Structure function extrapolation to LHC energies based on combined BK/DGLAP equations

Michael Lublinsky

DESY/Theory

Based on:

- M. L., E. Gotsman, E. Levin, and U. Maor, *Nucl. Phys.* **A 696** (2001) 851;
- E. Gotsman, E. Levin, M. L., and U. Maor, *Eur. Phys. J.* **C 27** (2003) 411.

Motivation

Main problems of the DGLAP evolution:

- DGLAP evolution predicts a steep growth of parton distributions at low x which will eventually violate the unitarity constraints;
- The twist OPE breaks down at low x, when the higher twists become of the same order as the leading one;
- DGLAP evolution is totally unable to describe low Q^2 data.

NLO corrections do not solve these problems.

Non-linear evolution is a solution to these problems!

- It accounts for the saturation effects due to high parton densities;
- It sums high twist contributions;
- It allows extrapolation to large distances.

New procedure for extrapolation of parton distributions

Two steps:

- Solve the BK non-linear evolution equation.
  - it takes into account high twist contributions;
  - but only in the leading $\ln(1/x)$ approximation of pQCD;
  - and without a correct short distance description.
- Introduce a correcting function for which a DGLAP-type linear equation is proposed and solved.

The full solution is

$$N\,(\,r_\perp\,,\,x\,;\,b\,)\;=\;\tilde{N}\,(\,r_\perp\,,\,x\,;\,b\,)\;+\;\Delta\,N\,(\,r_\perp\,,\,x\,;\,b\,)$$

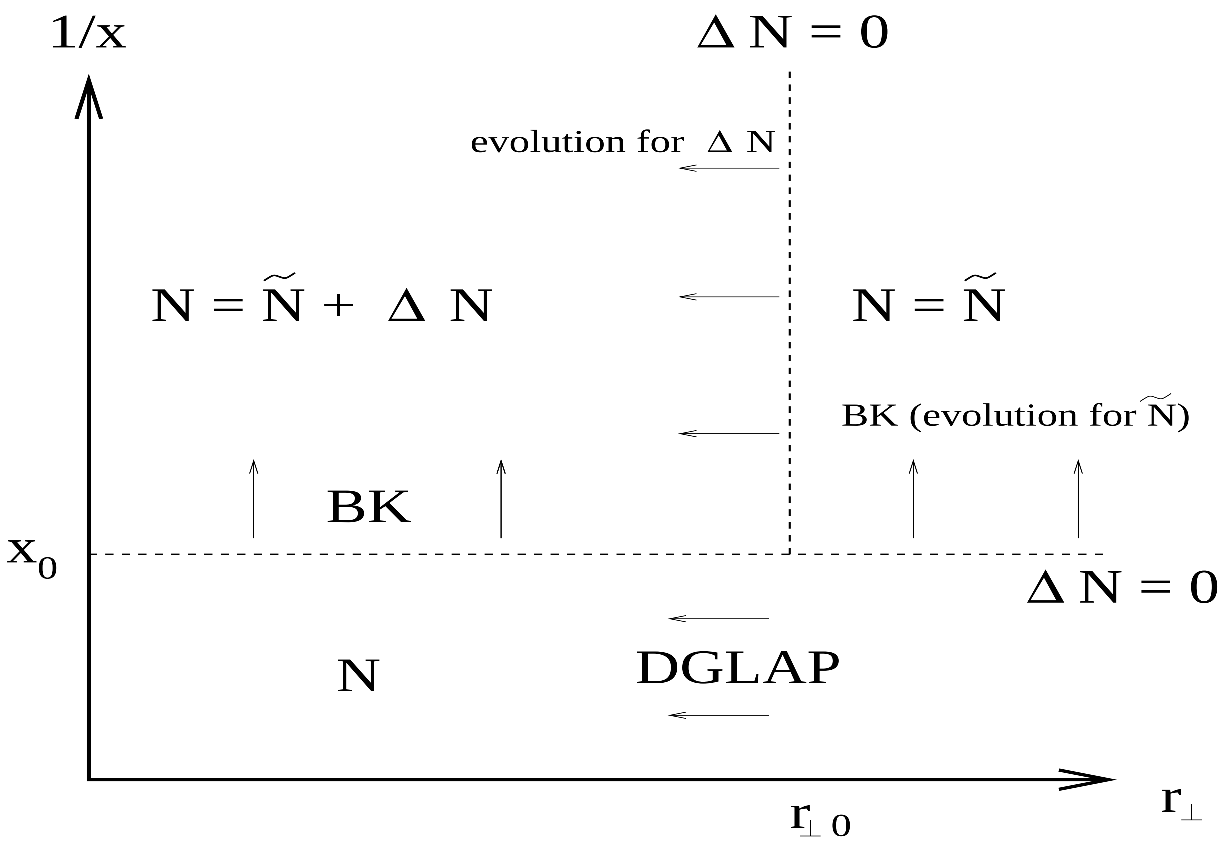
$\tilde{N}\,(\,r_\perp\,,\,x\,;\,b\,) \leftarrow$ BK non-linear equation;

