Large mixing in the CP violating Higgs sector in the decoupling limit

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Outline:

- **Introduction/motivation**
- **Models with two Higgs doublets**
  - **with CP conservation:** **conditions for CP**
  - **with CP violation:** **decoupling limit, large H-A mixing**
- **Conclusions and outlook**

Introduction:

The Standard Model: one Higgs doublet $\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$

- **The Higgs potential**

$$\mathcal{V} = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2$$

a nonzero vev $\langle \Phi^0 \rangle$ breaks the gauge symmetry spontaneously

$\Rightarrow$ one physical Higgs boson H, with its mass $m_H^2 = \lambda\, v^2$ not predicted

- **The Yukawa Lagrangian**

$$-\mathcal{L}_Y = \overline{Q}_L \Phi \eta^D D_R + \overline{Q}_L \tilde{\Phi} \eta^U U_R + \text{h.c.}, \quad \text{where } \tilde{\Phi} \equiv i\sigma_2 \Phi^*$$

$\Rightarrow$ provides fermion masses $m_f = \eta^f v/\sqrt{2}$ and no FCNC couplings

- **Hierarchy problem: Higgs mass not stable against radiative corrections**

$\Rightarrow$ motivation for beyond the SM physics

Supersymmetry employs two Higgs doublets

- at tree level the Higgs potential severly constrained

  $\Rightarrow$ quartic couplings are gauge couplings

  $\Rightarrow$ no CP violation

- however, loop corrections are very important and induce

  $\Rightarrow$ all possible quartic couplings
  $\Rightarrow$ and CP-violating effects

  in the effective Higgs potential

- if low-scale supersymmetry breaking by F-terms

  $\Rightarrow$ even the tree level effective Higgs potential may assume the most general form

Therefore

Consider the most general two-Higgs doublet model as a generic model to fully define and explore the decoupling limit of the CP-violating Higgs sector

The general model with two Higgs doublets

[Gunion, Haber; Ginzburg, Krawczyk, Osland; Dubinin, Semenov]

Let Φ_1 and Φ_2 denote two complex $Y = 1$, SU(2)$_L$ doublet scalar fields

- The most general gauge invariant scalar potential

$$\mathcal{V} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}]$$

$$+ \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}$$

- The Yukawa Lagrangian

$$-\mathcal{L}_Y = \overline{Q}_L \tilde{\Phi}_1 \eta_1^U U_R + \overline{Q}_L \Phi_1 \eta_1^D D_R + \overline{Q}_L \tilde{\Phi}_2 \eta_2^U U_R + \overline{Q}_L \Phi_2 \eta_2^D D_R + \text{h.c.}$$

- In many discussions: to avoid tree-level FCNC impose the Z_2 symmetry

$$\Phi_1 \to -\Phi_1 \quad \text{and} \quad D_R \to -D_R$$

  implying $m_{12} = \lambda_6 = \lambda_7 = 0$, and $\eta_1^U = \eta_2^D = 0$.

  Breaking Z_2 softly by $m_{12} \neq 0 \Longrightarrow$ finite Higgs-mediated FCNC at one loop.

- We take all terms nonzero.

In general $m_{12}^2 = |m_{12}^2| e^{i\theta_m}$, $\lambda_{5,6,7} = |\lambda_{5,6,7}| e^{i\theta_{5,6,7}}$, and the vev's complex

Conditions for CP-conservation:

- **explicit CP-violation:** consider $\Phi_1^\dagger \Phi_2 \to e^{-i\eta} \Phi_1^\dagger \Phi_2$
  - the η-dependent terms can removed if

$$\theta_m - \eta, \quad \theta_5 - 2\eta \quad \text{and} \quad \theta_{6,7} - \eta$$

are multiples of π, or in other words:

$$\text{Im}(m_{12}^4 \lambda_5^*) = 0, \qquad \text{Im}(m_{12}^2 \lambda_{6,7}^*) = 0$$

  - a useful convention: choose m_{12}^2 real $\Longrightarrow \lambda_5, \lambda_6$ and λ_7 also real.

