

BFKL dynamics in jet-physics

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Is there BFKL dynamics in jet emission?

Strong expectation (in last 20-30 years): No.

However

- BFKL equation for $Q\bar{Q}$ multiplicity
- Kovchegov equation for non-global observables

Multiplicity in hard events (e.g. e^+e^- at Q):

Resummation of **collinear** and **IR** logs, use DGLAP+coherence

$$N(Q, Q_0) = n(Q_0) \exp \left\{ \int_{Q_0}^Q \frac{dk_t}{k_t} \gamma_0(\alpha_s(k_t)) \right\} \sim e^{c \sqrt{\ln Q^2/\Lambda^2}}$$

Multiplicity of $Q\bar{Q}$ system: BFKL equation

Mueller&GM, PLB575(2003)37 [hep-ph/0308284]

$$e^+e^- \rightarrow p\bar{p} + \mathbf{k} q_1 \dots q_n, \quad \mathbf{k} \rightarrow Q\bar{Q}$$

- $v_k = |\vec{k}|/E_k < 1 \Rightarrow$ no collinear singularities
- $\mathbf{k} \ll Q \Rightarrow$ \mathbf{k} off-shell soft gluon
- $k^2 - 4M^2 \sim M^2 \Rightarrow$ factorization

$Q\bar{Q}$ -multiplicity emitted off ab -dipole

$$\frac{dN_{ab}}{dk^2} = I(\theta_{ab}, Q, k) \cdot D(k), \quad p_a p_b \rightarrow p'_a p'_b + \mathbf{k} q_1 q_2 \dots$$

Leading contribution from soft secondary gluons $\mathbf{k} q_1 \dots q_n$.

Born level: $a b \rightarrow a' b' + \mathbf{k}$

$$I^{(0)}(\theta_{ab}, v_k) = \int \frac{d\Omega_k}{4\pi} \frac{1 - \cos \theta_{ab}}{(1 - v_k \cos \theta_{ak})(1 - v_k \cos \theta_{kb})}$$

$v_k < 1$: No collinear singularities \Rightarrow only **IR** logs in $Q\bar{Q}$ -multiplicity

$Q\bar{Q}$ -inclusive distribution (k off-shell tagged soft gluon)

Real emission: $\rightarrow k + q$ additional soft gluon

$$\begin{aligned} w_{ab}^{\text{R}}(k; q) &= \frac{(ab)}{(aq)(qk)(kb)} + \frac{(ab)}{(ak)(kq)(qb)} \\ &= \Theta(q-k) w_{ab}(q) \cdot [w_{aq}(k) + w_{qb}(k)] \\ &\quad + \Theta(k-q) w_{ab}(k) \cdot [w_{ak}(q) + w_{kb}(q)] \end{aligned}$$

with
$$w_{ab}(k) = \frac{(ab)}{(ak)(kb)} = \frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{ak})(1 - \cos \theta_{kb})}$$

Virtual correction: $\rightarrow k + q$ virtual (on-shell) soft gluon

$$\begin{aligned} w_{ab}^{\text{V}}(k; q) &= -\Theta(q-k) w_{ab}(q) \cdot w_{ab}(k) \\ &\quad -\Theta(k-q) w_{ab}(k) \cdot [w_{ak}(q) + w_{kb}(q)] \end{aligned}$$

Sum $w_{ab}^{\text{R+V}}(k; q) = \Theta(q-k) w_{ab}(q) \cdot [w_{aq}(k) + w_{qb}(k) - w_{ab}(k)]$

$$I^{(1)}(\theta_{ab}, \tau) = \tau \cdot \int \frac{d\Omega_q}{4\pi} w_{ab}(q) \cdot [I^{(0)}(\theta_{aq}) + I^{(0)}(\theta_{qb}) - I^{(0)}(\theta_{aq})]$$

$$\tau = \int_k^Q \frac{dq_t}{q_t} \frac{N_c \alpha_s(q_t)}{\pi}, \quad I(\theta_{ab}, Q, k) = I(\theta_{ab}, \tau)$$

Branching probability $\left\{ \frac{dq_t}{q_t} \frac{N_c \alpha_s(q_t)}{\pi} \right\} \frac{d\Omega_q}{4\pi} w_{ab}(q)$

