

# **b quark fragmentation**

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- Motivation
- First look at available data / theories
- Plans

# Factorization for light quarks/hadrons

Light hadron production in  $e^+e^-$ :

$$e^+e^- \rightarrow hX$$

The light hadron cross section can be written as

$$\sigma_h(x, E) = \sum_i \int \frac{dz}{z} \sigma_i(z, E) D_{i \rightarrow h} \left( \frac{x}{z}, E \right) + \mathcal{O} \left( \frac{\Lambda}{E} \right)$$

where  $E = \sqrt{s}$ ,  $x = 2E_h/E$ ,  $\sigma_i(z, E)$  is the  $i^{\text{th}}$  parton cross-section and  $D_{i \rightarrow h}(z, E)$  is the fragmentation function

- $D_{i \rightarrow h}(z, E)$  is needed to cure collinear divergence,
- depends on the factorization scheme,
- evolves in  $E$  with GDLAP, initial condition fixed by expt. data,
- is process independent: take from  $e^+e^-$ , use for  $ep, pp$ .

Non Perturbative (NP) corrections are small:  $\mathcal{O}(\frac{\Lambda}{E}) \sim 0.2\%$  at the  $Z$ .

## Factorization for Heavy Quarks ?

The HQ mass  $m_Q$  is a cutoff for collinear divergence: no need for factorization  
Perturbative cross section:

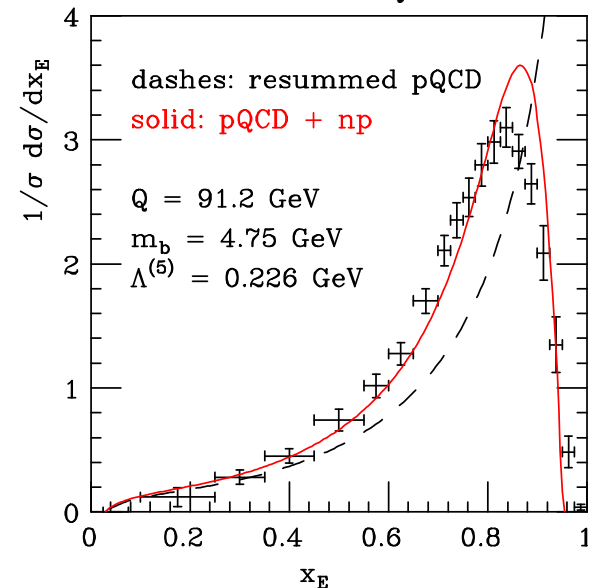
$$\sigma_h(x, E, m_Q) = \sigma_Q(x, E, m_Q) + \mathcal{O}\left(\frac{\Lambda}{m_Q}\right)$$

NP corrections may be large:  $\mathcal{O}(\Lambda/m_Q) \sim 5\%$  for  $b$  and  $15\%$  for  $c$

Convolute quark cross section with a NP fragmentation function  $D_{Q \rightarrow H}^{\text{NP}}(z)$ :

$$\sigma_H(x, E) = \sum_i \int \frac{dz}{z} \sigma_Q(z, m_Q, E) D_{Q \rightarrow H}^{\text{NP}}\left(\frac{x}{z}\right)$$

Is  $D_{Q \rightarrow H}^{\text{NP}}(z)$  process independent ?



Test the portability of  $D_{Q \rightarrow H}^{\text{NP}}(z)$  from  $e^+e^-$  to  $ep, pp$

## What can be done for the Workshop

- Fit  $D^{NP}(z)$  on  $e^+e^-$  data
- Evaluate uncertainty from  $D^{NP}(z)$  on  $ep$ ,  $pp$  cross sections
- Study how to measure  $D^{NP}(z)$  directly in  $ep$  or  $pp$

Before starting two points need to be understood:

1. which data ?
2. which theory ?

