The $a_0 - a_2$ pion scattering length from $K^+ \to \pi^+\pi^0\pi^0$ decay

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A “commercial ” for $K_+ \to \pi^+ \nu\bar\nu$ and $K_L \to \pi^0 \nu\bar\nu$

$K_+ \to \pi^+ \nu\bar\nu$ and $K_L \to \pi^0 \nu\bar\nu$: a unique physics opportunity

Insight on CKM and CP violation equivalent to BELLE + BABAR

$+$

Short distance reach comparable to LEP200

but

Independent of either programs!

A "commercial " for Neutral Hyperons

We can put the final word on Ξ^0 and Λ beta decay

And put the final word on V_{us}...

and flavor symmetry breaking

Summary

- Pion scattering lengths.
- The method.
- Theoretical precision.
- Further theoretical work.

N. Cabibbo, arXiv: hep-ph 0405001

Pion scattering lengths and chiral dynamics

Weinberg 1966 :

$$a_0 m_{\pi^+} = \frac{7m_{\pi^+}^2}{16\pi f_\pi^2} = 0.159$$

$$a_2 m_{\pi^+} = \frac{-m_{\pi^+}^2}{8\pi f_\pi^2} = -0.045$$

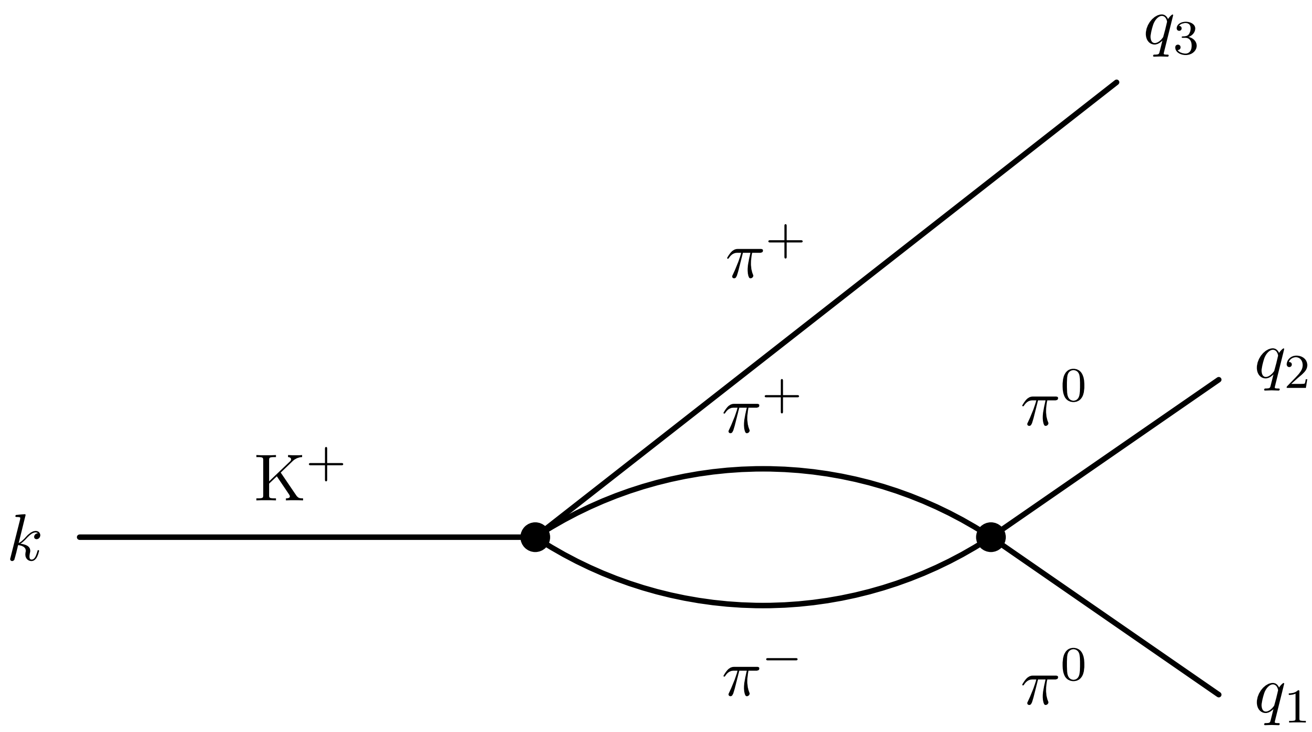
Colangelo et al. 2001 :

$$a_0 m_{\pi^+} = 0.220 \pm 0.005$$

$$a_2 m_{\pi^+} = -0.0444 \pm 0.0010$$

$$(a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004$$

Can we match this precision?



