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On the Rare $K \longrightarrow \pi \ell^+ \ell^-$ Decays

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Motivations

1- New experimental results for the rare decays

$$K_S \longrightarrow \pi^0 e^+ e^- \qquad K_L \longrightarrow \pi^0 \gamma\gamma$$ [NA48 '03]

Those results permit a

Re-analysis of the CP violating rare decay

$$K_L \longrightarrow \pi^0 e^+ e^-$$

2- Computation of the coupling constants of $\mathscr{L}_{em}^{\Delta S=1}$

to leading and next-to-leading orders in Large N_C QCD

$K_L \longrightarrow \pi^0 e^+ e^-$

- CP Conserving part $K_L \longrightarrow \pi^0 \gamma^* \gamma^* \longrightarrow \pi^0 e^+ e^-$
- CP Violating part
  - Direct $\not{CP}$ — Short Distance Contribution
  - Indirect $\not{CP}$ — directly from $K_S \longrightarrow \pi^0 e^+ e^-$
  - Interference Term
    - Constructive
    - Destructive

?

Chiral Lagrangians

The type of interferences is strongly linked to coupling constants of chiral Lagrangians

Precisely to w_1 and w_2 in their combination $\tilde{w} \doteq w_1 - w_2$

In fact the usual chiral electro-weak order p^4 Lagrangian is [EPdeR '87]

$$\mathscr{L}^{(4)}_{\Delta S=1,\,\mathrm{em}} = \frac{-ie}{2f_\pi^2} G_8 F^{\mu\nu} \left[w_1 \,\mathrm{Tr}\,(Q\lambda_{6-i7} L_\mu L_\nu) + w_2 \,\mathrm{Tr}\,(Q L_\mu \lambda_{6-i7} L_\nu) \right] + \text{ h.c.}$$

$$L_\mu = i f_\pi^2 U \partial_\mu U^\dagger$$

and can be rewritten in a Large N_C counting as

$$\mathscr{L}^{(4)}_{\Delta S=1,\,\mathrm{em}} = -\frac{ie}{6f_\pi^2} G_8 F^{\mu\nu} \left\{ \tilde{w}\ \mathrm{tr}\,(\lambda_{6-i7} L_\mu L_\nu) + 3 w_2 \,(L_\mu)_{13}\,(L_\nu)_{21} \right\} + \mathrm{h.c.}$$

