Chiral Phase Transition the Nambu-Jona-Lasinio model

F. GASTINEAU, E. BLANQUIER and J. AICHELIN

1. Motivation or what do we want to describe
2. The Nambu-Jona-Lasinio Model
3. Hadrons and cross sections within the NJL model
4. What happens close to T_c ?
5. Numerical results

Motivation (experimental)

The RHIC Au - Au experiments have shown

that

- the particle ratios are close to that expected from statistical model calculations
- radial velocities and v_2 values are large
- hydrodynamical models can describe even details ($v_2(p_t)$) of the reaction

↓

It seems that the system comes close to equilibrium

but

pQCD cross sections are small

all calculations for the entrance channel give $dE/dx \approx 1 GeV/fm$

too small to bring the system to equilibrium

How can these observations be reconciled?

Motivation (theoretical)

Smoking gun: We want to see direct information from the plasma

but

in the presently available approaches not possible

because they assume

sudden transition from high density (QGP) to low density (hadrons without inelastic collisions)

- Statistical models
  - global equilibrium of QGP until chemical freeze out
  - → all observables depend on μ_B and T only (not true: resonance production at RHIC
- Hydrodynamical models
  - local equilibrium
  - assume that all particles freeze out at the same surface with a given μ_B and T (Cooper Frye Formula) with the same collective velocity
  - → all particle ratios are fcts of μ_B and T only, spectra depend in addition on the collective velocity. Does not agree with RHIC ratios (QM04)
  - → the most one can get: an eq. of state; Sufficient?

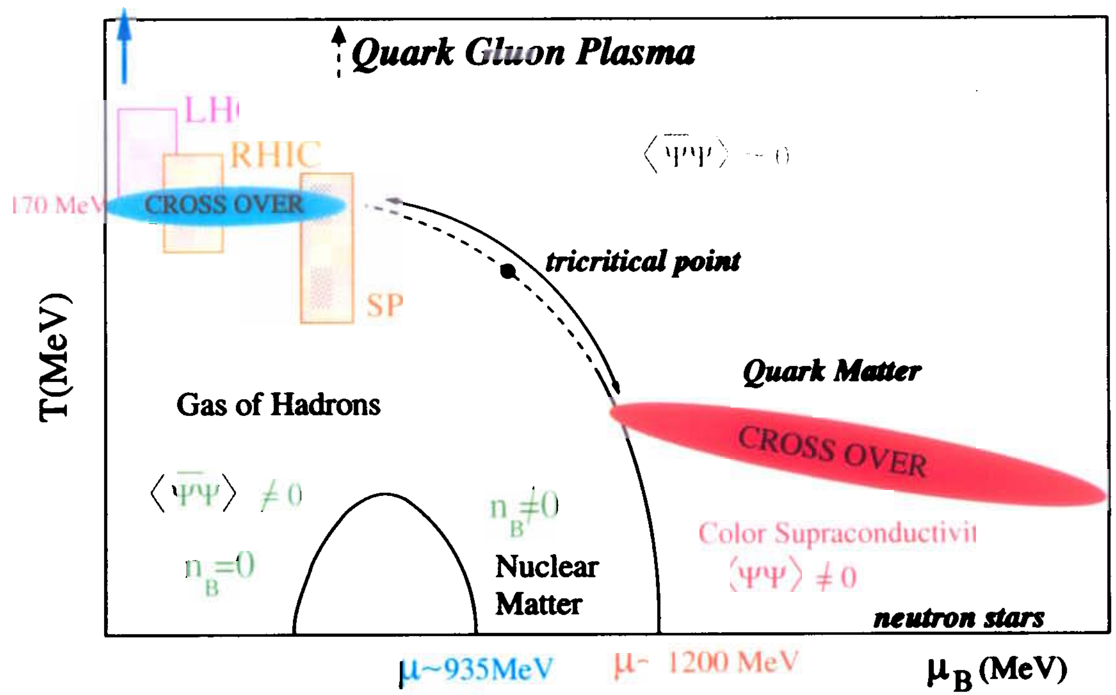
better approach to this phase transition is desirable but also very complicated

