CERN, 5th May 2004

Rare Kaon Decays Revisited

Samuel FRIOT

in collaboration with **D. GREYNAT** and **E. de RAFAEL**

arXiv:hep-ph/0404136 submitted to PLB

Motivations

1- New experimental results for the rare decays

$$K_S \longrightarrow \pi^0 e^+ e^- \qquad\qquad K_L \longrightarrow \pi^0 \gamma\gamma$$ [NA48 '03]

Those results permit a

Re-analysis of the CP violating rare decay

$$K_L \longrightarrow \pi^0 e^+ e^-$$

2- Computation of the coupling constants of $\mathscr{L}_{em}^{\Delta S=1}$

to leading and next-to-leading orders in Large N_C QCD

$$K_L \longrightarrow \pi^0 e^+ e^-$$

- CP Conserving part $K_L \longrightarrow \pi^0 \gamma^* \gamma^* \longrightarrow \pi^0 e^+ e^-$

- CP Violating part

  - Direct $\cancel{CP}$ Short Distance Contribution
  - Indirect $\cancel{CP}$ directly from $K_S \longrightarrow \pi^0 e^+ e^-$

  - Interference Term → Constructive / Destructive ?

Chiral Lagrangians

The type of interferences is strongly linked to coupling constants of chiral Lagrangians

Precisely to w_1 and w_2 in their combination $\tilde{w} \doteq w_1 - w_2$

In fact the usual chiral electro-weak order p^4 Lagrangian is [EPdeR '87]

$$\mathscr{L}^{(4)}_{\Delta S=1,\,\mathrm{em}} = \frac{-ie}{2f_\pi^2} G_8 F^{\mu\nu} [w_1 \,\mathrm{Tr}\,(Q\lambda_{6-i7} L_\mu L_\nu) + w_2 \,\mathrm{Tr}\,(Q L_\mu \lambda_{6-i7} L_\nu)] + \text{ h.c.}$$

$$L_\mu = i f_\pi^2 U \partial_\mu U^\dagger$$

and can be rewritten in a Large N_C counting as

$$\mathscr{L}^{(4)}_{\Delta S=1,\,\mathrm{em}} = -\frac{ie}{6f_\pi^2} G_8 F^{\mu\nu} \left\{ \tilde{w}\,\mathrm{tr}\,(\lambda_{6-i7} L_\mu L_\nu) + 3w_2\,(L_\mu)_{13}\,(L_\nu)_{21} \right\} + \text{h.c.}$$