$\pi^+\pi^- \rightarrow \pi^0\pi^0$ re-scattering from $K^+ \rightarrow \pi^+\pi^+\pi^-$.

$K^+ \to \pi^+\pi^0\pi^0 \quad \text{and } K^+ \to \pi^+\pi^+\pi^-$

The unperturbed amplitudes are well known — see PDG,
but with the best data we would like to fit our own.
A simple parametrization:

$$K^+ \to \pi^+\pi^0\pi^0 \quad : \quad \mathcal{M}_0(s_1, s_2, s_3) = A^0_{\text{av}}(1 + g^0(s_{\pi\pi} - s_0)/2m^2_{\pi^+})$$

$$K^+ \to \pi^+\pi^+\pi^- \quad : \quad \mathcal{M}_+(s_1, s_2, s_3) = A^+_{\text{av}}(1 + g^+(s_{\pi\pi} - s_0)/2m^2_{\pi^+}),$$

with g^0, g^+ the linear slope parameters.

Let us write:

$$\mathcal{M}(K^+ \to \pi^+\pi^0\pi^0) = \mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1$$

where $\mathcal{M}_1$ is the contribution of the re-scattering graph.

We must consider two cases, according to whether $M_{\pi\pi}$ is above or below the $\pi^+\pi^-$ threshold.

$$s_{\pi\pi} > 4m_{\pi^+}^2 \quad : \quad \mathcal{M}_1 = i2\frac{(a_0 - a_2)m_{\pi^+}}{3}\mathcal{M}_{+,\mathrm{thr}}\sqrt{(s_{\pi\pi} - 4m_{\pi^+}^2)/s_{\pi\pi}}$$

$$s_{\pi\pi} < 4m_{\pi^+}^2 \quad : \quad \mathcal{M}_1 = -2\frac{(a_0 - a_2)m_{\pi^+}}{3}\mathcal{M}_{+,\mathrm{thr}}\sqrt{(4m_{\pi^+}^2 - s_{\pi\pi})/s_{\pi\pi}}$$

where $\mathcal{M}_{+,\mathrm{thr}}$ is the value of $\mathcal{M}_+$ at the $\pi^+\pi^-$ threshold.

- Above threshold: $\mathcal{M}_1$ imaginary
- Below threshold: $\mathcal{M}_1$ real and negative

A technical clarification

I-spin is badly broken between the $\pi^0\pi^0$ and the $\pi^+\pi^-$ threshold which is just the region we are interested in!

How to define I=0 and I=2 scattering lengths?

Solution we have adopted:

Define $(a_0 - a_2)/3$ through:

$$M(\pi^+\pi^- \to \pi^0\pi^0)|_{\text{S-wave}} = imv_{\pi^+}(a_0 - a_2)/3 + O(v_{\pi^+}^2)$$

