Multiple Interaction in DIS from AGK rules

Evaluation within a Dipole Model

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Impact Parameter Dipole Saturation Model

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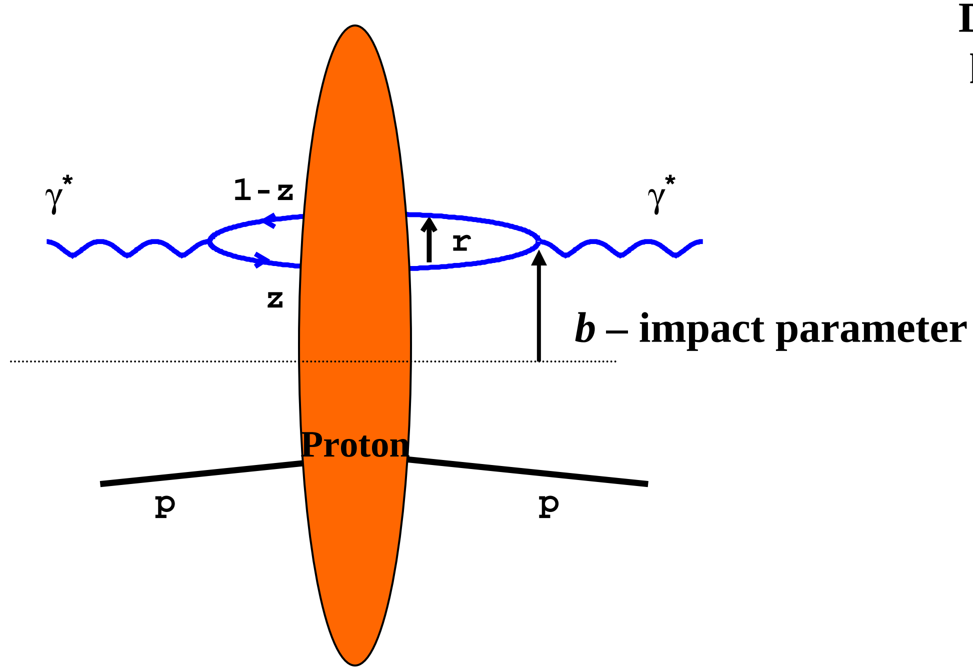
hep-ph/0304189



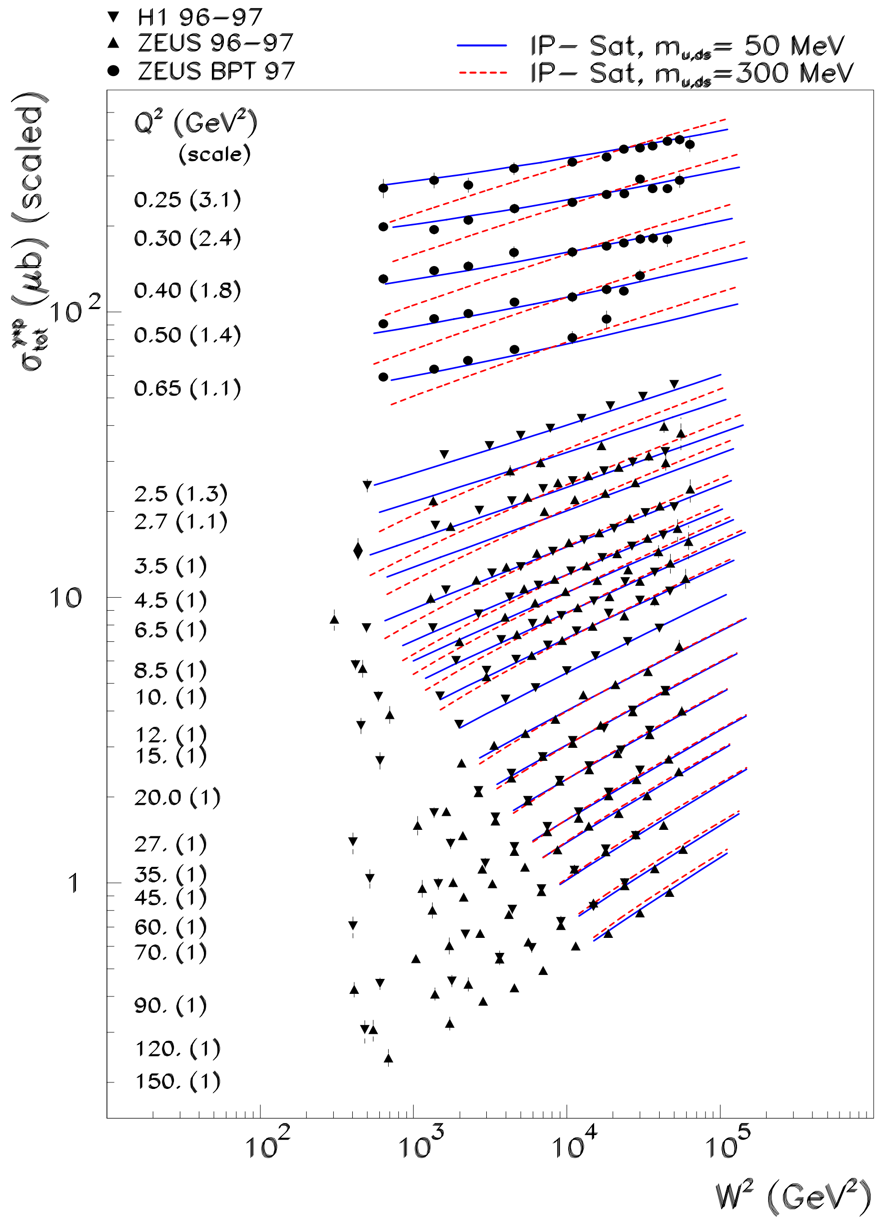
$$\sigma_{tot}^{\gamma^* p} = \int d^2\vec{b}\, d^2\hat{\vec{r}} \int_0^1 dz\, \Psi(Q^2, z, \vec{r})^* \sigma_{q\bar{q}}(x, r^2, b) \Psi(Q^2, z, \vec{r})$$

$$\frac{d\sigma_{qq}(x,r)}{d^2 b} = 2 \cdot \left\{ 1 - \exp\left(-\frac{\pi^2}{2 \cdot 3} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T(b) \right) \right\}$$

$$T(b) = \frac{1}{2\pi B_G} \exp(-b^2/2B_G)$$

***T(b)* - proton shape**

$$\sigma^{\gamma^* p} = \sum_f \int d^2\vec{r} \int d^2 b \int_0^1 dz \Psi_f^*(Q^2, z, \vec{r})\ 2\left\{1 - \exp\left(-\frac{\pi^2}{2\cdot 3} r^2 \alpha_s x g(x,\mu^2) T(b)\right)\right\} \Psi_f(Q^2, z, \vec{r})$$



$$\mu^2 = \frac{C}{r^2} + \mu_0^2$$

$$xg(x,\mu_0^2) = A_g \left(\frac{1}{x}\right)^{\lambda_g} (1-x)^{5.6}$$

$Q^2 > 0.25\ \mathrm{GeV}^2$
$m_u = 0.05$ GeV
$m_c = 1.30$ GeV

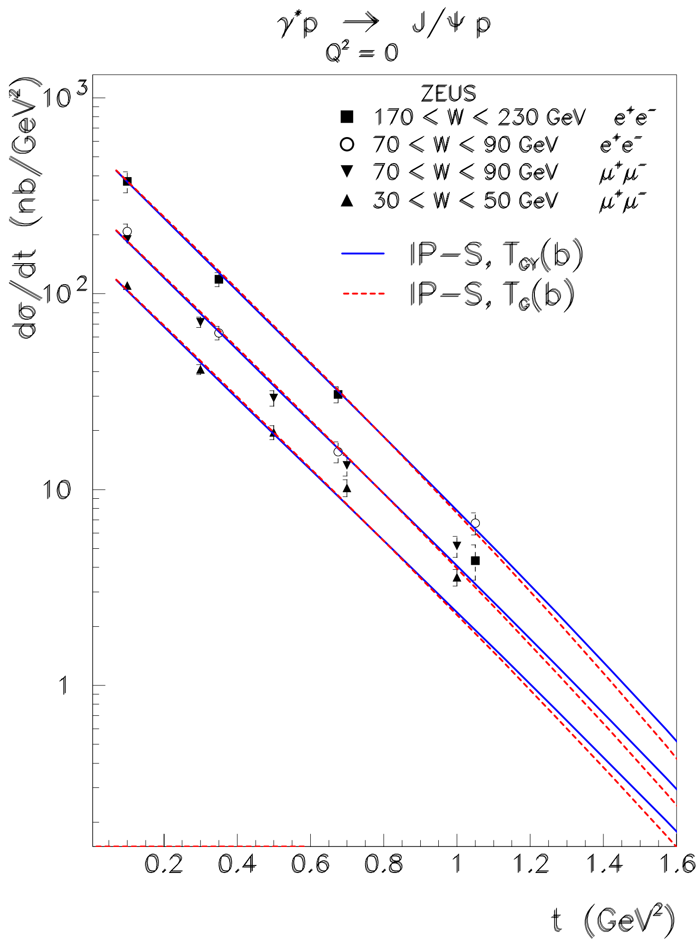
Fit parameters
$\lambda_g = -0.12$ $C = 4.0$
$Q_0^2 = 0.8\ \mathrm{GeV}^2$
$\chi^2/N = 0.8$

