Diffractive Higgs : Theory

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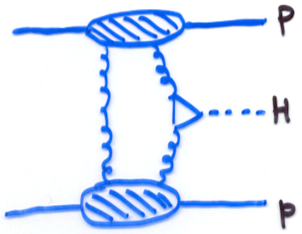
- Review of Khoze, Martin, Ryskin calculation



- Goals – quantify uncertainty in theoretical predictions
  - * Sudakov * Gap Survival
  - * Skewedness & gluon distribution.

  – and to improve the accuracy of background estimates
  - * Hadron level simulation (Monk, Pilkington)

Some history

- **1997**

$$\left.\frac{d\sigma}{dy}\right|_{y=0} \simeq 2 \times 10^{-2}\ fb \qquad (M_H = 100\ \mathrm{GeV})$$

$\Rightarrow$ "Exclusive Higgs production is only of academic interest" (KMR 1997)

- **2000**

$$\left.\frac{d\sigma}{dy}\right|_{y=0} \simeq 2\ fb \qquad (M_H = 100\ \mathrm{GeV})$$

(KMR 2000)

- Sudakov – inclusion of single logs $\times 3$

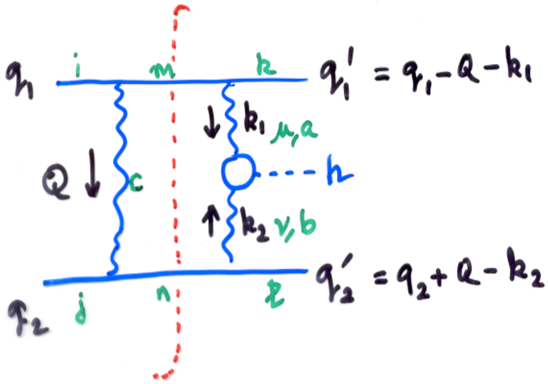
$$- \; e^{-S} f_g(x, Q_\perp^2)\, f_g(x', Q_\perp^2) \longrightarrow \frac{\partial\left(e^{-S/2} G(x, Q_\perp^2)\right)}{\partial \ln Q_\perp^2}$$

$\times 10$

$$\times \; \frac{\partial\left(e^{-S/2} G(x', Q_\perp^2)\right)}{\partial \ln Q_\perp^2}$$

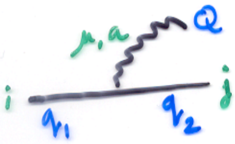
- Skewedness $\times 2$
- NLO K-factor $\times 1.5$

$qq \to q + H + q$



$s = 2 q_1 \cdot q_2$

- Compute imaginary part
- Use eikonal approximation



$$\approx 2 g \, q_1^\mu \, \tau^a_{ij} \, \delta_{\lambda\lambda'}$$

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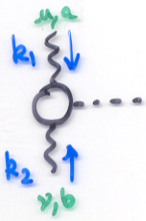


$$\equiv V^{ab}_{\mu\nu} = \delta^{ab} \left(g_{\mu\nu} - \frac{k_{2\mu} k_{1\nu}}{k_1 \cdot k_2} \right) V$$

$$\equiv \frac{M_H^2 \alpha_s}{4\pi v} F_s\left(\frac{M_H^2}{4 m_t^2}\right)$$

$$\operatorname{Im} A^{ik}_{j\ell} = \frac{1}{2} \int d(PS)_2 \, \delta\left((q_1 - Q)^2\right) \delta\left((q_2 + Q)^2\right)$$

$$\times 2 \times \frac{2 g q_1^\alpha \, 2 g q_{2\alpha}}{Q^2} \times \frac{2 g q_1^\mu}{k_1^2} \, \frac{2 g q_2^\nu}{k_2^2}$$

$$\times V^{ab}_{\mu\nu} \, \tau^c_{im} \, \tau^c_{jn} \, \tau^a_{mk} \, \tau^b_{n\ell}$$