Higher order resummation

$$\frac{\partial I(\theta_{ab}, \tau)}{\partial \tau} = \int \frac{d\Omega_q}{4\pi} w_{ab}(q) [I(\theta_{aq}, \tau) + I(\theta_{qb}, \tau) - I(\theta_{ab}, \tau)]$$

Toward BFKL equation

$$\frac{\partial I(\theta_{ab}, \tau)}{\partial \tau} = \int \frac{d\Omega_q}{4\pi} w_{ab}(q) \left[I(\theta_{aq}, \tau) + I(\theta_{qb}, \tau) - I(\theta_{ab}, \tau) \right]$$

Small angle limit: $\theta_{ab} = \theta \ll 1$ (boosted frame)

$$\int \frac{d\Omega_q}{4\pi} w_{ab}(q) \Rightarrow \int_{-\infty}^{\infty} \frac{d^2\theta'}{2\pi} \frac{\theta^2}{\theta'^2 (\vec{\theta}' - \vec{\theta})^2}$$

obtain

$$\partial_{\tau} I(\vec{\theta}, \tau) = \int \frac{d^2\theta'}{2\pi} \frac{\theta^2}{\theta'^2 (\vec{\theta} - \vec{\theta}')^2} \left[I(\vec{\theta}') + I(\vec{\theta} - \vec{\theta}') - I(\vec{\theta}) \right]$$

$$\tau = \int_{E_k}^E \frac{dq_t}{q_t} \frac{N_c \alpha_s(q_t)}{\pi}$$

Elastic T-matrix: $T(\vec{b}, \tau)$, \vec{b} = impact parameter, $\tau = \frac{N_c \alpha_s}{\pi} Y$

$$\partial_{\tau} T(\tau, \vec{b}) = \int \frac{d^2b'}{2\pi} \frac{b^2}{b'^2 (\vec{b} - \vec{b}')^2} \left[T(\vec{b}') + T(\vec{b} - \vec{b}') - T(\vec{b}) \right]$$

Same evolution equation (formally)

1) $\vec{b} \Rightarrow \vec{\theta}$

2) fixed coupling \Rightarrow running coupling

Same behaviour at large τ : $I(\vec{\theta}, \tau) \sim \frac{e^{\chi(0)\tau}}{\sqrt{\tau}}$

$$\chi(k) = 2\psi(1) - \psi\left(\frac{1}{2} + ik\right) - \psi\left(\frac{1}{2} - ik\right), \quad \chi(0) = 4 \ln 2$$

But...

Physics and mathematics differences

The connection between the two equations is formal:

- same multi-soft gluon-distributions
- different relevant kinematical configurations
 - ❖ **jet-physics:** θ_i of emitted gluons of same order, k_{ti} ordered
 - ❖ **small-x:** k_{ti} of exchanged gluons of same order, θ_i ordered

Consequences:

- $I(\rho, \tau)$ (jet-physics)
 - ❖ $\rho = \frac{1}{2}(1 - \cos \theta)$, $0 < \rho < 1$,
 - ❖ $\tau = \int_k^Q \frac{dq_t}{q_t} \frac{N_c \alpha_s(q_t)}{\pi}$, never too large
 - ❖ relevant region: ρ finite (e.g. $\rho = 1$ for e^+e^- in cm)
- $T(b, \tau)$ (small-x physics)
 - ❖ impact parameter b , $0 < b < \infty$
 - ❖ $\tau = \frac{N_c \alpha_s}{\pi} Y$ asymptotic behaviour relevant
 - ❖ relevant region: b small (short distance)

BFKL versus $Q\bar{Q}$ equations

Enrico Onofri & GM [hep-ph/0404242]

$$\partial_\tau T(b, \tau) = \int_0^1 \frac{d\eta}{1-\eta} \left(\frac{T(\eta b)}{\eta} - T(b) \right) + \int_0^1 \frac{d\eta}{1-\eta} \left(T\left(\frac{b}{\eta}\right) - T(b) \right)$$
$$\partial_\tau I(\rho, \tau) = \int_0^1 \frac{d\eta}{1-\eta} \left(\frac{I(\eta\rho)}{\eta} - I(\rho) \right) + \int_\eta^1 \frac{d\eta}{1-\eta} \left(I\left(\frac{\rho}{\eta}\right) - I(\rho) \right)$$