$x < 10^{-2}$

t-dependence of the diffractive cross sections determines the *b* distribution

$t = -\vec{\Delta}^2 \qquad \vec{\Delta}$ - transv. momentum (2-d) $\qquad \vec{b}$ - impact parameter (2-d)

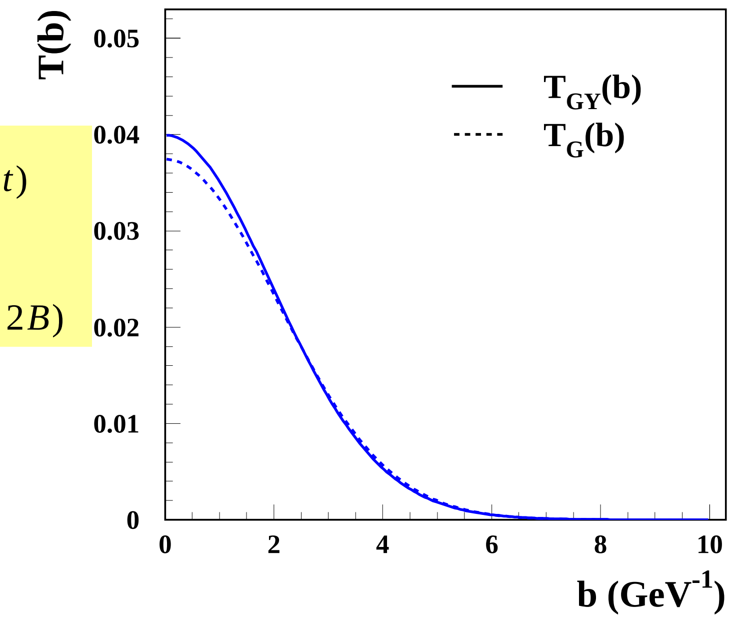
$$\frac{d\sigma_{VM}^{\gamma^* p}}{dt} = \frac{1}{16\pi} \left| \int d^2\vec{r} \int d^2 b e^{-i\vec{b}\vec{\Delta}} \int_0^1 dz \Psi_{VM}^*(Q^2, z, \vec{r}) \; 2\left\{1 - \exp(-\frac{\pi^2}{2\cdot 3} r^2 \alpha_s x g(x,\mu^2) T(b))\right\} \Psi(Q^2, z, \vec{r}) \right|^2$$



$$\frac{d\sigma^{diff}}{dt} \sim \exp(B \cdot t)$$

$\Rightarrow$

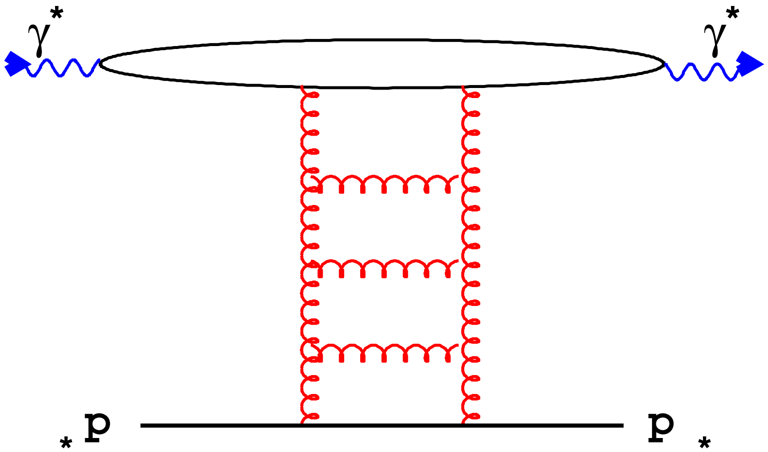
$$T(b) \sim \exp(-\vec{b}^2 / 2B)$$



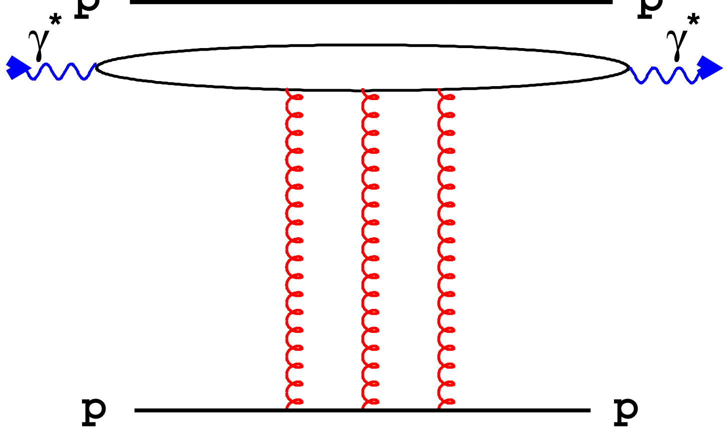
$$T_G(b) \propto \exp(-\vec{b}^2 / 2B_G) \qquad B_G = 4.25 \text{ GeV}^2$$

$$T_{GY}(b) \propto \int d^2 b' \exp(-(\vec{b} - \vec{b}')^2 / 2w_G) K_0(b'/w_E)$$

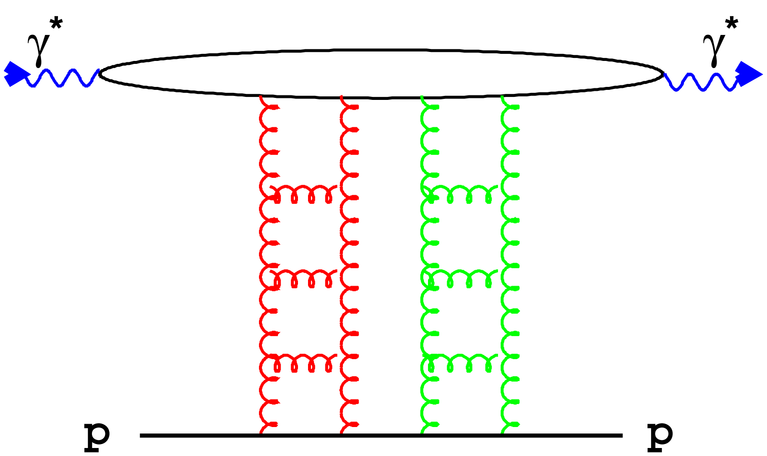
t-channel picture



Color singlet dominates over octet in the 2-gluon exchange amplitude at high energies



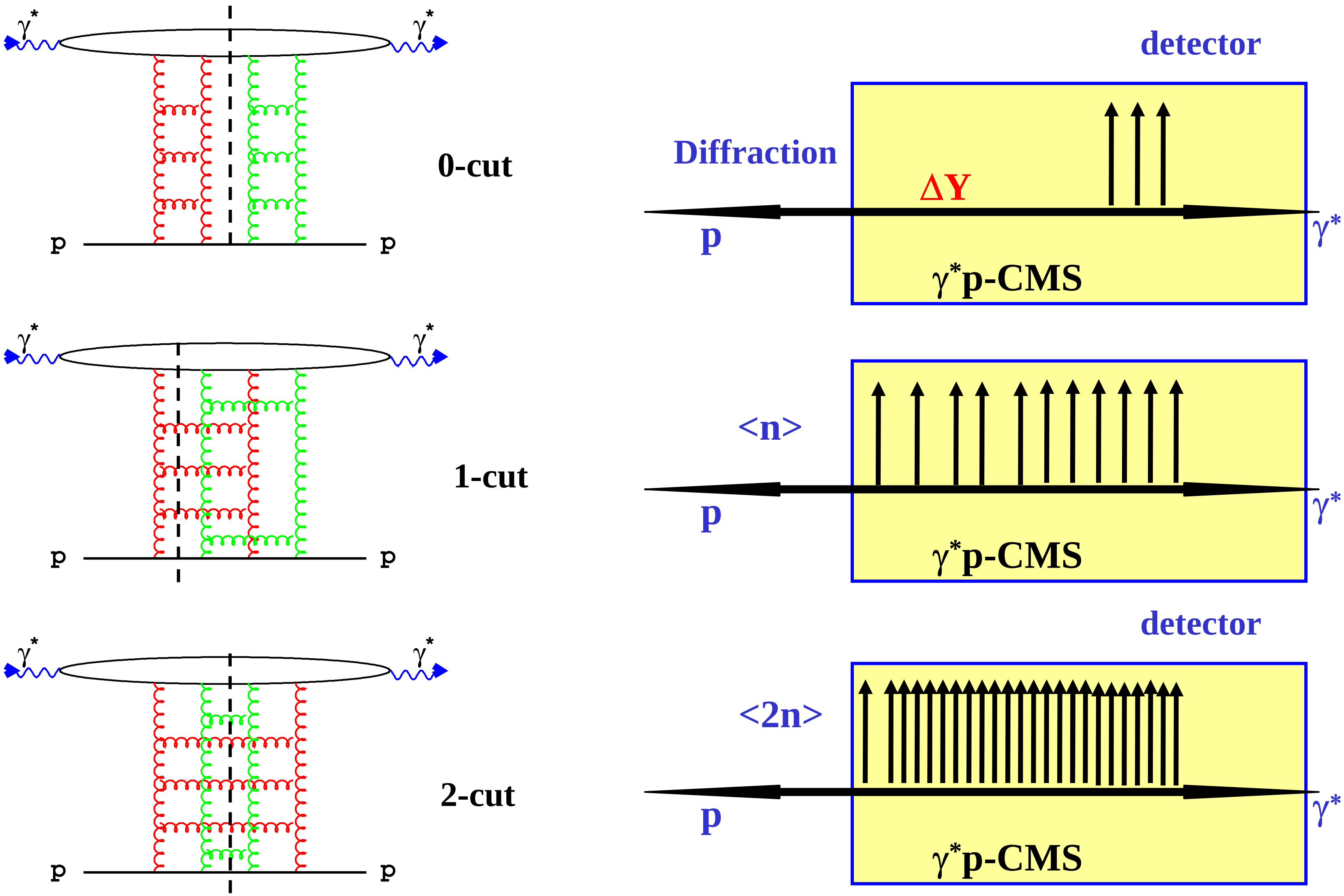
3-gluon exchange amplitude is suppressed at high energies