where w_2 is $\mathcal{O}(N_C)$

$\tilde{w} = w_1 - w_2$ is $\mathcal{O}(1)$

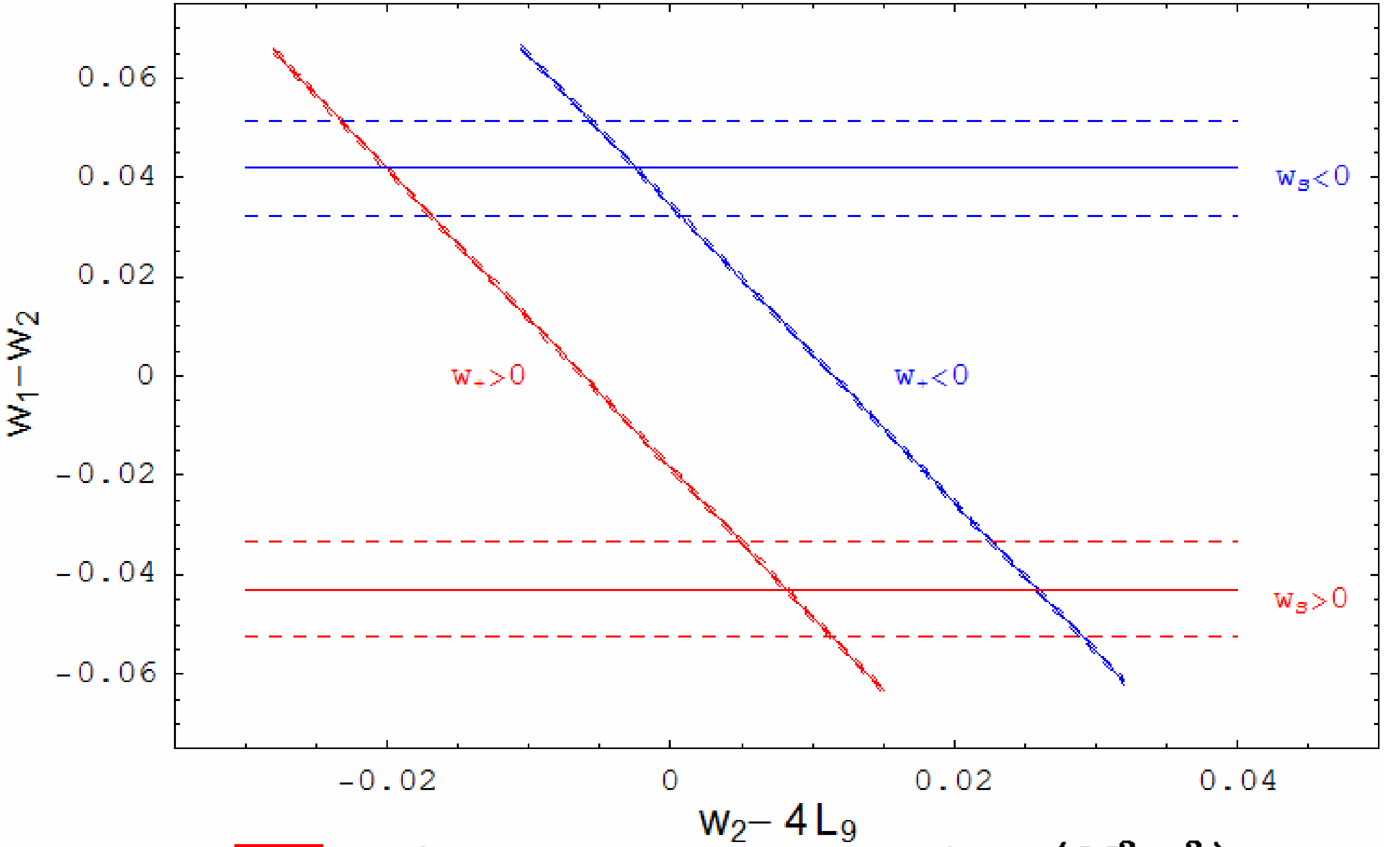
Phenomenological constraints on this coupling constants via

$$K^+ \longrightarrow \pi^+ e^+ e^- \quad \text{and} \quad K_S \longrightarrow \pi^0 e^+ e^-$$

Constraints from Branching Ratios

$$\mathrm{Br}\left(K^+ \longrightarrow \pi^+ e^+ e^-\right) \quad vs \quad \mathrm{Br}\left(K_S \longrightarrow \pi^0 e^+ e^-\right)$$

[PDG '02] [NA48 '03]



key-quantities

$$\boxed{w_+} = -\frac{1}{3}(4\pi)^2\left[\tilde{w} + 3(w_2 - 4L_9)\right] - \frac{1}{6}\log\left(\frac{M_K^2 m_\pi^2}{M_\rho^4}\right)$$

$$\boxed{w_s} = -\frac{1}{3}(4\pi)^2\,\tilde{w} - \frac{1}{3}\log\left(\frac{M_K^2}{M_\rho^2}\right)$$

Order p^4

Octet Dominance Hypothesis
and Large-N_C Predictions

- Octet Dominance Hypothesis [EPdR 87]: $w_2 = 4L_9$

- Large-N_c Predictions :

