSY}^2}\right)\right] + \frac{3(g_2^2-g_1^2)\left[h_b^2(|\mu|^2-|A_b|^2) + h_t^2(|\mu|^2-|A_t|^2)\right]}{128\,\pi^2 M_{\rm SUSY}^2} + \Delta\,\lambda_3^{field} + \frac{9g_2^4-11g_1^4}{384\,\pi^2}\,l,$$

$$\lambda_4 = -\frac{g_2^2}{2}\left[1 - \frac{3}{16\pi^2}\,(h_t^2+h_b^2)\,l\right] - \frac{3}{8\pi^2}\,h_t^2 h_b^2\left[l + \frac{1}{2}X_{tb}\right] + \frac{3}{96\pi^2}\,\frac{|\mu|^2}{M_{\rm SUSY}^2}\left[h_t^4\left(3 - \frac{|A_t|^2}{M_{\rm SUSY}^2}\right) + h_b^4\left(3 - \frac{|A_b|^2}{M_{\rm SUSY}^2}\right)\right] - \frac{3g_2^2\left[h_b^2(|\mu|^2-|A_b|^2) + h_t^2(|\mu|^2-|A_t|^2)\right]}{64\pi^2 M_{\rm SUSY}^2} + \Delta\,\lambda_4^{field} - \frac{3g_2^4}{64\pi^2}\,l\,,$$

where $l = \mathtt{log}\frac{M_{SUSY}^2}{m_{top}^2}$**,** $h_t = \frac{\sqrt{2}m_{top}}{v s_\beta}$, $h_b = \frac{\sqrt{2}m_b}{v c_\beta}$

$$X_{tb} \equiv \frac{|A_t|^2 + |A_b|^2 + 2\mathtt{Re}(A_b^* A_t)}{2M_{\rm SUSY}^2} - \frac{|\mu|^2}{M_{\rm SUSY}^2} - \frac{||\mu|^2 - A_b^* A_t|^2}{6M_{\rm SUSY}^4}\,.$$

$$\lambda_5 = -\Delta\lambda_5 \quad = \quad -\frac{3}{96\,\pi^2}\left(h_t^4\left(\frac{\mu A_t}{M_{\rm SUSY}^2}\right)^2 + h_b^4\left(\frac{\mu A_b}{M_{\rm SUSY}^2}\right)^2\right),$$

$$\lambda_6 = -\Delta\lambda_6 \quad = \frac{3}{96\,\pi^2}\left[h_t^4\,\frac{|\mu|^2\mu A_t}{M_{\rm SUSY}^4} - h_b^4\,\frac{\mu A_b}{M_{\rm SUSY}^2}\left(6 - \frac{|A_b|^2}{M_{\rm SUSY}^2}\right)\right.$$
$$\left. + (h_b^2 A_b - h_t^2 A_t)\,\frac{3\,\mu}{M_{\rm SUSY}^2}\,\frac{g_2^2 + g_1^2}{4}\right],$$

$$\lambda_7 = -\Delta\lambda_7 \quad = \frac{3}{96\,\pi^2}\left[h_b^4\,\frac{|\mu|^2\mu A_b}{M_{\rm SUSY}^4} - h_t^4\,\frac{\mu A_t}{M_{\rm SUSY}^2}\left(6 - \frac{|A_t|^2}{M_{\rm SUSY}^2}\right)\right.$$
$$\left. + (h_t^2 A_t - h_b^2 A_b)\,\frac{3\,\mu}{M_{\rm SUSY}^2}\,\frac{g_2^2 + g_1^2}{4}\right].$$

Wave-function renormalization

$$\Delta\,\lambda_1^{field} = \frac{1}{2}(g_1^2 + g_2^2)A'_{11}, \qquad \Delta\,\lambda_2^{field} = \frac{1}{2}(g_1^2 + g_2^2)A'_{22},$$

$$\Delta\,\lambda_3^{field} = -\,\frac{1}{4}(g_1^2 - g_2^2)(A'_{11} + A'_{22}), \qquad \Delta\,\lambda_4^{field} = -\,\frac{1}{2}g_2^2(A'_{11} + A'_{22}),$$

$$\Delta\,\lambda_5^{field} = 0,$$

$$\Delta\,\lambda_6^{field} = \frac{1}{8}(g_1^2 + g_2^2)(A'_{12} - {A'_{21}}^*) = 0, \;\; \Delta\,\lambda_7^{field} = \frac{1}{8}(g_1^2 + g_2^2)(A'_{21} - {A'_{12}}^*) = 0.$$

$$A'_{ij} = \left[h_U^2\begin{bmatrix} |\mu|^2 & -\mu^* A_U^* \\ -\mu A_U & |A_U|^2 \end{bmatrix} + h_D^2\begin{bmatrix} |A_D|^2 & -\mu^* A_D^* \\ -\mu A_D & |\mu|^2 \end{bmatrix}\right] \times$$
$$\times\left(-\frac{3}{96\pi^2 M_{SUSY}^2}\right)\left(1 - \frac{1}{2}\,l\right).$$

Numerical comparison of various corrections to the λ_i parameters at the scale m_{top} in the CP conserving limit $\varphi \equiv \mathtt{arg}(\mu A_{t,b}) = 0$.

i	1	2	3	4	5	6	7
$\mathcal{O}(h_t^4)$ [Q]	0.907	-0.203	0.057	0.057	0.227	-0.453	0.057
$\Delta\lambda_i$	0.860	-0.182	0.054	0.072	0.227	-0.442	0.046
1-loop [PW]	0.907	-0.191	0.064	0.043	0.227	-0.453	0.057
1+2-loop ·[PW]	0.761	-0.152	0.052	0.032	0.135	-0.371	0.044
2-loop [PW]	-0.146	0.039	-0.012	-0.011	-0.092	0.082	-0.013
1-loop(D+wfr)	-0.047	0.009	-0.010	0.028	0	0.011	-0.011
(D+wfr)/(2-loop)	0.32	0.23	0.83	-2.55	0	0.13	0.85
ALL	0.715	-0.143	0.042	0.061	0.135	-0.360	0.033

$\mathcal{O}(h_t^4)$	1-loop, only $\sim h_t^4$ terms
$\Delta\lambda_i$	our, 1-loop + 1-loop(D-terms+wfr)
[PW]	2-loop, A.Pilaftsis,C.E.M.Wagner,Nucl.Phys.B553 (1999) 3
D	1-loop, D-terms contribution
wfr	1-loop, wave-function renormalization contribution
(D+wfr)/(2-loop)	$\frac{\Delta\lambda(\text{D+wfr})}{\Delta\lambda(\text{2-loop})[\text{PW}]}$
ALL	1-loop[PW] + 2-loop[PW] + 1-loop(D-terms+wfr)

CPsuperH parameter set: $m_Z = 91.19\,\text{GeV}$, $m_b = 3\,\text{GeV}$, $m_t = 175\,\text{GeV}$, $m_W = 79.96\,\text{GeV}$, $g_2 = 0.6517$, $g_1 = 0.3573$, $v = 245.4\,\text{GeV}$, $G_F = 1.17 \cdot 10^{-5}\,\text{GeV}^{-2}$, $\alpha_S(m_t) = 0.1072$, $\sigma = m_t$. $M_{SUSY} = 500\,\text{GeV}$, $|A_t| = |A_b| = A = 1000\,\text{GeV}$, $|\mu| = 2000\,\text{GeV}$.