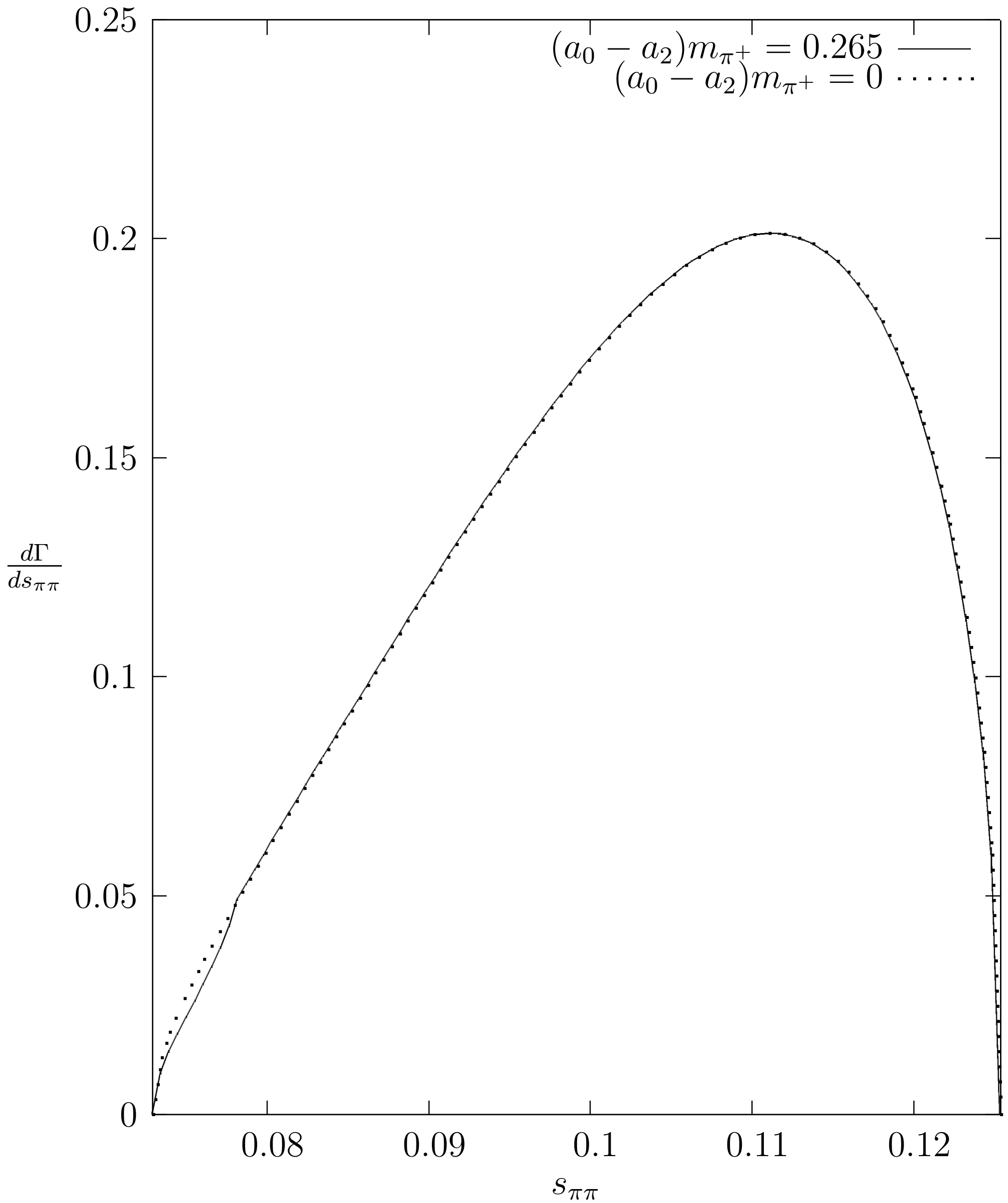
where $v_{\pi^+} = \sqrt{(s_{\pi\pi} - 4m_{\pi^+}^2)/s_{\pi\pi}}$

The key to the measurement

