V_{us}, K_{e3} decays, and radiative corrections: an update *

Geneva, 5 May 2004

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*based on hep-ph/0401173 and previous papers, in particular with V. Cirigliano, H. Neufeld, and M. Knecht

1. Introduction
2. Setting the scenery: amplitude, decay distribution, form factor
3. IR-safe decay distribution: a viable approach
4. Extraction of $|V_{us}|$ from K_{e3} data
5. Consistency check of K_{e3}^{0} and K_{e3}^{+} data
6. Summary

1 Introduction

Taking the PDG's values of the first-row CKM matrix entries,

$$\begin{aligned} |V_{ud}| &= 0.9734 \pm 0.0008, \\ |V_{us}| &= 0.2196 \pm 0.0026, \\ |V_{ub}| &= 0.0036 \pm 0.0010, \end{aligned}$$

one finds a $2.2\,\sigma$ deviation from the first-row unitarity relation:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0042 \pm 0.0019.$$

- $|V_{ud}|$ is remarkably precisely known
- $|V_{ub}|$ is of no importance at present precision
- $|V_{us}|$ comes from thirty-year-old data (and twenty-year-old theory)!

$\longrightarrow$ series of high-precision $K_{\ell 3}$ experiments: E865, NA48, KLOE, ISTRA, ...

First result from E865 (Sher et al. '03) for the K_{e3}^{+} decay leads to a branching ratio

$$BR(K_{e3(\gamma)}^{+}) = (5.13 \pm 0.02_{\mathrm{stat}} \pm 0.09_{\mathrm{sys}} \pm 0.04_{\mathrm{norm}})\%.$$

In order to minimize systematic uncertainties, the channels $K_{\pi 2}^{+}$, $K_{\mu 3}^{+}$, and $K_{\pi 3}^{+}$ were used as normalization sample; PDG BRs were used for the normalization, thus the normalization error. This translates into the following decay width

$$\Gamma(K_{e3(\gamma)}^{+}) = (4.14 \pm 0.08) \times 10^{6} s^{-1},$$

whereas the PDG quotes a decay rate that is $\sim 2.3\,\sigma$ off:

$$\Gamma(K_{e3(\gamma)}^{+}) = (3.93 \pm 0.05) \times 10^{6} s^{-1}.$$

$\longrightarrow$ $|V_{us}|$ would go up by about $2-3\%$

$\longrightarrow$ no deviation from unitarity anymore

$\longrightarrow$ inconsistency between old and new K_{e3}^{+} data

$\longrightarrow$ inconsistency between K_{e3}^{+} and K_{e3}^{0} data

2 Setting the scenery

Experiments are already **sensitive** to **subleading** effects in K_{e3} decays. These include **NNLO QCD** corrections and **isospin breaking** by quark masses and electromagnetism.

The calculations and numerical analyses presented here were undertaken within **chiral perturbation theory** with virtual photons and leptons. ChPT is the **effective low-energy** version of the **standard model** and works directly with pions and kaons as relevant degrees of freedom.

$$K(p_K) \to \pi(p_\pi)\, e^+(p_e)\, \nu_e(p_\nu)$$

Without rad. corr. a K_{e3} decay is **sufficiently** parametrized by one form factor $f_+^{(0)}(t)$:

$$\boxed{\mathcal{M} = C\,\frac{G_F}{\sqrt{2}}\,V_{us}^*\,l^\mu\, f_+^{(0)}(t)\,(p_K + p_\pi)_\mu}$$

where the form factor denotes the vector current transition matrix element

$$f_+^{(0)}(t)(p_K + p_\pi)_\mu = \langle \pi | V_\mu^4 - i V_\mu^5 | K \rangle.$$

We also have the weak leptonic current l^μ,

$$l^\mu = \bar{u}(p_\nu)\,\gamma^\mu\,(1-\gamma_5)\,v(p_e)\;,$$

and the coefficient $C = \begin{cases} 1 & \text{for } K^0_{e3} \\ 1/\sqrt{2} & \text{for } K^+_{e3} \end{cases}$. $\qquad t = (p_K - p_\pi)^2$.