Diagonalization in the minimum performed in two stages. First make the transformation to the CP conserving limit

$$\lambda_1 = \frac{1}{2v^2}[(\frac{s_\alpha}{c_\beta})^2 m_h^2 + (\frac{c_\alpha}{c_\beta})^2 m_H^2 - \frac{s_\beta}{c_\beta^3}\mathtt{Re}\mu_{12}^2] + \frac{1}{4}(\mathtt{Re}\lambda_7 \mathtt{tg}^3\beta - 3\mathtt{Re}\lambda_6 \mathtt{tg}\beta),$$

$$\lambda_2 = \frac{1}{2v^2}[(\frac{c_\alpha}{s_\beta})^2 m_h^2 + (\frac{s_\alpha}{s_\beta})^2 m_H^2 - \frac{c_\beta}{s_\beta^3}\mathtt{Re}\mu_{12}^2] + \frac{1}{4}(\mathtt{Re}\lambda_6 \mathtt{ctg}^3\beta - 3\mathtt{Re}\lambda_7 \mathtt{ctg}\beta),$$

$$\lambda_3 = \frac{1}{v^2}[2m_{H\pm}^2 - \frac{\mathtt{Re}\mu_{12}^2}{s_\beta c_\beta} + \frac{s_{2\alpha}}{s_{2\beta}}(m_H^2 - m_h^2)] - \frac{\mathtt{Re}\lambda_6}{2}\mathtt{ctg}\beta - \frac{\mathtt{Re}\lambda_7}{2}\mathtt{tg}\beta,$$

$$\lambda_4 = \frac{1}{v^2}(\frac{\mathtt{Re}\mu_{12}^2}{s_\beta c_\beta} + m_A^2 - 2m_{H\pm}^2) - \frac{\mathtt{Re}\lambda_6}{2}\mathtt{ctg}\beta - \frac{\mathtt{Re}\lambda_7}{2}\mathtt{tg}\beta,$$

$$\mathtt{Re}\lambda_5 = \frac{1}{v^2}(\frac{\mathtt{Re}\mu_{12}^2}{s_\beta c_\beta} - m_A^2) - \frac{\mathtt{Re}\lambda_6}{2}\mathtt{ctg}\beta - \frac{\mathtt{Re}\lambda_7}{2}\mathtt{tg}\beta,$$

$$\mu_1^2 = \lambda_1 v_1^2 + (\lambda_3 + \lambda_4 + \mathtt{Re}\lambda_5)\frac{v_2^2}{2} - \mathtt{Re}\mu_{12}^2 \mathtt{tg}\beta + \frac{v^2 s_\beta^2}{2}(3\mathtt{Re}\lambda_6 \mathtt{ctg}\beta + \mathtt{Re}\lambda_7 \mathtt{tg}\beta),$$

$$\mu_2^2 = \lambda_2 v_2^2 + (\lambda_3 + \lambda_4 + \mathtt{Re}\lambda_5)\frac{v_1^2}{2} - \mathtt{Re}\mu_{12}^2 \mathtt{ctg}\beta + \frac{v^2 c_\beta^2}{2}(\mathtt{Re}\lambda_6 \mathtt{ctg}\beta + 3\mathtt{Re}\lambda_7 \mathtt{tg}\beta).$$

M.D., A. Semenov, hep-ph/9812246, Eur.Phys.J C28 (2003) 223, F. Boudjema, A. Semenov, Phys.Rev. D66 (2002) 095007, J. Gunion, H. Haber, Phys.Rev. D67 (2003) 075019

Masses of the CP-conserving limit expressed in terms of lambdas

$$\lambda_{1,2} \equiv \lambda_{1,2}^{SUSY} - \Delta\lambda_{1,2}/2, \quad \lambda_{3,4} \equiv \lambda_{3,4}^{SUSY} - \Delta\lambda_{3,4}, \quad \lambda_{5,6,7} \equiv -\Delta\lambda_{5,6,7},$$

$$\mathtt{tg}2\alpha = \frac{s_{2\beta}(m_A^2 + m_Z^2) + v^2((\Delta\bar\lambda_3 + \Delta\bar\lambda_4)s_{2\beta} + 2c_\beta^2 \Delta\mathtt{Re}\bar\lambda_6 + 2s_\beta^2 \mathtt{Re}\Delta\bar\lambda_7)}{c_{2\beta}(m_A^2 - m_Z^2) + v^2(\Delta\bar\lambda_1 c_\beta^2 - \Delta\bar\lambda_2 s_\beta^2 - \mathtt{Re}\Delta\bar\lambda_5 c_{2\beta} + (\mathtt{Re}\Delta\bar\lambda_6 - \mathtt{Re}\Delta\bar\lambda_7)s_{2\beta})}$$

$$\begin{aligned}
m_H^2 \quad &= \quad c_{\alpha+\beta}^2 m_Z^2 + s_{\alpha-\beta}^2 m_A^2 \\
&-v^2(\Delta\bar\lambda_1 c_\alpha^2 c_\beta^2 + \Delta\bar\lambda_2 s_\alpha^2 s_\beta^2 + 2(\Delta\bar\lambda_3 + \Delta\bar\lambda_4)c_\alpha c_\beta s_\alpha s_\beta \\
&+\mathtt{Re}\Delta\bar\lambda_5(c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2)) + 2s_{\alpha+\beta}(\mathtt{Re}\Delta\bar\lambda_6 c_\alpha c_\beta + \mathtt{Re}\Delta\bar\lambda_7 s_\alpha s_\beta)
\end{aligned}$$

$$\begin{aligned}
m_h^2 \quad &= \quad s_{\alpha+\beta}^2 m_Z^2 + c_{\alpha-\beta}^2 m_A^2 \\
&-v^2(\Delta\bar\lambda_1 s_\alpha^2 c_\beta^2 + \Delta\bar\lambda_2 c_\alpha^2 s_\beta^2 - 2(\Delta\bar\lambda_3 + \Delta\bar\lambda_4)c_\alpha c_\beta s_\alpha s_\beta \\
&+\mathtt{Re}\Delta\bar\lambda_5(c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2)) - 2c_{\alpha+\beta}(\mathtt{Re}\Delta\bar\lambda_6 s_\alpha c_\beta - \mathtt{Re}\Delta\bar\lambda_7 c_\alpha s_\beta)
\end{aligned}$$

$$m_{H\pm}^2 \quad = \quad m_W^2 + m_A^2 + \frac{v^2}{2}(\Delta\bar\lambda_4 - \mathtt{Re}\Delta\bar\lambda_5)$$