## Which data ?

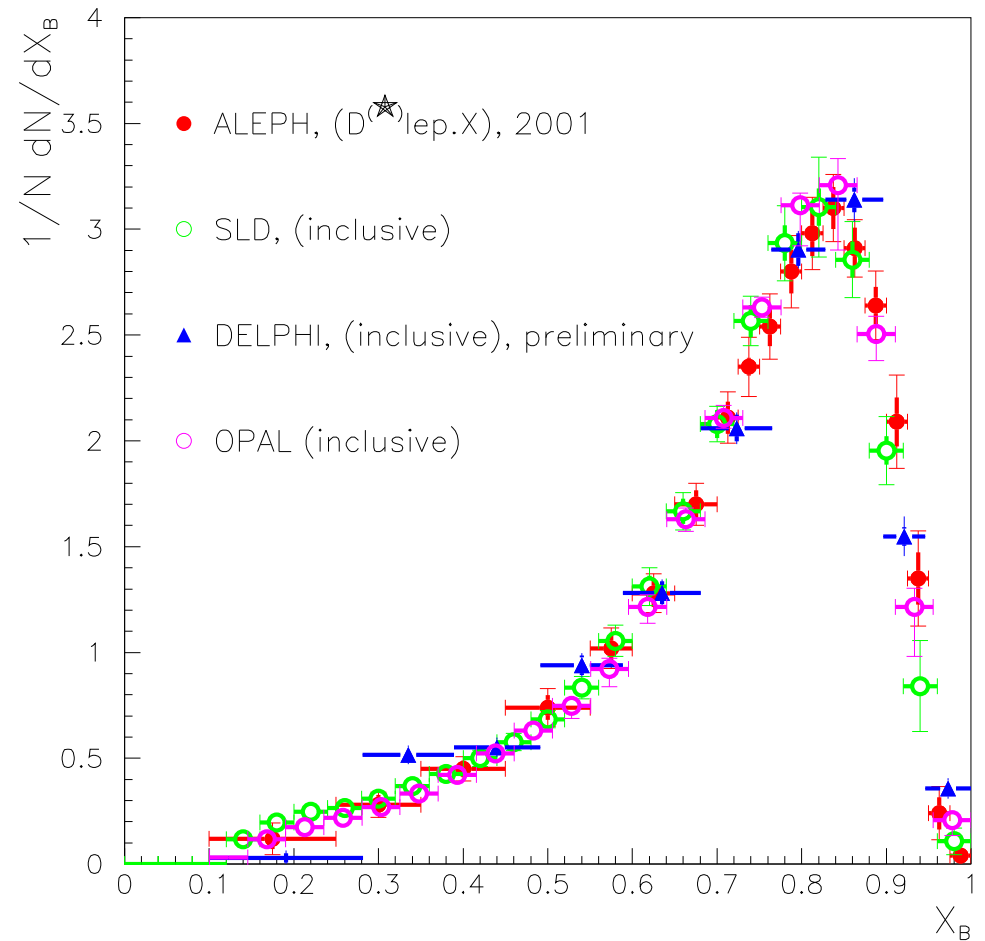
All the available measurements are from  $e^+e^-$

Observable:  $x_E = E_H/E_{\text{beam}}$  (H is the weakly decaying hadron)