The spin-averaged **decay distribution** obtained from this reads

$$\boxed{\rho^{(0)}(y,z) \;=\; \mathcal{N}\,A_1^{(0)}(y,z)\,|\,f_+^{(0)}(t)|^2}$$

with these definitions

$$\mathcal{N} = C^2\frac{G_F^2\,|V_{us}|^2 M_K^5}{128\pi^3}\,,\quad z = \frac{2E_\pi}{M_K}\,,\quad y = \frac{2E_e}{M_K}\,,\quad t = M_K^2(1-z+r_\pi)\;.$$

The simple kinematical density $A_1^{(0)}(y,z)$ is given by

$$A_1^{(0)}(y,z) \;=\; 4(z+y-1)(1-y)+r_e(4y+3z-3)-4r_\pi+r_e(r_\pi-r_e)\;,$$

where these squared mass ratios are used:

$$r_e=\frac{m_e^2}{M_K^2}\;,\quad r_\pi=\frac{m_\pi^2}{M_K^2}\;.$$

How does the **presence of electromagnetism** affect the form factor? Virtual photon exchange introduces **2nd variable** and causes an **IR divergence**:

$$\boxed{f_+^{(0)}(t)\to F_+(t,u,M_\gamma)}$$

with $u=(p_K-p_e)^2$ for K^+, and $u=(p_\pi+p_e)^2$ for K^0.

We chose to regularize the IR singularities with a fictitious photon mass M_γ.

To order α, the new form factor $F_+(t,u,M_\gamma)$ may be most conveniently written as product of **universal** LD corrections times a new **effective** form factor:

$$\boxed{F_+(t,u,M_\gamma) = \left[1 + \frac{\alpha}{4\pi}\Gamma(u,m_e^2,m_\pi^2;M_\gamma)\right] \times\ f_+(t)}$$

Note that the second variable u has been shifted into the LD factor! Note also the close formal resemblance of the effective form factor to the original one!

For later use:

$$f_+(t) = f_+(0)\left[1 + \lambda_+ \frac{t}{m_{\pi^\pm}^2}\right]$$

or even beyond the linear approximation (as soon as experiments become sensitive enough).

A few comments on what enters into the **effective** form factor $f_+(t)$:

- **isospin breaking** by the quark masses, e.g. $\pi^0 - \eta$ mixing effects (up to $O((m_u - m_d)p^2)$)
- isospin conserving contributions from $SU(3)$ **breaking** (up to $O(p^6)$)
- both manifest themselves as QCD **loop** and **local** counterterm corrections
- isospin breaking by **local** effects of electromagnetism ($O(e^2 p^2)$)

Don't forget that there are also **non-local** electromagnetic effects; these, however, are confined within the **LD correction** factor (and won't be shown here)!

Let's focus on what goes beyond the classical analysis of Gasser, Leutwyler, Roos: **local** $O(p^6)$ and **leptonic** corrections to $f_+(t)$, since this is where the theoretical uncertainties come from!

- the common **isospin conserving** corr. of order p^6 at zero momentum transfer:

$$\widetilde{f}_+^{K\pi}(0)\Big|_{p^6} \quad = \quad -8\left(\frac{M_K^2 - M_\pi^2}{F_\pi^2}\right)^2 \left[C_{12}^r(\mu) + C_{34}^r(\mu)\right] + \Delta_{\text{loops}}(\mu)\Big|_{t=0}$$

Post, Schilcher; Bijnens, Talavera '02, '03

- the **local electromagnetic** term for $K^0 \to \pi^- e^+ \nu_e$:

$$\widehat{f}_+^{K^0\pi^-} \;=\; 4\pi\alpha \left[2K_{12}^r(\mu) + \tfrac{4}{3}X_1 - \tfrac{1}{2}\widetilde{X}_6^r(\mu) - \frac{1}{32\pi^2}\left(3 + \log\frac{m_e^2}{M_{\pi^\pm}^2} + 3\log\frac{M_{\pi^\pm}^2}{\mu^2} \right) \right]$$