$$g_1^2 + g^2 = g^2\, m_Z^2/m_W^2, \quad g^2 - g_1^2 = g^2(2 - \, m_Z^2/m_W^2)$$

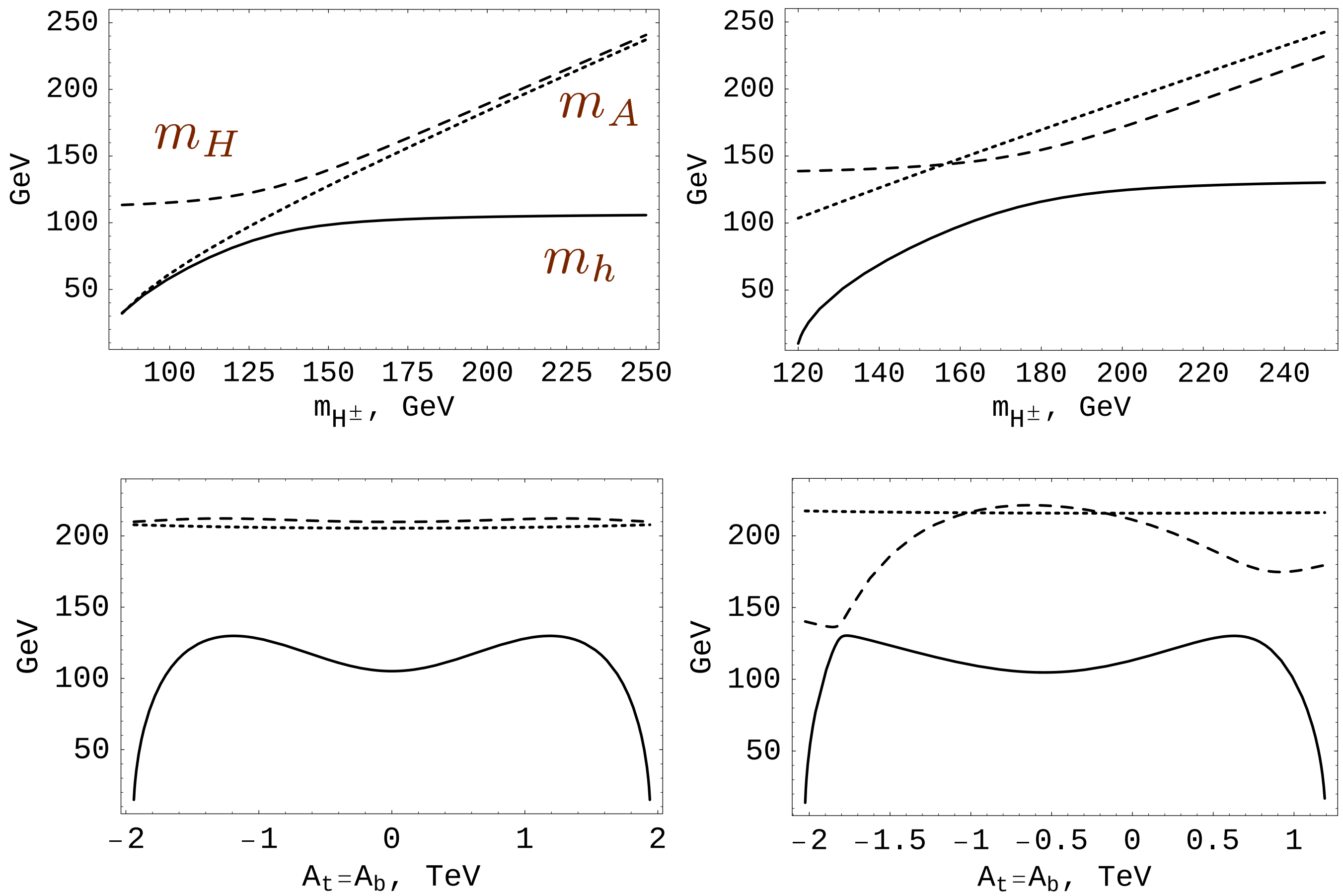


Figure 1: Neutral Higgs boson masses h, H, A versus $m_{H^\pm}$ and the trilinear parameters A_t, A_b in the CP conserving limit. Solid line denotes the m_h mass, short dashed – m_A, long-dashed – m_H. (a) tgβ =5, M_{SUSY} =0.5 TeV, $A_t = A_b = \mu$ =0; (b) tgβ =5, M_{SUSY} =0.5 TeV, $A_t = A_b$ =0.9 TeV, $\mu = -1.5$ TeV; (c) tgβ =5, M_{SUSY} =0.5 TeV, $m_{H^\pm}$ =220 GeV, μ =0, $A_t = A_b$; (d) tgβ =4, M_{SUSY} =0.5 TeV, $m_{H^\pm}$ =220 GeV, $\mu = -2$ TeV, $A_t = A_b$.

$$U(\varphi_1,\varphi_2) = c_0 A + c_1 hA + c_2 HA + \frac{m_h^2}{2}H^2 + \frac{m_H^2}{2}H^2 + \frac{m_A^2}{2}A^2 + m_{H^\pm}^2 H^+H^- + \texttt{third and fourth order terms in } h,\, H,\, A,\, H^\pm$$

$$c_0 = -v\,(\mathtt{Im}\mu_{12}^2 - \frac{v^2}{2}s_\beta c_\beta \mathtt{Im}\bar{\lambda}_5) + \frac{v^3}{2}(c_\beta^2\mathtt{Im}\bar{\lambda}_6 + s_\beta^2\mathtt{Im}\bar{\lambda}_7)$$

in the classical minimum c_0 =0

$$c_1 = \frac{v^2}{2}(s_\alpha s_\beta - c_\alpha c_\beta)\mathtt{Im}\bar{\lambda}_5 + v^2\,(s_\alpha c_\beta \mathtt{Im}\bar{\lambda}_6 - c_\alpha s_\beta \mathtt{Im}\bar{\lambda}_7)$$

$$c_2 = -\frac{v^2}{2}(s_\alpha c_\beta + c_\alpha s_\beta)\mathtt{Im}\bar{\lambda}_5 - v^2\,(c_\alpha c_\beta \mathtt{Im}\bar{\lambda}_6 + s_\alpha s_\beta \mathtt{Im}\bar{\lambda}_7)$$

Then perform orthogonal rotation a_{ij} (i,j =1,2,3) in h, H, A sector to diagonalize

$$(h, H, A)\; M^2 \begin{pmatrix} h \\ H \\ A \end{pmatrix} = (h_1, h_2, h_3)\; a_{ik}^T\; M_{kl}^2\; a_{lj} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

with the mass matrix

$$M^2 = \frac{1}{2}\begin{pmatrix} m_h^2 & 0 & c_1 \\ 0 & m_H^2 & c_2 \\ c_1 & c_2 & m_A^2 \end{pmatrix}$$

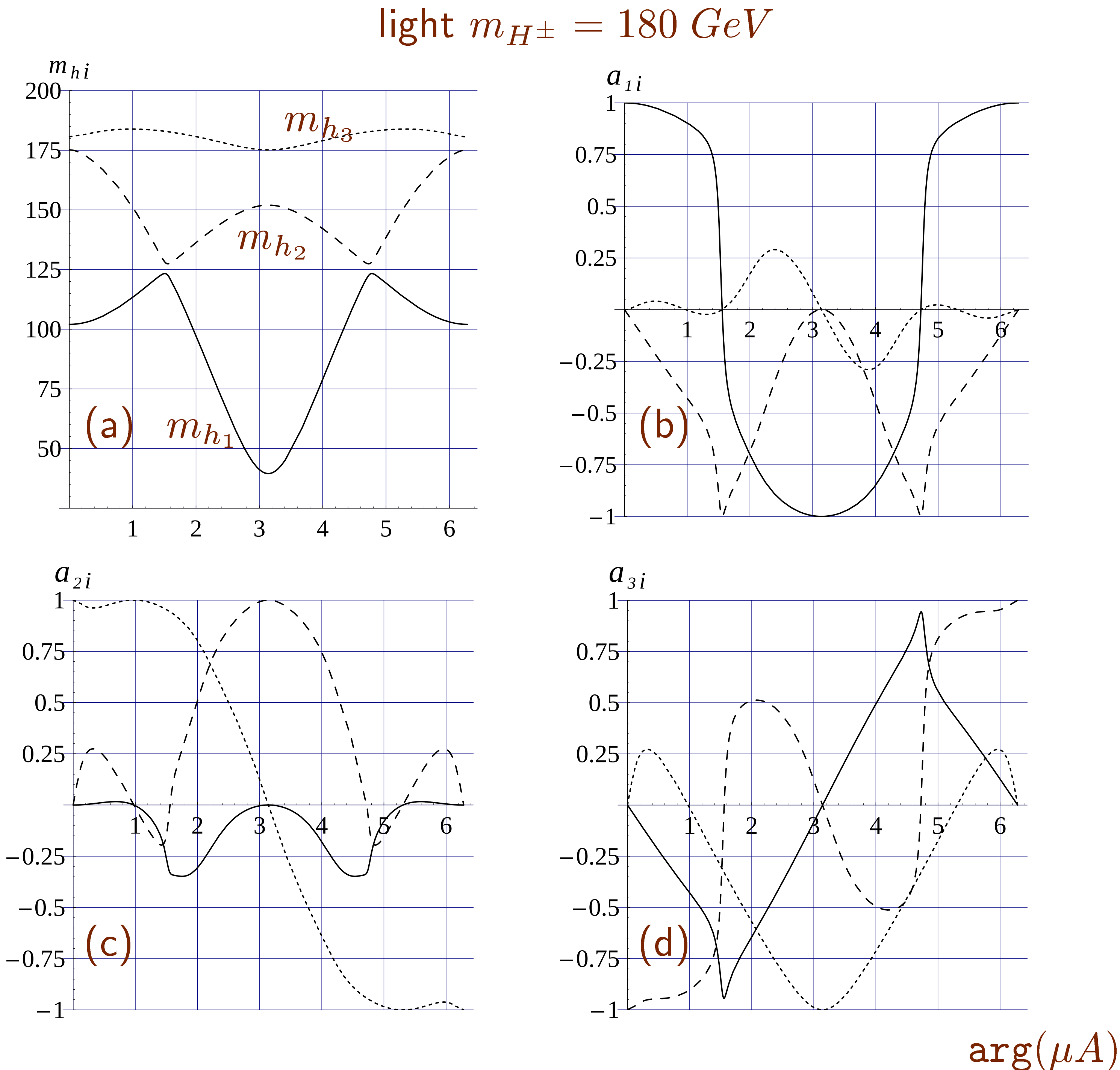


Figure 2: (a) Neutral Higgs boson masses, (b),(c),(d) the matrix elements a_{ij} versus the phase $\varphi = \arg(\mu A)$ at the parameter values tgβ =5, $m_{H^\pm}$ =180 GeV, M_{SUSY} =0.5 TeV, $A_t = A_b$ =1 TeV, μ =2 TeV. Solid line denotes $i = 1$, long dashed – $i = 2$ and short dashed – $i = 3$.