$$Q \equiv \alpha q_1 + \beta q_2 + Q_T$$

$$(q_1 - Q)^2 = 0 \Rightarrow \beta \simeq Q_T^2/s \quad < 0$$

$$(q_2 + Q)^2 = 0 \Rightarrow \alpha \simeq -Q_T^2/s \quad > 0$$

$$\Big\} \; Q^2 \simeq Q_T^2$$

$$Q_T^2 = -\underline{Q}_T^2$$

$$\int d(PS)_2 = \int \frac{d^4 Q}{(2\pi)^2} = \frac{s}{2} \frac{1}{(2\pi)^2} \int d\alpha \, d\beta \, d^2 \underline{Q}_T$$

$$\operatorname{Im} A^{ik}_{j\ell} = \int \frac{d^2 \underline{Q}_T}{(2\pi)^2} \frac{2s}{\underline{k}_1^2 \underline{k}_2^2 \underline{Q}_T^2} g^4 \Big(1 - \cancel{\frac{q_1 \cdot k_2 \; q_2 \cdot k_1}{k_1 \cdot k_2 \; q_1 \cdot q_2}}\Big)$$

negligible

$$\times \underbrace{\tau^c_{im} \tau^c_{jn} \tau^a_{mk} \tau^b_{n\ell}}_{} \; V$$

$\rightarrow \dfrac{N_c^2 - 1}{4 N_c^2}$ after averaging over colour.

Putting $g^2 = 4\pi\alpha_s$, $F_s \simeq 2/3$ (ie $4m_t^2/m_H^2 \to \infty$), $1/v = (\sqrt{2} G_F)^{1/2}$

$$\operatorname{Im} A = \frac{N_c^2 - 1}{N_c^2} \, 2s \, \alpha_s^2 \int \frac{d^2 \underline{Q}_T}{\underline{Q}_T^2} \frac{1}{\underline{k}_1^2 \underline{k}_2^2} \frac{2}{3} \frac{M_H^2 \alpha_s}{4\pi} (\sqrt{2} G_F)^{\frac{1}{2}}$$

$$d\sigma(qq \to qHq) = \frac{1}{2s} \frac{d^3 \underline{q}_1'}{(2\pi)^3} \frac{d^3 \underline{q}_2'}{(2\pi)^3} \underbrace{\frac{1}{2E_1'\,2E_2'}}_{\simeq 2s} |A|^2$$

$$\times \frac{d^3 q_H}{(2\pi)^3} \frac{1}{2E_H} (2\pi)^4 \delta^{(4)}(q_1 + q_2 - q_1' - q_2' - q_H)$$

$$d^3\underline{q}_1'\, d^3\underline{q}_2'\, d^3\underline{q}_H\, \delta^{(4)}(\ldots) = d^2\underline{q}_1'\, d^2\underline{q}_2'\, dy_H\, E_H$$

$$\Rightarrow \frac{d\sigma}{d^2\underline{q}_1'\, d^2\underline{q}_2'\, dy_H} = \left(\frac{N_c^2 - 1}{N_c^2}\right)^2 \frac{\alpha_s^6}{(2\pi)^5} \frac{G_F}{\sqrt{2}} \left[\int \frac{d^2 \underline{Q}_T}{\underline{Q}_T^2} \frac{\underline{k}_1 \cdot \underline{k}_2}{\underline{k}_1^2\, \underline{k}_2^2} \cdot \frac{2}{3}\right]^2$$

$(2\,\underline{k}_1 \cdot \underline{k}_2 \simeq M_H^2)$

final result for $qq \to qHq$ in lowest order.

To go from $qq \to qHq$ to $pp \to pHp$ KMR assume

(a) $\frac{4}{3}\frac{\alpha_s}{\pi} \longrightarrow f(x, Q_\perp^2) = \frac{\partial G(x, Q_\perp^2)}{\partial \ln Q_\perp^2}$

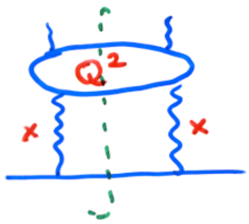
(b) $d\sigma \longrightarrow d\sigma\, e^{b(q_1'^2 + q_2'^2)}$ $\rightsquigarrow \int d^2\underline{q}'\, e^{-b\underline{q}'^2} = \frac{\pi}{b}$

$$\Rightarrow \frac{d\sigma}{dy_H} = \frac{1}{256\pi b^2} \frac{\alpha_s^2 G_F \sqrt{2}}{9} \left[\int \frac{d^2 Q_T}{\underline{Q}_T^4} f(x_1, Q_T^2)\, f(x_2, Q_T^2)\right]^2$$

