A re-analysis of radiative K^0_{e3} decays

Jürg Gasser, Bastian Kubis[1]

Universität Bern

Nello Paver

Università di Trieste & I.N.F.N. Trieste

Michela Verbeni

Universidad de Granada

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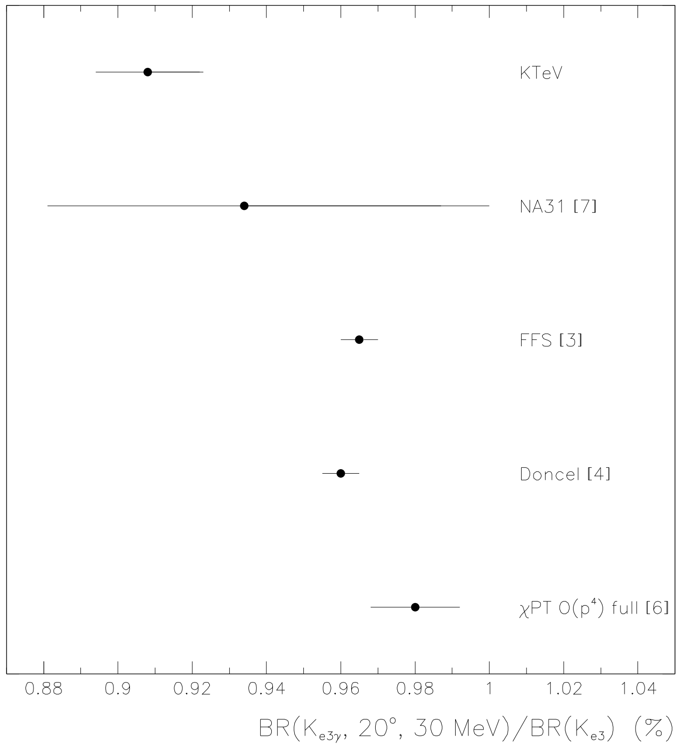
$$\boxed{K_L^0 \;\to\; \pi^- \, e^+ \, \nu_e \, \gamma \quad \left[K^0_{e3\gamma}\right]}$$

- Motivation : An experimental result
- Introduction : ChPT and Low's theorem
- Formalism : K_{e3}, $K_{e3\gamma}$ amplitudes
- Observables : decay widths, the ratio R
- Electromagnetic corrections
- Structure dependent terms from differential rates
- Summary

Motivation



⇒ "...significantly lower than all published theoretical predictions."

KTeV Coll., Phys. Rev. D64 (01) 11204

⇒ new measurements planned (or done actually):

NA48 (see M.M. Velasco's talk),

KLOE(@DAΦNE)

note : "χPT $\mathcal{O}(p^4)$ full" in this figure is not purely theoretical, but "processed" by KTeV!

Introduction

Chiral Perturbation Theory

- expansion in small masses (M_π, M_K) and momenta
- equivalent to loop expansion; $K \to \pi\, \ell^+ \nu_\ell\, \gamma$ $[K_{\ell 3\gamma}]$ up to one loop:

J. Bijnens, G. Ecker, J. Gasser, Nucl. Phys. B396 (1993) 81

Low's Theorem

- radiative processes $X \to Y\gamma$ involving charged particles are infrared divergent for photon momentum $q \to 0$:

$$T(X \to Y\gamma) \;=\; \underbrace{\frac{C_{-1}}{q} + C_0}_{\text{inner bremsstrahlung (IB)}} \;+\; \underbrace{C_1\, q \;+\; \ldots}_{\text{structure dependent (SD)}}$$

- Low's theorem : C_{-1}, C_0 given in terms of the non-radiative amplitude (and derivatives thereof):

$$C_{-1},\ C_0\ \propto\ T(X\to Y),\ \partial T(X\to Y)$$

F.E. Low, Phys. Rev. 110 (1958) 974

Strategy: ChPT + Low's Theorem

- chiral representation fulfils Low's theorem
- expect bremsstrahlung to dominate due to infrared singularity
  $\Rightarrow$ improve on chiral representation by using
  physical amplitudes / data for C_{-1}, C_0
- ChPT for structure dependent contributions