- LEP/SLD: precise measurements of

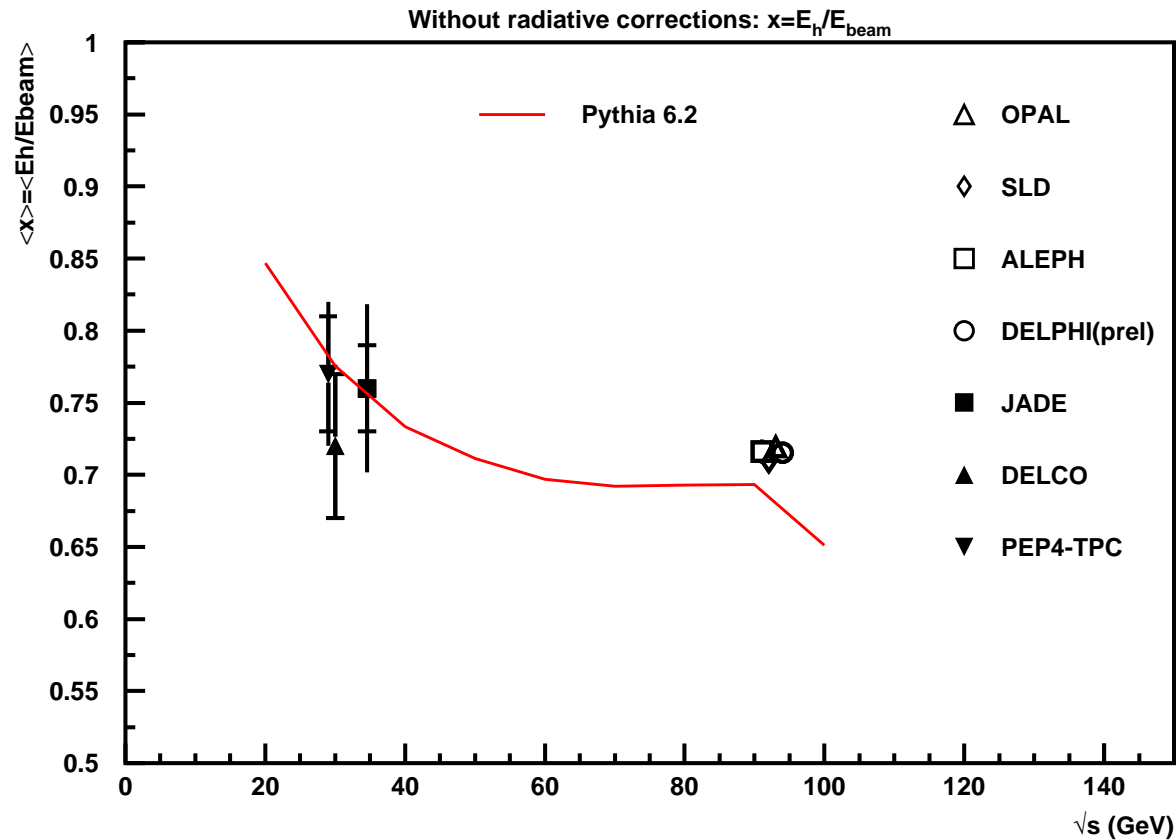
$$\frac{1}{\sigma} \frac{d\sigma}{dx_E}$$

- Full error matrix available
- $\langle x_E \rangle$  available
- data in agreement (at first sight)



## Which data (2)

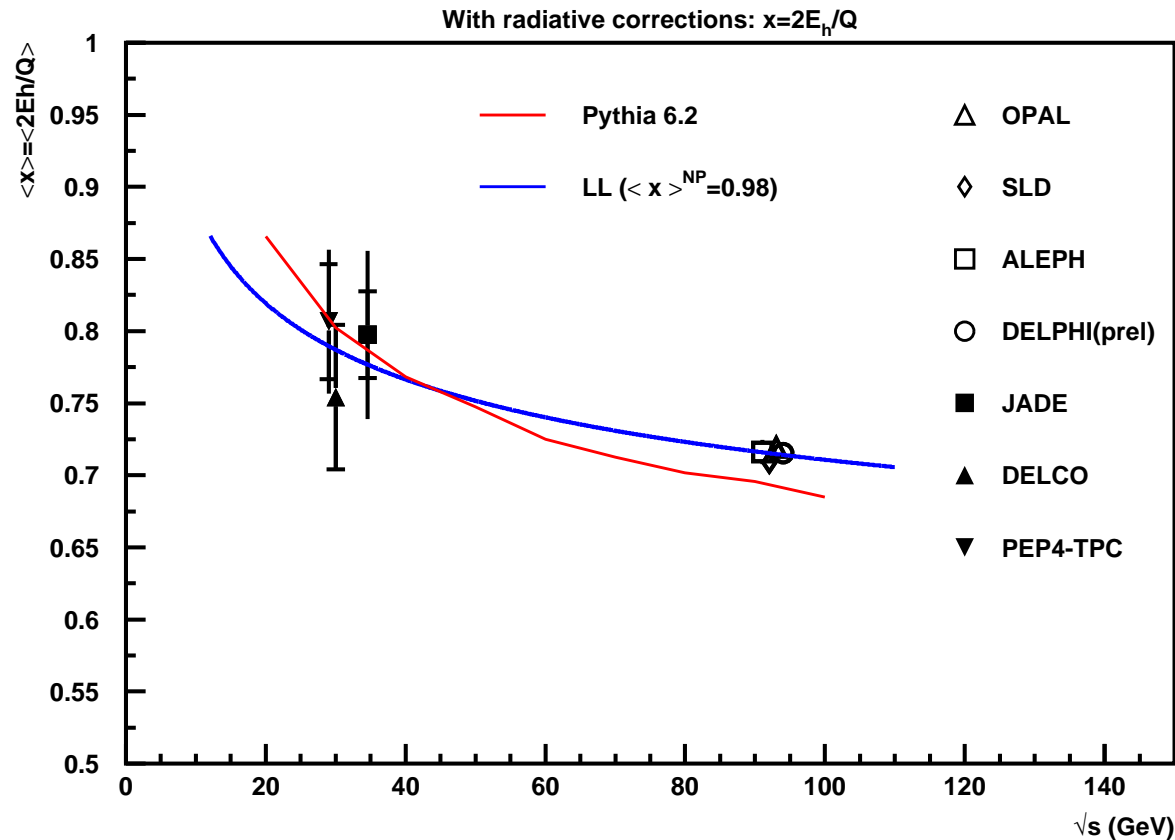
Measurement of  $\langle x_E \rangle$  also from PEP, PETRA, TOPAZ