$\Delta\,N\,(\,r_\perp\,,\,x\,;\,b\,) \leftarrow$ DGLAP-type linear equation;

where

$$N\,(\,r_\perp\,,\,x\,;\,b\,)\;=\;Im\;a^{el}_{dipole}\,(\,r_\perp\,,\,x\,;\,b\,)\,.$$

Strategy



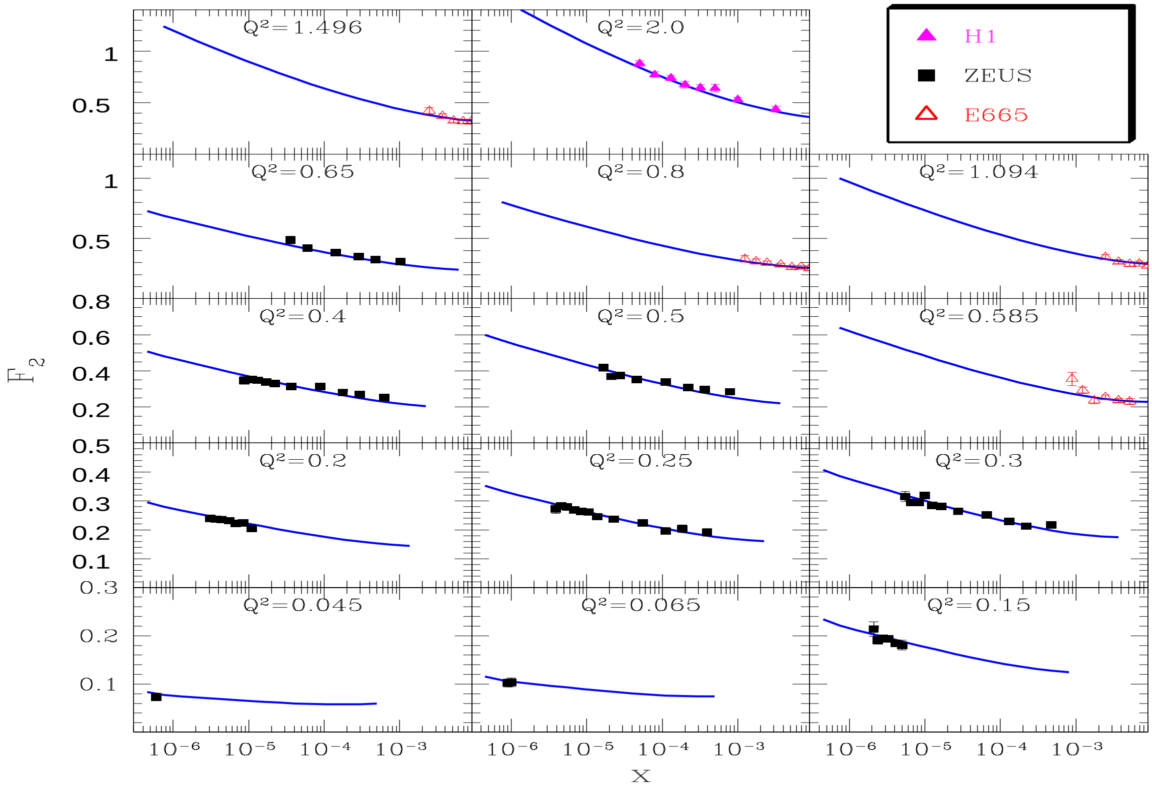
Initial conditions: Glauber - Mueller formula at $x_0 = 10^{-2}$:

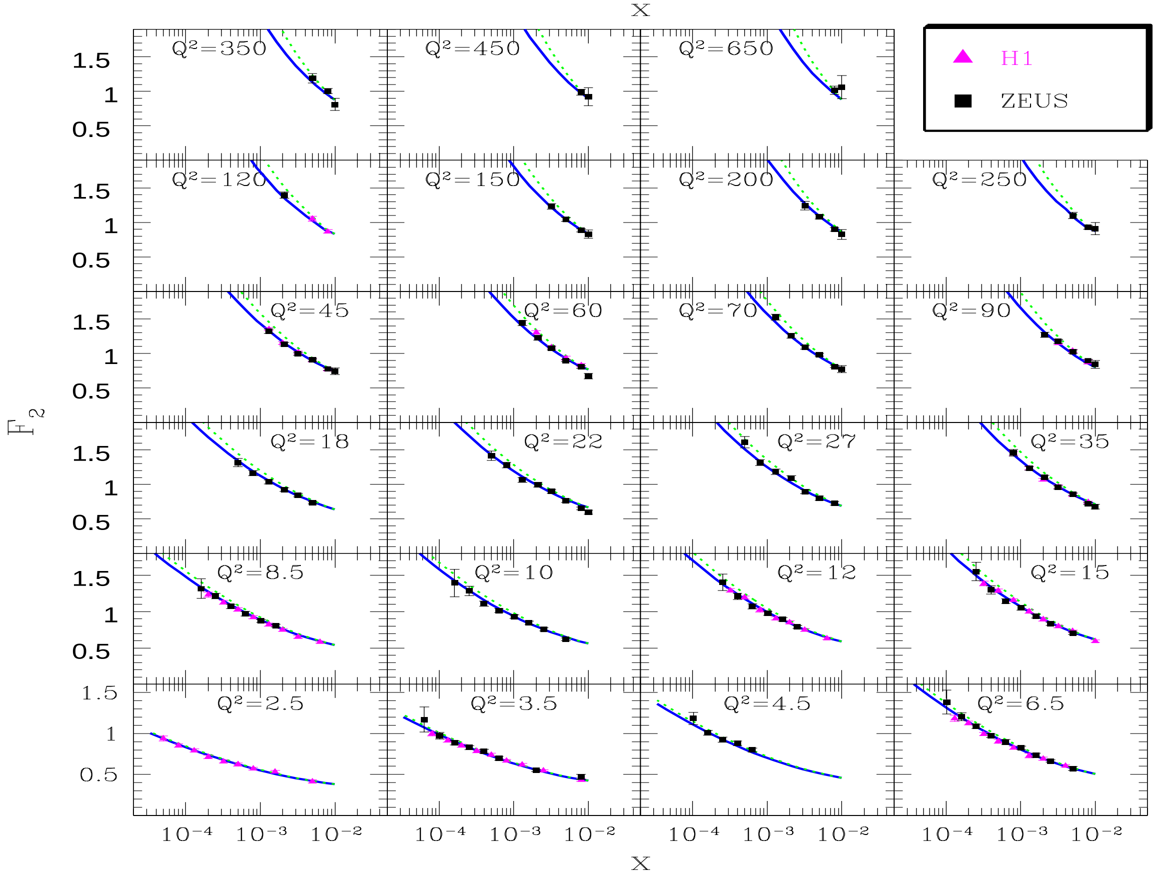
$$\tilde{N}(r_\perp, x_0; b) \;=\; 1 \;-\; \exp\left[-\frac{\alpha_S \pi r_\perp^2}{2\, N_c\, R^2}\, xG^{DGLAP}(x_0, 4/r_\perp^2)\, S(b)\right]$$

Fit of F_2 data in two steps:

- $\tilde{N}$ is fitted to describe the low Q^2 data; R^2 is a fitting parameter.
- ΔN is added to fit the large Q^2 data; $r_{\perp 0}$ is a fitting parameter.