- **spontaneous CP violation:** write $\langle \Phi_1^\dagger \Phi_2 \rangle = \frac{1}{2} v_1 v_2 e^{i\xi}$
  - minimization condition $\frac{\partial \mathcal{V}}{\partial \cos\xi} = 0$, $\frac{\partial^2 \mathcal{V}}{\partial (\cos\xi)^2} > 0$
  - spontaneous CP violation if $\xi \neq 0, \pi/2$ or π at the potential minimum
  - otherwise CP is conserved.

The CP-violating 2HDM

Now m_{12}^2, λ_5, λ_6 and λ_7 are complex

- write $m_{12}^2 = m_{12}^{2\,R} + i m_{12}^{2\,I}\,, \quad \lambda_{5,6,7} = \lambda_{5,6,7}^R + i\lambda_{5,6,7}^I$
- choose the phases of Higgs fields so the minimum of the potential is for

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \tan\beta = \frac{v_2}{v_1}$$

- v_i are assumed to be real and positive $\Longrightarrow 0 \le \beta \le \pi/2$.

The corresponding potential minimum conditions are:

$$\begin{aligned}
m_{11}^2 &= m_{12}^{2\,R} t_\beta - \frac{1}{2}v^2\left[\lambda_1 c_\beta^2 + \lambda_{345} s_\beta^2 + 3\lambda_6^R s_\beta c_\beta + \lambda_7^R s_\beta^2 t_\beta\right], \\
m_{22}^2 &= m_{12}^{2\,R} t_\beta^{-1} - \frac{1}{2}v^2\left[\lambda_2 s_\beta^2 + \lambda_{345} c_\beta^2 + \lambda_6^R c_\beta^2 t_\beta^{-1} + 3\lambda_7^R s_\beta c_\beta\right], \\
m_{12}^{2\,I} &= \frac{1}{2}v^2\left[\lambda_5^I c_\beta s_\beta + \lambda_6^I c_\beta^2 + \lambda_7^I s_\beta^2\right],
\end{aligned}$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5^R$

- rotate Φ_1 and Φ_2 to the so-called Higgs basis $(\tan\beta = v_2/v_1)$

$$\begin{aligned} \Phi_a &= c_\beta \Phi_1 + s_\beta \Phi_2 \,, \\ \Phi_b &= -s_\beta \Phi_1 + c_\beta \Phi_2 \,, \end{aligned}$$

in which

$$\Phi_a = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h_a + iG^0\right) \end{pmatrix}, \quad \Phi_b = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(h_b + ia\right) \end{pmatrix},$$

i.e. only Φ_a develops a vev and provides $G^\pm$ and G^0

- re-express the entire Higgs potential in this basis

$$\begin{aligned} \mathcal{V} = \; & m_{aa}^2 \Phi_a^\dagger \Phi_a + m_{bb}^2 \Phi_b^\dagger \Phi_b - [(m_{ab}^2 + i m_{12}^{2\,I}) \Phi_a^\dagger \Phi_b + \text{h.c.}] \\ & + \tfrac{1}{2}\lambda (\Phi_a^\dagger \Phi_a)^2 + \tfrac{1}{2}\lambda_V (\Phi_b^\dagger \Phi_b)^2 + (\lambda_T + \lambda_F)(\Phi_a^\dagger \Phi_a)(\Phi_b^\dagger \Phi_b) \\ & + (\lambda - \lambda_A - \lambda_F)(\Phi_a^\dagger \Phi_b)(\Phi_b^\dagger \Phi_a) \\ & + \Big\{ (\tfrac{1}{2}\lambda - \tfrac{1}{2}\lambda_A + i p)(\Phi_a^\dagger \Phi_b)^2 \\ & - \big[(\widehat{\lambda} - i n)(\Phi_a^\dagger \Phi_a) + (\lambda_U + i q)(\Phi_b^\dagger \Phi_b) \big] \Phi_a^\dagger \Phi_b + \text{h.c.} \Big\} \,, \end{aligned}$$