- Measurements that don't provide a direct observable such as  $\langle x_E \rangle$  but a fragmentation parameter of some MC model are not considered
- TOPAZ (VENUS) data have too low precision, not considered
- EW radiative corrections are needed to compare with QCD models

## Which data (3)

$e^+e^-$  data with EW correction (taken from Pythia)



**Red:** Pythia 6.2 with ISR radiation from  $e$  off

**Blue:** Leading Log formula (Nason, Mele, PLB 245 (1990) 635)

- Evidence for scaling violation of  $\langle x_E \rangle$
- Low energy data consistent with LEP/SLD within (large) uncertainties
- Default Pythia a bit low at LEP/SLD energy

## Which theory ? (1)

$D^{\text{NP}}(z)$  is not an observable, depends on the theoretical model  
Need to use consistent theory for  $e^+e^-$  and for  $ep, pp$

Possible theories:

- MC (Pythia)
  - + NP fragmentation is tunable
  - + fits to  $e^+e^-$  data already done by LEP/SLD experiments
  - + usable for any observable in  $ep, pp$
  - + detailed model for fragmentation
  - cross sections accurate to LO
- Fixed Order (NLO) calculations
  - fits to recent  $e^+e^-$  to be done (fit to old ALEPH data in Nason, Oleari Nuc.Phys.B 565 (2000) 245)
  - + programs for  $ep, pp$  available for any observable
  - + cross sections accurate to NLO
  - large log resummation missing, important at LEP/SLD, LHC, not at HERA



## Which theory ? (2)

- Fixed Order plus resummation of Leading Logs (FONLL)

- + some fit to  $e^+e^-$  already available (Corcella, Mitov Nucl. Phys. B 565 (2000) 245 )
- calculation for  $ep$ ,  $pp$  only for single differential distributions
- + accurate to NLO and NLL

would be very good if a simple program for any observable was available

- MC⊙NLO

- ? NP fragmentation partially tunable (Herwig)

- missing for  $e^+e^-$ , use Herwig ?
- + available for  $pp$
- missing for  $ep$
- + accurate to NLO and LL
- + detailed model for fragmentation

would be ideal if  $ep$  (and  $ee$ ) implemented

- mixed solution (for the moment):

take  $D^{\text{NP}}(z)$  from FONLL fit to LEP/SLD and apply to FO

- + FO~FONLL for  $b$  at HERA, should be OK at HERA
- not good for LHC

## Available Fits to $e^+e^-$

**Fits to Single Parameter functions:**

<b>Kartvelishvili</b>	$f(x) = (1 - x)x^\alpha$
<b>Peterson</b>	$f(x) = \frac{1}{x} \left(1 - \frac{1}{x} - \frac{\epsilon}{(1-z)}\right)^{-2}$

**FONLL**

Corcella, Mitov Nucl. Phys. B 565 (2000) 245  
(without soft log resummation)

Parameter	ALEPH	SLD
$\alpha$	$15.0 \pm 0.4$	$14.6 \pm 0.4$
$\epsilon$	$0.0029 \pm 0.0002$	$0.0023 \pm 0.0002$

**FO and FONLL**

Nason, Oleari Nuc.Phys.B 565 (2000) 245

Theory	Parameter	ALEPH (old)
<b>FO(LO)</b>	$\epsilon$	0.0069
<b>FO(NLO)</b>	$\epsilon$	0.0033
<b>FONLL</b>	$\epsilon$	0.0023