Formalism

- amplitude for K^0_{e3} described in terms of single form factor $f_+(t)$, parametrisation $f_+(t) = f_+(0)\left\{1 + \lambda_+ \frac{t}{M_\pi^2}\right\}$ see H. Pichl's talk
- amplitude for $K^0_{e3\gamma}$ $\left(K^0_L(p) \to \pi^-(p')\, e^+(p_e)\, \nu_e(p_\nu)\, \gamma(q) \right)$:

$$
\begin{aligned}
T(K^0_{e3\gamma}) \;=\; & G_F\, e\, V^*_{us}\, \epsilon^\mu(q)^* \Big[\left(V_{\mu\nu} - A_{\mu\nu}\right) \underbrace{\bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5)\, v(p_e)}_{\text{leptonic weak current}} \\
& + \underbrace{f_+(t) \frac{(p+p')_\nu}{2 p_e q}\, \bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5)\, (m_e - \not{p}_e - \not{q})\, \gamma_\mu\, v(p_e)}_{\text{photon radiation from electron}} \Big]
\end{aligned}
$$

- 5 independent variables, e.g. E_γ, E_π, E_e, $\theta_{e\gamma}$, W^2 $(W = p_e + p_\nu)$

Characterisation of $V_{\mu\nu},\ A_{\mu\nu}$:

- decomposition $V_{\mu\nu} = V_{\mu\nu}^{\mathsf{IB}} + V_{\mu\nu}^{\mathsf{SD}}$,
  $V_{\mu\nu}^{\mathsf{IB}} \simeq$ photon radiation from pion, depends *only* on f_+
  H.W. Fearing, E. Fischbach, J. Smith, Phys. Rev. D2 (1970) 542

- Ward identities ($\rightarrow$ gauge invariance):

$$q^\mu\, V_{\mu\nu}^{\mathsf{IB}} = f_+(t)\,(p+p')_\nu \quad , \qquad q^\mu\, V_{\mu\nu}^{\mathsf{SD}} = q^\mu\, A_{\mu\nu} = 0$$

- $V_{\mu\nu}^{\mathsf{SD}}$, $A_{\mu\nu}$ decomposed in terms of 8 functions V_{1-4} , A_{1-4} :
  $\Rightarrow$ V_3 , A_3 suppressed by $m_e^2/M_K^2 \approx 10^{-6}$, not observable in $K^0_{e3\gamma}$
  $\Rightarrow$ V_4 , A_4 suppressed by two orders in the chiral expansion
  $\Rightarrow$ (essentially) 4 functions $\boxed{V_{1/2}\,,\ A_{1/2}}$ for SD contribution

Chiral one-loop prediction for structure dependent part:

$$
\begin{aligned}
V_1 &= -\frac{8}{F_\pi^2} L_9 + (\pi, K, \eta)\text{−loops} \\
V_2 &= \frac{4}{F_\pi^2}\left(L_9 + L_{10}\right) + (\pi, K, \eta)\text{−loops} \\
A_1 &= 0 \\
A_2 &= -\frac{1}{8\pi^2 F_\pi^2}
\end{aligned}
$$

- low-energy constants: $L_9 \leftrightarrow \langle r^2 \rangle_\pi^V$ or λ_+, $\quad L_{10} \leftrightarrow (\pi \to e\nu\gamma)$
- axial form factors A_i given in terms of chiral anomaly

J. Wess, B. Zumino, Phys. Lett. B37 (1971) 95

E. Witten, Nucl. Phys. B223 (1983) 422

Observables: decay widths

- for the non-radiative width:

$$\Gamma\left(K^0_{e3}\right) = \frac{M_K^5 G_F^2 |V_{us}|^2}{128\pi^3} f_+(0)^2 \times I$$