- the real invariants $m_{aa}^2, m_{bb}^2, m_{ab}^2, \lambda, \lambda_V, \lambda_T, \lambda_F, \lambda_A, \lambda_U$ and $\widehat{\lambda}$ are expressed in terms of real parts of $\lambda_1 ... \lambda_7$ [Gunion, Haber '02]
- the imaginary parts combine to

$$
\begin{aligned}
n &= 2m_{12}^{2\,I}/v^2 = \lambda_5^I s_\beta c_\beta + \lambda_6^I c_\beta^2 + \lambda_7^I s_\beta^2 \,, \\
p &= \tfrac{1}{2}\lambda_5^I (c_\beta^2 - s_\beta^2) - s_\beta c_\beta (\lambda_6^I - \lambda_7^I) \,, \\
q &= \lambda_5^I s_\beta c_\beta - \lambda_6^I s_\beta^2 - \lambda_7^I c_\beta^2
\end{aligned}
$$

- the mass of the physical charged Higg fields $H^\pm$ is

$$
m_{H^\pm}^2 = m_A^2 + \tfrac{1}{2} v^2 \lambda_F \,.
$$

where an auxilliary quantity m_A^2 is defined by the relation

$$
m_{12}^{2\,R} = m_A^2 s_\beta c_\beta + v^2 \lambda_5^R s_\beta c_\beta + \tfrac{1}{2} v^2 (\lambda_6^R c_\beta^2 + \lambda_7^R s_\beta^2) \,.
$$

- the mass matrix for the three neutral Higgs fields, h_a, h_b and a, takes the form

$$
M^2 = v^2 \begin{bmatrix} \lambda & -\widehat{\lambda} & -n \\ -\widehat{\lambda} & m_A^2/v^2 + \lambda - \lambda_A & -p \\ -n & -p & m_A^2/v^2 \end{bmatrix}
$$

- in the CP-conserving case, $n = p = q = 0$

diagonalising the 2×2 submatrix $\Longrightarrow$ the two CP-even h and H are

$$\begin{aligned} H &= h_a \cos\gamma + h_b \sin\gamma \\ h &= -h_a \sin\gamma + h_b \cos\gamma \end{aligned}$$

$$\text{with} \;\; \tan 2\gamma = \frac{2\widehat{\lambda}}{\lambda_A - m_A^2/v^2}$$

and $a = A$ is the CP-odd with mass m_A^2

- in the CP-violating case

the mass matrix for h, H, A takes the form

$$v^2 \begin{bmatrix} \lambda + (m_A^2/v^2 - \lambda_A) c_\gamma^2 c_{2\gamma}^{-1} & 0 & -n s_\gamma - p c_\gamma \\ 0 & \lambda - (m_A^2/v^2 - \lambda_A) s_\gamma^2 c_{2\gamma}^{-1} & -n c_\gamma + p s_\gamma \\ -n s_\gamma - p c_\gamma & -n c_\gamma + p s_\gamma & m_A^2/v^2 \end{bmatrix}$$

i.e. h, H, A all mix $\Longrightarrow$ $H_1,\; H_2,\; H_3$ mass-eigenstates

- if $n = p = 0, q \neq 0$

mixing only via triple and quartic couplings

The decoupling limit

In general the mass matrix $\mathcal{M}^2$ can only be diagonalised numerically.

However, in the decoupling limit defined as

$$m_A^2 \gg |\lambda_i| v^2 \qquad \text{with} \;\; |\lambda_i| < \mathcal{O}(1)$$

the mixing matrix can be solved to order $\mathcal{O}(v^2/m_A^2)$.

- the light H_1 must be indistinguishable from the SM Higgs – must be CP-even
- but mixing between H and A can be finite and large

[Gunion, Haber, JK, in prep.]