Results





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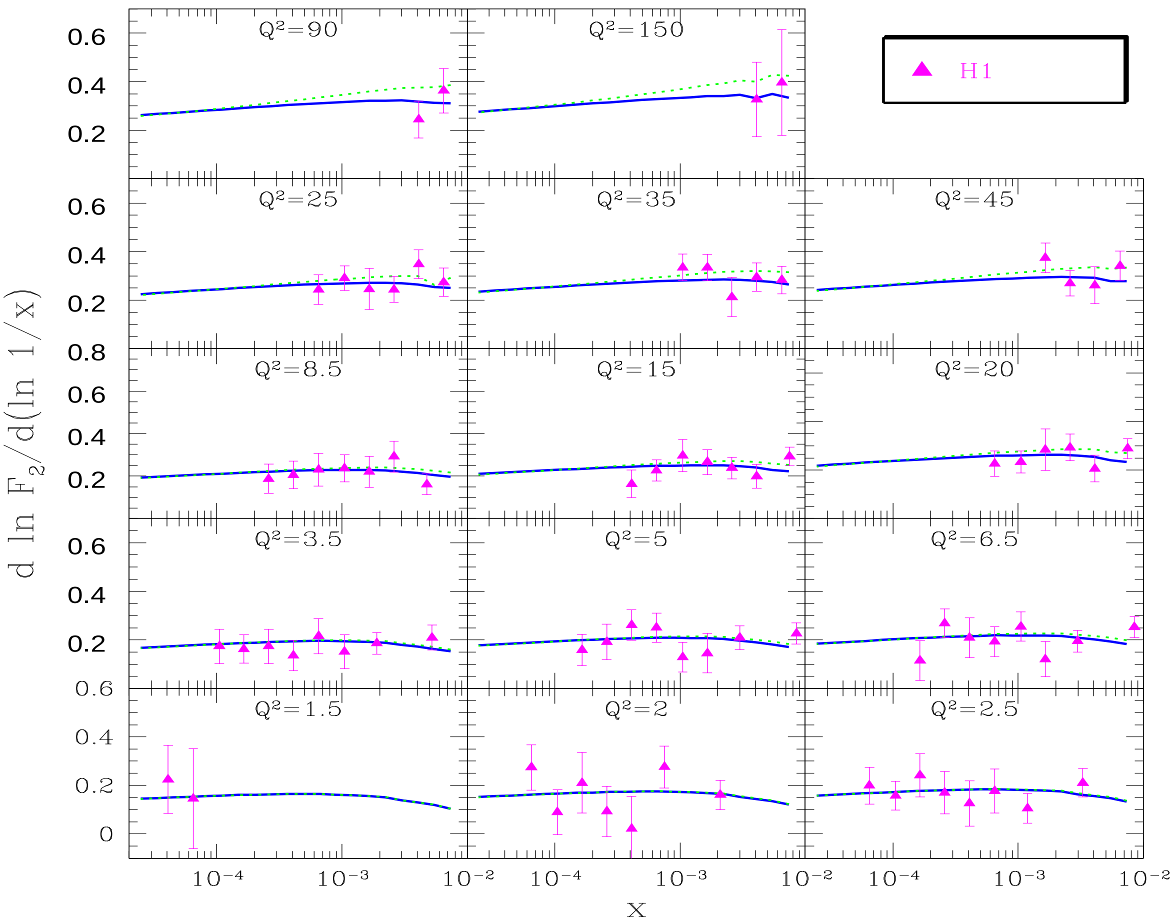
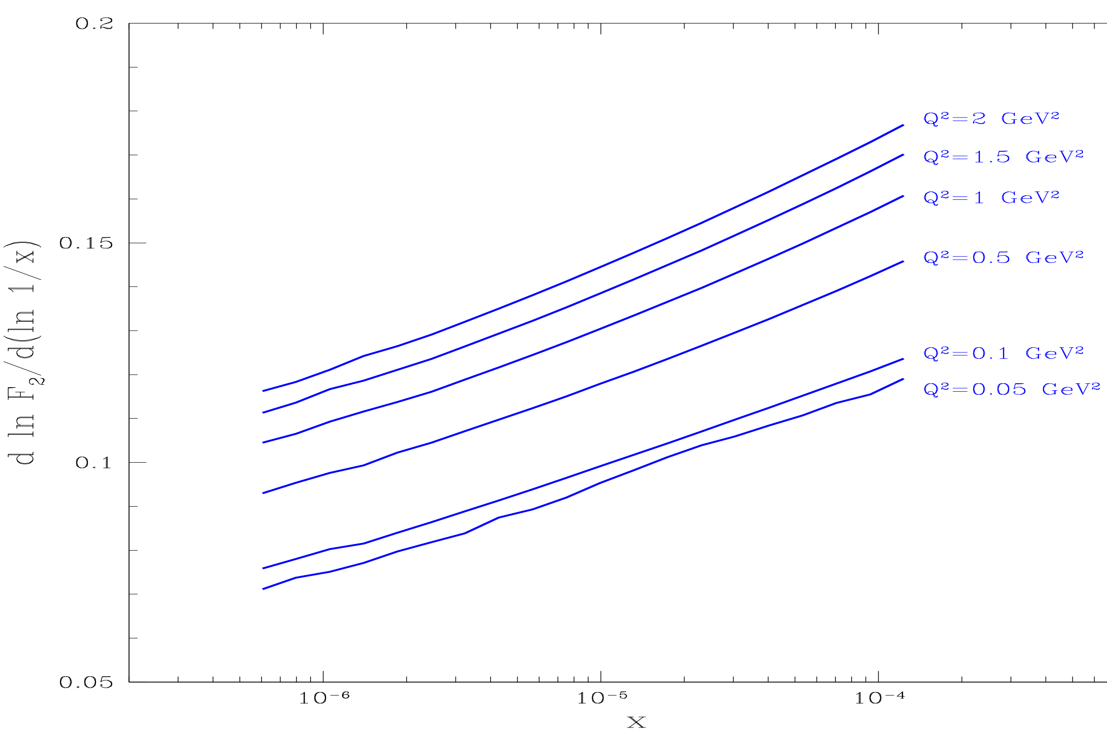
$$\lambda = \partial \ln \mathbf{F_2} / \partial \ln (\mathbf{1} / \mathbf{x})$$