where

$$w_2 \quad \text{is} \quad \mathcal{O}(N_C)$$

$$\tilde{w} = w_1 - w_2 \quad \text{is} \quad \mathcal{O}(1)$$

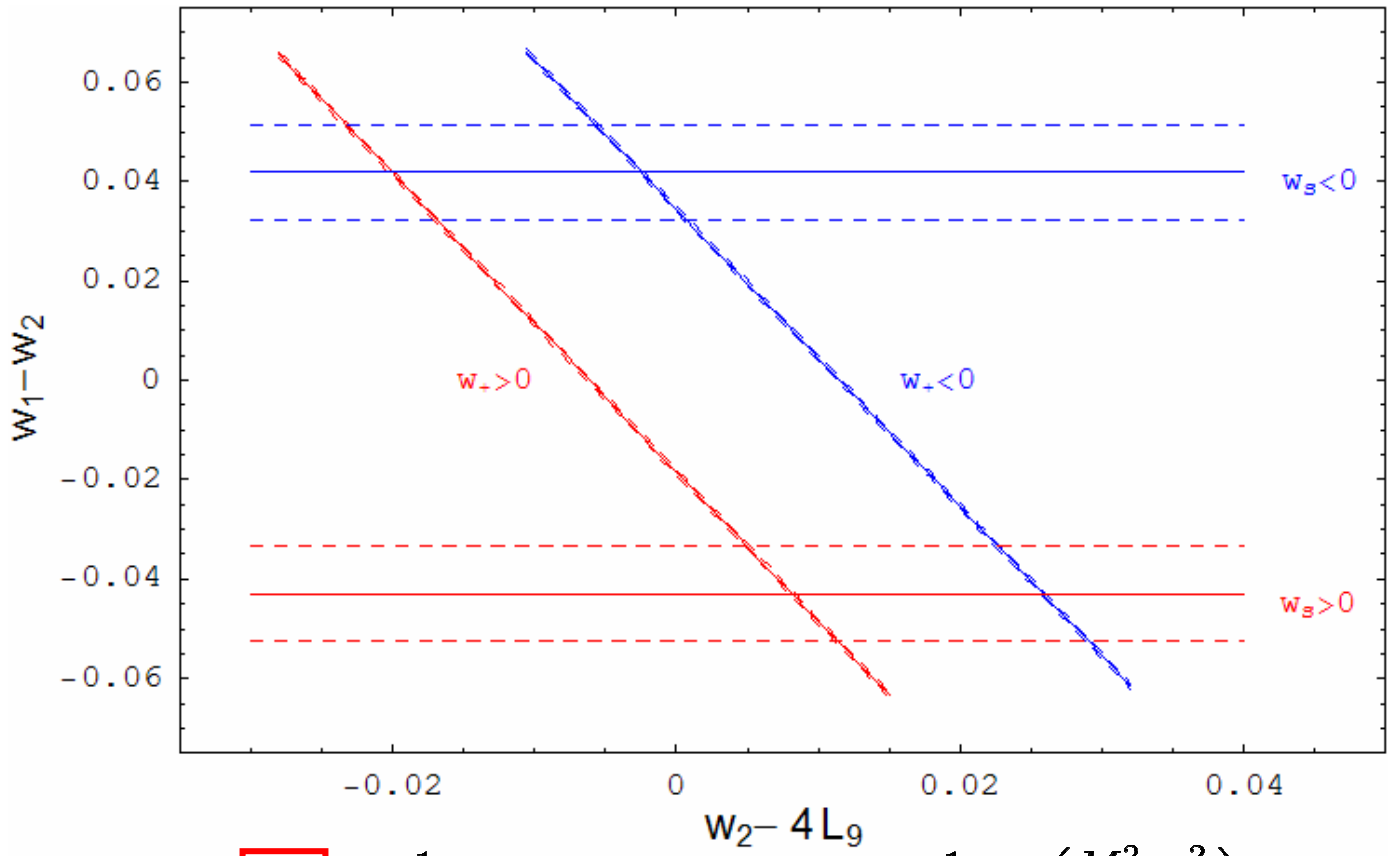
Experimental constraints on this coupling constants via

$$K^+ \longrightarrow \pi^+ e^+ e^- \quad \text{and} \quad K_S \longrightarrow \pi^0 e^+ e^-$$

Constraints from Branching Ratios

$$\mathrm{Br}\left(K^+ \longrightarrow \pi^+ e^+ e^-\right) \quad vs \quad \mathrm{Br}\left(K_S \longrightarrow \pi^0 e^+ e^-\right)$$

[PDG '02] [NA48 '03]



key-quantities

$$\boxed{w_+} = -\frac{1}{3}(4\pi)^2\left[\tilde{w} + 3(w_2 - 4L_9)\right] - \frac{1}{6}\log\left(\frac{M_K^2 m_\pi^2}{M_\rho^4}\right)$$

$$\boxed{w_s} = -\frac{1}{3}(4\pi)^2\,\tilde{w} - \frac{1}{3}\log\left(\frac{M_K^2}{M_\rho^2}\right)$$

Order p^4

Octet Dominance Hypothesis and Large-N_C Predictions

- Octet Dominance Hypothesis [EPdR 87]: $w_2 = 4L_9$

- Large-N_c Predictions :

Through the bosonization of the factorized part of the operator

$$Q_2 = 4\left(\bar{s}_L\gamma^\mu u_L\right)\left(\bar{u}_L\gamma_\mu d_L\right)$$