heavy $m_{H^\pm} = 300\ GeV$

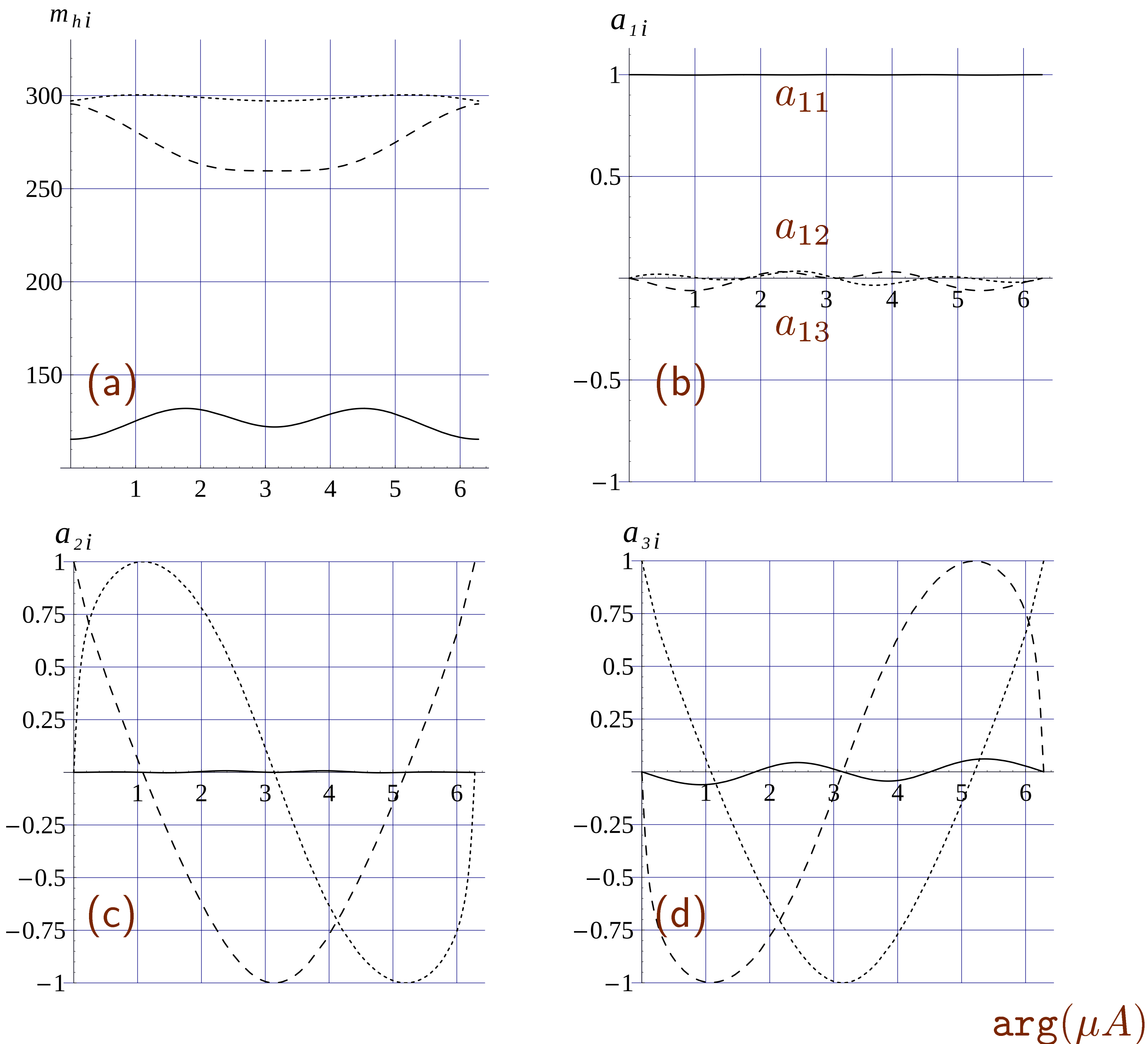


Figure 3: (a) Neutral Higgs boson masses (GeV), (b),(c),(d) the matrix elements a_{ij} versus the phase $\varphi = \mathtt{arg}(\mu A)$ at the parameter values $\mathrm{tg}\beta$ =5, $m_{H^\pm}$ =300 GeV, M_{SUSY} =0.5 TeV, $A_t = A_b$ =1 TeV, μ =2 TeV. Solid line denotes $i = 1$, long dashed – $i = 2$ and short dashed – $i = 3$.

Our approach is algorithmized in the form of CompHEP 41 model format. Possible to calculate automatically complete tree-level sets of diagrams for decays and collisions (any initial states) with the following generation of unweighted events.

The output is compared with CPsuperH and FeynHiggs packages, based on diagrammatic approaches. α-rotation of $(h,\ H)$ is not introduced, direct transformation of $SU(2)$ eigenstates to mass eigenstates $H_1,\ H_2,\ H_3$ is performed.

	$\varphi=0$	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
m_{h_1},	115.4	118.7	125.9	131.4	130.7	125.2	122.0
m'_{h_1}	112.1	114.4	119.7	124.2	125.0	123.0	121.6
m_{H_1} [26]	115.8	118.8	125.5	130.2	123.2	98.2	78.0
m_{H_1} [16]	106.8	109.0	113.9	117.4	114.9	105.7	99.4
m_{h_2}	295.5	289.6	279.7	269.3	262.2	259.8	259.6
m'_{h_2}	294.4	291.0	283.9	276.2	270.6	268.1	267.6
m_{H_2} [26]	295.6	290.0	279.1	264.3	249.2	239.7	236.9
m_{H_2} [16]	302.2	297.8	290.9	282.2	273.9	268.3	264.4
m_{h_3}	297.1	299.5	300.4	299.9	298.8	297.6	297.1
m'_{h_3}	298.2	299.1	299.2	298.2	296.7	295.1	294.4
m_{H_3} [26]	297.6	300.0	301.1	301.3	300.9	300.4	300.2
m_{H_3} [16]	302.3	304.4	305.0	304.5	303.5	302.4	302.0
$\Gamma_{h_1\to gg}\times 10^4$	1.378	1.529	1.907	2.220	2.101	1.707	1.516
$\Gamma'_{h_1\to gg}\times 10^4$	1.283	1.381	1.624	1.841	1.846	1.687	1.597
$\Gamma''_{h_1\to gg}\times 10^4$	2.128	2.373	2.991	3.496	3.266	2.593	2.277
$\Gamma_{H_1\to gg}\times 10^4$ [26]	2.040	2.187	2.462	2.225	0.863	0.037	0.110
$\Gamma_{H_1\to gg}\times 10^4$ [16]	1.878	1.964	2.107	1.961	1.262	0.503	0.263
$\Gamma_{h_1\to\gamma\gamma}\times 10^6$	7.703	8.593	10.981	13.313	12.953	10.645	9.508
$\Gamma'_{h_1\to\gamma\gamma}\times 10^6$	6.887	7.447	8.896	10.369	10.683	9.935	9.460
$\Gamma''_{h_1\to\gamma\gamma}\times 10^6$	7.582	8.484	10.925	13.306	12.945	10.594	9.438
$\Gamma_{H_1\to\gamma\gamma}\times 10^6$ [26]	6.373	7.058	9.038	11.217	9.983	5.336	3.021
$\Gamma_{H_1\to\gamma\gamma}\times 10^6$ [16]	5.796	6.287	7.605	8.996	8.969	7.223	6.101
$\Gamma_{h_1\to\mu\bar\mu}\times 10^{-5}$	0.212	0.204	0.179	0.166	0.218	0.304	0.341
$\Gamma_{H_1\to\mu\bar\mu}\times 10^{-5}$ [16]	0.157	0.152	0.141	0.137	0.175	0.240	0.269
$\Gamma_{h_1\to\tau\bar\tau}\times 10^{-3}$	0.591	0.567	0.498	0.461	0.607	0.848	0.950
$\Gamma_{H_1\to\tau\bar\tau}\times 10^{-3}$ [16]	0.435	0.423	0.391	0.382	0.485	0.668	0.746
$\Gamma_{h_1\to d\bar d}\times 10^{-7}$	0.202	0.194	0.170	0.158	0.208	0.290	0.325
$\Gamma_{H_1\to d\bar d}\times 10^{-7}$ [16]	0.193	0.187	0.171	0.167	0.212	0.297	0.335
$\Gamma_{h_1\to s\bar s}\times 10^{-5}$	0.744	0.713	0.626	0.580	0.764	1.066	1.195
$\Gamma_{H_1\to s\bar s}\times 10^{-5}$ [16]	0.709	0.687	0.629	0.612	0.780	1.089	1.230
$\Gamma_{h_1\to c\bar c}\times 10^{-3}$	0.083	0.086	0.093	0.097	0.095	0.088	0.083
$\Gamma_{H_1\to c\bar c}\times 10^{-3}$ [16]	0.101	0.103	0.108	0.111	0.107	0.096	0.089
$\Gamma_{h_1\to b\bar b}\times 10^{-2}$	0.504	0.483	0.424	0.393	0.518	0.724	0.810
$\Gamma_{H_1\to b\bar b}\times 10^{-2}$ [16]	0.481	0.469	0.426	0.414	0.528	0.737	0.832