(Assuming $\underline{q}_1' - \underline{q}_1 \simeq 0$ so $\underline{k}_1^2 \simeq \underline{k}_2^2 \simeq \underline{k}_1 \cdot \underline{k}_2 \simeq \underline{Q}_T^2$)

Assumption

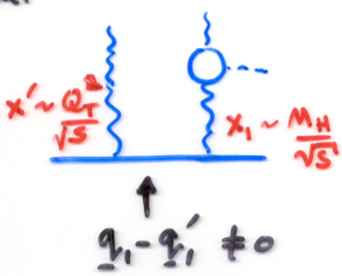
(a) Would be exact if process were not skewed → it follows from DGLAP assuming $q(x,Q^2) = \delta(1-x)$



$\Big\}\, G(x,Q^2)$

ie, $\dfrac{\partial G(x,Q^2)}{\partial \ln Q^2} \simeq \dfrac{\alpha_s}{\pi} C_F \leftarrow P_{gq}(z)\big|_{z\to 0}$

But we have



$x_1 > \dfrac{M_H^2}{s}$

$x_1 \neq x'$, $x_1 \gg x'$

(sim. for x_2 s.t. $x_1 x_2 s \simeq M_H^2$ hence x_1 & x_2 fixed by y_H)

- Really need skewed parton densities

for $x \gg x'$ and $x \ll 1$ skewed gluon is fixed by diagonal gluon

ie, $$\frac{f(x,Q_T^2)}{f(x,x',Q_T^2)} \approx \left(\frac{2^{2\lambda+3}}{\sqrt{\pi}}\,\frac{\Gamma(\lambda+5/2)}{\Gamma(\lambda+4)}\right)^{-1}$$ Shuvaev et al.

$G(x,Q^2) \sim x^{-\lambda}$

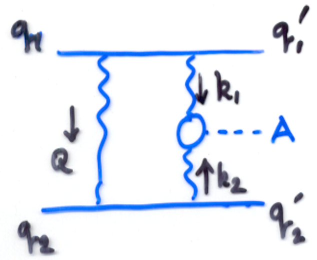
$\Rightarrow$ $\boxed{\text{Enhancement by a factor } \approx (1.2)^4 \simeq 2}$ (LHC) ← $\lambda \simeq 0.2$

PSEUDOSCALAR HIGGS

- As for 0^+ case except $\propto \varepsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}$

$$\underline{k}_1 \cdot \underline{k}_2 \longrightarrow (\underline{k}_1 \times \underline{k}_2) \cdot \underline{n}$$

$\leftarrow$ unit vector along beam axis



if $\underline{q}_1 = \underline{q}_2 = 0$

then $\underline{q}_1' = -\underline{Q} - \underline{k}_1$

$\underline{q}_2' = +\underline{Q} - \underline{k}_2$

Have
$$\int \frac{d^2 Q}{Q^2} \frac{1}{k_1^2} \frac{1}{k_2^2} \left\{ (\underline{q}_1' + \underline{Q}) \times (\underline{q}_2' - \underline{Q}) \right\} \cdot \underline{n}$$

$$\Rightarrow \int \frac{dQ_T^2}{Q_T^2} \frac{1}{Q_T^4} (\underline{q}_1' \times \underline{q}_2') \cdot \underline{n}$$

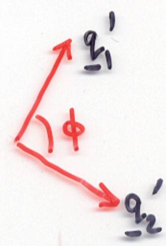
$\longleftarrow$ in limit $|\underline{q}_1'|, |\underline{q}_2'| \ll |\underline{Q}_T|$

Suppressed by $\sim \dfrac{|t|}{\langle Q_T^2 \rangle}$ relative to 0^+

more IR sensitive

outgoing protons like to be at right angles in azimuth

$\sim \sin^2 \phi$



ie can enhance 0^- rate* by cutting on $|\underline{q}_1'|, |\underline{q}_2'| > q_{cut}$

(*Relative to 0^+ rate)

Aside if CP violated in Higgs sector then can form a CP asymmetry
$$\frac{\sigma(\phi < \pi) - \sigma(\phi > \pi)}{\sigma(\phi < \pi) + \sigma(\phi > \pi)}$$
can be formed.