Through the bosonization of the factorized part of the operator

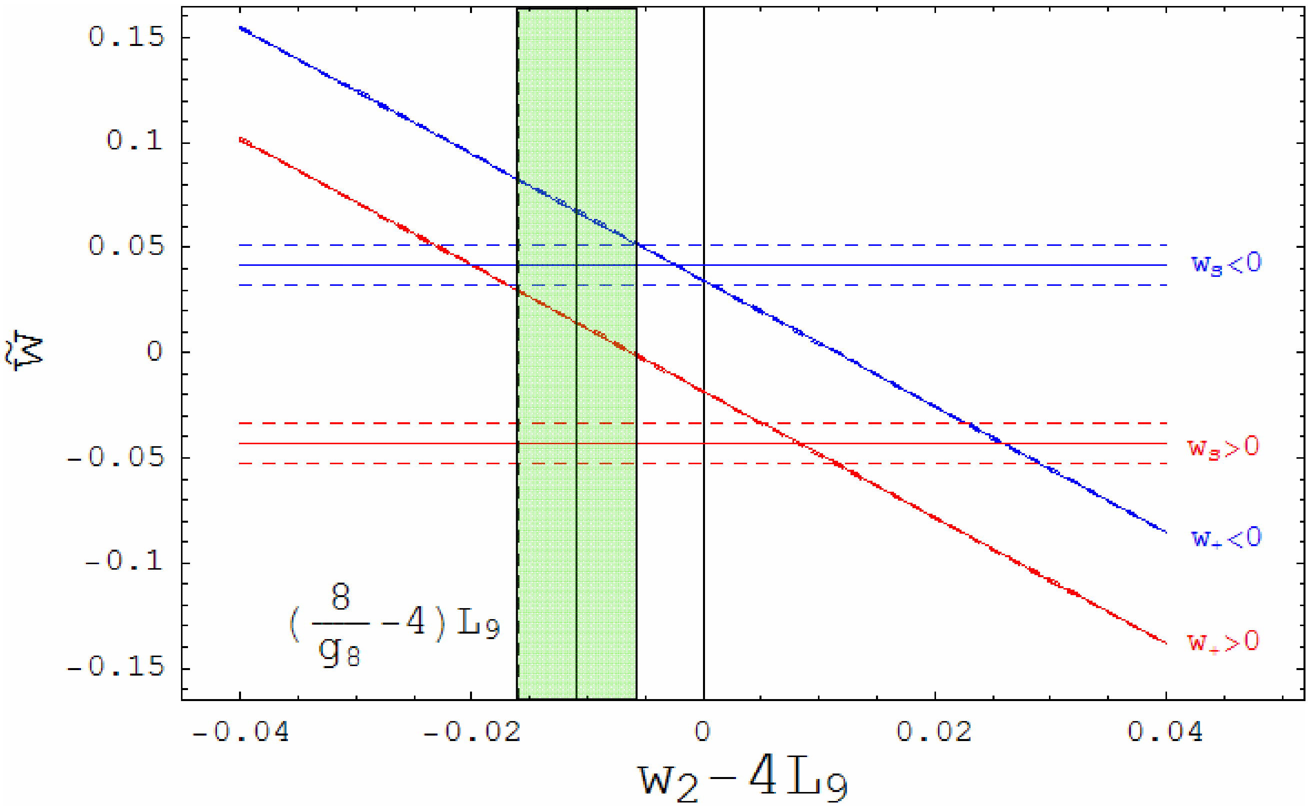
$$Q_2 = 4\left(\bar{s}_L\gamma^\mu u_L\right)\left(\bar{u}_L\gamma_\mu d_L\right)$$

matched to our order p^4 chiral Lagrangian we find

$$g_8 w_2 = 8L_9 + \mathcal{O}\left(N_C^0\right)$$
$$g_8 \tilde{w} = 0 + \mathcal{O}\left(N_C^0\right)$$

See also [Bruno Prades '93]