Table 5: The parameter set $\alpha_{EM}(m_Z)=0.7812\cdot 10^{-2}$, $\alpha_S(m_Z)$ =0.1172, $G_F=1.174\cdot 10^{-5}\,\mathrm{GeV}^{-2}$, m_b =3 GeV $\mathtt{tg}\beta = \mathtt{5}$, $M_{SUSY}=500\,\mathrm{GeV}$, $|A_t|=|A_b|=A=1000\,\mathrm{GeV}$, $|\mu|=2000\,\mathrm{GeV}$, $m_{H^\pm}=300\,\mathrm{GeV}$. Our results togrther with CPsuperH [16] and FeynHiggs [26] with options 2003011100 (the one-loop regime). m'_h, Γ' two-loop effective λ_i, Γ/Γ'' in our case sparticles not involved/included.

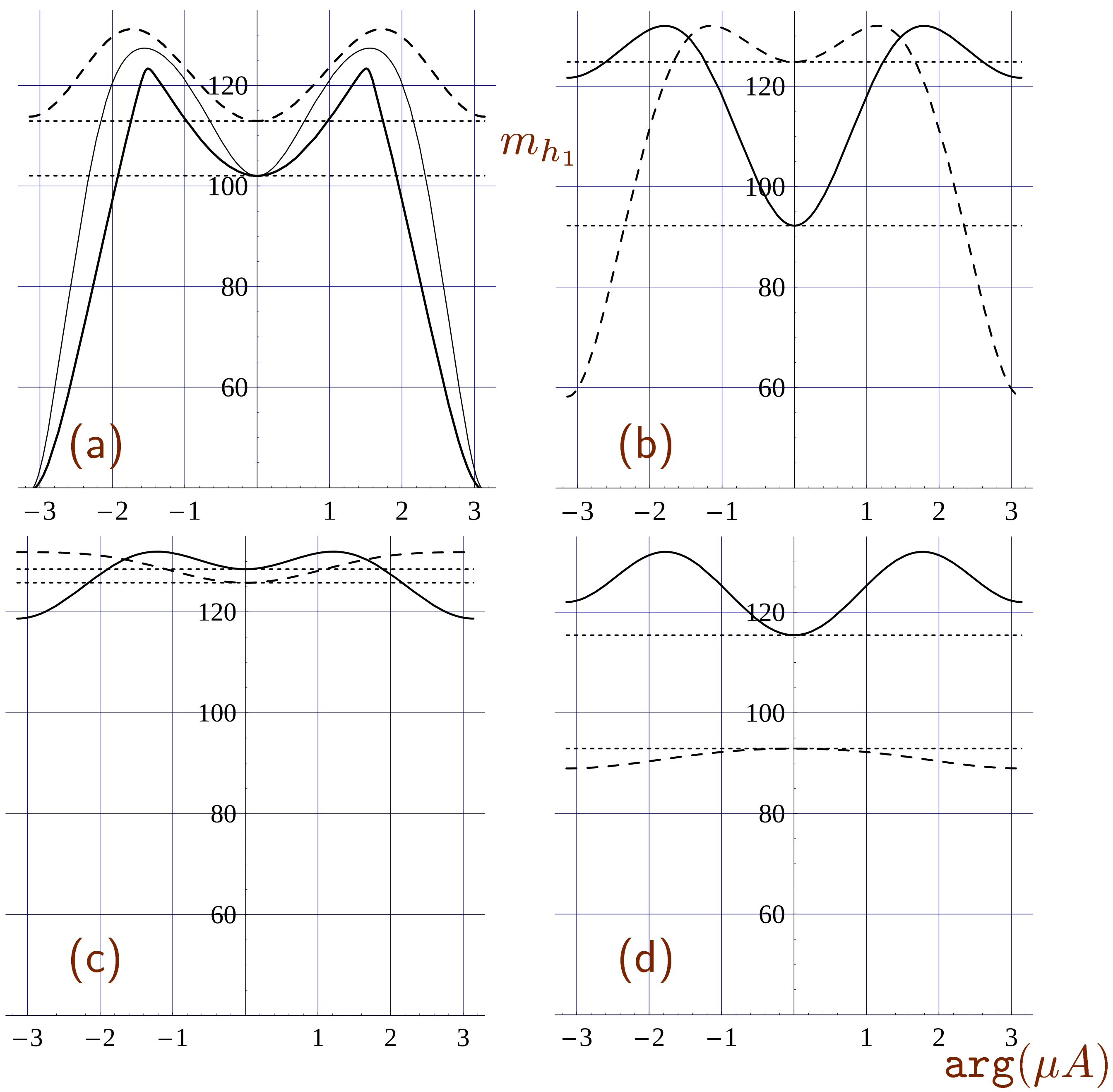


Figure 4: Light Higgs boson mass $m_{h_1}(\varphi)$ (GeV) vs $\mathtt{Arg}\mu A$ in various regions of the MSSM parameter space. Horisontal dotted lines indicate the h_1 mass in the CP-conserving limit ($m_{h_1} = m_h$). (a) $\mathrm{tg}\beta$ =5, M_{SUSY} =0.5 TeV, $A_t = A_b$ =1 TeV, μ =2 TeV. Solid line $m_{H^\pm}$ =180 GeV, dashed $m_{H^\pm}$ =250 GeV. Thin solid line denotes $m_h(\varphi)$. (b) $\mathrm{tg}\beta$ =5, $m_{H^\pm}$ =300 GeV, μ =2 TeV, solid line $A_t = A_b$ =-1.2 TeV, dashed $A_t = A_b$ =1.3 TeV. (c) $\mathrm{tg}\beta$ =5, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line μ =-1.6 TeV, dashed μ =0.7 TeV (d) μ =2 TeV, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line $\mathrm{tg}\beta$ =5, dashed $\mathrm{tg}\beta$ =40.