**Fits within Pythia**

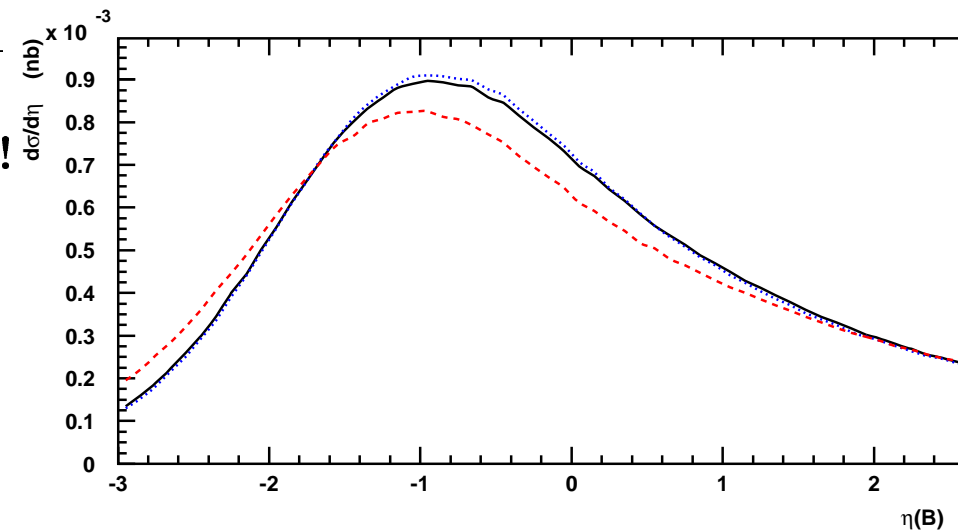
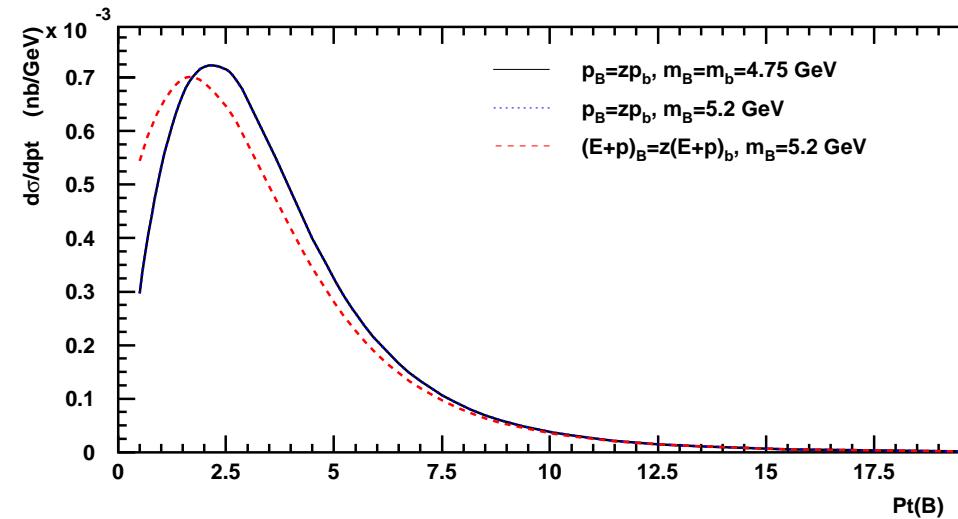
Parameter	ALEPH	SLD	OPAL
$\epsilon$	$0.0031 \pm 0.0006$	0.0055	$0.0041 \pm 0.0004$
$\alpha$	$13.7 \pm 1.3$	10.0	$11.9 \pm 0.5$

# Effects at HERA

First look using FO calculation for HERA (FMNR)

Use  $D^{\text{NP}}(z)$  to rescale quark momentum and obtain  $B$  hadron differential cross-sections

- Peterson with  $\epsilon = 0.0035$
- $B$ -hadron momentum obtained by:  
**Black:** rescaling  $\vec{p}^b$  in the frame  $p_Z^b = p_Z^{\bar{b}}$  (FMNR default)  
**Blue:** as above but on  $B$  hadron mass shell  
**Red:** rescaling  $(E + P)^b$
- Dependence on scaling variable at low  $p_T$ !