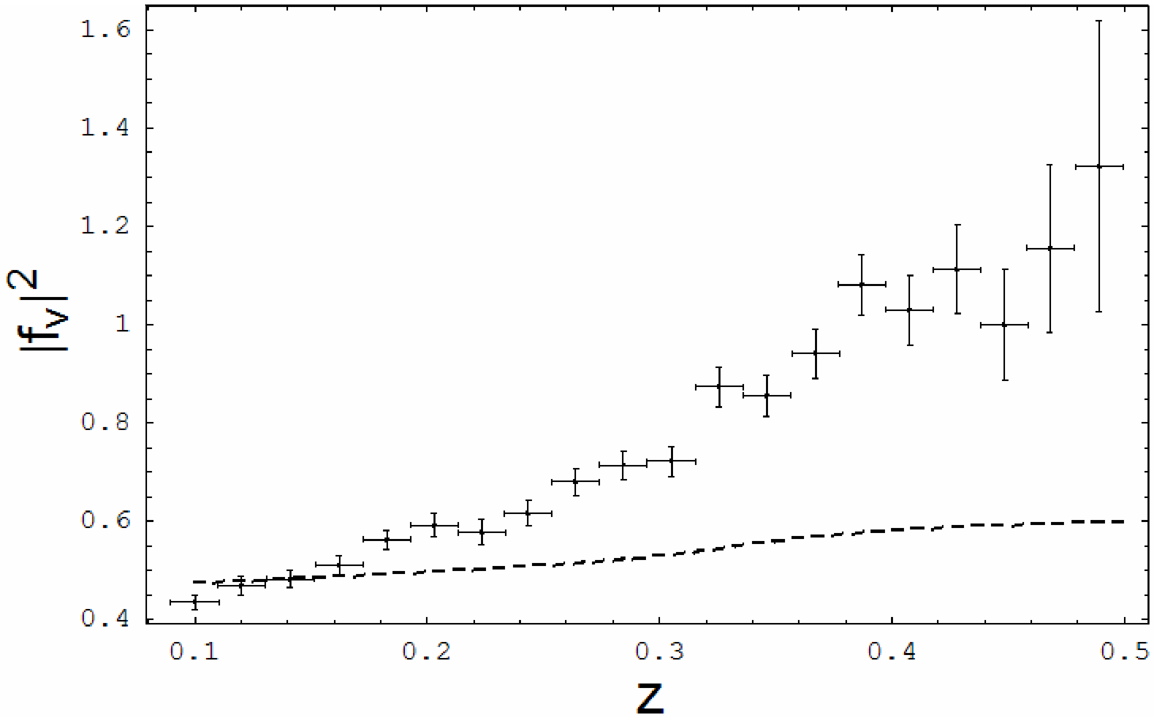
Results



Conclusion: $w_S < 0$ in both cases then CONSTRUCTIVE INTERFERENCES

But the sign of w_+ is not clearly predicted: Need for a Large N_C next-to-leading order calculation

Constraints from the $K^+ \longrightarrow \pi^+ e^+ e^-$ form factor



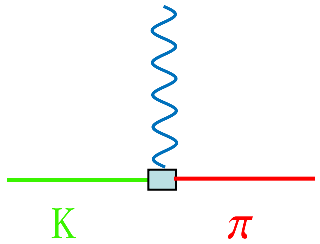
Order p^4:

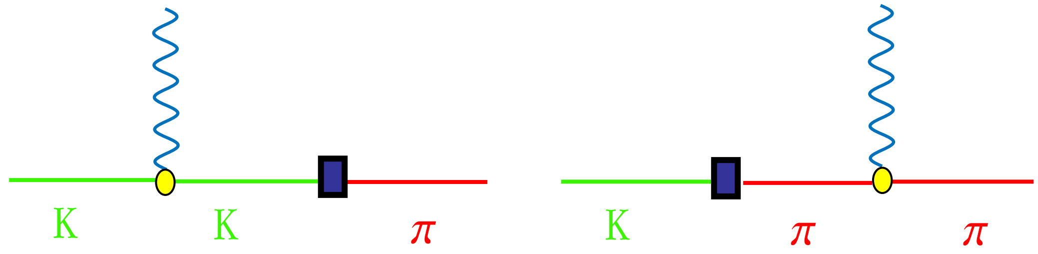
$$|f_V(z)| = \left| \frac{G_8}{G_F} \left\{ \frac{1}{3} - w_+ - \frac{1}{60} z - \chi(z) \right\} \right|$$

where $\chi(z)$ is the function of the pion chiral loop.