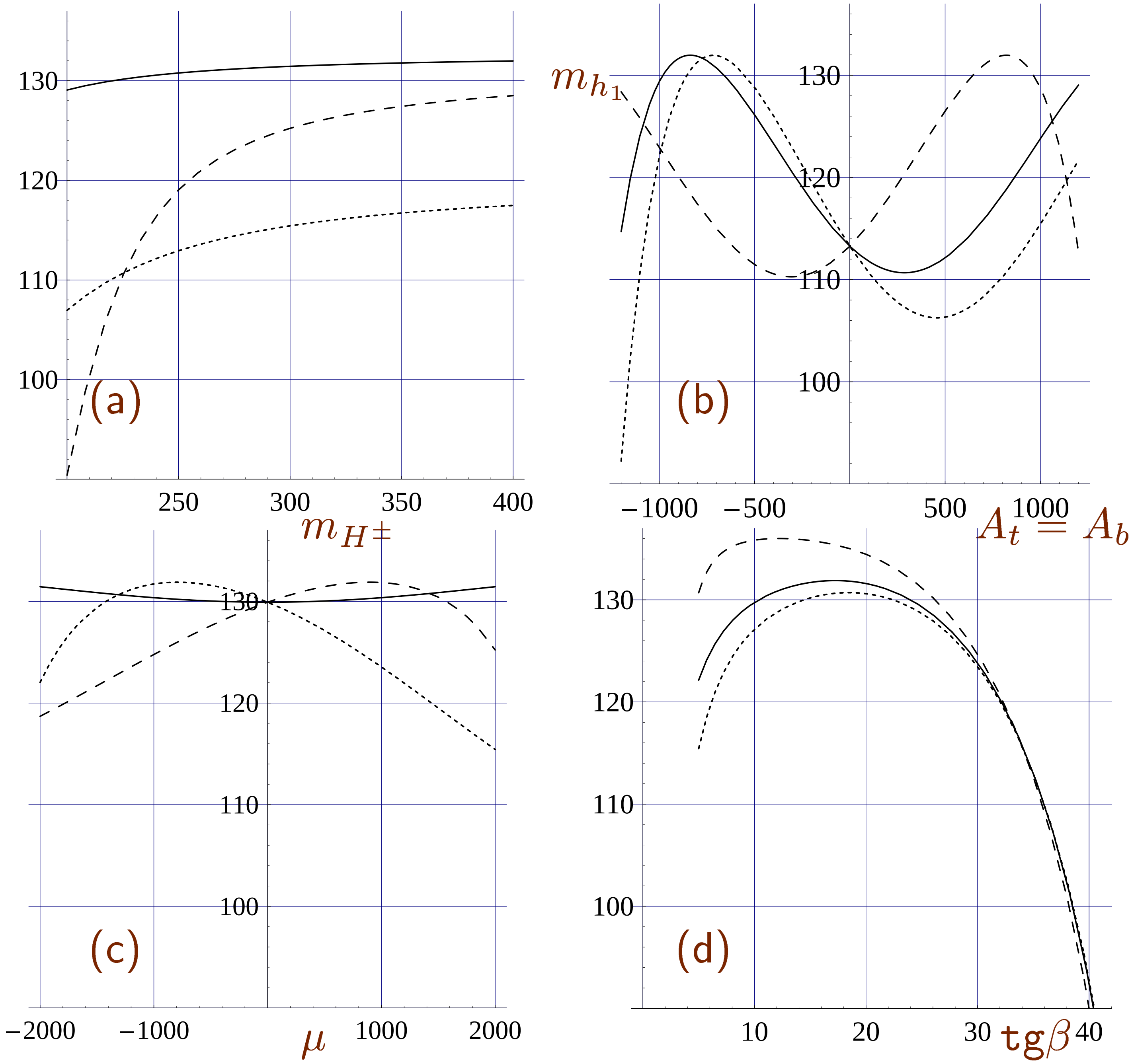


Figure 5: Light Higgs boson mass m_{h_1} (GeV) at M_{SUSY}=0.5 TeV. For illustrative purposes we show by dotted line $m_h(\varphi = 0)$ in the CP conserving limit. (a) tgβ =5, $A_t = A_b$ =1 TeV, μ =2 TeV, solid line $\varphi = \pi/2$, dashed $\varphi = 5\pi/6$, (b) tgβ =5, $m_{H^\pm}$ =300 GeV, μ =2 TeV, solid line $\varphi = 0.9$, dashed $\varphi = 2.3$, (c) tgβ =5, $m_{H^\pm}$ =300 GeV, μ =2 TeV, solid line $\varphi = \pi/2$, dashed $\varphi = 5\pi/6$, (d) μ =2 TeV, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line $\varphi = \pi/4$, dashed $\varphi = 2\pi/3$.

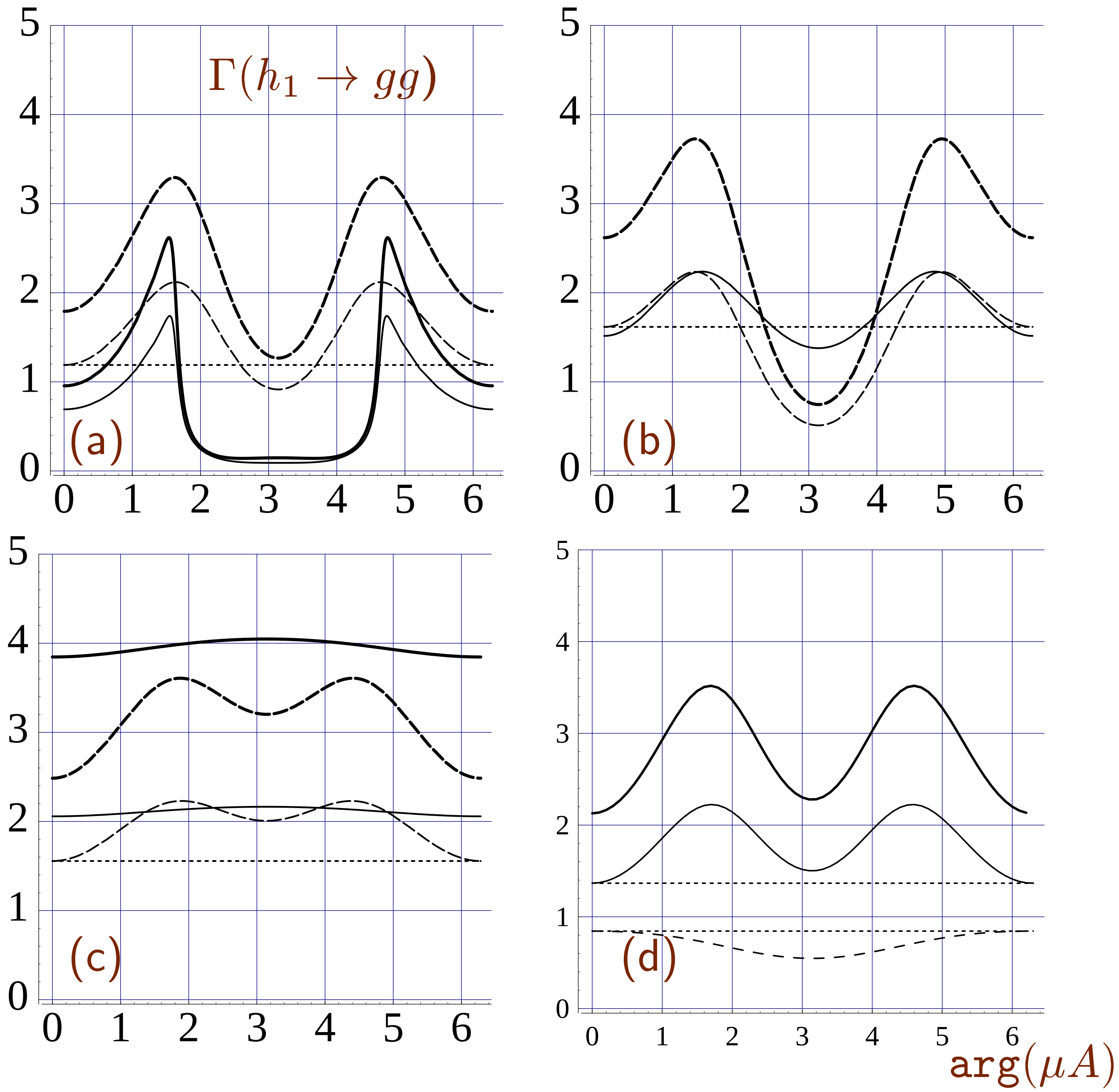


Figure 6: The decay width $\Gamma(h_1 \to gg) \times 10^4$ (GeV) at M_{SUSY} =500 GeV. Dotted lines show $m_h(\varphi = 0)$ in the CP conserving limit. Thin solid/dashed lines are only for SM contributions, thick solid/dashed lines show SM and sparticle contrbutios with K-factor included. (a) $\mathrm{tg}\beta$ =5, $A_t = A_b$ =1 TeV, μ =2 TeV, solid line $m_{H^\pm}$ =190 GeV, dashed $m_{H^\pm}$ =250 GeV, (b) $\mathrm{tg}\beta$ =5, $m_{H^\pm}$ =300 GeV, μ =2 TeV, solid line $A_t = A_b$ =-1.0 TeV, dashed $A_t = A_b$ =1.2 TeV, (c) $\mathrm{tg}\beta$ =5, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line μ =0.2 TeV, dashed μ =1.5 TeV, (d) μ =2 TeV, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line $\mathrm{tg}\beta$ =5, dashed $\mathrm{tg}\beta$ =40