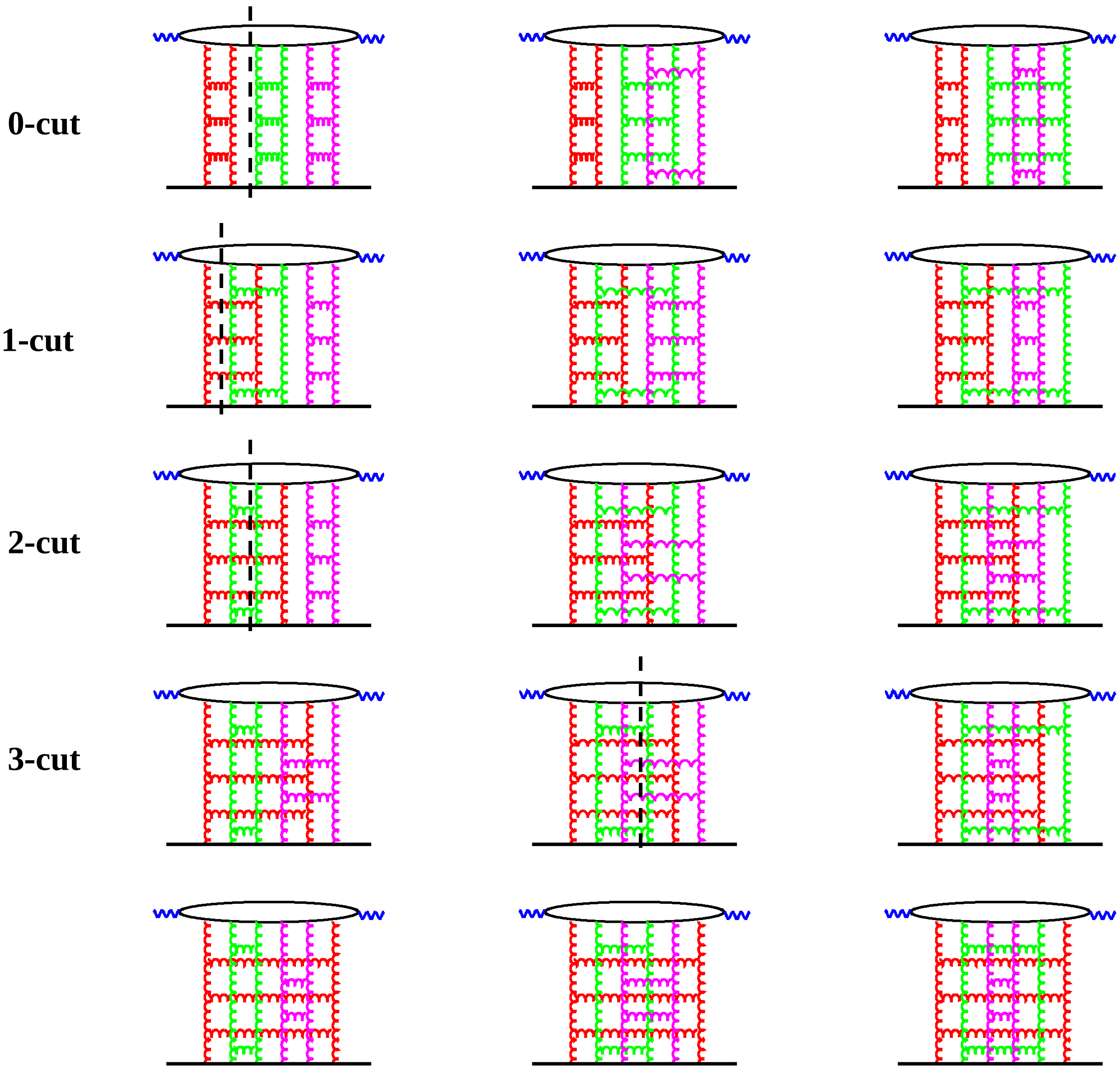


2-gluon pairs in color singlet (Pomerons) dominate the multi-gluon QCD amplitudes at high energies

2-Pomeron exchange in QCD

Final States (naïve picture)



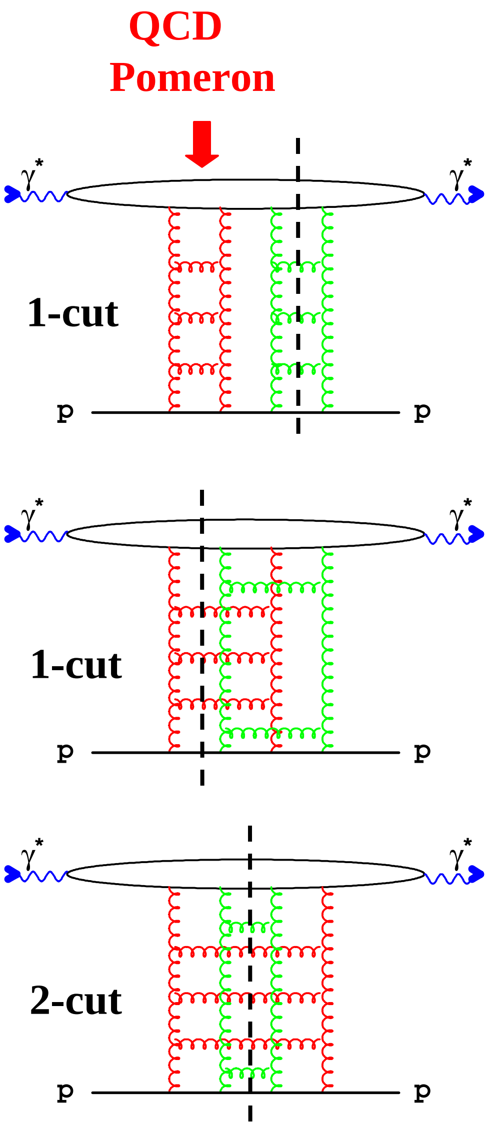

0-cut
1-cut
2-cut
3-cut

AGK Rules

The cross-section for k-cut pomerons:
Abramovski, Gribov, Kancheli
Sov. ,J., Nucl. Phys. 18, p308 (1974)

$$\sigma_k = \sum_{m=k}^{\infty} (-1)^{m-k} 2^m \frac{m!}{k!(m-k)!} F^{(m)}$$

$F^{(m)}$ **– amplitude for the exchange of** ***m*** **Pomerons**



AGK Rules in the Dipole Model

Total cross section **Mueller-Salam (NP B475, 293)**

$$\sigma_{tot} = 2\sum_{m=1}^{\infty}(-1)^{m-1}F^{(m)}$$

Dipole cross section

$$\frac{d\sigma}{d^2b} = 2(1-\exp(-\Omega/2)) = 2\sum_{m=1}^{\infty}(-1)^{m-1}\left(\frac{\Omega}{2}\right)^m \frac{1}{m!}$$

Amplitude for the exchange of *m* pomerons in the dipole model

$$F^{(m)} = \left(\frac{\Omega}{2}\right)^m \frac{1}{m!} \qquad \Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) xg(x,\mu^2) T(b)$$ **KT model**

AGK rules

$$\frac{d\sigma_k}{d^2b} = \sum_{m=k}^{\infty} (-1)^{m-k} 2^m \frac{m!}{k!(m-k)!} F^{(m)}$$