The hermitian mass matrix M^2 must be supplemented with the anti-hermitian $M\Gamma$, built up by loops in the propagator matrix

The complex matrix is given by the Weisskopf-Wigner sum

$$\mathcal{M}_c^2 = M^2 - iM\Gamma$$

[Ellis, Lee, Pilaftsis '04]
[Choi, Liao, Zerwas, JK, in prep]

In the decoupling limit we consider 2×2 system: H-A only

- **by CPT invariance the complex mass matrix is symmetric**

$$\mathcal{M}_c^2 = \begin{bmatrix} m_H^2 - i m_H \Gamma_H & \delta m_{HA}^2 \\ \delta m_{HA}^2 & m_A^2 - i m_A \Gamma_A \end{bmatrix}$$

- **and can be diagonalized as** **[Güsken, Kühn, Zerwas ’85]**

$$\mathcal{M}^2 = C\, \mathcal{M}_c^2\, C^{-1}$$

with the complex rotation matrix $C = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

- **the complex mixing angle is given by**

$$X \equiv \frac{1}{2}\tan 2\theta = \frac{\delta m_{HA}^2}{m_H^2 - m_A^2 - i(\Gamma_H - \Gamma_A)}$$

i.e. mixing can be large if masses and decay widths of H and A are almost degenerate

- the mixing shifts the masses and decay widths

$$m^2_{H_3} - m^2_A - i(\Gamma_{H_3} - \Gamma_A) = [m^2_H - m^2_A - i(\Gamma_H - \Gamma_A)]\frac{1}{2}[1 - \sqrt{1+4X^2}]$$

and similarly for

$$m^2_{H_2} - m^2_H - i(\Gamma_{H_2} - \Gamma_H) = [m^2_A - m^2_H - i(\Gamma_A - \Gamma_H)]\frac{1}{2}[1 - \sqrt{1+4X^2}]$$

- the mass eigenstates H_2 and H_3 of $\mathcal{M}^2$ are no longer orthogonal

instead

$$|H_2 >= \cos\theta|H > + \sin\theta|A >, \qquad < \tilde{H}_2| = \cos\theta < H| + \sin\theta < A|$$

$$|H_3 >= -\sin\theta|H > + \cos\theta|A >, \quad < \tilde{H}_3| = -\sin\theta < H| + \cos\theta < A|$$