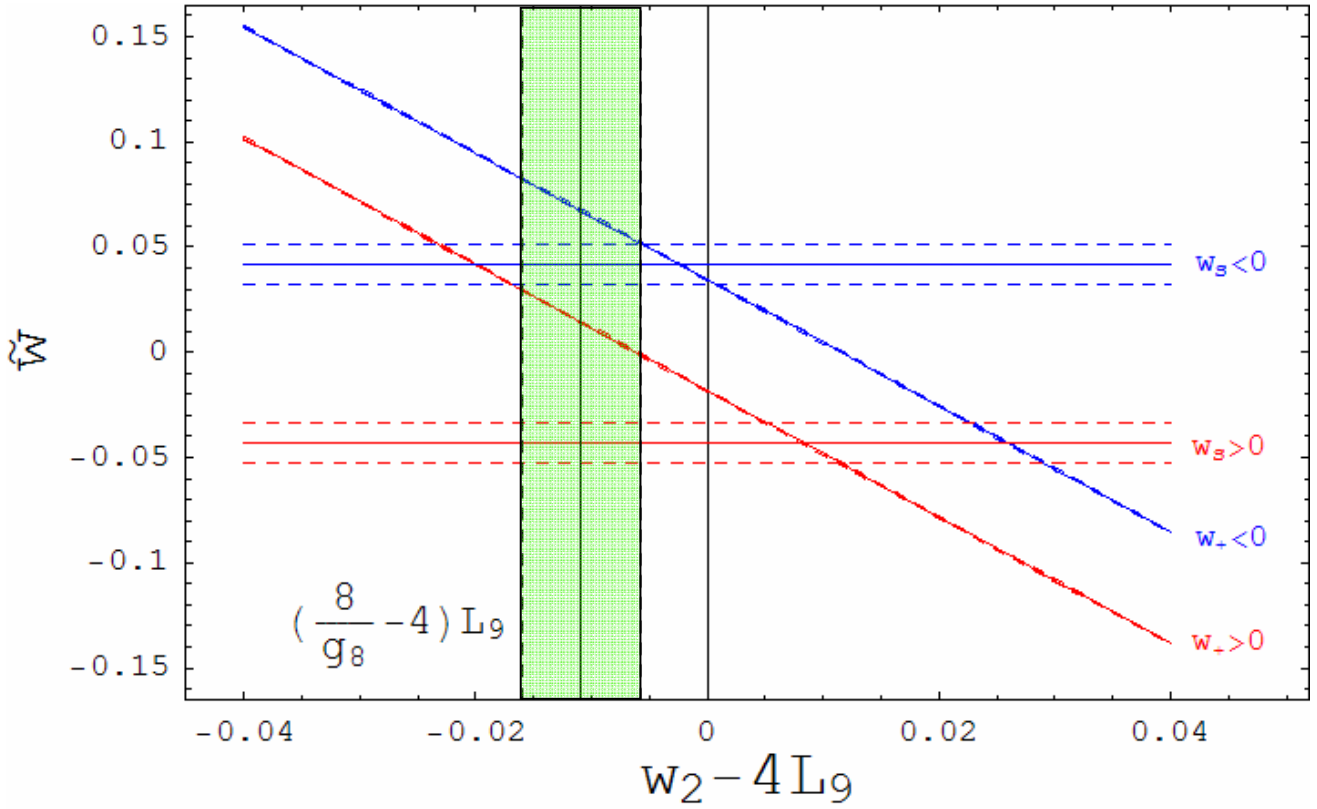
matched to our order p^4 chiral Lagrangian we find

$$g_8 w_2 = 8L_9 + \mathcal{O}\left(N_C^0\right)$$

$$g_8\tilde{w} = 0 + \mathcal{O}\left(N_C^0\right)$$

See also [Bruno Prades '93]