Dipole model

$$\frac{d\sigma_k}{d^2b} = \sum_{m=k}^{\infty} (-1)^{m-k} 2^m \frac{m!}{k!(m-k)!} \left(\frac{\Omega}{2}\right)^m \left(\frac{1}{m!}\right) = \frac{\Omega^k}{k!} \sum_{m=k}^{\infty} (-1)^{m-k} \frac{\Omega^{m-k}}{(m-k)!}$$

$$\Rightarrow \quad \frac{d\sigma_k}{d^2b} = \frac{\Omega^k}{k!} \exp(-\Omega)$$

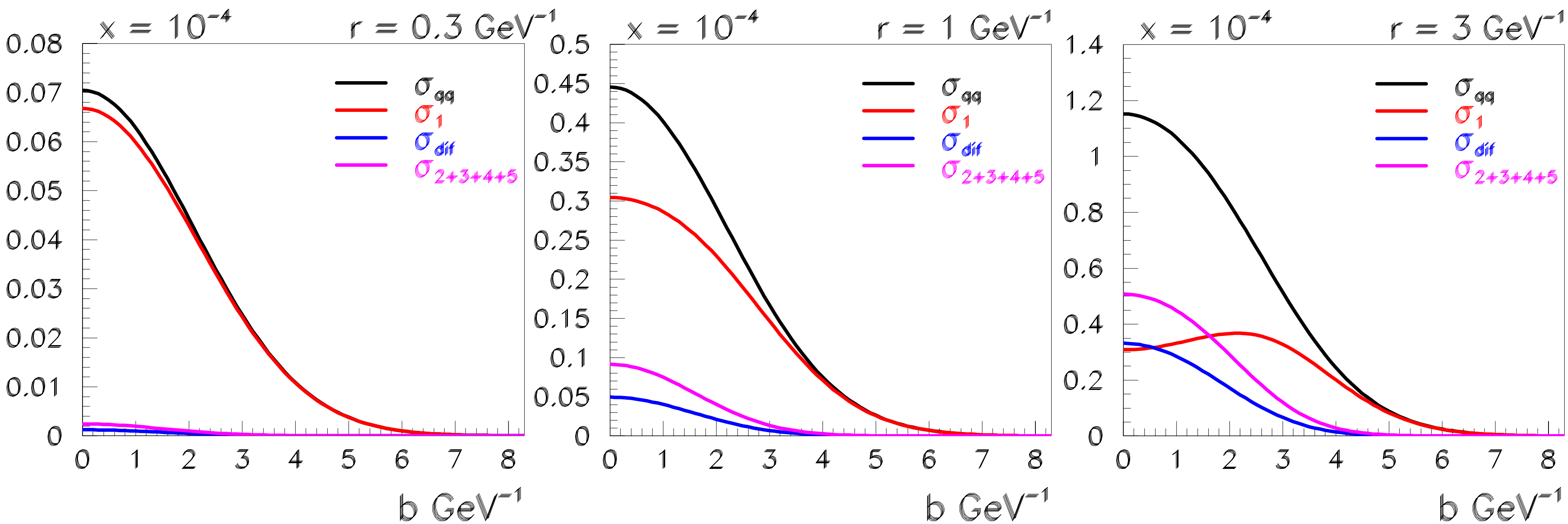
Diffraction from AGK rules

$$\frac{d\sigma_{diff}}{d^2b} = \frac{d\sigma_{qq}}{d^2b} - \sum_{k=1}^{\infty} \frac{d\sigma_k}{d^2b} = 2(1-\exp(-\Omega/2)) - (1-\exp(-\Omega))$$

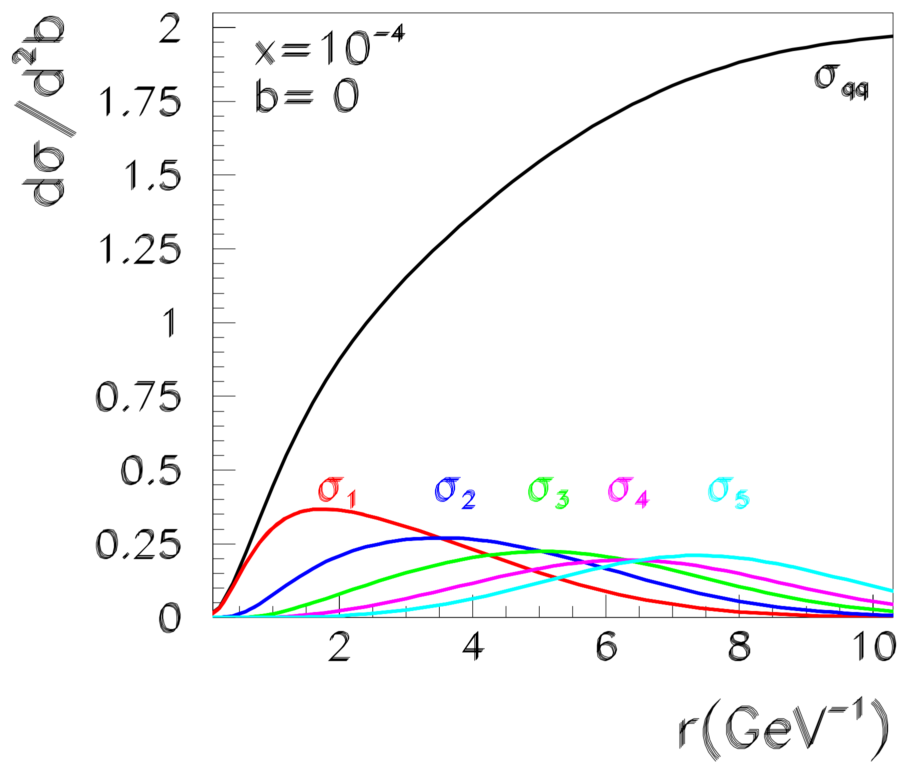
$$= 1 - 2\exp(-\Omega/2) + \exp(-\Omega) = (1-\exp(-\Omega/2))^2$$

$$\frac{d\sigma_{qq}}{d^2b} = 2\cdot\left\{1-\exp\left(-\frac{\Omega}{2}\right)\right\}$$

$$\frac{d\sigma_k}{d^2b} = \frac{\Omega^k}{k!}\exp(-\Omega)$$



$$\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) x g(x,\mu^2) T(b)$$

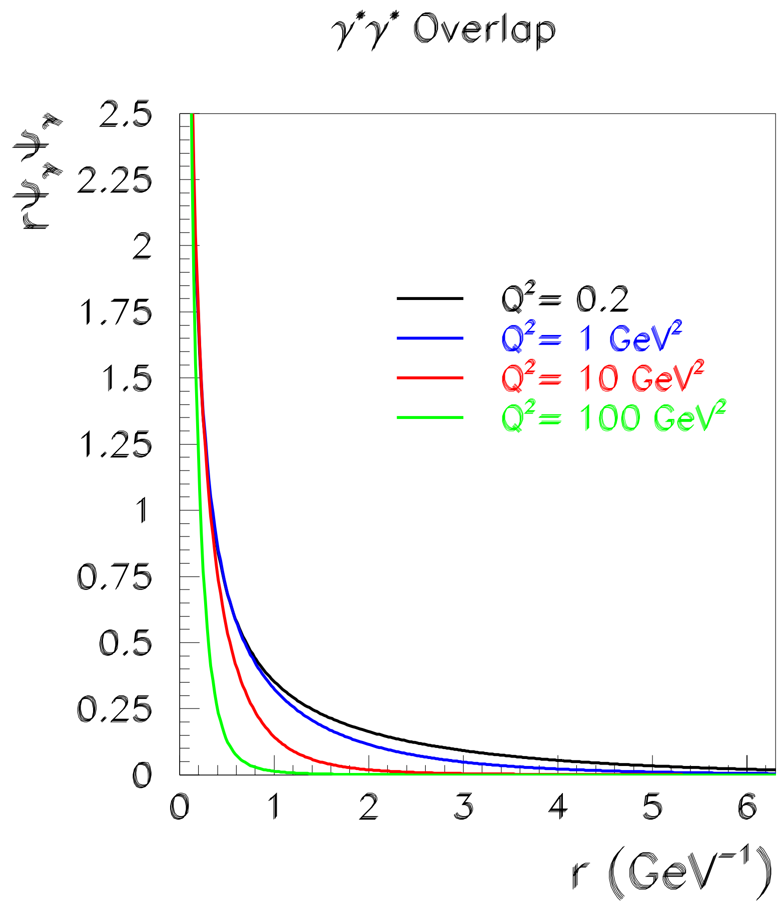


dσ/d²b
x=10⁻⁴
b= 0
σ_qq
σ₁
σ₂
σ₃
σ₄
σ₅
r(GeV⁻¹)
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
2
4
6
8
10

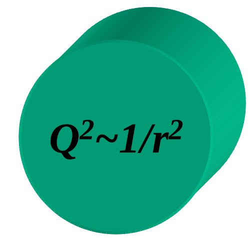
$$\sigma_{T,L}^{\gamma^* P}(x,Q^2) = \int_0^\infty d^2\vec{r} \int_0^1 dz \sum_f \left|\Psi_{T,L}^f(\vec{r},z,Q^2)\right|^2 \sigma_{qq}(x,r)$$

$$\left|\Psi_T^f(\vec{r},z,Q^2)\right|^2 = \frac{3\alpha_{em}}{2\pi^2} e_q^2 \{[z^2+(1-z)^2]\varepsilon^2 K_1^2(\varepsilon r) + m_q^2 K_0^2(\varepsilon r)\}$$