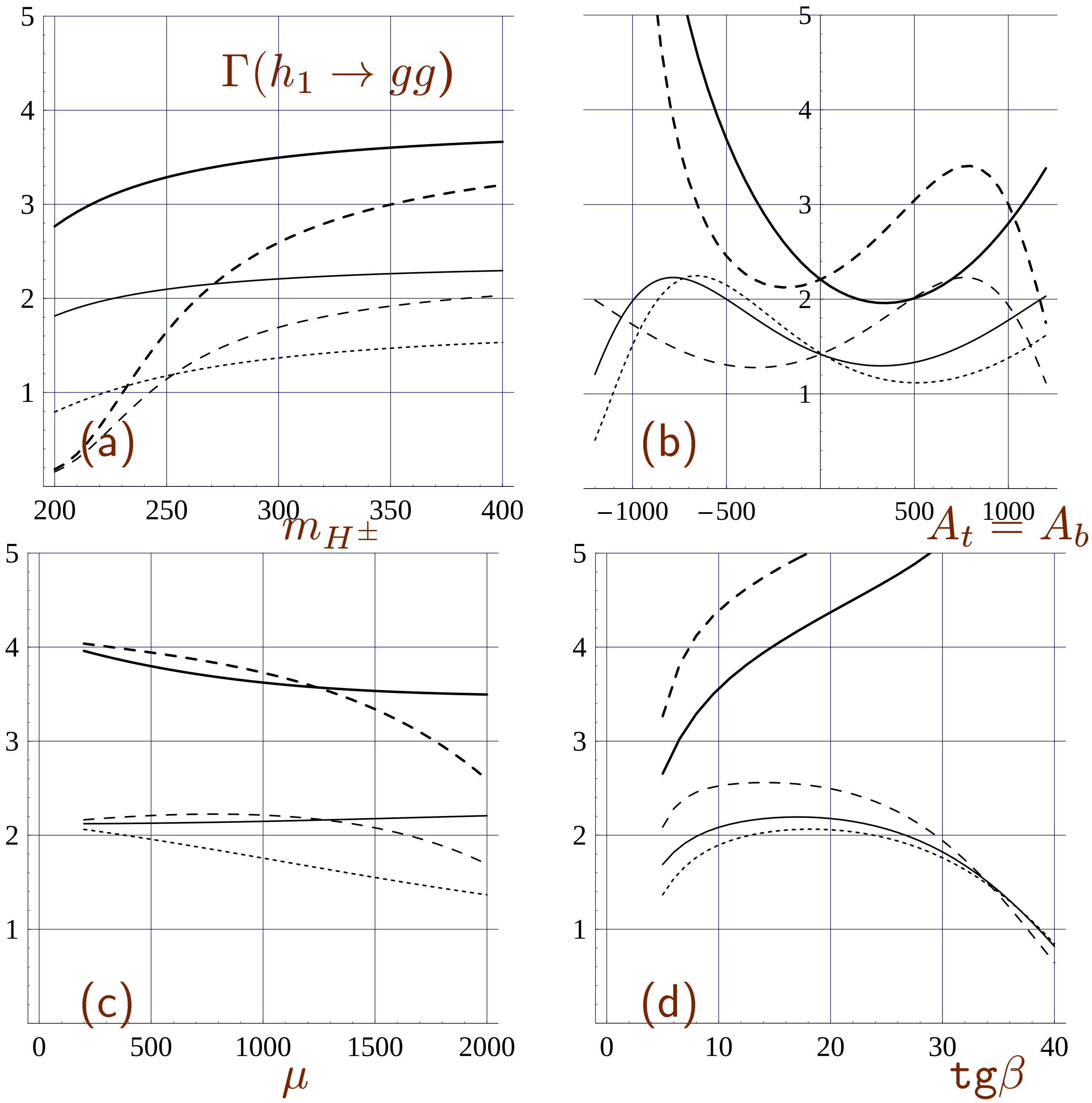


Figure 7: The decay width $\Gamma(h_1 \to gg) \times 10^4$ (GeV) at M_{SUSY} =500 GeV. Dotted lines show $m_h(\varphi = 0)$ in the CP conserving limit. Thin solid/dashed lines are only for SM contributions, thick solid/dashed lines show SM and sparticle contrbutios with K-factor included. (a) $\mathrm{tg}\beta$ =5, $A_t = A_b$ =1 TeV, μ =2 TeV, solid line $\varphi = \pi/2$, dashed $\varphi = 5\pi/6$, (b) $\mathrm{tg}\beta$ =5, $m_{H^\pm}$ =300 GeV, μ =2 TeV, solid line $\varphi = 0.9$, dashed $\varphi = 2.3$, (c) $\mathrm{tg}\beta$ =5, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line $\varphi = \pi/2$, dashed $\varphi = 5\pi/6$, (d) μ =2 TeV, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line $\varphi = \pi/4$, dashed $\varphi = 2\pi/3$.