Phase diagram

Lattice calculation of QCD presently limited to static properties at $\mu_B = 0$

Therefore: phenomenological models are the only candidates

What every phenomenological model should at least reproduce:



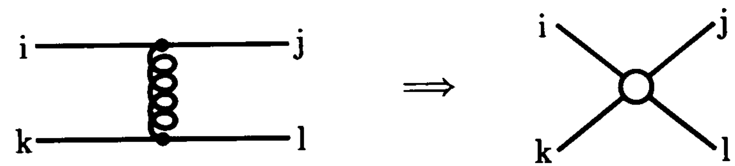
One of the models which does this: Nambu Jona-Lasinio (NJL)

A Good Candidate: NJl model

Can be directly obtained from QCD

$$\mathcal{L}_{QCD} \quad [(i\partial_\mu \quad T^a A_\mu \quad)\gamma^\mu \quad m_0]\psi \quad \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a}$$

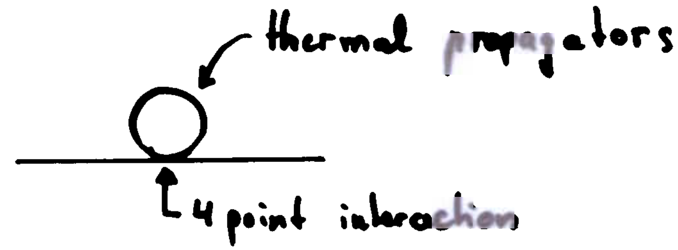
- Integration over gluonic degrees of freedom
- Point like interaction (No gluons !)
- Using a Fierz Transformation one obtains:



$$\mathcal{L}_{NJL} = \bar\psi(i\partial_\mu\gamma^\mu - m_0)\psi + G_S\left[(\bar\psi\lambda_F\psi)^2 + (\bar\psi i\gamma^5\lambda_F\psi)^2\right] \text{ (Singlet } \rightarrow \text{ mesons)}$$
$$-G_D\left[(\bar\psi^C\lambda_F^A\lambda_C^{A'}\psi)(\bar\psi\lambda_F^A\lambda_C^{A'}\psi^C)\right] \text{ (Octet diquarks baryons)} + U_A(1) \text{ breaking}$$

- ↗ Good description of the low energy physics (long history)
- ↗ Good description of meson and baryon properties
- ↘ Cut-off in momentum space (model non-renormalizable)
- ↘ Lack of confinement

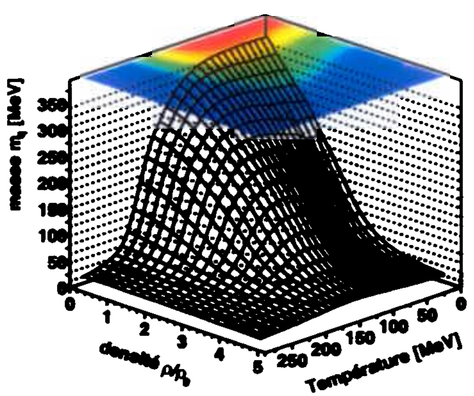
Quark Masses

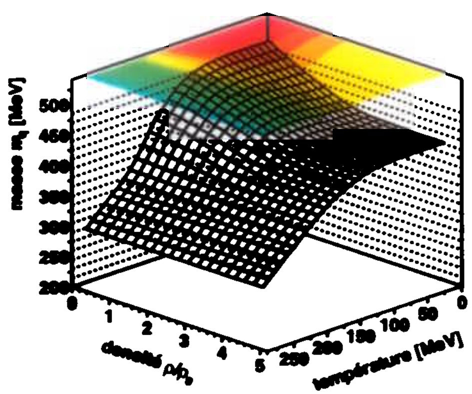


Whereas meson masses increase with μ and T the quark masses decrease from the curren
he cons ituent quark mass

In the mean field approximation:

- $\langle\langle \bar{q}q \rangle\rangle = \frac{-M_f N_c}{\pi^2} \int dp \frac{p^2}{E_p} \left[1 - f(E+\mu,T) - f(E-\mu,T)\right]$
- $M_i = m_{0,i} - 4G_S \langle\langle \bar{i}i \rangle\rangle + 2G_D \langle\langle \bar{k}k \rangle\rangle \langle\langle \bar{j}j \rangle\rangle$ with $i \neq j \neq k$
- phase transition order : 1, 2, none ?