- analogously for the radiative width:

$$\Gamma(K^0_{e3\gamma}) = \frac{\alpha M_K^5 G_F^2 |V_{us}|^2}{16\pi^7} f_+(0)^2 \times I^\gamma(E_\gamma^{\text{cut}}, \theta_{e\gamma}^{\text{cut}})$$

in the following: “standard cuts” $E_\gamma^{\text{cut}} = 30$ MeV, $\theta_{e\gamma}^{\text{cut}} = 20^\circ$

- I, I^γ depend only on λ_+: $\quad I^{(\gamma)} = a_0^{(\gamma)} + a_1^{(\gamma)}\lambda_+ + a_2^{(\gamma)}\lambda_+^2$,

$$\frac{I^{(\gamma)}\left(\lambda_+ = \lambda_+^{\text{exp}} \approx 0.03\right)}{I^{(\gamma)}\left(\lambda_+ = 0\right)} - 1 \sim 0.1$$

The ratio R

- accessible in experiments:

$$R = \frac{\Gamma(K^0_{e3\gamma}, E_\gamma > E_\gamma^{\text{cut}}, \theta_{e\gamma} > \theta_{e\gamma}^{\text{cut}})}{\Gamma(K^0_{e3})} = \frac{8\alpha}{\pi^4} \frac{I^\gamma}{I}$$

$\Rightarrow$ all sorts of constants cancel in the ratio

- R depends on λ_+, but

$$\frac{R\left(\lambda_+ = \lambda_+^{\text{exp}} \approx 0.03\right)}{R\left(\lambda_+ = 0\right)} - 1 = \mathcal{O}\left(10^{-4}\right)$$

$\Rightarrow$ the form factor dependence cancels completely in R!

- in ChPT: only visible one-loop effect from structure dependent contributions:

$$\frac{R\,(\mathrm{IB}+\mathrm{SD})}{R\,(\mathrm{IB})}-1 \;\approx\; -0.01$$

$\Rightarrow$ inner bremsstrahlung completely dominant !

$\boxed{\text{we have a tree prediction that is accurate at the 1\%-level!}}$

- assume 30% uncertainty on structure dependent terms

$\Rightarrow$ our estimate Low + ChPT (preliminary):

$$\boxed{R = (0.952 \pm 0.004) \times 10^{-2}}$$

Electromagnetic corrections

Hadronic prediction of $\sim 1\%$ accuracy – radiative corrections?

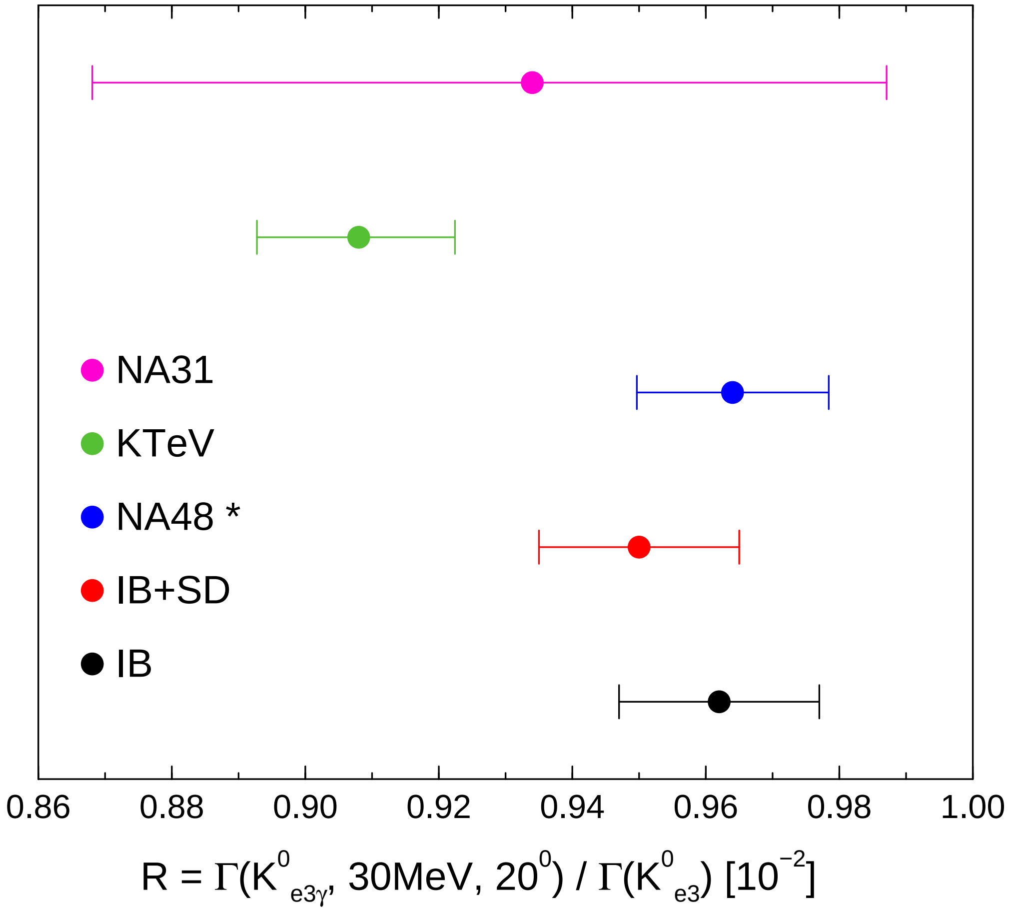
$$\mathcal{R} = \frac{\Gamma_{\mathsf{incl}}(K^0_{e3\gamma}, E_\gamma > E_\gamma^{\mathsf{cut}}, \theta_{e\gamma} > \theta_{e\gamma}^{\mathsf{cut}})}{\Gamma_{\mathsf{incl}}(K^0_{e3})}$$