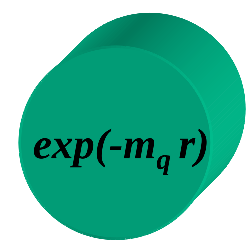
$$\left|\Psi_L^f(\vec{r},z,Q^2)\right|^2 = \frac{3\alpha_{em}}{2\pi^2} e_q^2 \{4Q^2 z^2 (1-z)^2 K_0^2(\varepsilon r)\} \qquad \varepsilon^2 = z(1-z)Q^2 + m_q^2$$



$K_1(\varepsilon r) = 1/\varepsilon r$ for $\varepsilon r << 1$



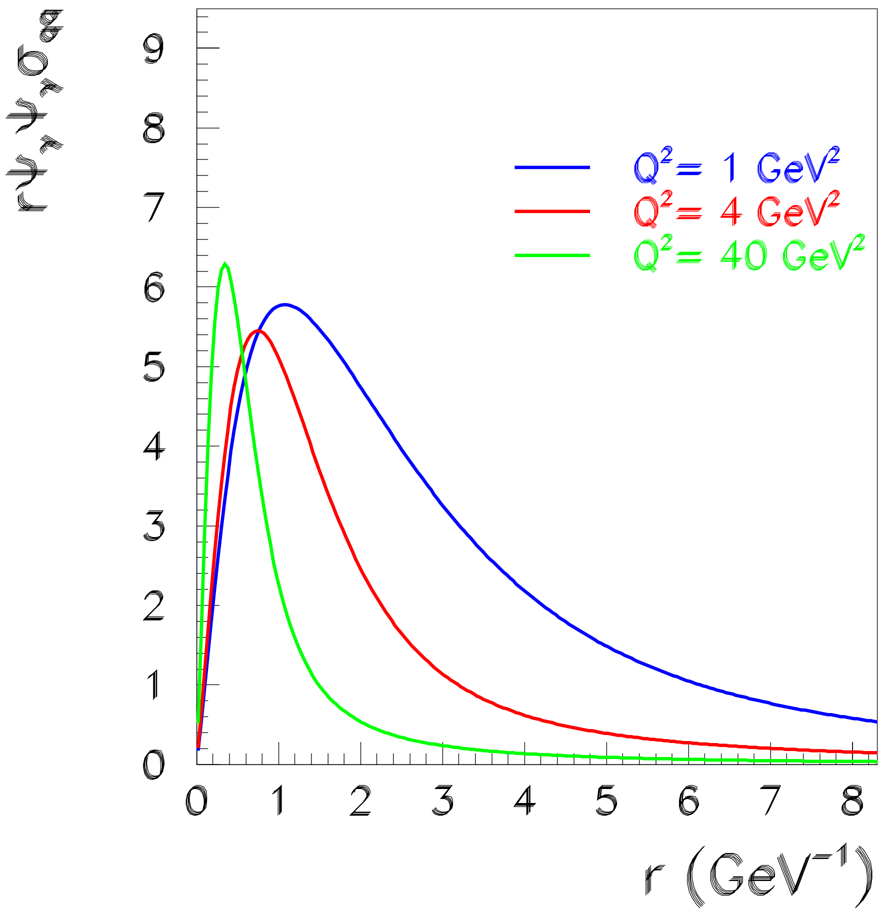
$K_1(\varepsilon r) = \sqrt{\pi/2x}\exp(-\varepsilon r)$ for $\varepsilon r >> 1$



All quarks

$$2\pi r\int_0^1 dz\sum_f \left|\Psi^f_{T,L}(\vec{r},z,Q^2)\right|^2 \sigma_{qq}(x,r)$$

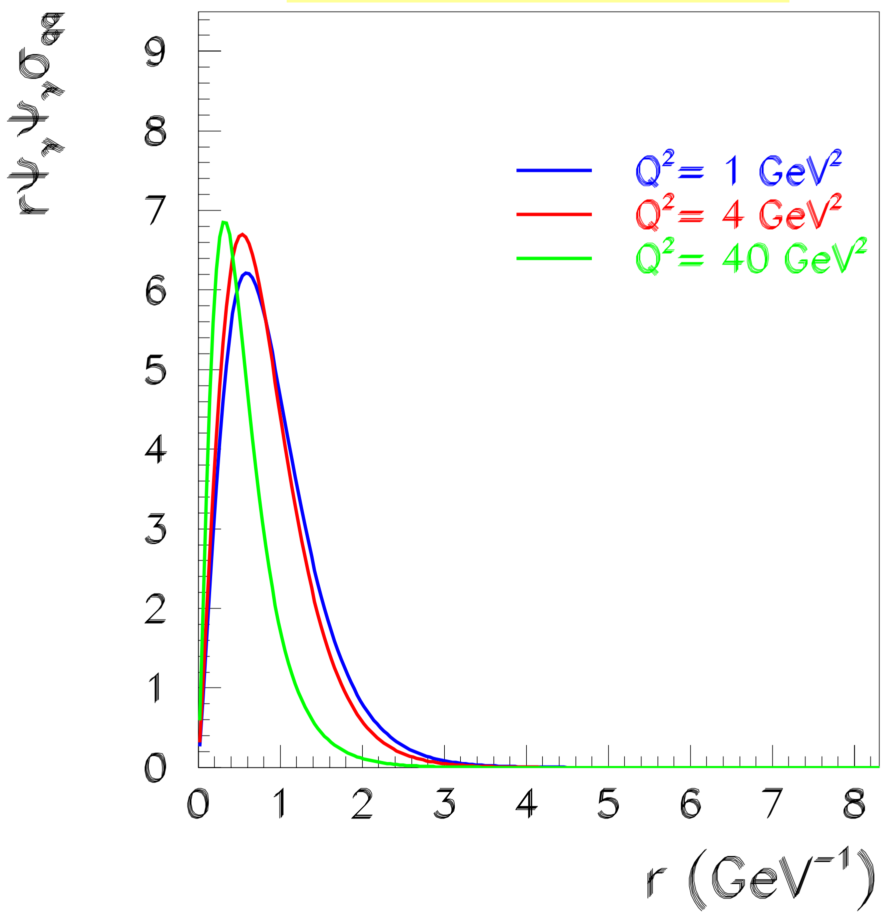
$m_{u,d,s} = 100\ \mathrm{MeV} \qquad m_c = 1.3\ \mathrm{GeV}$