BFKL case

$$b = e^{-x}, \quad -\infty < x < \infty, \quad \text{translation invariance}$$

$$T(b, \tau) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \tilde{T}(k) e^{(ik - \frac{1}{2})x} e^{\chi(k)\tau} \simeq \tilde{T}(0) \frac{e^{4 \ln 2 \tau} e^{-\frac{x^2}{2D\tau}}}{\sqrt{2\pi D\tau}}$$

Conservation

$$\partial_\tau \int_{-\infty}^{\infty} dx T(e^{-x}, \tau) e^{-4 \ln 2 \tau} = 0$$

$Q\bar{Q}$ -multiplicity

$$\rho = \frac{1 - \cos \theta}{2} = e^{-x}, \quad 0 < x < \infty, \quad \text{no translation inv.}$$

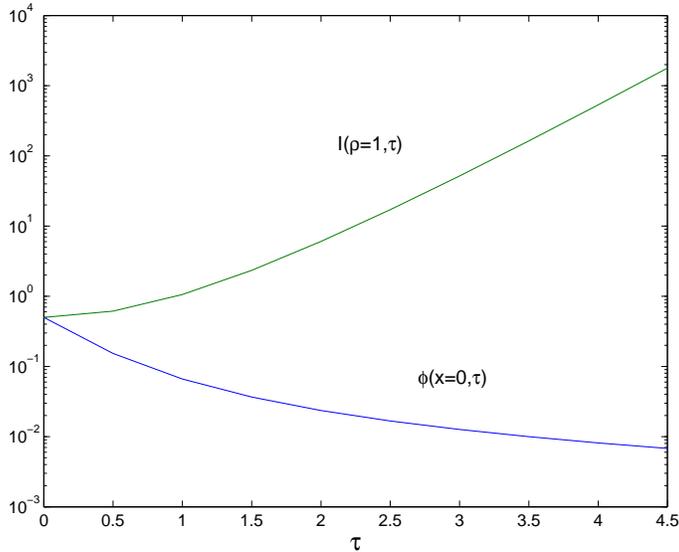
$$I(\rho, \tau) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \tilde{I}(k) u(k, x) e^{\chi(k)\tau} \sim \frac{(x + x_0) e^{4 \ln 2 \tau} e^{-\frac{x^2}{2D\tau}}}{\tau \sqrt{2\pi D\tau}}$$

Absorption

$$\partial_\tau \int_0^\infty dx I(e^{-x}, \tau) e^{-4 \ln 2 \tau} < 0$$

Solution of $Q\bar{Q}$ equations

Enrico Onofri&GM [hep-ph/0404242]

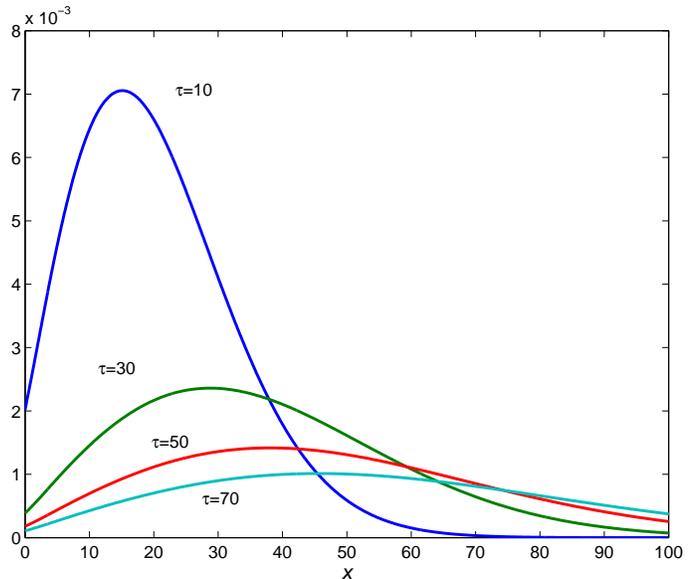
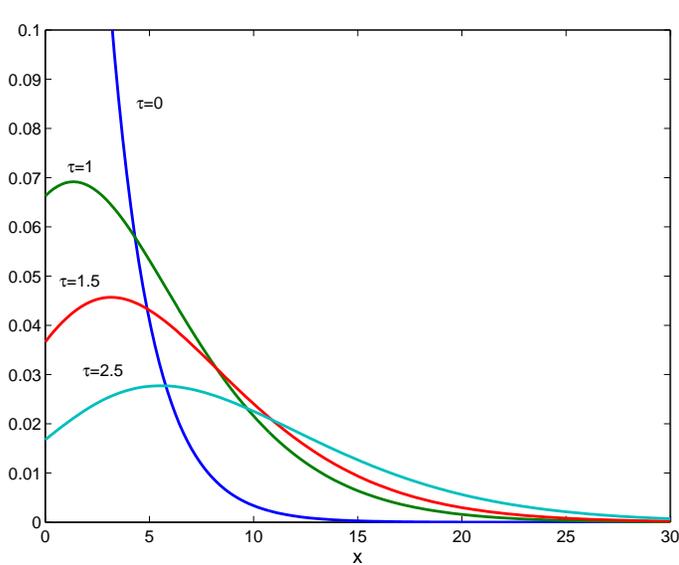