- denominator: see H. Pichl's talk, net effect: I changed by 0.3%
- numerator: can we expect same net effect for $K^0_{e3\gamma}$?
  (only bremsstrahlung important $\Rightarrow$ given by K^0_{e3} $\Rightarrow$ cancellation??)

$\Rightarrow$ too simple: what is Ward identity in the presence of photons??

$\Rightarrow$ assume $\pm 1.5\%$ $\Rightarrow$ dominant uncertainty !

Comparison experiment/theory:



* NA48 data point according to M.M. Velasco's talk, added after the workshop

Structure dependent terms

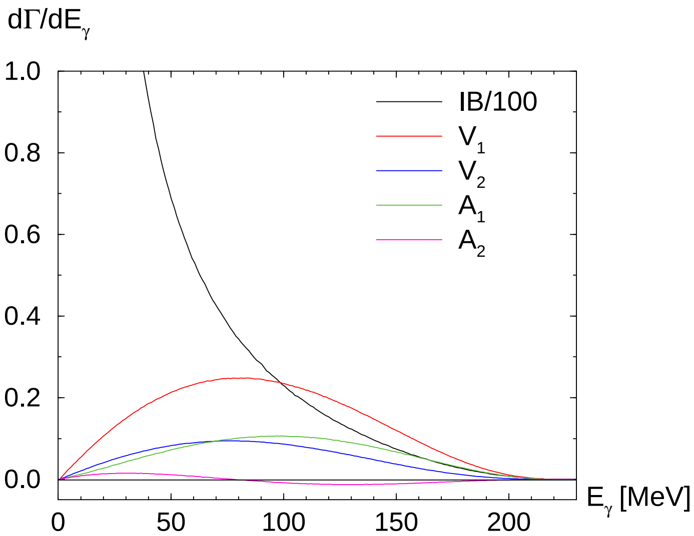
- Idea: access information independent on branching ratios by studying (unnormalised) differential rates $\Rightarrow$ $V_{1/2}$, $A_{1/2}$
- $d\Gamma/dE_\gamma$ most promising due to special role of E_γ in IB vs. SD
- KTeV : study $d\Gamma/dE_\gamma$ with
  1. real and constant structure functions
  2. two SD terms, neglect other two in "soft kaon approximation"