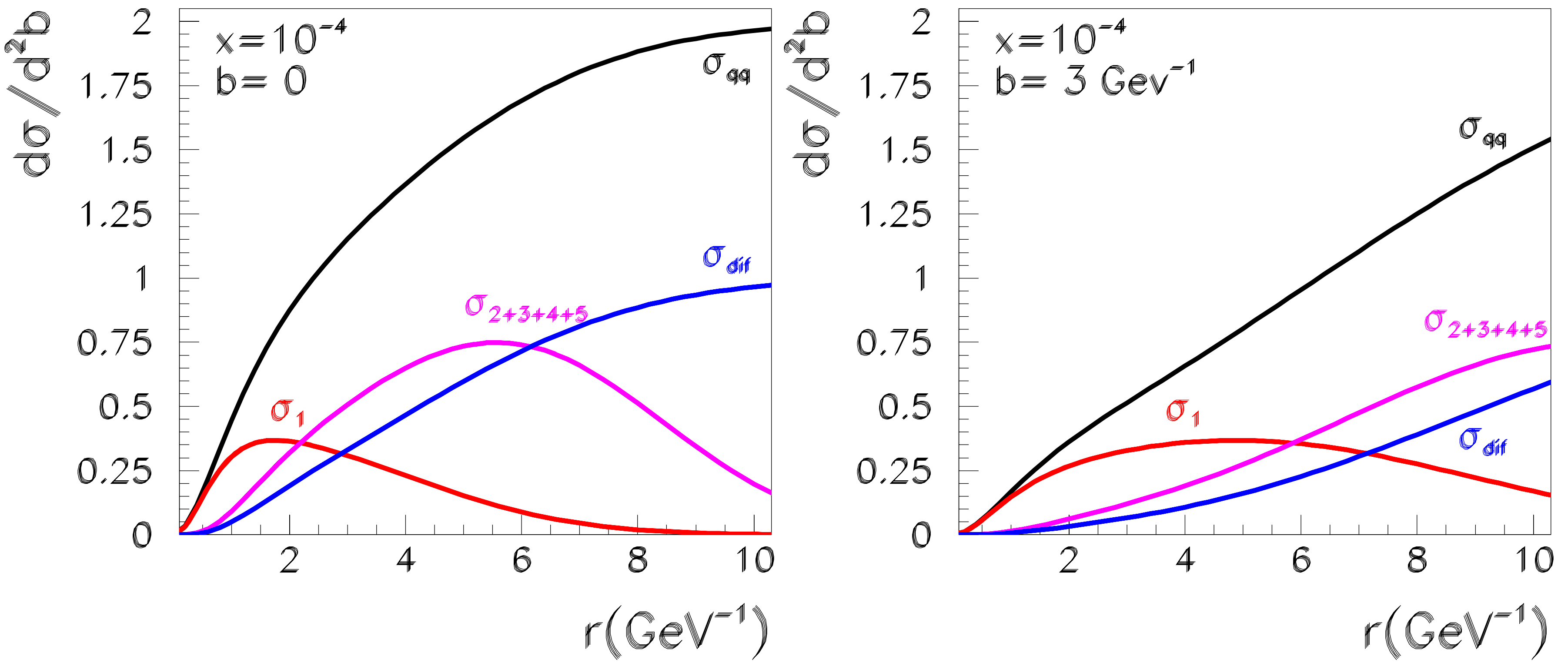


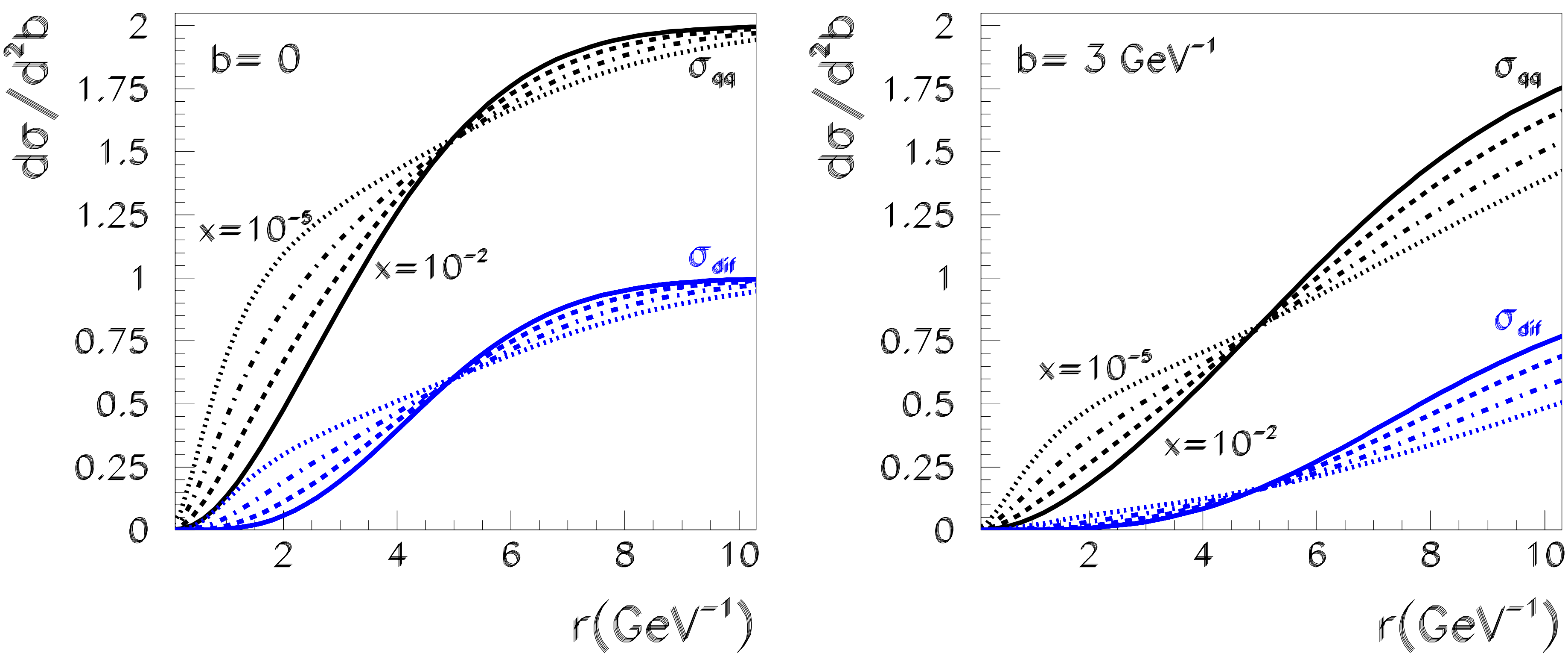
Charmed quark

$$2\pi r\int_0^1 dz\,\sigma_{qq}(x,r)\Psi^c_{T,L}(\vec{r},z,Q^2)^2$$

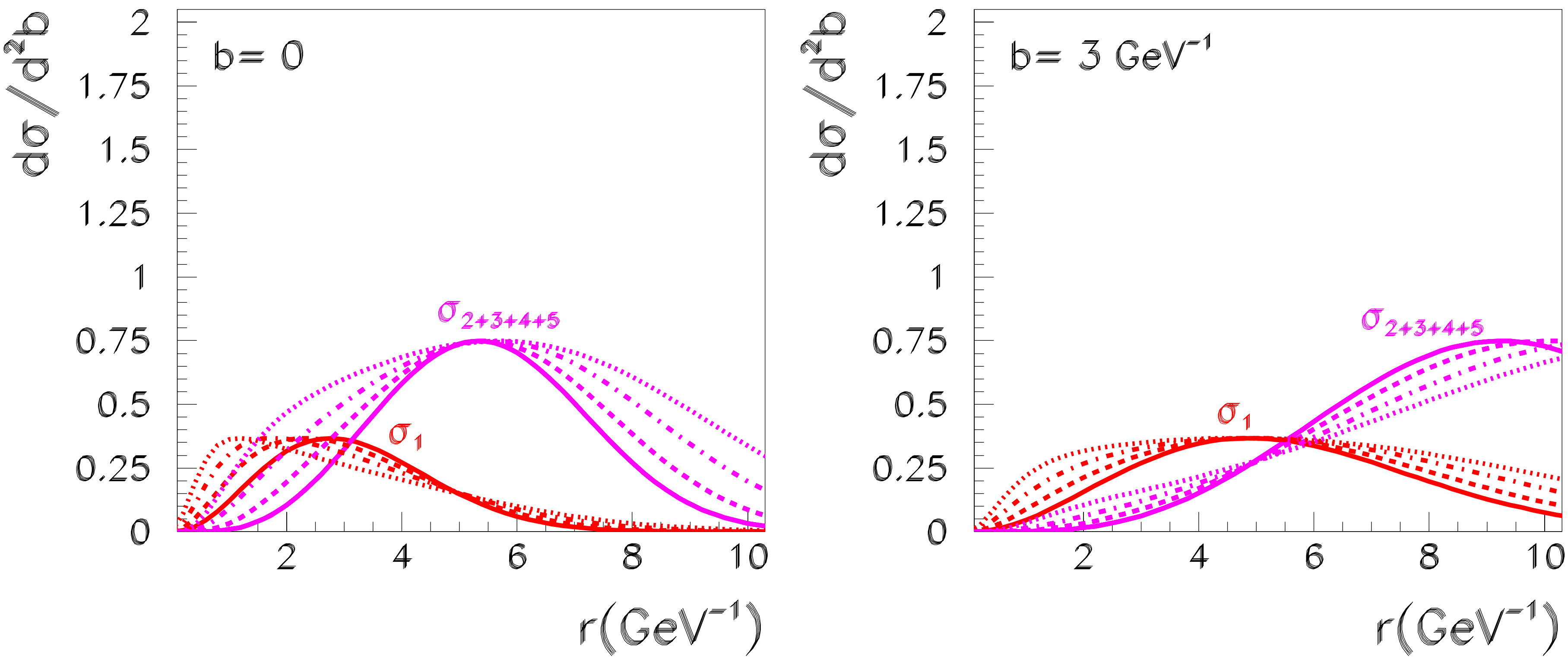
$m_c = 1.3\ \mathrm{GeV}$




dσ/d²b
x=10⁻⁴
b= 0
σ_qq
σ_dif
σ_2+3+4+5
σ_1
r(GeV⁻¹)
x=10⁻⁴
b= 3 Gev⁻¹
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
2
4
6
8
10