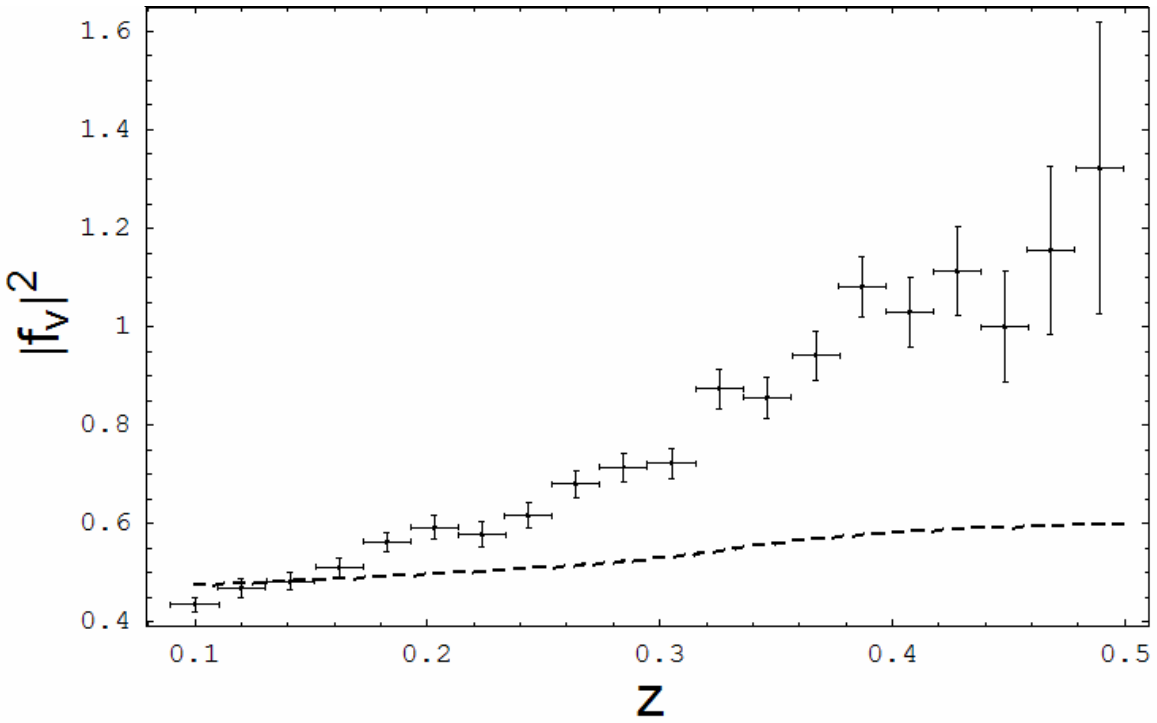
Results



Conclusion: $w_S < 0$ in both cases then CONSTRUCTIVE INTERFERENCES

But the sign of w_+ is not clearly predicted: Need for a Large N_C next-to-leading order calculation

Constraints from the $K^+ \longrightarrow \pi^+ e^+ e^-$ form factor

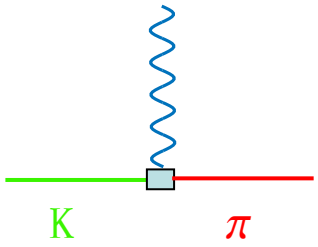


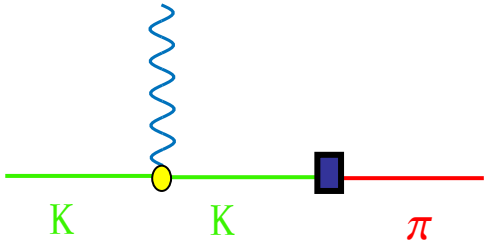
Order p^4 : $$|f_V(z)| = \left| \frac{G_8}{G_F} \left\{ \frac{1}{3} - w_+ - \frac{1}{60} z - \chi(z) \right\} \right|$$

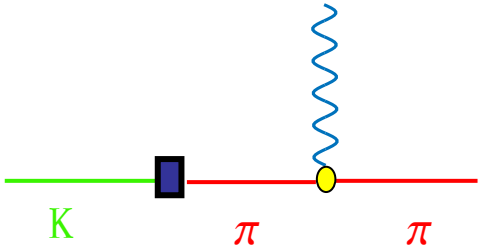
where $\chi(z)$ is the function of the pion chiral loop.

Clearly, the order p^4 is not sufficient

Narrow resonances dynamical framework







$$1 - \frac{\mathbf{w_2}}{2F_\pi^2}Q^2$$

$$-\left(1 - \frac{\mathbf{2L_9}}{F_\pi^2}Q^2\right)$$

NRD Hyp.

MHA

$$\alpha\frac{M_\rho^2}{M_\rho^2+Q^2} + \beta\frac{M_{K^*}^2}{M_{K^*}^2+Q^2}$$

$$-1\,\frac{M_\rho^2}{M_\rho^2+Q^2}$$

$$\alpha + \beta = 1$$

chiral limit

$$\mathbf{w_2 = 4L_9}$$

Confirmation of results: Octet Dominance [EPdeR '87]