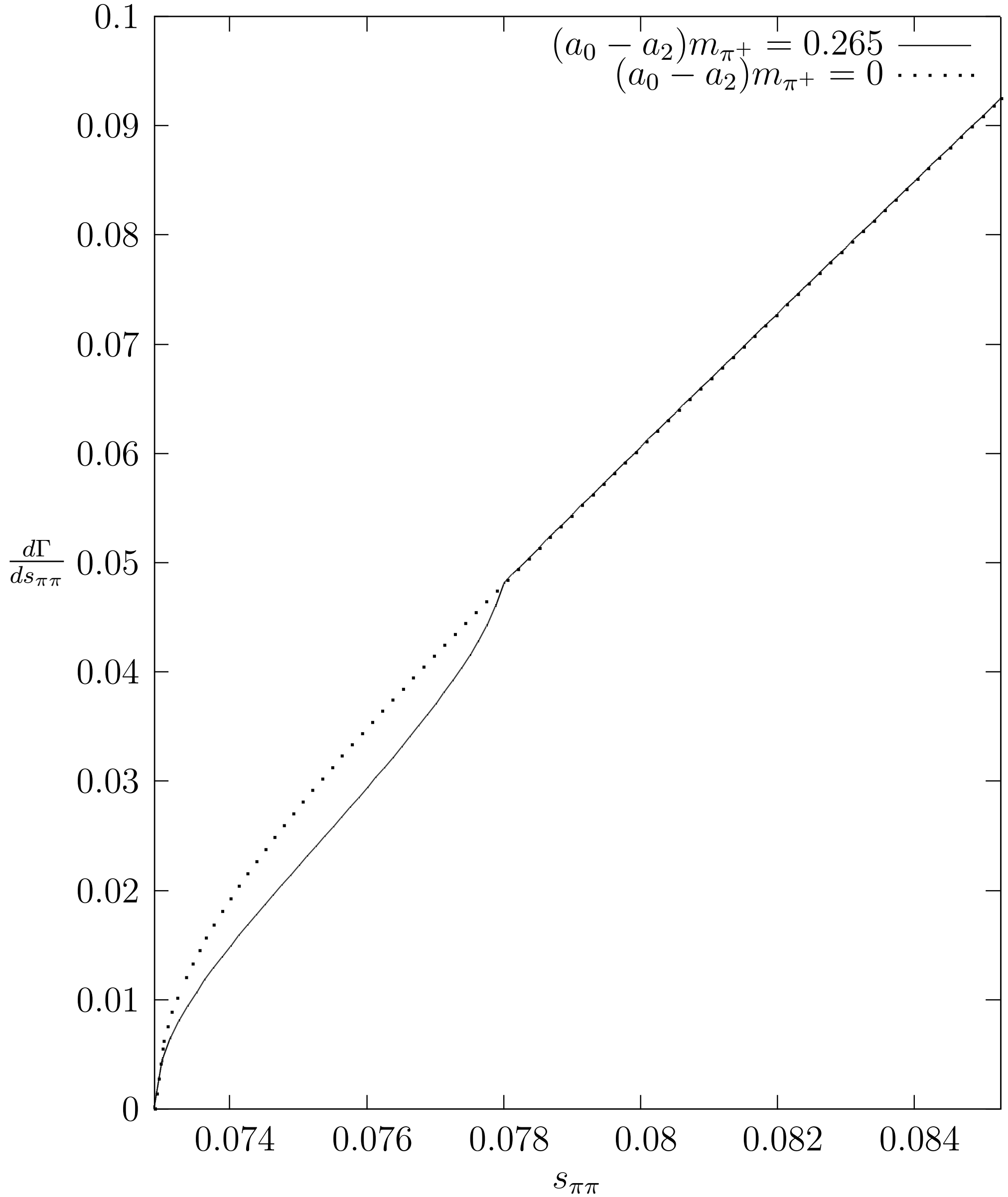
Below threshold there is an interference term
which is absent above the threshold:

$$|\mathcal{M}|^2 = \begin{cases} (\mathcal{M}_0)^2 + (\mathcal{M}_1)^2 + 2\mathcal{M}_0\mathcal{M}_1 & : \quad s_{\pi\pi} < 4m_{\pi^+}^2 \\ (\mathcal{M}_0)^2 + |\mathcal{M}_1|^2 & : \quad s_{\pi\pi} > 4m_{\pi^+}^2 \end{cases}$$

The interference term
is proportional to $a_0 - a_2$



The $s_{\pi\pi}$ invariant mass distribution
with/without the re-scattering correction, in arbitrary units.



The $s_{\pi\pi}$ invariant mass distribution
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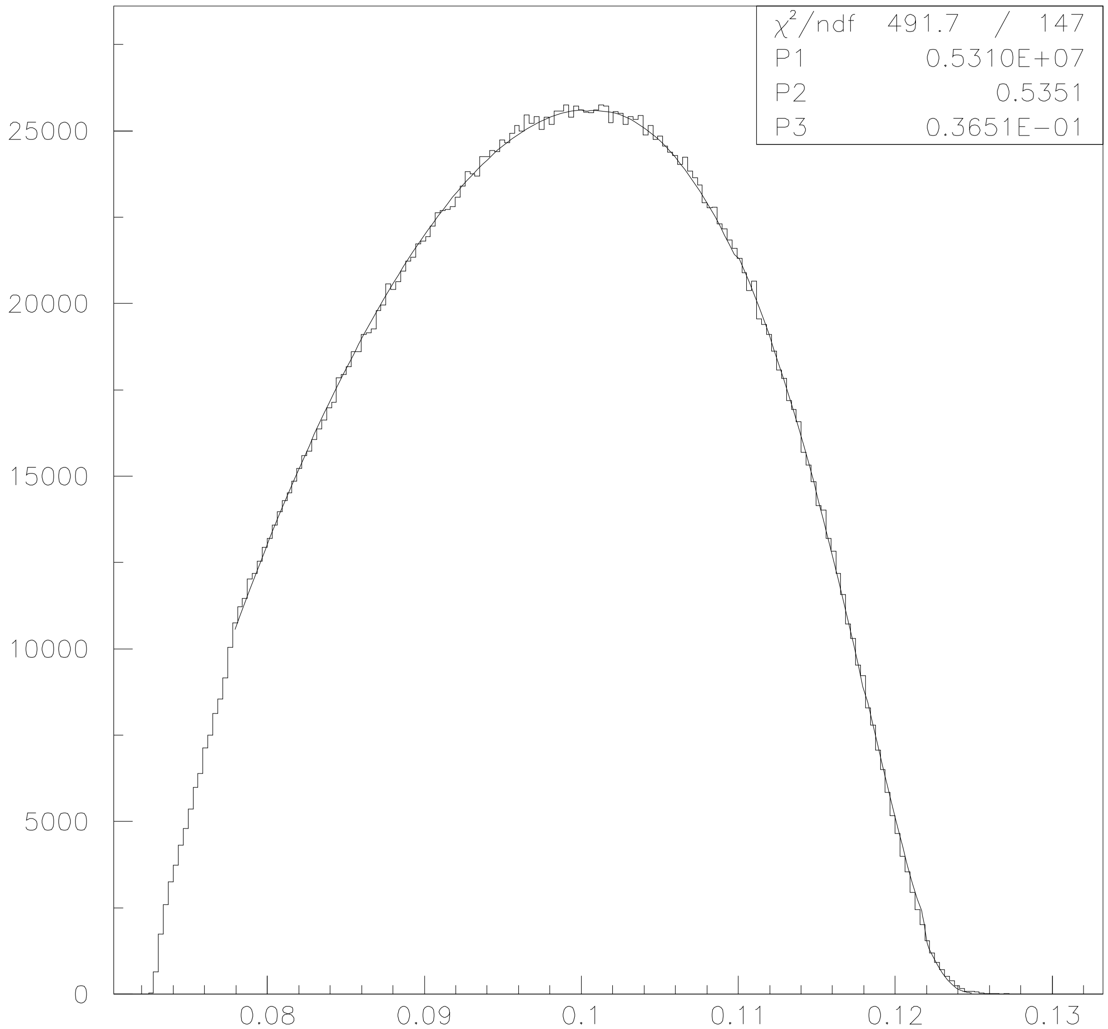
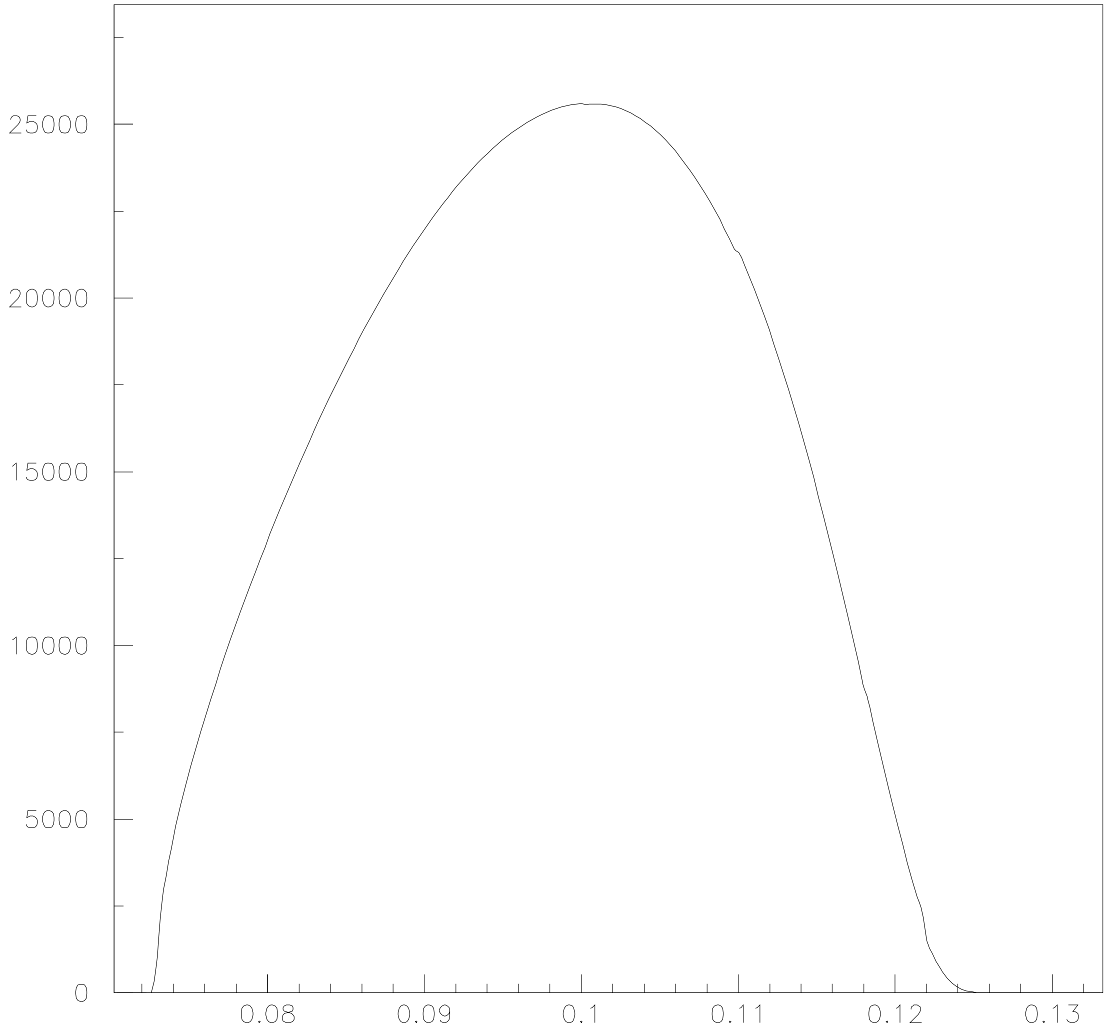
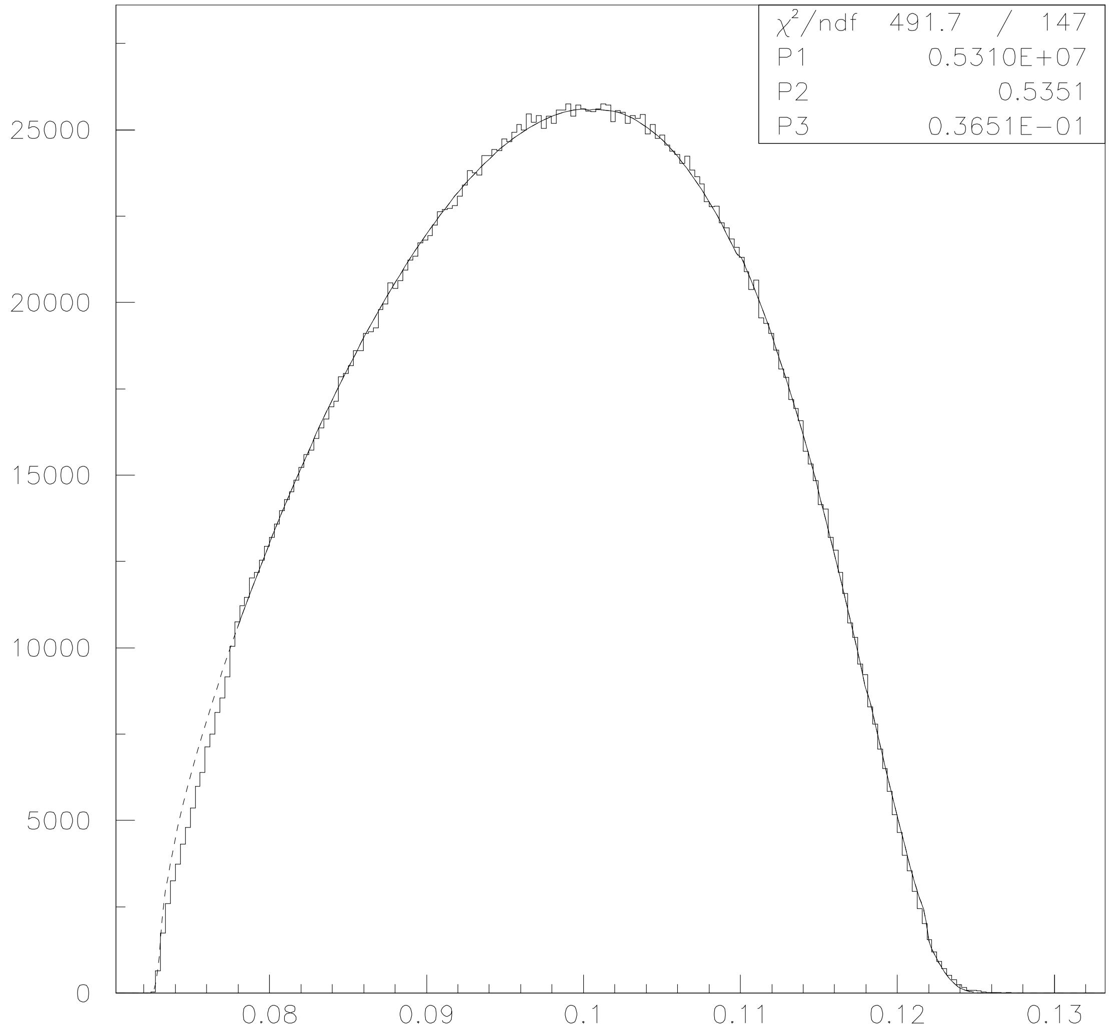