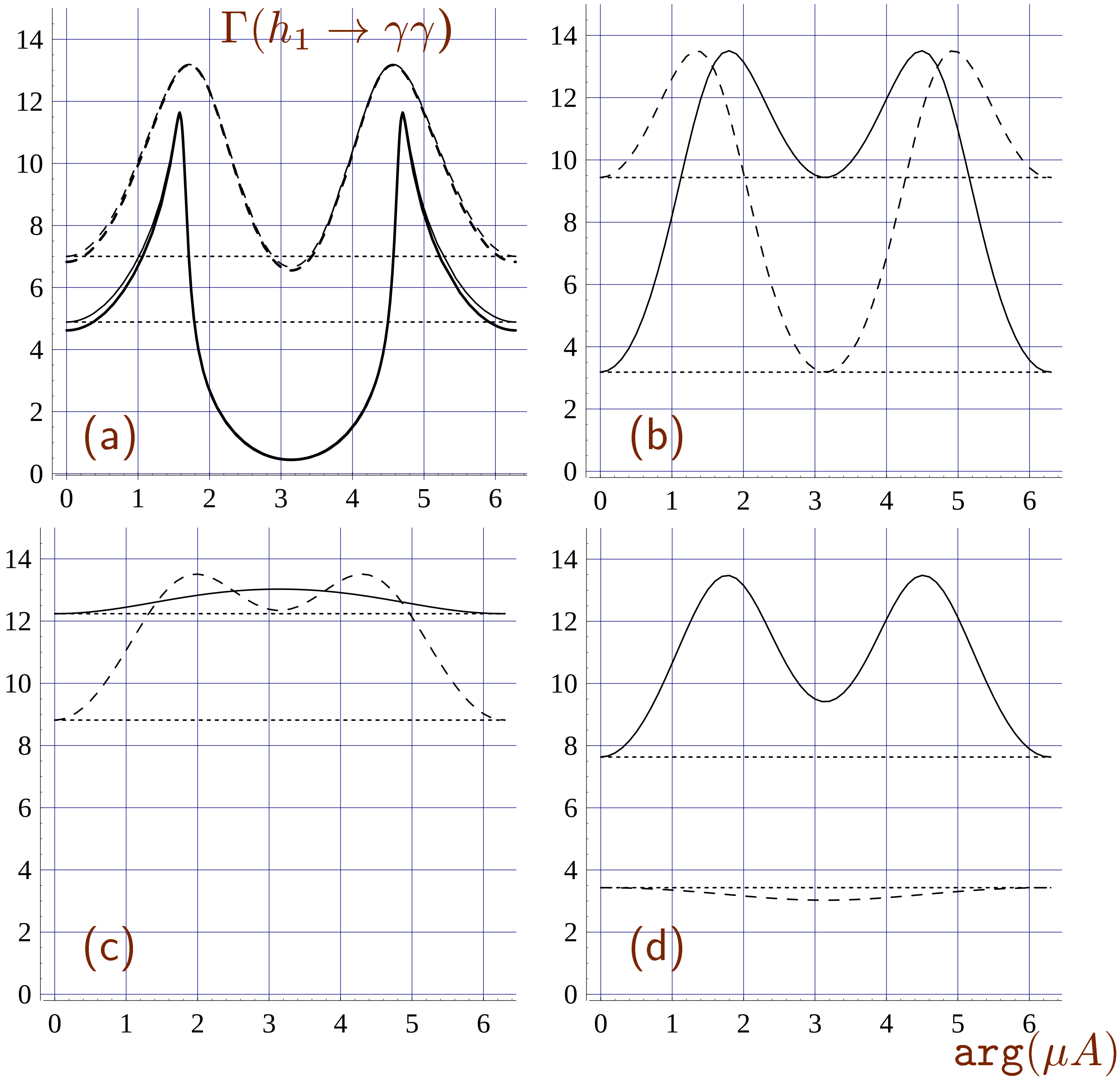


Figure 8: The decay width $\Gamma(h_1 \to \gamma\gamma) \times 10^6$ (GeV) at M_{SUSY} =500 GeV. Dotted lines show $m_h(\varphi = 0)$ in the CP conserving limit. Thin solid/dashed lines are only for SM contributions, thick solid/dashed lines show SM and sparticle contrbutios with K-factor included. (a) $\mathrm{tg}\beta$ =5, $A_t = A_b$ =1 TeV, μ =2 TeV, solid line $m_{H^\pm}$ =190 GeV, dashed $m_{H^\pm}$ =250 GeV, (b) $\mathrm{tg}\beta$ =5, $m_{H^\pm}$ =300 GeV, μ =2 TeV, solid line $A_t = A_b$ =-1.0 TeV, dashed $A_t = A_b$ =1.2 TeV, (c) $\mathrm{tg}\beta$ =5, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line μ =0.2 TeV, dashed μ =1.5 TeV, (d) μ =2 TeV, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line $\mathrm{tg}\beta$ =5, dashed $\mathrm{tg}\beta$ =40

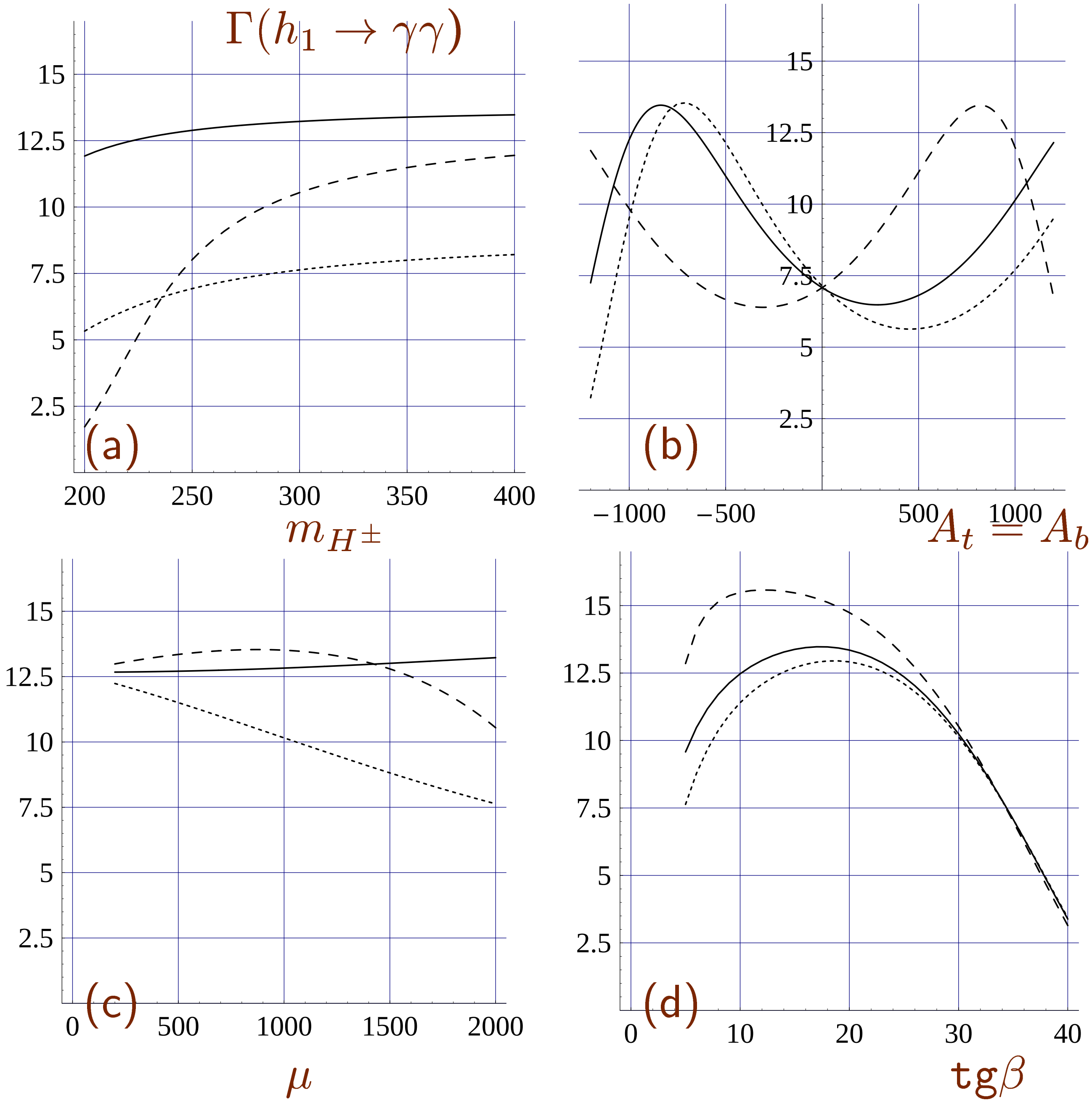


Figure 9: The decay width $\Gamma(h_1 \to \gamma\gamma) \times 10^6$ (GeV) at M_{SUSY} =500 GeV. Dotted lines show $m_h(\varphi = 0)$ in the CP conserving limit. Thin solid/dashed lines are only for SM contributions, thick solid/dashed lines show SM and sparticle contrbutios with K-factor included. (a) tgβ =5, $A_t = A_b$ =1 TeV, μ =2 TeV, solid line $\varphi = \pi/2$, dashed $\varphi = 5\pi/6$, (b) tgβ =5, $m_{H^\pm}$ =300 GeV, μ =2 TeV, solid line $\varphi = 0.9$, dashed $\varphi = 2.3$, (c) tgβ =5, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line $\varphi = \pi/2$, dashed $\varphi = 5\pi/6$, (d) μ =2 TeV, $m_{H^\pm}$ =300 GeV, $A_t = A_b$ =1 TeV, solid line $\varphi = \pi/4$, dashed $\varphi = 2\pi/3$.

- The case of real effective parameters in the two-doublet Higgs sector implicitly assumes the fine-tuning of the phases without a clear physical motivation. In the MSSM complex parameters naturally appear if we introduce CP noninvariant mixings in the Higgs boson-squark sector. The effects may be large (mass splittings 10-100 GeV, changes of branchings several - 100 times) and very important for phenomenology

- The effective field theory calculations demonstrate good qualitative agreement with CPsuperH and Feyn-Higgs packages based on diagrammatic methods. However, substantial deviations are observed at large phases and small $m_{H^\pm}$ that may be explaned by both the theoretical uncertainties and not identical definitions of the Higgs boson states. Mass of the light Higgs state h_1 is sensitive to threshold corrections in effective λ_i, as demonstrated by explicit calculations of nonleading D-terms and the wave-function renormalization terms, which can be of the order of two-loop contributions.