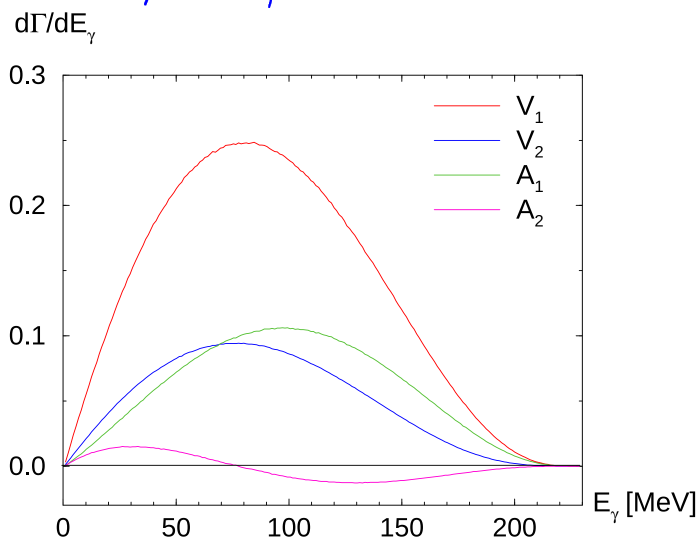
$\Rightarrow$ ChPT : first assumption reasonable
(imaginary part + momentum dependence suppressed to two-loop)

$\Rightarrow$ check validity of the second assumption

$\Rightarrow$ study also other distributions

Structure dependent contributions to $d\Gamma/dE_\gamma$



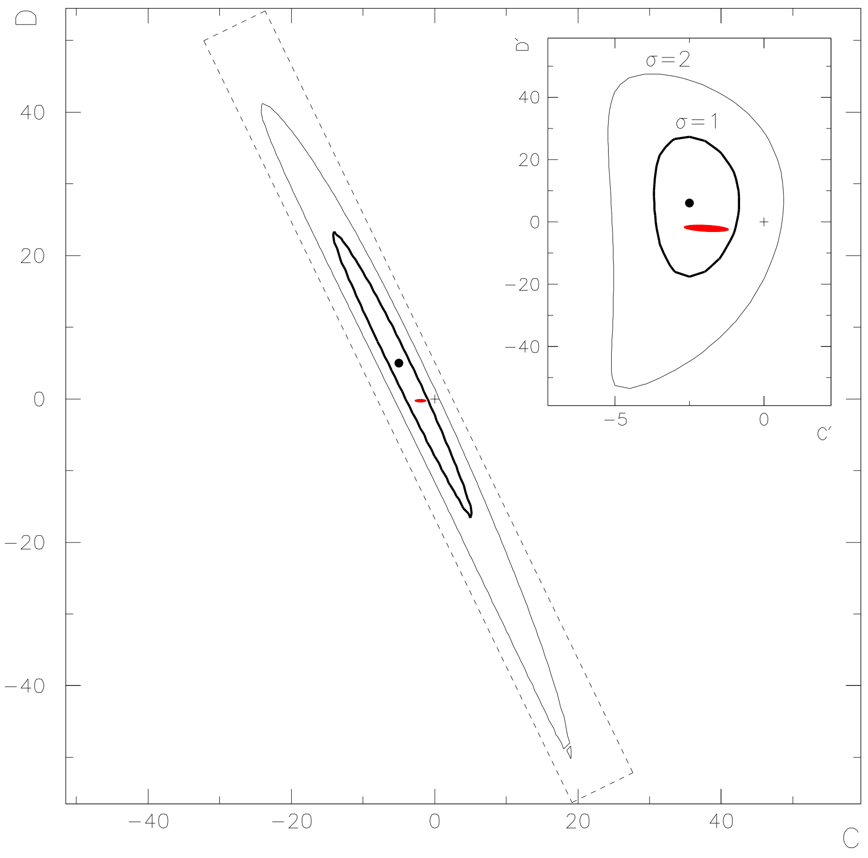


$\Rightarrow$ $d\Gamma/dE_\gamma$ is essentially sensitive to *one* combination

$$
\begin{aligned}
C' &= 0.9 \times V_1 + 0.4 \times V_2 + 0.4 \times A_1 \\
&= 0.9 \times \left(V_1 - V_2\right) + \underbrace{1.3 \times V_2}_{\text{neglected}} + 0.4 \times \left(A_1 + A_2\right) - \underbrace{0.4 \times A_2}_{\text{neglected}}
\end{aligned}
$$

$\Rightarrow$ "soft kaon" not a good approximation!