- the **local electromagnetic** term for $K^+ \to \pi^0 e^+ \nu_e$:

$$\widehat{f}_+^{K^+\pi^0} \;=\; 4\pi\alpha \left[2K_{12}^r(\mu) - \tfrac{8}{3}X_1 - \tfrac{1}{2}\widetilde{X}_6^r(\mu) - \frac{1}{32\pi^2}\left(3 + \log\frac{m_e^2}{M_{K^\pm}^2} + 3\log\frac{M_{K^\pm}^2}{\mu^2} \right) \right]$$

Cirigliano et al. ’02

3 IR-safe decay distribution

$F_+(t,u,M_\gamma)$ being IR singular $\longrightarrow$ consider **real photon emission**

$$\boxed{\rho(y,z) = \rho^{(0)}(y,z)\Big|_{F_+(t,u,M_\gamma)} + \rho^\gamma(y,z,M_\gamma)\,,}$$

where $\rho^\gamma(y,z,M_\gamma)$ is obtained from the related radiative decay

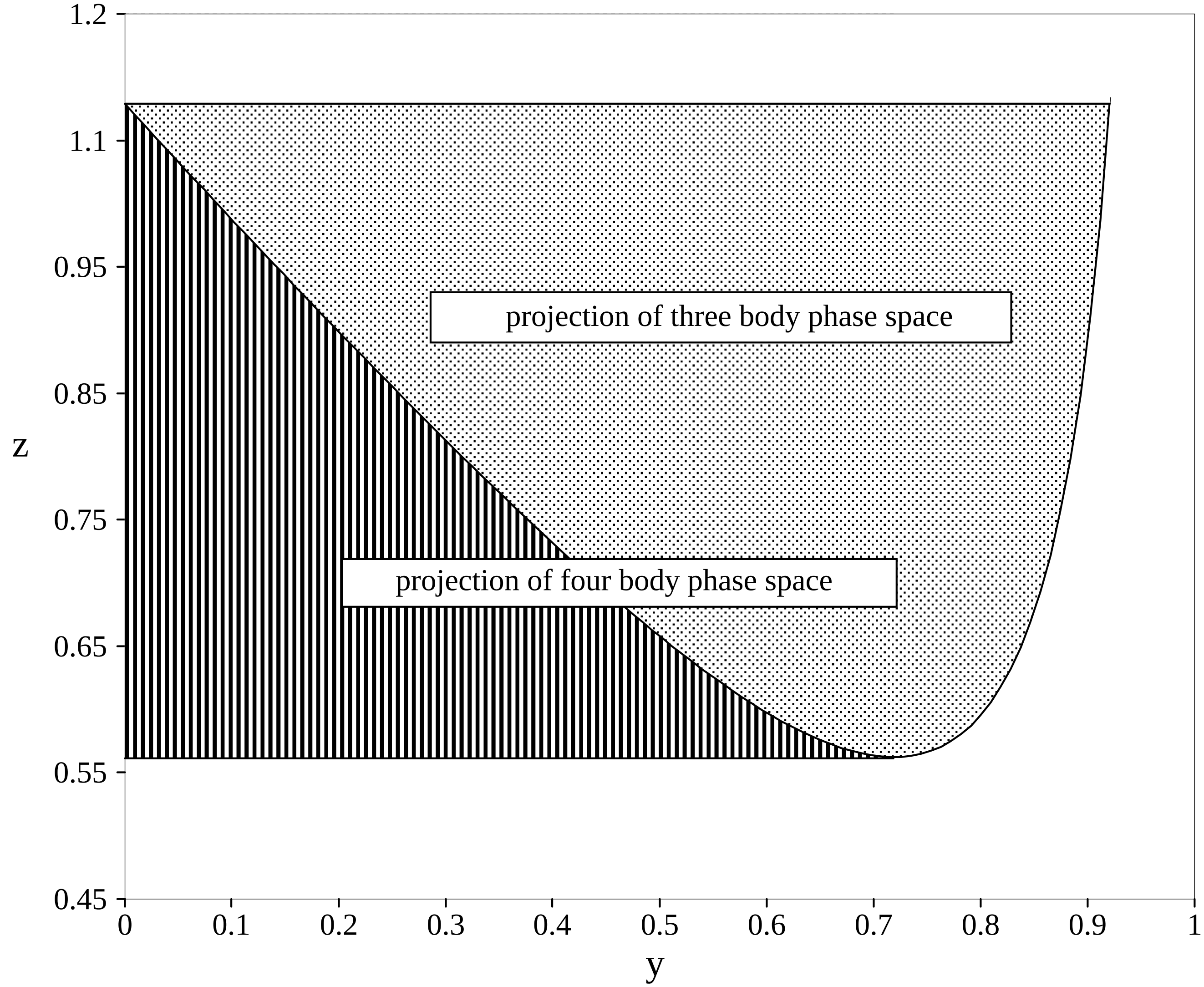
$$K(p_K) \to \pi(p_\pi)\, e^+(p_e)\, \nu_e(p_\nu)\, \gamma(p_\gamma)\,.$$