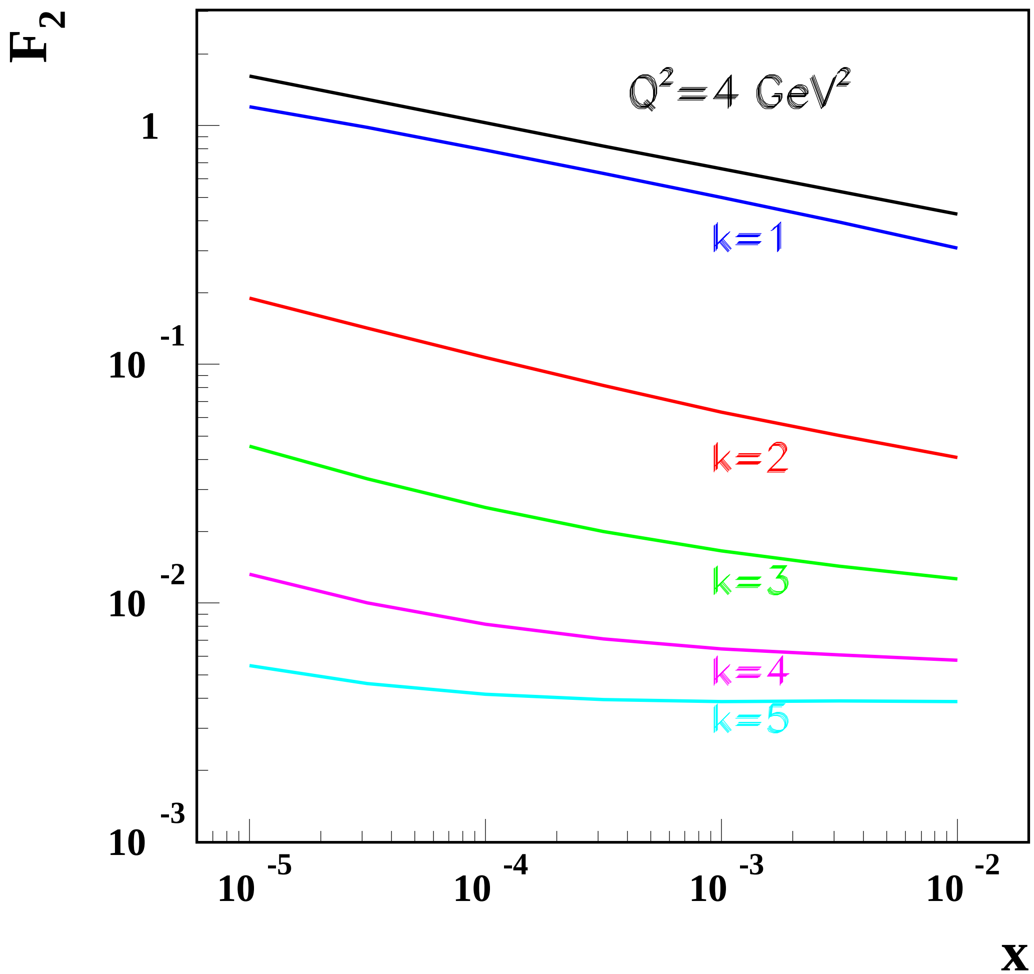
b= 0
b= 3 GeV⁻¹
$d\sigma/d^2b$
σ_{qq}
σ_{dif}
$x=10^{-5}$
$x=10^{-2}$
$r(GeV^{-1})$
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2
4
6
8
10

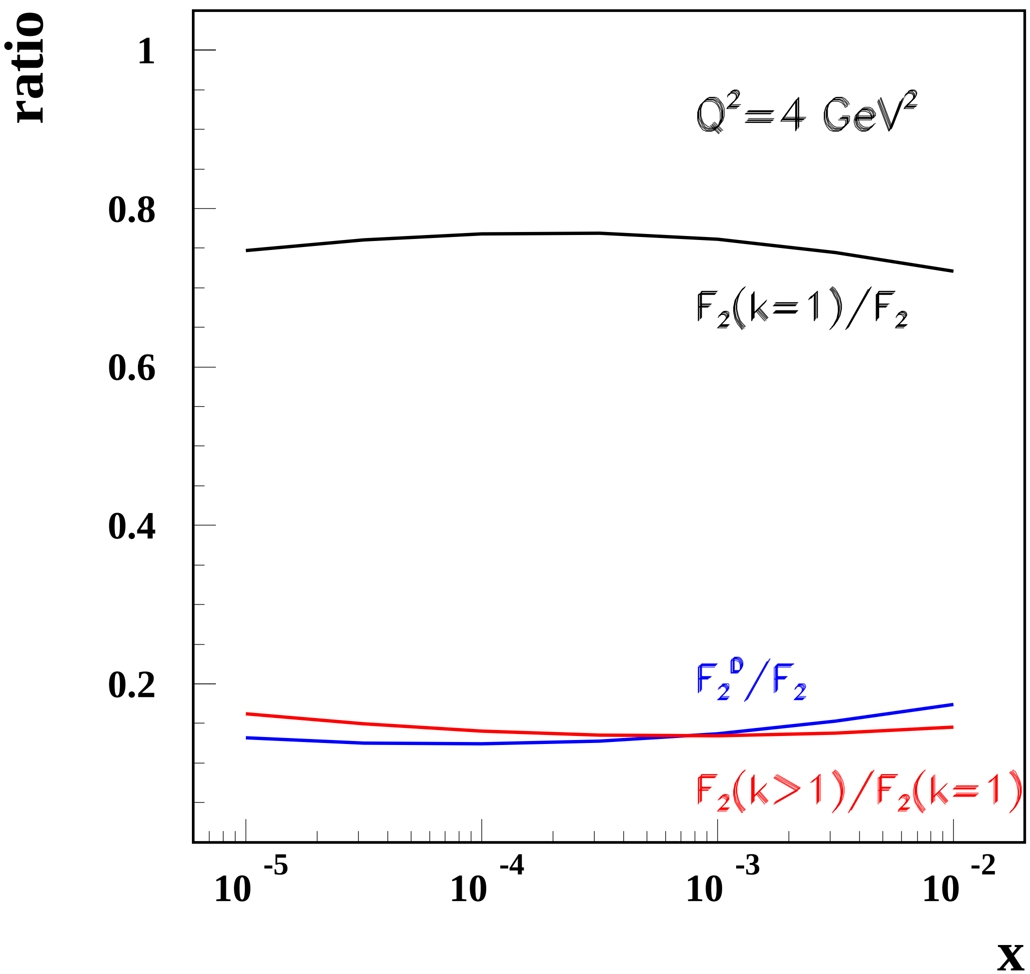


b= 0
b= 3 GeV⁻¹
dσ /d²b
r(GeV⁻¹)
σ 2+3+4+5
σ 1
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2
4
6
8
10

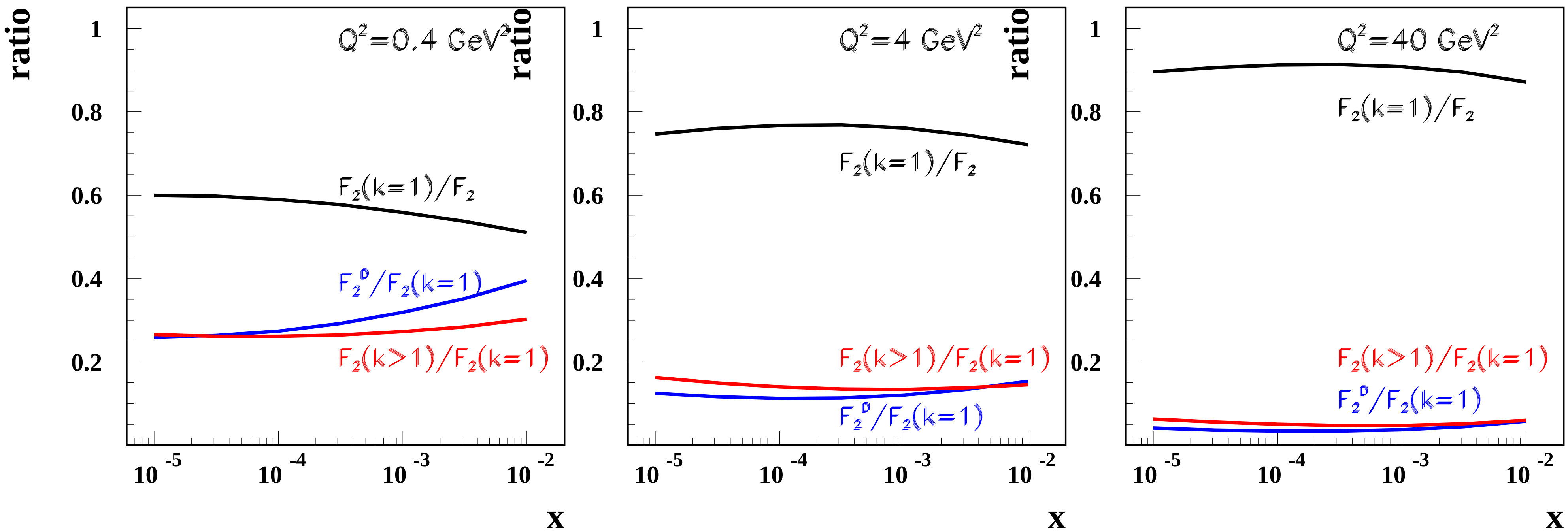
$$\sigma^{\gamma^* p} = \sum_f \int d^2\vec{r} \int d^2 b \int_0^1 dz \Psi_f^*(Q^2, z, \vec{r}) \; 2\left\{1 - \exp(-\frac{\Omega}{2})\right\} \Psi_f(Q^2, z, \vec{r})$$

$$\sigma_k^{\gamma^* p} = \sum_f \int d^2\vec{r} \int d^2 b \int_0^1 dz \Psi_f^*(Q^2, z, \vec{r}) \; \frac{\Omega^k}{k!} \exp(-\Omega) \Psi_f(Q^2, z, \vec{r})$$