$$I(\rho, \tau)$$

$$\phi(x, \tau) = e^{-4 \ln^2 \tau} I(\rho, \tau)$$

$$\rho = e^{-x} = \frac{1 - \cos \theta}{2} = 1$$

$$\text{Initial condition } I(\rho, 0) = \frac{1}{2}\rho$$



$$\phi(x, \tau) = e^{-4 \ln^2 \tau} I(e^{-x}, \tau)$$

Large τ behaviour

$$I(\rho, \tau) \approx A \frac{(x + x_0) e^{-(x+x_0)^2/2D(\tau-\tau_0)}}{(\tau - \tau_0)^{3/2}}, \quad D = 28\zeta(3)$$

Kovchegov equation in jet-physics

New discovery in jet-physics: non global logs

M. Dasgupta and G.Salam, JHEP 08(02) 032; JHEP 03(02) 3311

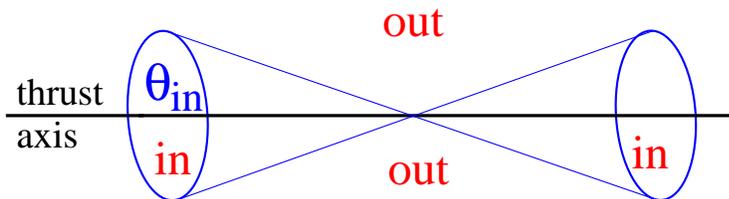
Jet-shape with only **part** of phase space involved. **Examples:**

- Stermann-Weinberg distribution (energy in a cone)
- photon isolation
- away from jet radiation
- rapidity cuts in hadron-hadron (e.g. pedestal dist.)
- DIS jet in current hemisphere

Relevant configuration: large angle soft emission

General features: M.Dasgupta,G.Salam; C.Berger,T.Kúcs,G.Stermann;
A.Banfi,G.Smye&GM; Yuri Dokshitzer &GM.

Simplest case e^+e^- : Soft emission off $q\bar{q}$ -dipole in **out**-region



$$\Sigma_{e^+e^-}(E_{out}) = \sum_n \int \frac{d\sigma_n}{\sigma_T} \Theta \left(E_{out} - \sum_{out} q_{ti} \right)$$

Basis for the analysis: multi-soft gluon emission (large N_c)

Bassetto,Ciafaloni&GM, Phys.Rep.100(1983)201

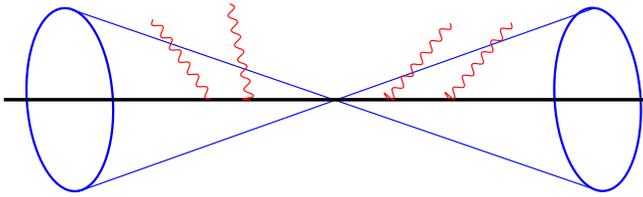
Multi-soft gluon emission off a ab -dipole

$$W_{ab}(q_1 \cdots q_n) = \frac{(ab)}{(aq_1) \cdots (q_nb)}$$

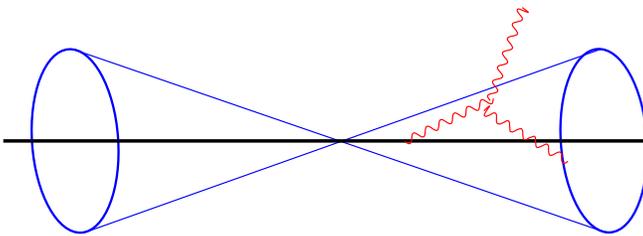
$$\partial_\tau \Sigma_{ab} = -(\partial_\tau R_{ab}) \Sigma_{ab} + \int_{\text{in}} \frac{d\Omega_q}{4\pi} w_{ab}(q) [\Sigma_{aq} \cdot \Sigma_{qb} - \Sigma_{ab}]$$

$$R_{ab} = \tau \int_{\text{out}} \frac{d\Omega_q}{4\pi} w_{ab}(q), \quad \tau = \int_{E_{\text{out}}}^Q \frac{dq_t}{q_t} \frac{N_c \alpha_s(q_t)}{\pi}$$