Compare KTeV result to ChPT prediction:



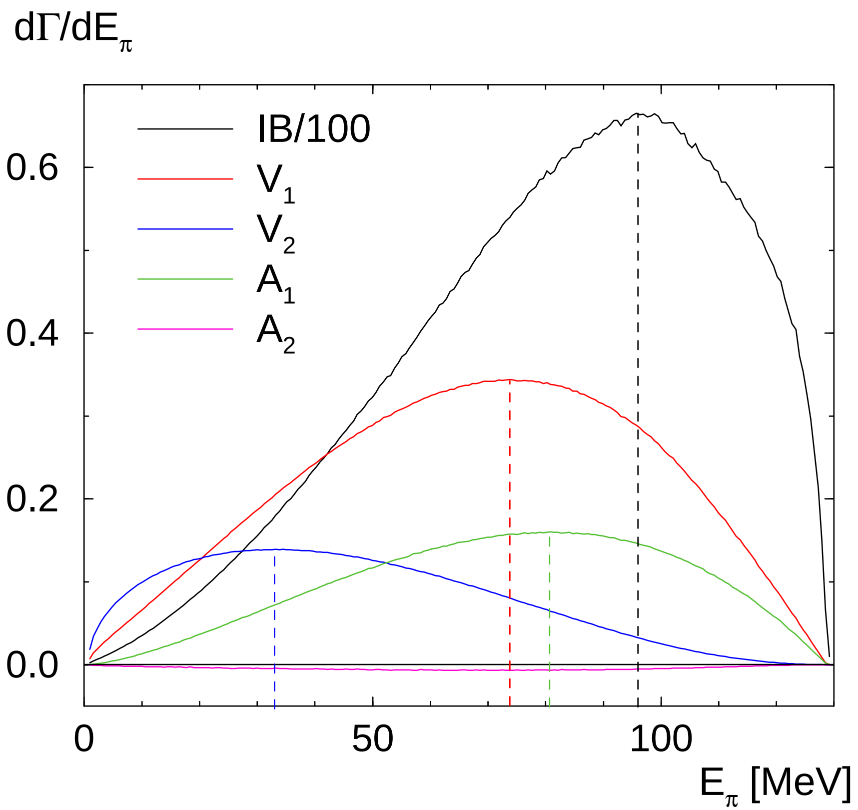
- KTeV: $C' = -2.5^{+1.5}_{-1.0} \pm 1.5$
- ChPT: $C' = -1.9 \pm 0.7$
  (preliminary)

$\Rightarrow$ excellent numerical agreement within 1-σ error

$\Rightarrow$ interpretation of C' in terms of structure functions different

$\Rightarrow$ serious constraints on SD terms feasible !