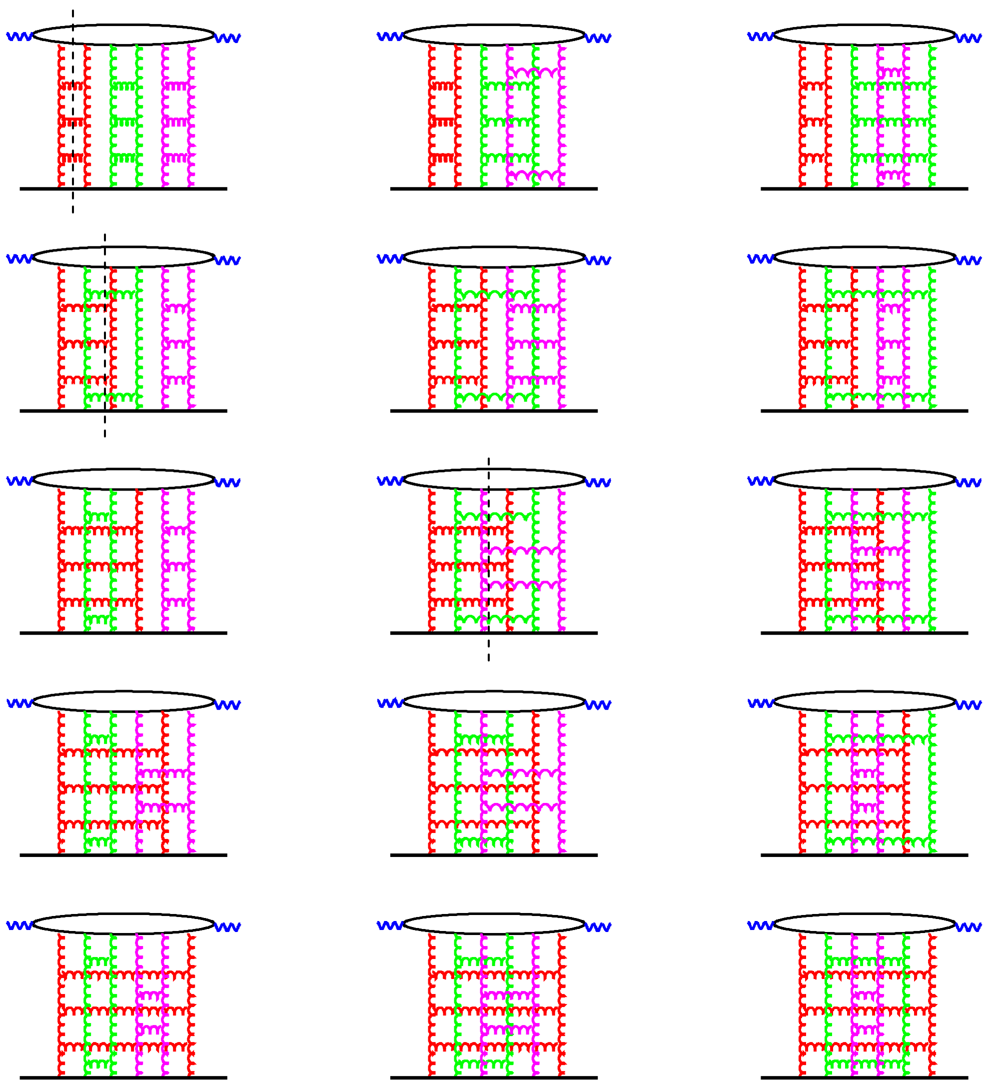
ratio
1
0.8
0.6
0.4
0.2
$Q^2=4$ GeV^2
$F_2(k=1)/F_2$
F_2^D/F_2
$F_2(k>1)/F_2(k=1)$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
x



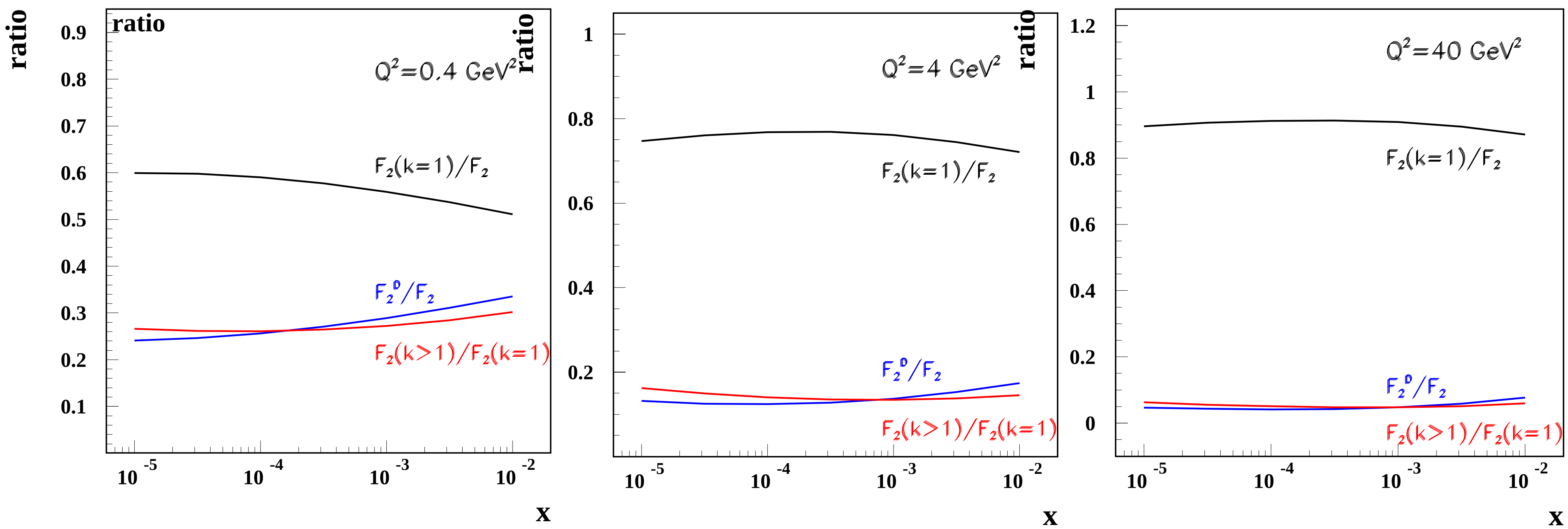
Note: AGK rules underestimate the amount of diffraction in DIS

Outlook: Monte Carlo Feynman diagrams for 3, 4, 5 cut-pomerons

Sum up contribution of uncut-pomerons to infinity



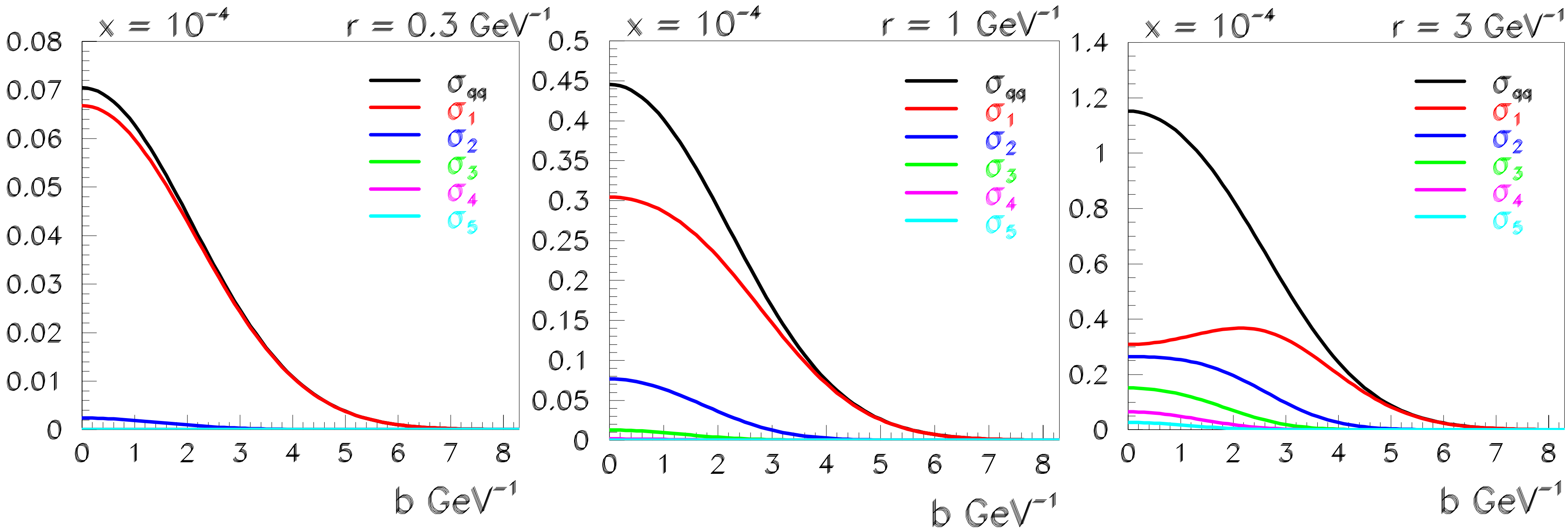
END



Note: AGK rules underestimate the amount of diffraction in DIS

$$\frac{d\sigma_{qq}}{d^2b} = 2 \cdot \left\{ 1 - \exp(-\frac{\Omega}{2}) \right\}$$

$$\frac{d\sigma_k}{d^2b} = \frac{\Omega^k}{k!}\exp(-\Omega)$$



$$\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)$$