The $s_{\pi\pi}$ invariant mass distribution (1/4 of existing NA48 data) with fit above the $\pi^+\pi^-$ threshold.

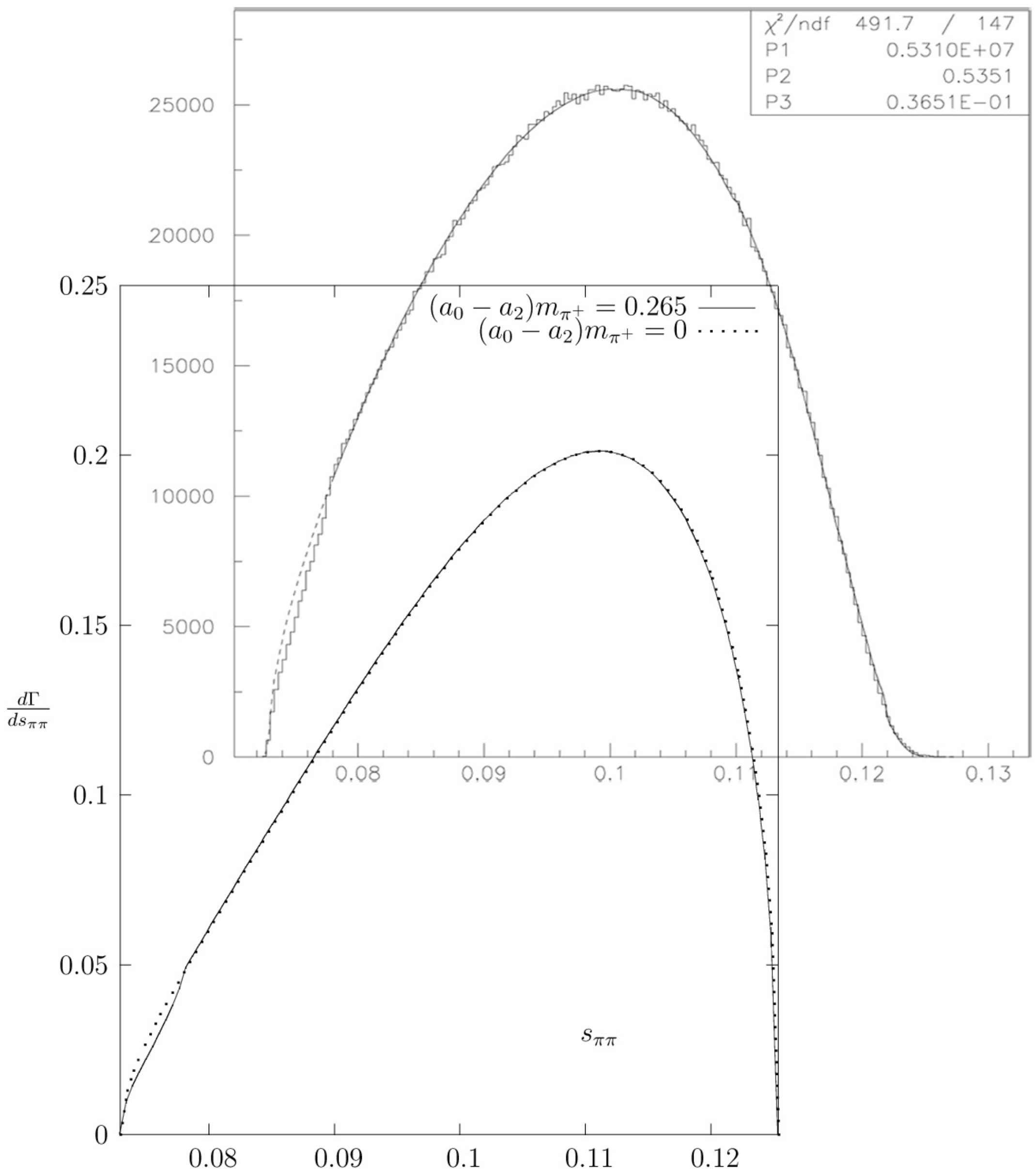
As analyzed by Italo Mannelli.



The fit to NA48 data



It does not look bad!