SUDAKOV

- So far all cross-sections are DIVERGENT

$$\int_0 \frac{dQ_T^2}{Q_T^4} \quad \left(\text{and} \quad \int_0 \frac{dQ_T^2}{Q_T^6}\right)$$

{ not quite so bad due to anomalous dimension of gluon density $\sim (Q_T^2)^\gamma$ }

But there exist Sudakov logarithms

$P_T < Q_T$: soft gluon screens emission

$$\text{Emission probability} \approx C_A \int_{Q_T^2}^{M_H^2/4} \frac{dP_T^2}{P_T^2} \frac{\alpha_s(P_T^2)}{\pi} \int_{P_T}^{\frac{1}{2}M_H} \frac{dE}{E}$$

(soft & collinear approximation i.e. double log approx.)

$$\left(\sim \frac{\alpha_s C_A}{\pi} \frac{1}{4} \ln^2 \frac{M_H^2}{4Q_T^2}\right)$$

exponentiating generates a factor in amplitude of

$$\exp(-S) = \exp\left(-\frac{C_A}{\pi} \int_{Q_T^2}^{\frac{1}{4}M_H^2} \alpha_s \frac{dP_T^2}{P_T^2} \int_{P_T}^{M_H/2} \frac{dE}{E}\right) \longleftarrow \text{double logs}$$

$$= \exp\left(-\int_{Q_T^2}^{M_H^2/4} \frac{\alpha_s(P_T^2)}{2\pi} \frac{dP_T^2}{P_T^2} \int_0^{1-\Delta} \left\{z P_{gg}(z) + \sum_q P_{qg}(z)\right\} dz\right)$$

double and single logs

Collinear AND soft logs if

$$\Delta = \frac{P_T}{(P_T + 0.62 M_H)}$$

$$\boxed{\text{As } Q_T \to 0 \text{ so the screening gluon fails to screen and } P_T \approx 0 \text{ emission is allowed. Hence } e^{-S} \text{ vanishes faster than any power of } Q_T.}$$

⇒ Sudakov factor ensures all cross-sections are formally convergent

But Typical Q_T is still small.

Saddle point in $\int \frac{dQ^2}{Q^n} e^{-S(Q^2, M_H^2)} f(x_1, Q^2) f(x_2, Q^2)$

occurs at $\log \frac{M_H^2}{4Q^2} = \frac{2\pi}{N_c \alpha_s(Q^2)} \left(\frac{n}{2} - 1 - 2\gamma \right)$

assuming $f \sim (Q^2)^\gamma$

So $Q \sim \frac{M_H}{2} \exp\left(- \frac{2\pi}{N_c \alpha_s} \frac{\left[\frac{n}{2} - 1 - 2\gamma\right]}{2} \right)$

$\alpha_s = 0.2$, $M_H = 100$ GeV, $n = 4$, $\gamma = 0.2$ → 2 GeV

(" " $n = 6$ " → $\ll 1$ GeV)

⇒ $Q \simeq 2$ GeV for 0^+ ie just in domain of pQCD.

$$\boxed{\text{It is crucial to sum the single logarithms since double log approx. overestimates Sudakov suppression by a factor} \sim 3}$$

More on Sudakov

- There is no leading log Sudakov suppression from the screening gluon

- Rather than $e^{-S} f(x_1, Q_T^2)\, f(x_2, Q_T^2)$ one ought to use $\frac{\partial}{\partial \ln Q_T^2}\left(G(x_1, Q_T^2)\, e^{-S/2}\right) \times \frac{\partial}{\partial \ln Q_T^2}\left(G(x_2, Q_T^2)\, e^{-S/2}\right)$

Kimber, Martin, Ryskin, Watt

in diagonal case KMRW claim

$$f(x, k_t^2, \mu^2) = \frac{\partial}{\partial \ln k_t^2}\left[G(x, k_t^2)\, e^{-S}\right]$$

becomes $e^{-S/2}$ due to skewedness ie, non-emission off screening gluon.

nb as $x \to 0$

$$f(x, k_t^2, \mu^2) \to \frac{\partial}{\partial \ln k_t^2}\left[G(x, k_t^2)\right] \quad \text{(BFKL)}$$

(since no $\log 1/x$ in S $\;e^{-S} \simeq 1$)

$$\boxed{\text{This last modification is very important and leads to a factor } \sim 10 \text{ enhancement}}$$

(ie $\frac{\partial}{\partial \ln Q_T^2} e^{-S/2}$ is crucial.)