We (Ginsberg '67 - '70, cleaned from errors in Cirigliano et al. '02, '03, '04) propose to:

- accept all photon energies
- accept all angles between pion and positron
- accept **ONLY** pion and positron energies within the original 3-body Dalitz plot

$\longrightarrow$ inclusive rate obtained by integrating over the original domain

The situation is best explained by this plot:



Finally, the **IR-safe density** $\rho(y,z)$ is in close analogy to the original density without e.m.

$$\boxed{\rho(y,z) = \mathcal{N}\, S_{EW}(m_\rho, m_Z)\, A_1(y,z)\, |\, f_+(t)|^2}$$

The **new** kinematical density A_1 reads

$$A_1(y,z) = A_1^{(0)}(y,z)\,\left[1 + \Delta^{\mathrm{IR}}(y,z)\right] + \Delta_1^{\mathrm{IB}}(y,z)\,.$$

$\Delta^{\mathrm{IR}}(y,z)$ contains IR divergences from **LD effects** in the form factor, and from the emission of **soft real photons**. The combination, however, is **finite**.

$\Delta_1^{\mathrm{IB}}(y,z)$ contains IR-finite corr. from $|\mathcal{M}_\gamma|^2$.

$S_{EW} = 1.0232$ takes care of electroweak rad. corr. from M_Z up to the scale of the mass of the ρ. Then, we are hopefully in the domain where resonance exchange takes over. Marciano, Sirlin '93

4 Extraction of $|V_{us}|$ from K_{e3} data

$$\boxed{|V_{us}| = \sqrt{\left[\frac{128\,\pi^3\,\Gamma(K_{e3(\gamma)})}{C^2\,G_F^2\,M_K^5\,S_{EW}(m_\rho,m_Z)\,I_K}\right]} \times \frac{1}{f_+^{K\pi}(0)}}$$

$$I_K = \int_{\mathcal{D}} dy\,dz\,A_1(y,z)\left[1+\lambda_+\frac{t}{m_{\pi^\pm}^2}\right]^2 = a_0 + a_1\,\lambda_+ + a_2\,\lambda_+^2\ .$$

$\mathcal{D}$ ... dotted area in the (y,z) plane.

To extract $|V_{us}|$ from a measured (inclusive) rate, we need to provide

- the phase-space factor I_K (i.e., the a_i and the **slope**)
- the form factor at zero-momentum transfer.