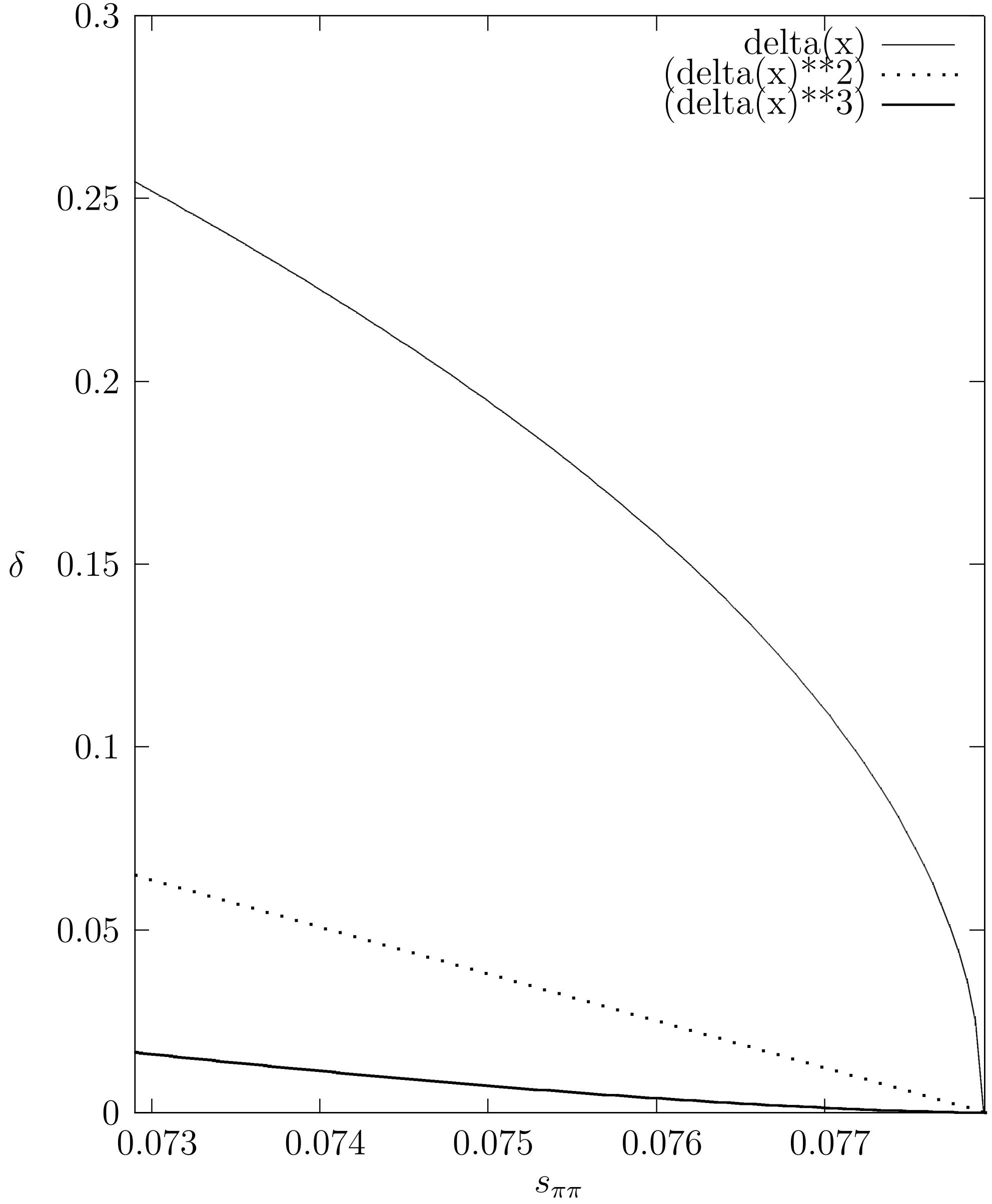


I could not resist!

Precision

- Below threshold define $\delta = \sqrt{\frac{4m_{\pi^+}^2 - s_{\pi\pi}}{4m_{\pi^+}^2}}$
- The differential rate is predicted with errors $\sim \delta^3$
- The theoretical error on $a_0 - a_2$ is $\sim \delta_{max}^2$

One can strike a balance between theoretical uncertainty and statistical error.



Theoretical precision can be improved by restricting the range of $s_{\pi\pi}$.

Space for improvement

- Experimental fit to terms $O(\delta^3)$ in differential rate.
- Compute these terms in Chiral Perturbation Theory
- Compute radiative corrections

The theoretical error can be made very small.