- correspondingly, the transition amplitudes are modified

for example, for the $I \longrightarrow H \longrightarrow F$ process

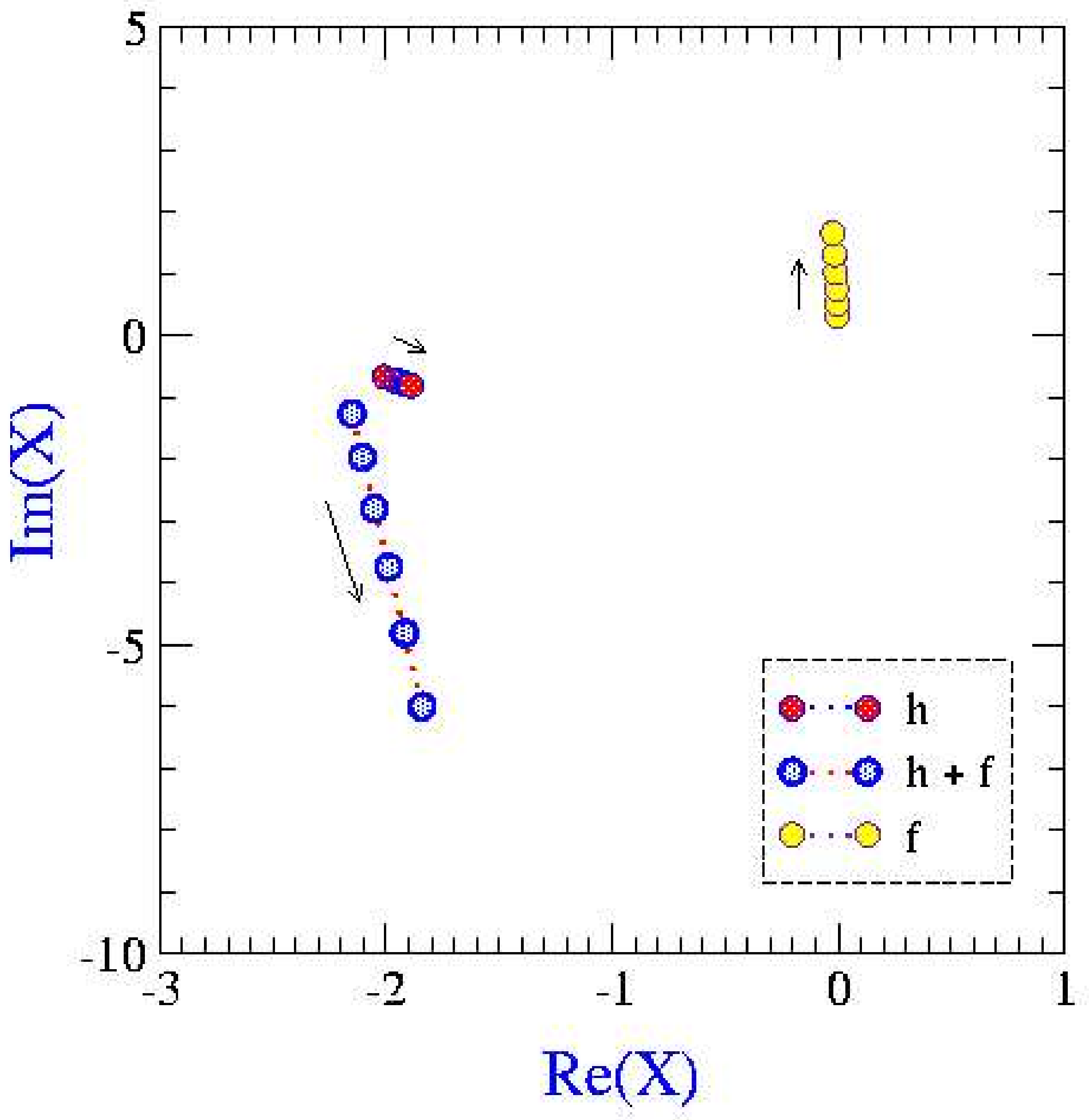
$$< F|H|I >= \sum_{i=2,3} < F|H_i > \frac{1}{s - m^2_{H_i} + i m_{H_i}\Gamma_{H_i}} < \tilde{H}_i|I >$$