With these coefficients of the K^0_{e3} phase-space integral,

	a_0	a_1	a_2
$\alpha = 0$	0.09390	0.3245	0.4485
$\alpha \neq 0$	0.09358	0.3241	0.4475

and this slope parameter,

$$\lambda_+^{K^0\pi^-} = 0.0291 \pm 0.0018\ ,$$

we obtain

$$I_{K^0} = 0.10339 \pm 0.00063, \quad \text{shifted by rad. corr. by} \quad -0.32\ \%$$

Similarly, with the coefficients of the K^+_{e3} phase-space integral,

	a_0	a_1	a_2
$\alpha = 0$	0.09653	0.3337	0.4618
$\alpha \neq 0$	0.09533	0.3287	0.4535

and the corresponding slope parameter,

$$\lambda_+^{K^+\pi^0} = 0.0278 \pm 0.0019,$$

we obtain

$$I_{K^+} = 0.10482 \pm 0.00067, \ \text{shifted by rad. corr. by} \ -1.27\%$$

- Up to NLO, $f_+(0) = \widetilde{f}_+(0) + \widehat{f}_+$ in the K^0_{e3} channel:

$$\underbrace{\widetilde{f}_+(0)|_{p^4} = 0.97699 \pm 0.00002}_{\text{only meson masses and tiny } \varepsilon^{(2)}} \quad \text{and} \quad \underbrace{\widehat{f}_+ = 0.0046 \pm 0.0009}_{K^r_{12},\ \text{dim. anal. for } X_1, \widetilde{X}^r_6}$$

- Up to NLO, $f_+(0) = \widetilde{f}_+(0) + \widehat{f}_+$ in the K^+_{e3} channel:

$$\underbrace{\widetilde{f}_+(0)|_{p^4} = 1.0002 \pm 0.0022}_{L_7, L^r_8, K^r_3, K^r_4, K^r_5, K^r_6, \varepsilon^{(2)}} \quad \text{and} \quad \underbrace{\widehat{f}_+ = 0.0032 \pm 0.0016}_{K^r_{12},\ \text{dim. anal. for } X_1, \widetilde{X}^r_6}$$

The order p^6 contribution to $f_+^{K\pi}(0)$ necessitates a few more words:

$$\Delta_{\text{loops}}(M_\rho)\Big|_{t=0} = 0.0146 \pm 0.0064$$ Bijnens, Talavera '03

The local order p^6 correction is quite **controversial** at the moment.

Leutwyler and Roos estimated in '84 (in the language of overlapping wave functions) the local NNLO ($O(m_q^2)$) effects to be

$$-0.016 \pm 0.008\,.$$

Bijnens and Talavera suggest to identify the local order p^6 correction with this number; however, there is the **question of scale** (see Cirigliano et al. '04):

$$-8\left(\frac{M_K^2 - M_\pi^2}{F_\pi^2}\right)^2 \left[C_{12}^r(M_\rho) + C_{34}^r(M_\rho)\right] = -0.016 \pm 0.008\,.$$

Most recently, Bećirević et al. '04 presented a (quenched) lattice study of $K \to \pi$ form factors at zero momentum transfer. With

$$f_+^{K\pi}(0) = 1 + f_+^{K\pi}(0)\Big|_{p^4} + \Delta f_+^{K\pi} ,$$

they find for the 'complete' NNLO correction

$$\Delta f_+^{K\pi} = -0.017 \pm 0.005_{\text{stat}} \pm 0.007_{\text{sys}} .$$

In the two scenarios, we find at zero-momentum transfer

LO+NLO QCD e.m. rad.corr. NNLO QCD

$$\begin{aligned}
f_+^{K^0\pi^-}(0) &= [0.97699 \pm 0.00002] + [0.0046 \pm 0.0008] - [0.001 \pm 0.010] = 0.981 \pm 0.010 . \\
f_+^{K^+\pi^0}(0) &= [1.00020 \pm 0.00220] + [0.0032 \pm 0.0016] - [0.001 \pm 0.010] = 1.002 \pm 0.010 .
\end{aligned}$$

$$\begin{aligned}
f_+^{K^0\pi^-}(0) &= [0.97699 \pm 0.00002] + [0.0046 \pm 0.0008] - [0.017 \pm 0.009] = 0.965 \pm 0.009. \\
f_+^{K^+\pi^0}(0) &= [1.00020 \pm 0.00220] + [0.0032 \pm 0.0016] - [0.017 \pm 0.009] = 0.986 \pm 0.010 .
\end{aligned}$$