Clearly, the order p^4 is not sufficient

Narrow resonances dynamical framework





$$1 - \frac{\mathbf{w_2}}{2F_\pi^2}Q^2 \qquad\qquad -\left(1 - \frac{\mathbf{2L_9}}{F_\pi^2}Q^2\right)$$

NRD Hyp. MHA

$$\alpha\frac{M_\rho^2}{M_\rho^2+Q^2} + \beta\frac{M_{K^*}^2}{M_{K^*}^2+Q^2} \qquad\qquad -\boxed{1}\frac{M_\rho^2}{M_\rho^2+Q^2}$$

$$\boxed{\alpha+\beta=1}$$

chiral limit

$$\boxed{\mathbf{w_2} = \mathbf{4L_9}}$$

Confirmation of results: Octet Dominance [EPdeR '87]

Minimal Narrow resonances Saturation of the Form Factor

$$L_9 = \frac{F_\pi^2}{2M_\rho^2}$$

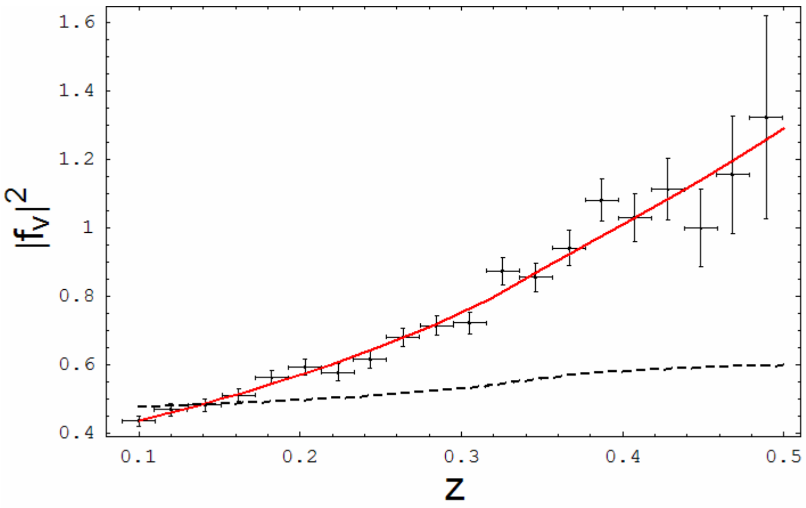
$$w_2 = \frac{2F_\pi^2}{M_\rho^2}\left[1 + \beta\left(\frac{M_\rho^2}{M_{K^*}^2} - 1\right)\right]$$