Minimal Narrow Resonances Saturation of the Form Factor

$$L_9 = \frac{F_\pi^2}{2M_\rho^2}$$

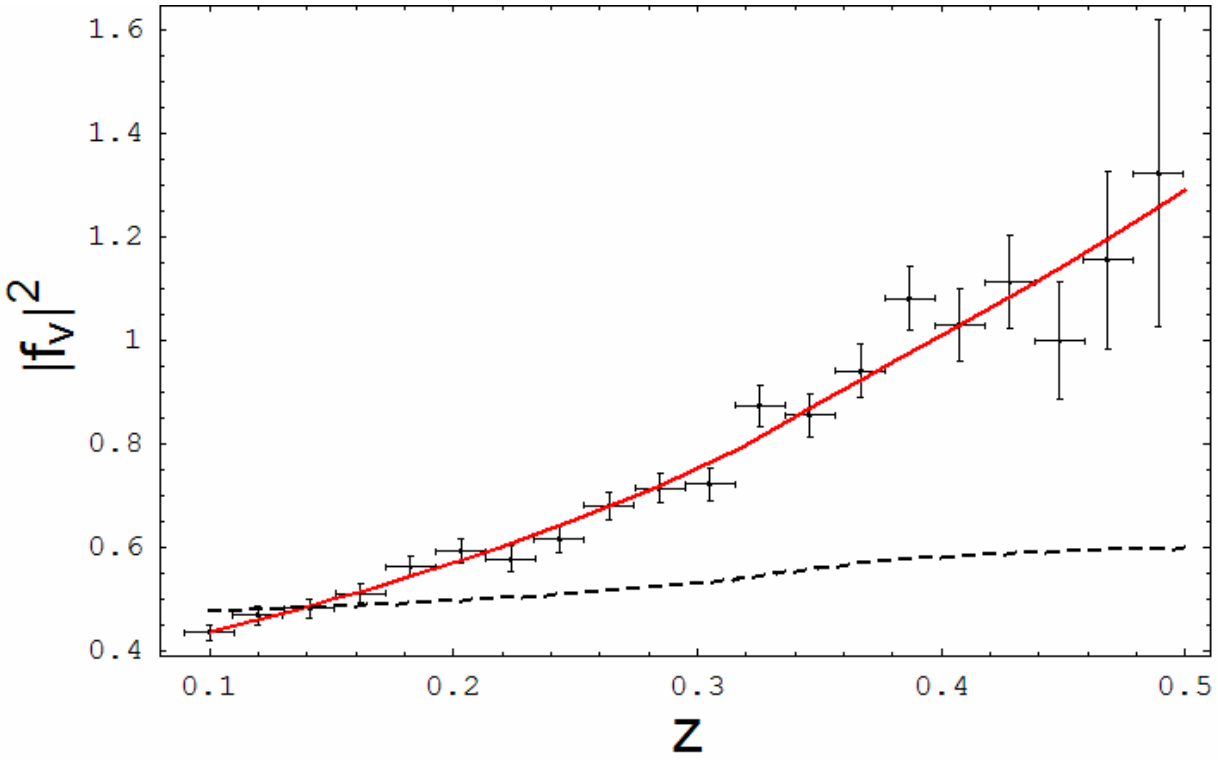
$$w_2 = \frac{2F_\pi^2}{M_\rho^2}\left[1+\beta\left(\frac{M_\rho^2}{M_{K^*}^2}-1\right)\right]$$

The form factor is now

$$|f_V(z)| = \left|\frac{G_8}{G_F}\left\{\frac{(4\pi)^2}{3}\left[\tilde{\mathbf{w}}\frac{M_\rho^2}{M_\rho^2-M_K^2 z}+6F_\pi^2\beta\frac{M_\rho^2-M_{K^*}^2}{\left(M_\rho^2-M_K^2 z\right)\left(M_{K^*}^2-M_K^2 z\right)}\right]\right.\right.$$

$$\left.\left.+\frac{1}{6}\ln\left(\frac{M_K^2 m_\pi^2}{M_\rho^4}\right)+\frac{1}{3}-\frac{1}{60}z-\chi(z)\right\}\right|$$