Example: extraction of $|V_{us}|$ from the Brookhaven K_{e3}^{+} rate.

With our values of $f_{+}^{K^{+}\pi^{0}}(0)$ and $I_{K^{+}}$, we arrive at

$$\begin{aligned} |V_{us}| &= 0.2238 \pm 0.0022 \pm 0.0007 \pm 0.0023 \\ &= 0.2238 \pm 0.0033, \end{aligned}$$

$$\begin{aligned} |V_{us}| &= 0.2274 \pm 0.0022 \pm 0.0008 \pm 0.0023 \\ &= 0.2274 \pm 0.0033. \end{aligned}$$

These numbers are in good/excellent agreement with CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0024 \pm 0.0021,$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0008 \pm 0.0021.$$

5 Consistency check of K_{e3}^{0} and K_{e3}^{+} data

$$\boxed{r_{+0} := f_{+}^{K^{+}\pi^{0}}(0) \Big/ f_{+}^{K^{0}\pi^{-}}(0)}$$

Theory tells us

$$r_{+0}^{\mathrm{th}} = 1 + \sqrt{3}\left(\varepsilon^{(2)} + \varepsilon_{\mathrm{S}}^{(4)} + \varepsilon_{\mathrm{EM}}^{(4)}\right) - \frac{\alpha}{4\pi}\log\frac{M_{K^{\pm}}^{2}}{M_{\pi^{\pm}}^{2}} - 16\pi\alpha X_{1}$$