GAP SURVIVAL

- Want to compute

  $P(pHp \mid gaps)$

  Apart from Sudakov there are other ways to fill in gaps
  "soft rescattering", "multiparton interactions"

- Assume that $P(pHp \mid gaps) = P(pHp) \times \underbrace{P(\text{gap not filled})}_{= S^2}$

  "gap survival factor"

- Simplest ansatz is that rescattering ~~is~~ ~~as~~ ~~single~~ can be described by Poisson statistics

  let $\Omega(b)$ be the mean number of rescattering events for a pp collision at impact parameter b

  then $P_n = \frac{\Omega(b)^n}{n!} \exp(-\Omega(b))$ is the probability of having n rescatterings. $P_0 = \exp(-\Omega(b))$

  Hence

$$S^2 \simeq \frac{\int d^2\underline{b}\; e^{-\Omega(b)} \left| M_{p+H+p}(b) \right|^2}{\int d^2\underline{b}\; \left| M_{p+H+p}(b) \right|^2}$$

- In such a "single channel" approach

$$\sigma_{inelastic} = \int d^2\underline{b}\,\left(1 - e^{-\Omega(b)}\right)$$

Hence $\sigma_{elastic} = \int d^2\underline{b}\,\left(1 - e^{-\Omega/2}\right)^2$

and $\sigma_{tot} = 2\int d^2b\,\left(1 - e^{-\Omega/2}\right)$

such that $\sigma_{tot} = \sigma_{elastic} + \sigma_{inelastic}$

$\Rightarrow$ $\Omega \ll 1$

$$\sigma_{tot} \simeq \int d^2b\, \Omega(b)$$ ← identify with single $\mathbb{P}$ exchange

eg $$\Omega(b) = \underbrace{\sigma_0 \left(\frac{s}{s_0}\right)^{\Delta}}_{=\sigma_{tot}} \times \frac{1}{2\pi B} \exp\left(-b^2/2B\right)$$

fit σ_0, Δ & $B(s)$ to σ_{tot} and $\sigma_{elastic}$ data. (Block & Halzen)

~~And~~

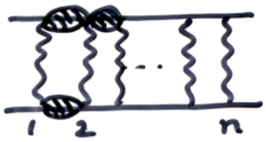
- To compute S^2 need b dependence of $|M_{p+H+p}|^2$

if $$|M_{p+H+p}|^2 \sim \left| \int e^{-b_0 q_t^2/2}\, e^{i\underline{q}\cdot\underline{b}}\, d^2\underline{b} \right|^2$$

$b_0 \simeq 5.5\,\text{GeV}^{-2}$ ($\pm$?)

$$\sim e^{-b^2/2b_0}$$

- KKMR use a two channel eikonal model.

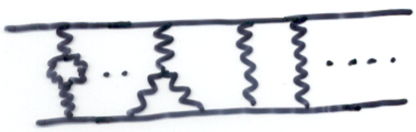
$\sum_n$ 

takes account of diffraction

Assume 

($\gamma \simeq 0.4$)

Also include high mass diffraction via triple regge



ie $\Omega = \Omega_{\mathbb{P}} + \Omega_D$

And use a modified $\alpha_{\mathbb{P}}(t)$ to account for π loops at low t ie, $\alpha_{\mathbb{P}} = \alpha(0) + \alpha'(t) - \Delta\left(m_\pi^2/t\right)$

Most (all?) eikonal models yield similar predictions for S^2 provided they are tuned to σ_{tot} & $\sigma_{elastic}$

eg, for central diffraction at LHC $S^2 \simeq 2\text{-}3\%$

FUTURE

* Attempt to estimate uncertainty of theoretical calculations (KMR estimate a factor 2.5 uncertainty for a 120 GeV 0^+ Higgs)

* Backgrounds – need more detailed study

* Dijets at Tevatron (crucial to test theory)

* Alternative approaches – Comparison ?