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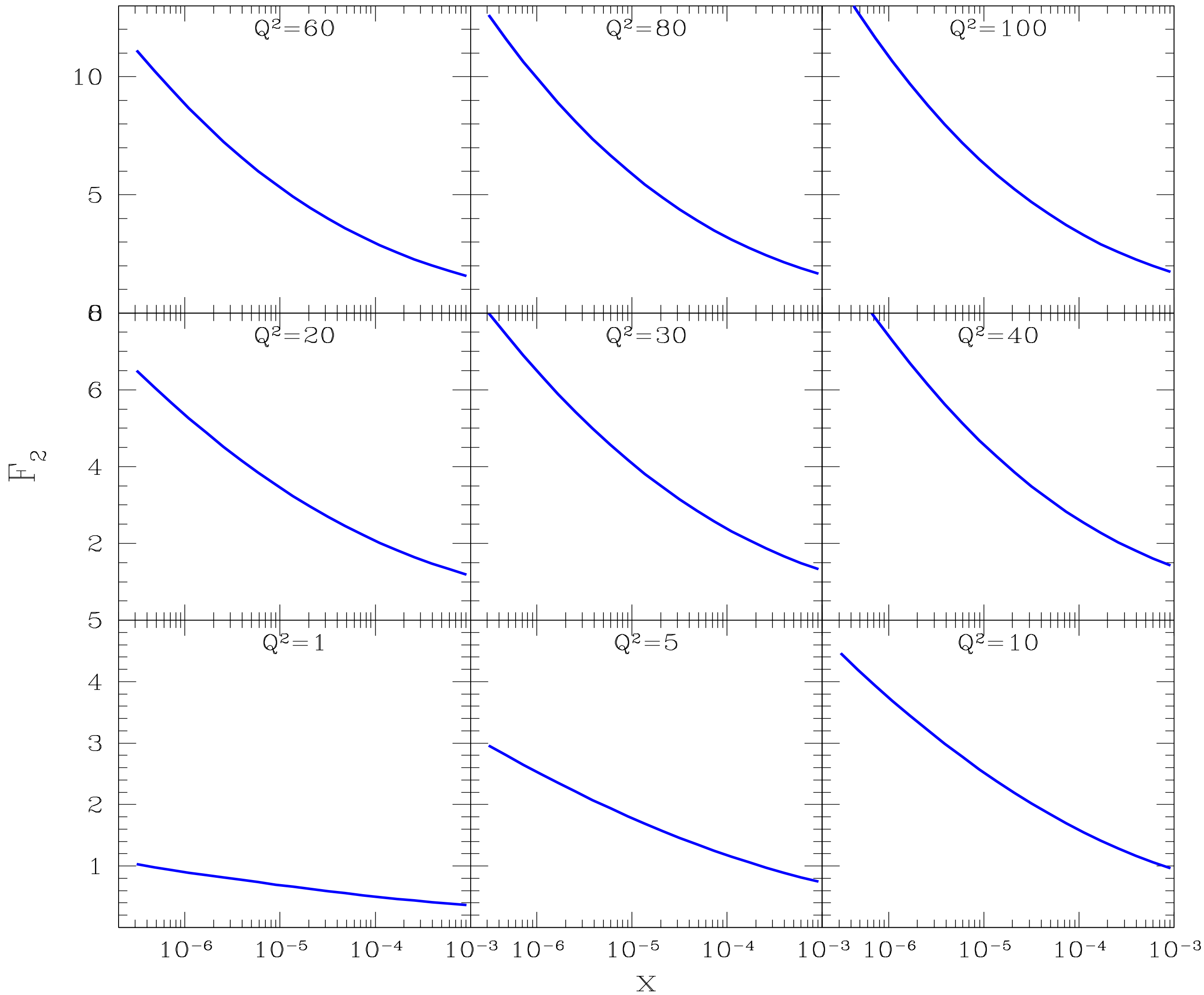
F_2 @ LHC