With simple dim. analysis for X_1 we find

$$\boxed{1.017 \leq r_{+0}^{\mathrm{th}} \leq 1.027}$$

Experimentally

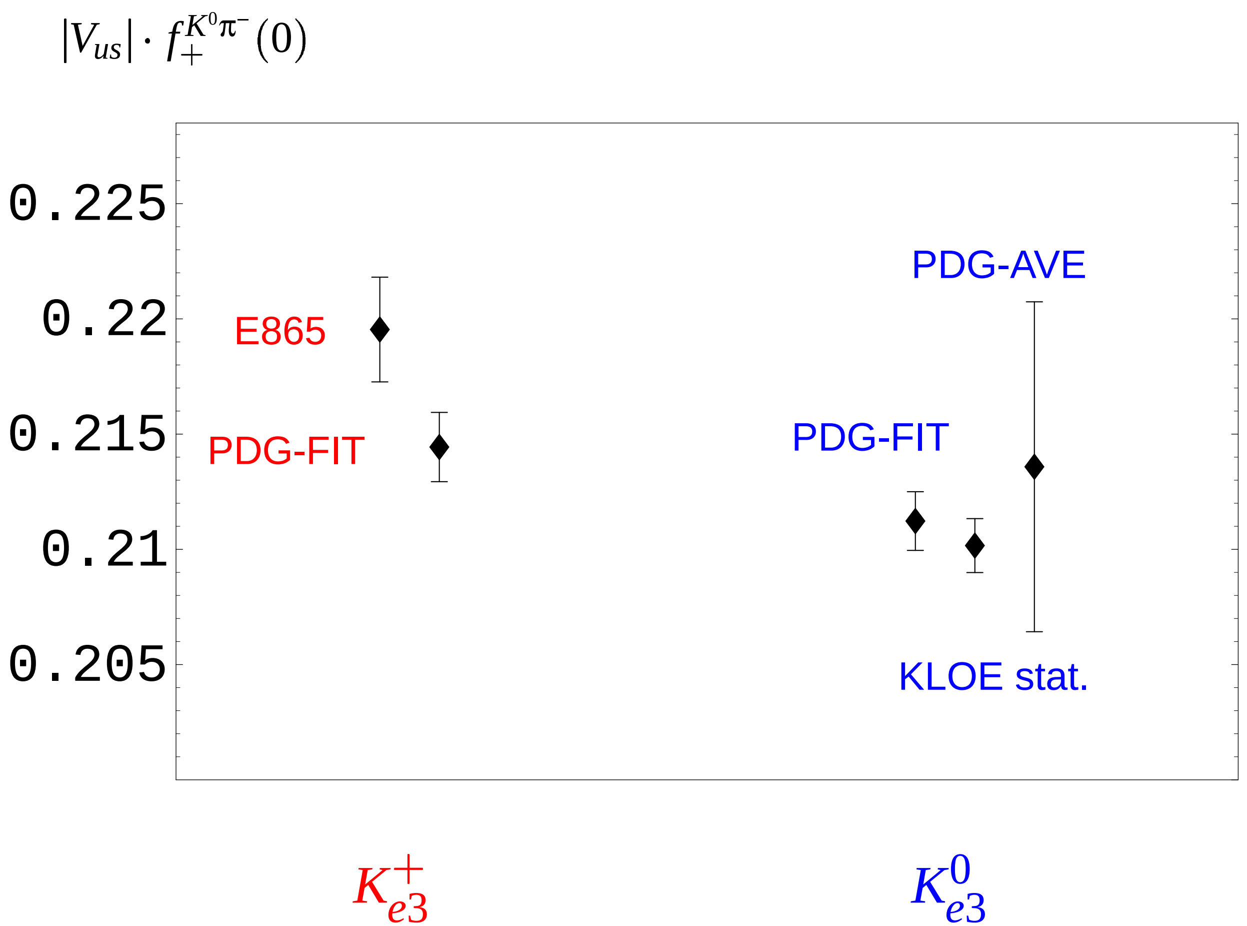
$$\boxed{r_{+0}^{\exp} = \left(\frac{2\,\Gamma(K^{+}_{e3(\gamma)})\,M^{5}_{K^{0}}\,I_{K^{0}}}{\Gamma(K^{0}_{e3(\gamma)})\,M^{5}_{K^{+}}\,I_{K^{+}}}\right)^{1/2}}$$

In order to arrive at a **meaningful** number, both in the exp. determination of the rate and in the calculation of the phase-space factor the same prescription regarding real photons must be used!

Assuming that this is the case, we find, for example, for the PDG fit of $\Gamma(K^{0}_{e3(\gamma)})$ and for E865's $\Gamma(K^{+}_{e3(\gamma)})$

$$r_{+0}^{\exp} = 1.062 \pm 0.013.$$

We may also use r^{th}_{+0} to normalize **independent** measurements of $|V_{us}| \cdot f_+^{K\pi}(0)$ to $f_+(0)^{K^0\pi^-}$! If we, for the time being, forget about the likely real-photon-scheme dependence of older data, we can summarize our results about $|V_{us}|$ from K_{e3} decays in this plot:



6 Summary

- **ChPT with virtual γ, ℓ:** one consistent low-energy framework to incorporate rad. corr.

- clear treatment of real rad. corr. established and cleaned from old errors

- **consistency** among exp. data **checkable** by comparison of decay rates with the theoretical band for the ratio r^{th}_{+0}

- question of order p^6 correction to the form factor still controversial, but there is a new lattice result; ChPT allows for a check of the lattice number once slope and curvature of form factors are measured

- present exp. status of $|V_{us}|$ is still quite confusing!

- new experiments are on the way to measure and hopefully clarify the status of $|V_{us}|$: NA48, KLOE