## Effects at HERA (2)

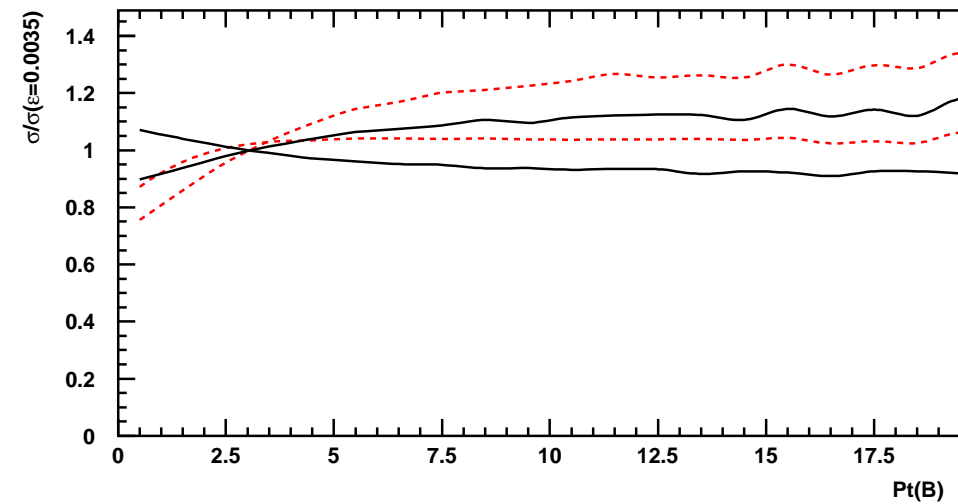
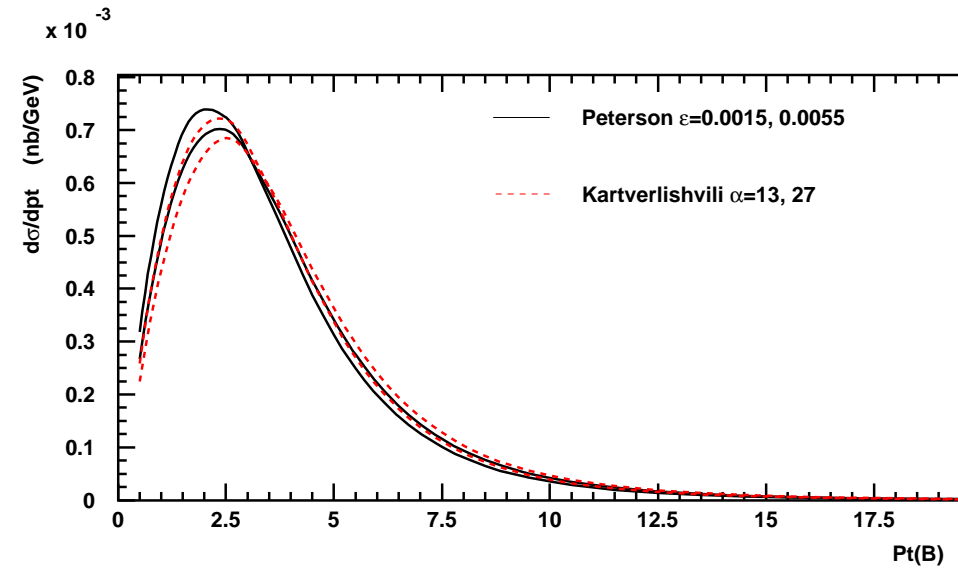
Try different  $D^{\text{NP}}(z)$

- Large variation range:

Black: Peterson with  $\epsilon = 0.0015 - 0.0055$

Red: Kartvelishvili with  $\alpha = 13 - 27$

- Effect of  $\pm 10\%$  at large  $p_T$



## Conclusions

- Universality of NP Fragm. Function needs to be tested
- Precise data available from LEP/SLD
- No perfect theoretical framework:
  - Try FONLL for  $ee/pp$  and  $FO$  for HERA
  - MC@NLO would be ideal if  $ep$  available
- effects of different choices of fragmentation at HERA  $\sim 10\%$

## Possible Developments (Plans)

- Fit LEP/SLD data using FONLL
- see effect on HERA observables (use HZTOOL ?)
- study how to measure fragmentation at HERA