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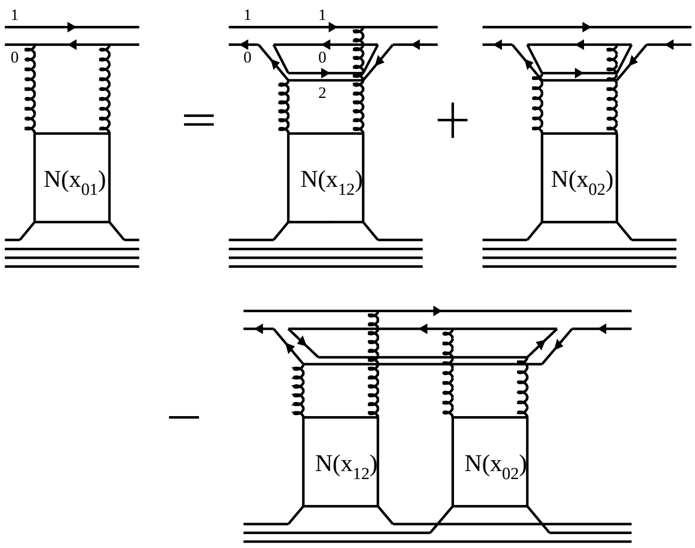
Non-linear evolution



GLR (81)
Mueller & Qiu (86)
....
Balitsky (95)
Kovchegov (99)
Braun (2000)
Iancu, Leonidov & McLerran (2000)

$$\frac{d\tilde{N}\,(\,x_{01}\,,\;y\,;\;b\,)}{dy} \;=\; -\;\frac{2\,C_F\,\alpha_S}{\pi}\;\tilde{N}\,(\,x_{01}\,,\;y\,;\;b\,)\;\ln\frac{x^2_{01}}{\rho^2}\;+$$

$$\frac{C_F\,\alpha_S}{\pi^2}\;\times\;\int_{\rho}\,d^2\,x_2\,\frac{x^2_{01}}{x^2_{02}\,x^2_{12}}\;\times$$

$$\Big(\,2\,\tilde{N}\,(\,x_{02}\,,\;y\,;\;b\,)\;-\;\tilde{N}\,(\,x_{02}\,,\;y\,;\;b\,)\,\tilde{N}\,(\,x_{12}\,,\;y\,;\;b\,)\Big)$$

Approximations: LLog $1/x$; $N_c \longrightarrow \infty$; α_s - const,; b - large.

Correcting Function ΔN

•

$$\tilde{n} \equiv \tilde{N}/(\alpha_s\, r_\perp^2)\,; \quad n \equiv N/(\alpha_s\, r_\perp^2)\,; \quad \Delta n \equiv \Delta N/(\alpha_s\, r_\perp^2)\,.$$

•

$$\frac{\partial \Delta n(r_\perp, x)}{\partial\, \ln(1/r_\perp^2)} = \frac{C_F \alpha_S}{\pi} \int_{x/x_0}^{1} P_{g\to g}(z)\; \Delta n(r_\perp, \frac{x}{z})\, dz -$$

$$\frac{2\, C_F\, \alpha_s}{\pi} \int_{x/x_0}^{1} \frac{dz}{z}\, \tilde{N}(r_\perp, \frac{x}{z}) \Delta n(r_\perp, \frac{x}{z}) \; +$$

$$\frac{C_F\, \alpha_s}{\pi} \int_{x/x_0}^{1} \left(P_{g\to g}(z) - \frac{2}{z} \right) \tilde{n}(r_\perp, \frac{x}{z})\, dz \; -$$