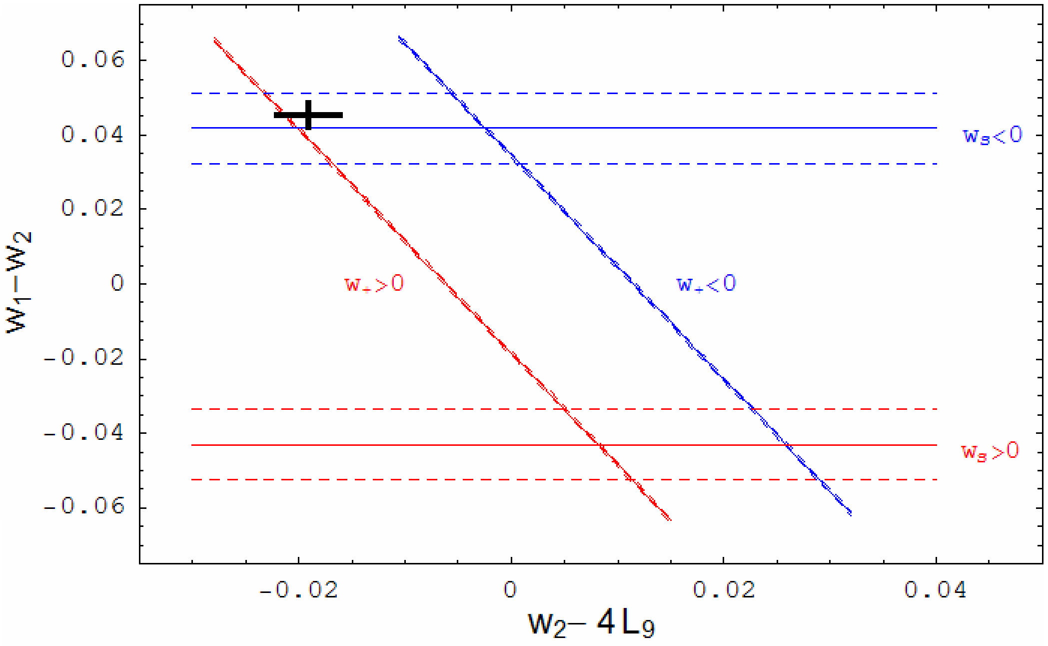
The form factor is now

$$|f_V(z)| = \left|\frac{G_8}{G_F}\left\{\frac{(4\pi)^2}{3}\left[\tilde{\mathbf{w}}\frac{M_\rho^2}{M_\rho^2 - M_K^2 z} + 6F_\pi^2\beta\frac{M_\rho^2 - M_{K^*}^2}{\left(M_\rho^2 - M_K^2 z\right)\left(M_{K^*}^2 - M_K^2 z\right)}\right]\right.\right.$$

$$\left.\left. + \frac{1}{6}\ln\left(\frac{M_K^2 m_\pi^2}{M_\rho^4}\right) + \frac{1}{3} - \frac{1}{60}z - \chi(z)\right\}\right|$$

Fit





$$\tilde{\mathbf{w}} = 0.045(3) \qquad \mathbf{w_2} - 4L_9 = -0.019(3)$$

$$\beta = 2.8(1) \qquad \Longrightarrow \qquad \mathbf{w_s} = -2.1(2)$$

Predictions

	Results	Experiments
$\mathrm{Br}\left(K^+ \longrightarrow \pi^+ e^+ e^-\right)$	$(3.0 \pm 1.1) \times 10^{-7}$	$(2.88 \pm 0.13) \times 10^{-7}$
$\mathrm{Br}\left(K^+ \longrightarrow \pi^+ \mu^+ \mu^-\right)$	$(8.7 \pm 2.8) \times 10^{-8}$	$(7.6 \pm 2.1) \times 10^{-8}$
$\mathrm{Br}\left(K_S \longrightarrow \pi^0 e^+ e^-\right)$	$(7.7 \pm 1.0) \times 10^{-9}$	$(5.8^{+2.8}_{-2.3} \pm 0.8) \times 10^{-9}$
$\mathrm{Br}\left(K_S \longrightarrow \pi^0 \mu^+ \mu^-\right)$	$(1.7 \pm 0.2) \times 10^{-9}$	$(2.9^{+1.4}_{-1.2} \pm 0.2) \times 10^{-9}$

Prediction for $\mathrm{Br}\left(K_L \longrightarrow \pi^0 e^+ e^-\right)$ and Outlook

- CP Conserving Part [Buchalla et al. '03] ≈ 0
- CP Direct Violating Part [Buchalla et al. '03] 0.44×10^{-11}
- CP Indirect Violating Part [NA48 '03] 2.31×10^{-11}
- Interference term [this work] 1.03×10^{-11}

$$\boxed{\mathbf{Br}\left(\mathbf{K_L} \longrightarrow \pi^0 \mathbf{e}^+ \mathbf{e}^-\right) = (\mathbf{3.7} \pm \mathbf{0.4}) \times \mathbf{10^{-11}}}$$

AIM

Next-to-leading order calculation in Large-N_C QCD of

w_2 **and** $\tilde{w}$

In the framework of [Hambye et al. '03]