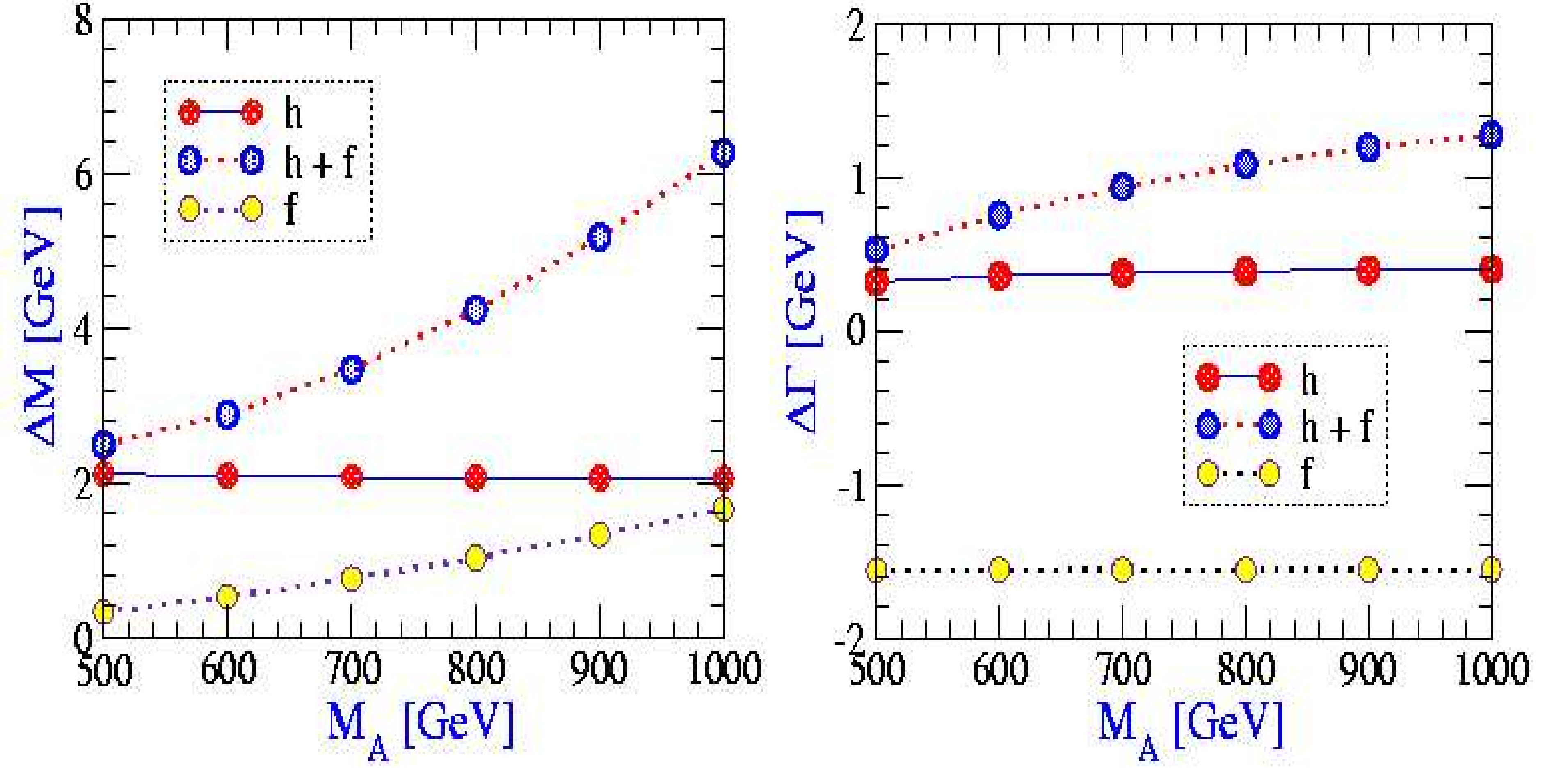
Example: **[Choi, Liao, Zerwas, JK]**

$\tan\beta = 3$, all $|\lambda| = 0.2$, phases of $\lambda_5, \lambda_6, \lambda_7 = \pi/2$

$m_h = 170$ GeV, and run m_A from 500 GeV to 1000 GeV

calculate the mixing parameter X and the shifts of masses and decay widths


5
0
-5
-10
-3
-2
-1
0
1
Im(X)
Re(X)
h
h + f
f



8
6
4
2
0
ΔM [GeV]
h
h + f
f
500
600
700
800
900
1000
M_A [GeV]
2
1
0
-1
-2
ΔΓ [GeV]
h
h + f
f
500
600
700
800
900
1000
M_A [GeV]

Conclusions:

The decoupling limit of a general CP-violating model studied:

- the lightest H_1 is a CP-even SM-like Higgs
- the mixing matrix, spectra and couplings calculated analytically to the first order in v^2/m_A^2
- the mixing of heavy neutral Higgses can be finite and large
- an example of large mixing shown

Outlook:

- results derived at tree-level; can be generalised to radiatively corrected couplings
- work on $\gamma\gamma \longrightarrow H \longrightarrow F$ in progress
- to be analysed in SUSY with different scenarios
- precision measurements of couplings important

The lesson of the decoupling limit: a SM-like Higgs provides very little information about the nature of electroweak breaking dynamics.