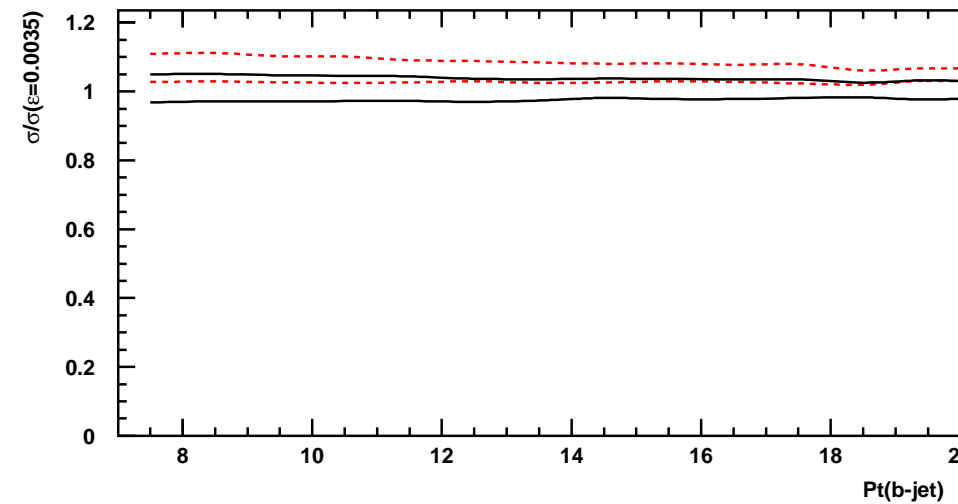
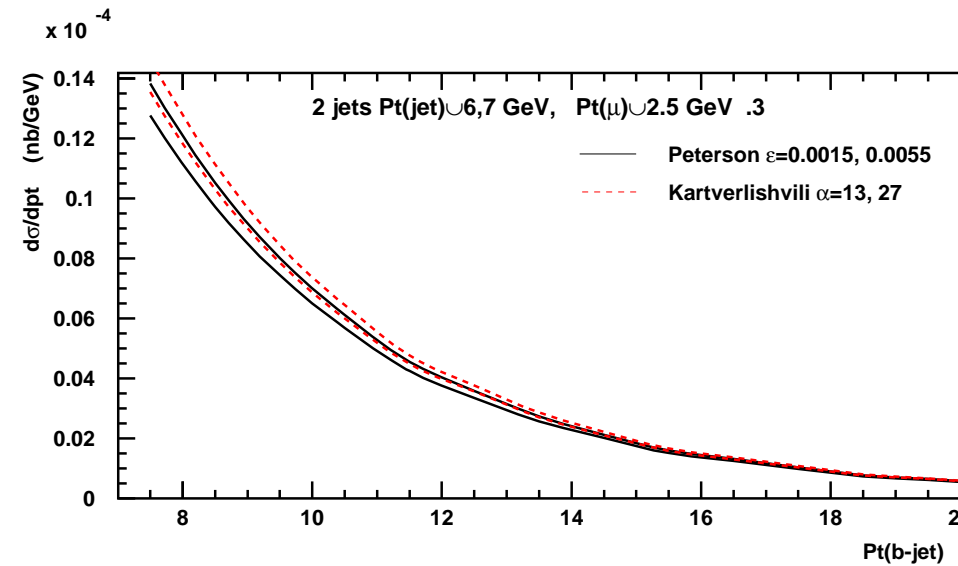
## Effects at HERA (3)

Much reduced dependence on fragmentation when using jets

**Black:** Peterson with  $\epsilon = 0.0015 - 0.0055$

**Red:** Kartvelishvili with  $\alpha = 13 - 27$

Effect of  $\pm 3\%$  at large  $p_T$



## HQ Perturbative Fragmentation Functions

HQ perturbative FF are introduced to resum large logs:

When  $E \gg m_Q$  the cross section is finite but large terms like  $\alpha_S^n \log^n(E^2/m_Q^2)$  appear.

Large logarithms can be resummed to all orders using perturbative FF:

$$\sigma_h(x, E, m_Q) = \int \frac{dz}{z} \sigma_Q^{\text{massless}}(z, Q) D_{Q \rightarrow H}^{\text{pert}}(x/z, E, m_Q) + O\left(\frac{m_Q}{E}\right) + O\left(\frac{\Lambda}{m_Q}\right)$$

So this is not related to the universality of  $D^{NP}(z)$  !