Two QCD components:



Bremsstrahlung component:
SL Sudakov factor: $S_{ab} = e^{-R_{ab}}$
Linear evolution (DGLAP type)



Soft branching inside Jet region
correlation function C_{ab}
SL only for non-global obs.
beyond SL for global obs.

$$\Sigma_{ab} = S_{ab} \cdot C_{ab}, \quad C_{ab} \simeq e^{-\frac{c}{2}\tau^2}, \quad c = 4.8834 \dots \text{ large } \tau$$

- Large buffer ([Dasgupta&Salam](#)):
branching in region **in** around p (or \bar{p})

$$\theta_{pq} < \theta_{\text{crit}}(\tau), \quad \theta_{\text{crit}} \simeq \theta_0 e^{-\frac{c}{2}\tau}$$

- Puzzle: connection to small x -physics (BFKL)!
[Mueller&GM,PLB575\(2003\)37 \[hep-ph/0308284\]](#)

Away-from-jet versus Kovchegov equation

- Away-from-jet energy flow $\Sigma(\tau, \vec{\theta})$ at small angle θ

$$\partial_\tau \Sigma(\tau, \vec{\theta}) = \int \frac{d^2 \theta'}{2\pi} \frac{\theta^2}{\theta'^2 (\vec{\theta} - \vec{\theta}')^2} \left[\Sigma(\vec{\theta}') \Sigma(\vec{\theta} - \vec{\theta}') - \Sigma(\vec{\theta}) \right]$$

$$\tau = \int_{E_{\text{out}}}^Q \frac{dq_t}{q_t} \frac{N_c \alpha_s(q_t)}{\pi}$$

- Kovchegov eq. for elastic S-matrix $S(\tau, \vec{b})$

$$\partial_\tau S(\tau, \vec{b}) = \int \frac{d^2 b'}{2\pi} \frac{b^2}{b'^2 (\vec{b} - \vec{b}')^2} \left[S(\vec{b}') S(\vec{b} - \vec{b}') - S(\vec{b}) \right]$$

$$\tau = \frac{N_c \alpha_s}{\pi} Y$$

Same evolution equation (formally)

- 1) $\vec{b} \Rightarrow \vec{\theta}$
- 2) fixed coupling \Rightarrow running coupling

Same behaviour at large τ

- 1) saturation in K equation

$$S(\tau, \vec{b}) \sim e^{-\frac{c}{2}\tau^2}, \quad \text{for } |\vec{b}| \ll Q_s^{-1}(\tau) \sim e^{-\frac{c}{2}\tau}$$

- 2) buffer in jet equation

$$\Sigma(\tau, \vec{\theta}) \sim e^{-\frac{c}{2}\tau^2}, \quad \text{for } |\vec{\theta}| \ll \theta_{\text{crit}}(\tau) \sim e^{-\frac{c}{2}\tau}$$

with $c = 4.88\dots$ determined by BFKL function $\chi(k)$

Small-x versus jet physics (without collinear logs)

Jet physics with only soft logs

- Soft-gluon **emission**: $a b \rightarrow a' b' + q_1, \dots, q_n$
- relevant kinematics:
 - ❖ θ_{q_i} of same order (hedgehog configuration)
 - ❖ q_{ti} (strongly) ordered
- coupling running in the ordered variable $\alpha_s(q_t)$
- evolution variable τ never too large
- relevant region θ finite

S-matrix basic object

- Soft-gluon **exchange**: $a b \rightarrow a' b' + q_1, \dots, q_n$
- relevant kinematics:
 - ❖ q_{ti} of same order
 - ❖ θ_{q_i} (strongly) ordered
- running coupling in frozen variable $\alpha_s(b^{-1}) \Rightarrow \alpha_s$ fixed
- evolution variable τ is large
- relevant region b small