Fit



$g_8 = 3.3$

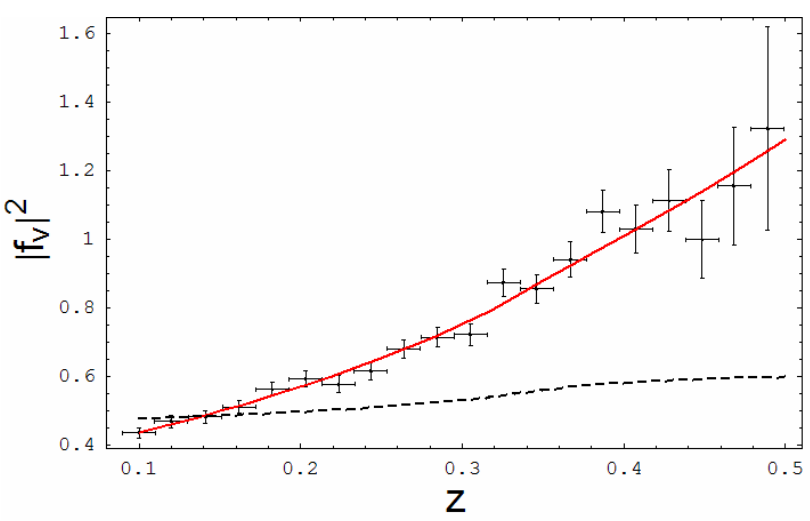
$F_0 = 92.4\text{MeV}$

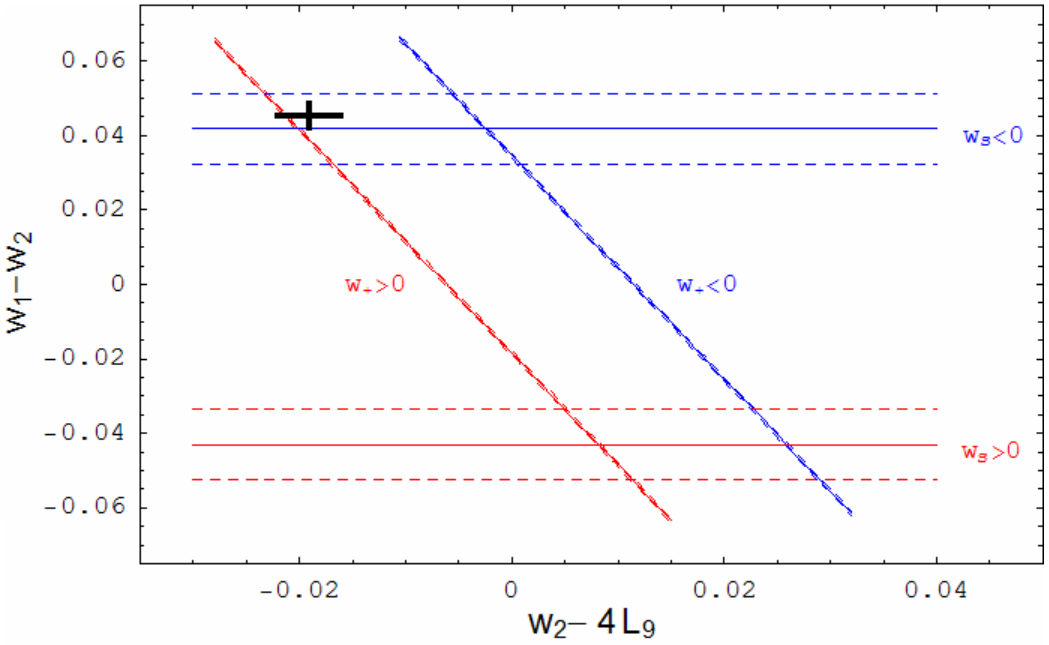
$\chi^2_{\text{min.}} = 13.0/18\text{d.f.}$

$\tilde{\mathbf{w}} = 0.045(3)$

$\beta = 2.8(1)$

Fit





$$\tilde{\mathbf{w}} = 0.045(3) \qquad \mathbf{w_2} - 4L_9 = -0.019(3)$$

$$\beta = 2.8(1) \quad \Longrightarrow \quad \mathbf{w_s} = -2.1(2)$$

Predictions

	Results	Experiments
$\mathrm{Br}\left(K^+ \longrightarrow \pi^+ e^+ e^-\right)$	$(3.0 \pm 1.1) \times 10^{-7}$	$(2.88 \pm 0.13) \times 10^{-7}$
$\mathrm{Br}\left(K^+ \longrightarrow \pi^+ \mu^+ \mu^-\right)$	$(8.7 \pm 2.8) \times 10^{-8}$	$(7.6 \pm 2.1) \times 10^{-8}$
$\mathrm{Br}\left(K_S \longrightarrow \pi^0 e^+ e^-\right)$	$(7.7 \pm 1.0) \times 10^{-9}$	$(5.8^{+2.8}_{-2.3} \pm 0.8) \times 10^{-9}$
$\mathrm{Br}\left(K_S \longrightarrow \pi^0 \mu^+ \mu^-\right)$	$(1.7 \pm 0.2) \times 10^{-9}$	$(2.9^{+1.4}_{-1.2} \pm 0.2) \times 10^{-9}$

Prediction for $\mathrm{Br}\left(K_L \longrightarrow \pi^0 e^+ e^-\right)$ and Outlook

- CP Conserving Part [Buchalla et al. '03] ≈ 0
- CP Direct Violating Part [Buchalla et al. '03] 0.44×10^{-11}
- CP Indirect Violating Part [NA48 '03] 2.31×10^{-11}
- Interference term [this work] 1.03×10^{-11}

$$\mathbf{Br\left(K_L \longrightarrow \pi^0 e^+ e^-\right) = (3.7 \pm 0.4) \times 10^{-11}}$$

AIM

Next-to-leading order calculation in Large-N_C QCD of

w_2 and $\tilde{w}$

In the framework of [Hambye et al. '03]