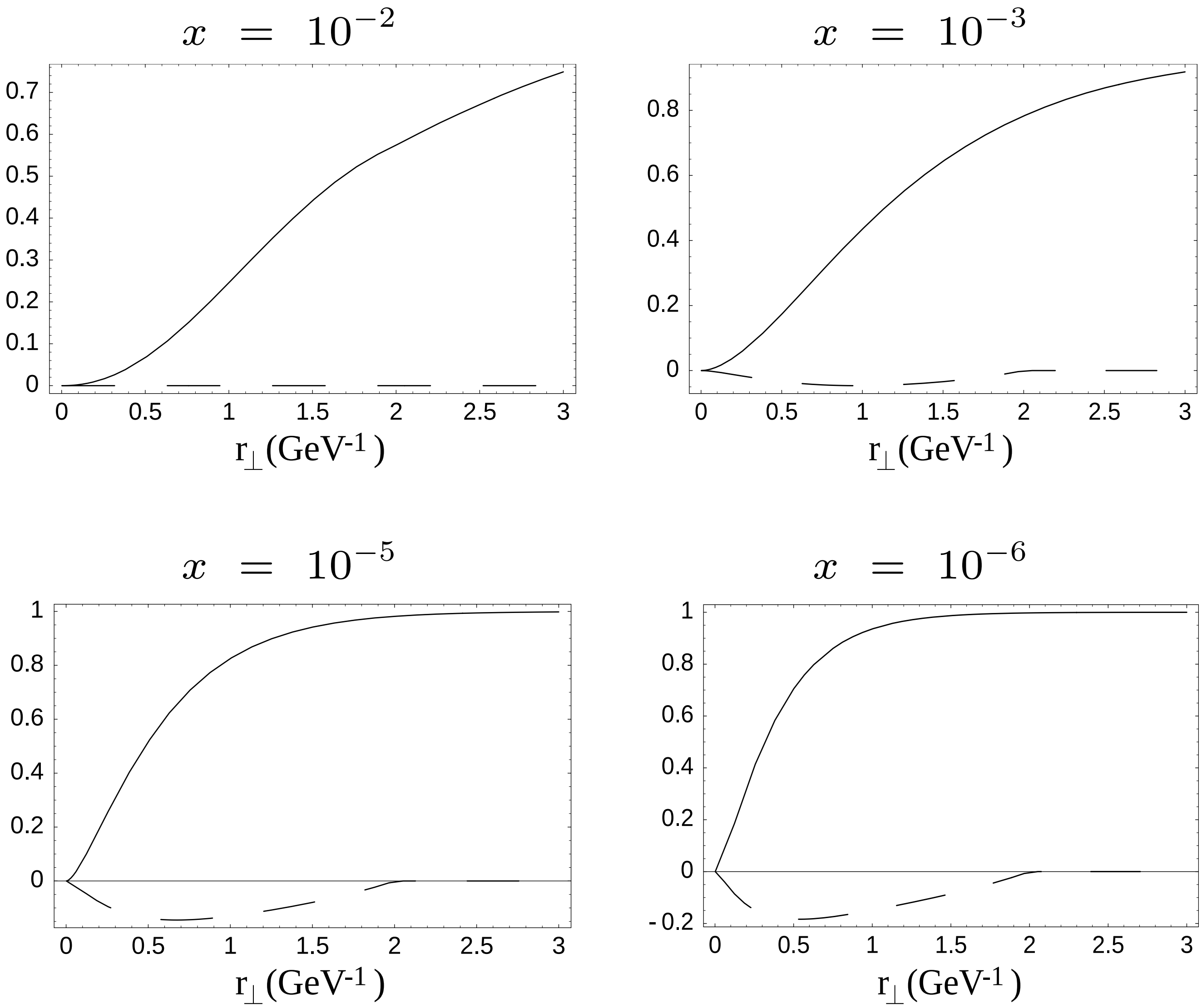
$$\frac{\partial \tilde{n}(r_\perp, x_0)}{\partial\, \ln(1/r_\perp^2)} \; + \; \frac{C_F\, \alpha_s}{\pi} \int_{x}^{x/x_0 -} P_{g\to g}(z)\; n(r_\perp, \frac{x}{z})\, dz\,.$$

• where

$$P_{g\to g}(z) = 2 \left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) + \left(\frac{11}{12} - \frac{n_f}{18} \right) \delta(1-z) \right]$$