Conclusion

There is a new type of (non-Abelian) soft logs

They enter observables with only soft singularities: a new QCD avenue

Soft emission at large angle are required (large N_c)

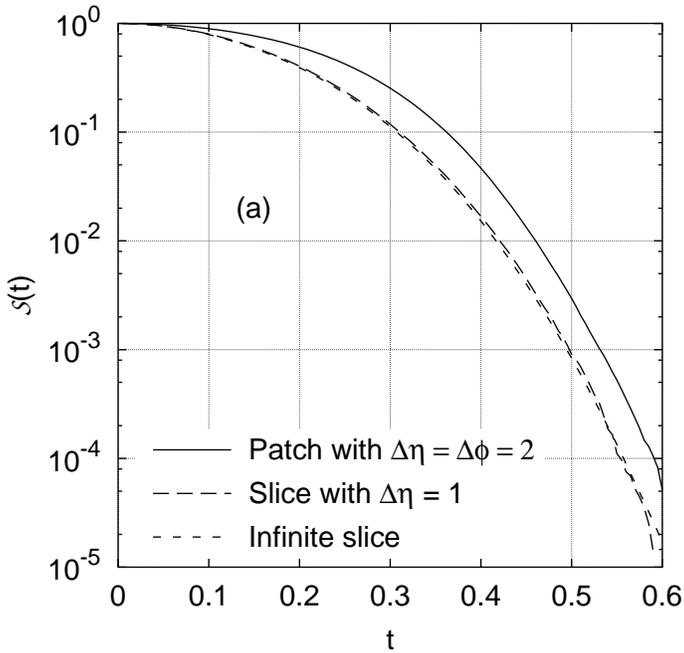
- $Q\bar{Q}$ -multiplicity
- Non-global observables
 - ❖ Serman-Weinberg distribution (energy in a cone)
 - ❖ rapidity cuts in hadron-hadron and DIS (LHC, Tevatron, Hera)
 - ❖ photon isolation
 - ❖ inter jet string/drag effects
 - ❖ profile of a separate jet (e.g. in DIS current hemisphere jet)

Non-Abelian soft logs resummed by **formally** similar small-x equations

The connection between the two equations is formal:

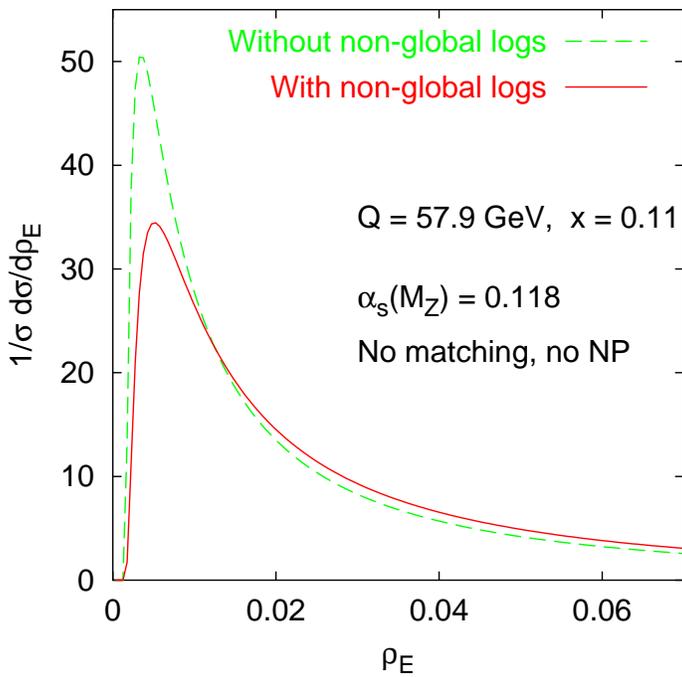
- same multi-soft gluon-distributions
- different relevant kinematical configurations
 - ❖ jet-physics: θ_i of emitted gluons of same order, k_{ti} ordered
 - ❖ small-x: k_{ti} of exchanged gluons of same order, θ_i ordered

Impact of non-global logs



Correlation function

$$C_{e^+e^-}(\tau) \text{ with } \tau = N_c t$$



Current-hemisphere
jet mass in DIS

M. Dasgupta, G. Salam, PLB512(2001)323, JHEP0203:017:2002