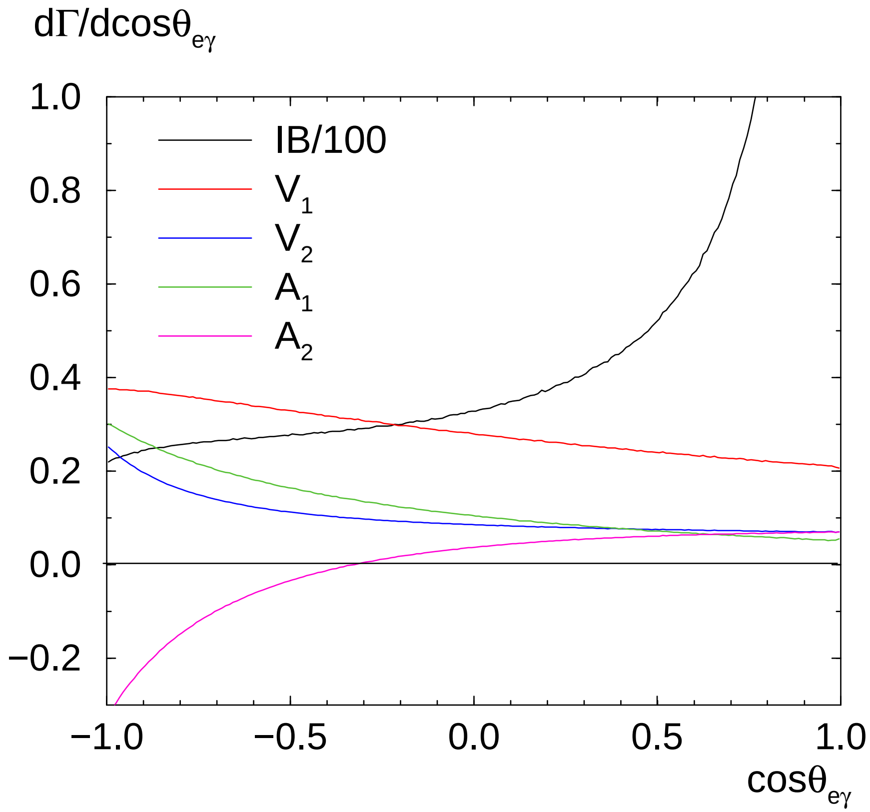
Other distributions: $d\Gamma/dE_\pi$



$\Rightarrow$ potentially sensitive to V_2 :

structure	E_π peak
IB	96 MeV
V_1	74 MeV
V_2	34 MeV
A_1	81 MeV

And the anomaly? $d\Gamma/d\cos\theta_{e\gamma}$



$\Rightarrow$ remember prediction $A_1 = 0$

$\Rightarrow$ here A_2 is (relatively) strong

$\Rightarrow$ potentially dominates the *variation* of $d\Gamma/d\cos\theta_{e\gamma}$ in backward direction

$\Rightarrow$ no serious statistical feasibility estimate yet

Summary (1)

Combination of Low's theorem and ChPT allows for an extremely precise prediction of $R = \Gamma(K^0_{e3\gamma})/\Gamma(K^0_{e3}) = (0.952 \pm 0.004) \times 10^{-2}$

- R completely insensitive to details of K^0_{e3} form factor
- structure dependent terms very small
- constants like G_F or $|V_{us}|$ cancel

Precision limited by radiative corrections :

$$\boxed{\mathcal{R} = \Gamma(K^0_{e3\gamma})/\Gamma(K^0_{e3}) = (0.950 \pm 0.004_{\text{hadr}} \pm 0.015_{\text{em}}) \times 10^{-2}}$$

vs. $\mathcal{R} = (0.908 \pm 0.008_{\text{stat}}\,{}^{+\,0.013}_{-\,0.012}{}_{\text{syst}}) \times 10^{-2}$ (KTeV)

$\mathcal{R} = (0.964 \pm 0.008_{\text{stat}} \pm 0.012_{\text{syst}}) \times 10^{-2}$ (NA48)

Summary (2)

Experimental extraction of structure dependent terms from $d\Gamma/dE_\gamma$ shown to be feasible by KTeV collaboration:

$$C' = -2.5\,^{+1.5}_{-1.0}\,{}_{\text{stat}} \pm 1.5_{\text{syst}} \quad \text{(KTeV)} \quad \text{vs.} \quad C' = -1.9 \pm 0.7_{p^6} \quad \text{(ChPT)}$$

Tentative ideas for experimentalists:

- extract V_2 from $d\Gamma/dE_\pi$ in order to disentangle V_1 and V_2 in C'
- chiral anomaly in A_2 at most accessible in $d\Gamma/d\cos\theta_{e\gamma}$

Obvious idea for theorists:

- perform same study for other $K_{\ell 3\gamma}$ channels!