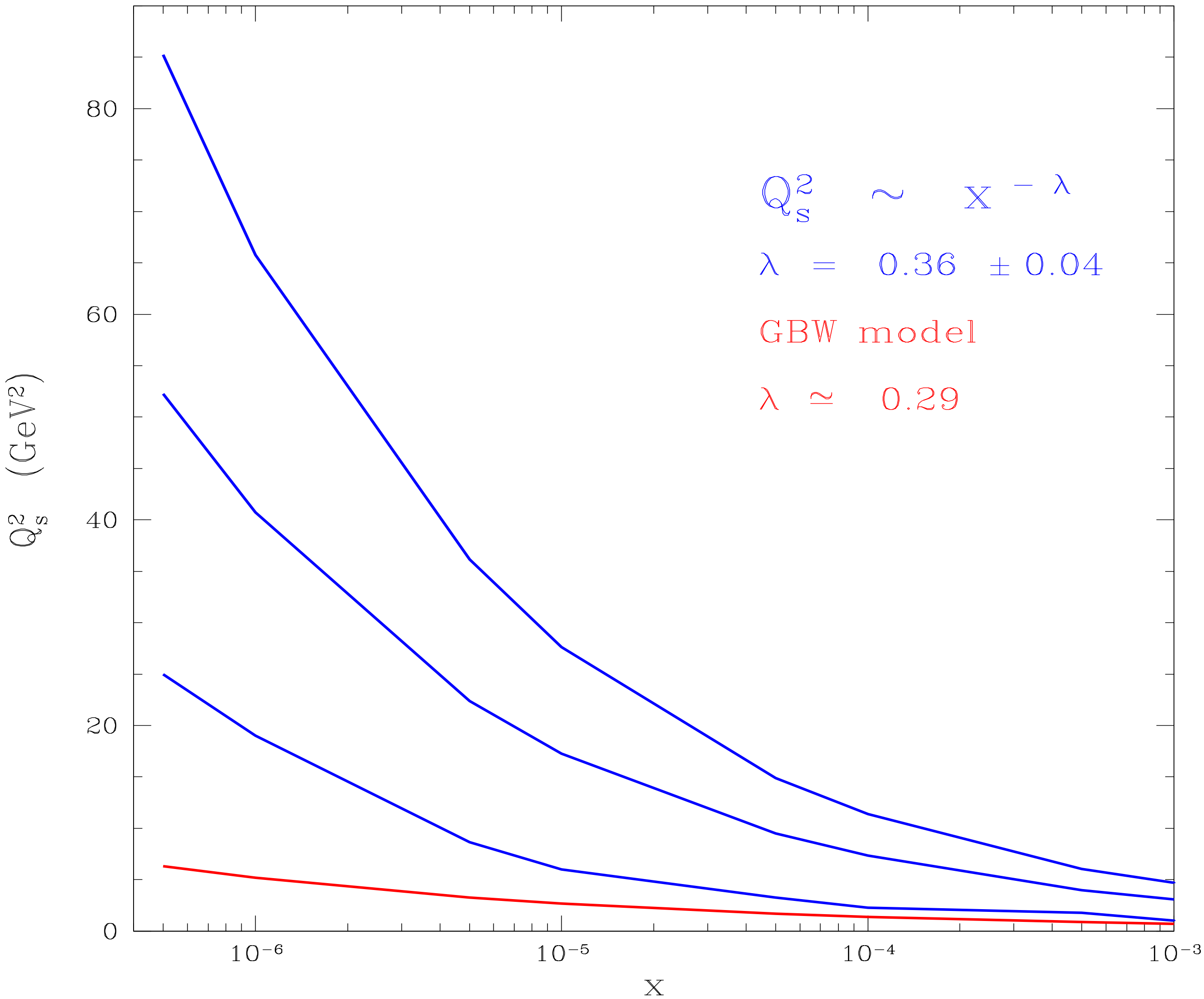
- Large $\ln(1/r_\perp^2)$ approximation
- $\Delta N \;\leq\; 0$; $\quad |\Delta N| < \tilde{N}$
- $\Delta N \;\rightarrow\; -\; \tilde{N} \rightarrow 0$ at $r_\perp \;\rightarrow\; 0$

Solutions of the equations ($b = 0$)



- α_s - running $\alpha_s^{max} \simeq 0.5$
- $\mathbf{xG^{DGLAP}} \leftarrow$ LO CTEQ6
- $\mathbf{R^2 = 3.1}\ (\mathrm{GeV}^{-2})$
- $\mathbf{r_{\perp\,0} = 2}\ (\mathrm{GeV}^{-1})$

Saturation Scale



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Summary

- A new approach to global QCD analysis based on combined BK/DGLAP equations is under development;

- Low x data on the F_2 structure function is reproduced using 2 fitting parameter; Resulting $\chi^2/ndf = 1$;

- The method allows extrapolation of the parton distributions to the LHC energies as well as very low photon virtualities $Q^2 \ll 1\,GeV^2$;

- Further steps to be done: a). eliminate the model dependence due to impact parameter; b) include high x domain + quarks; c) NLO.