- Left : Light quark mass $M_Q(T, \rho/\rho_0)$
- Right : Strange quark mass $M_S(T, \rho/\rho_0)$

Hadrons within the NJL model

Why should we studied hadron: n depending on the quark condensat the hadron carry nformation on QCD at inite temperatur and density.

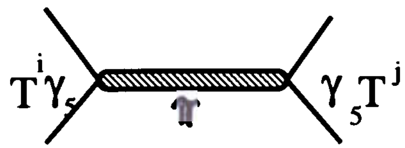
- ✘ Hadrons bound states of quarks and/or antiquarks.
- ✘ Mesons have been since long successfully described as bound states of a $q\bar{q}$ pairs (cf Lutz, Klevansky, Rehberg,....)
- ✘ Baryons can be considered as diquark-quark bound states in the extended NJL.
- ✘ Only preliminary studies (zero or finite T) : Ishii,Tjong, Bentz, Reinhardt,...

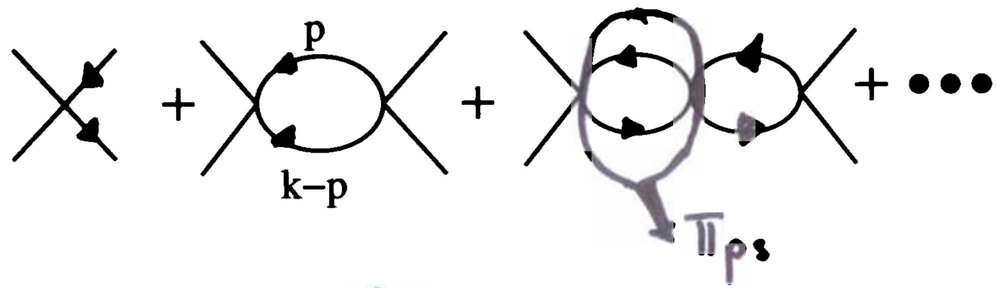
NJL may provide

- ❑ Mass of hadrons at any T and μ.
- ❑ Coupling constants between hadrons and his constituents.

How to get Mesons

We use the equivalence between a quark-antiquark scattering by mesons exchange and the scattering in a RPA approach.





$$iM^{ij}_{PS} = i\gamma_5 T^i \frac{g^2_{\pi q\bar{q}}}{k^2 - m^2_\pi} i\gamma_5 T^j$$

$$iU^{ij}_{PS} = i\gamma_5 T^i \frac{g^2}{1-g^2\Pi^{q\bar{q}}_{PS}} i\gamma_5 T^j$$

with

$$\Pi^{q\bar{q}}_{PS}(k) = \int \frac{d^4p}{(2\pi)^4} Tr \quad i\gamma_5 T^i S_F(p) i\gamma_5 T^j S_F(k-p)$$

Identifying the two poles $iM^{ij}_{PS} = iU^{ij}_{PS}$ one finds for m_π

$$1 \quad g^2\Pi^{q\bar{q}}_{PS}(k^2 = m^2_\pi) \quad 0$$

Taylor expansion around the pole

$$iU^{ij}_{PS} = i\gamma_5 T^i \underbrace{[\frac{\partial \Pi^{q\bar{q}}_{PS}}{\partial k^2}]^{-1}_{(k^2=m_\pi^2)}}_{g^2_{\pi q\bar{q}}} \frac{-i}{k^2 - m_\pi^2} i\gamma_5 T^j$$

gives the $g^2_{\pi q\bar{q}}$ coupling constant.

At finite density and finite temperature one uses thermal propagators

$$m_\pi = m_\pi(T, \mu)$$

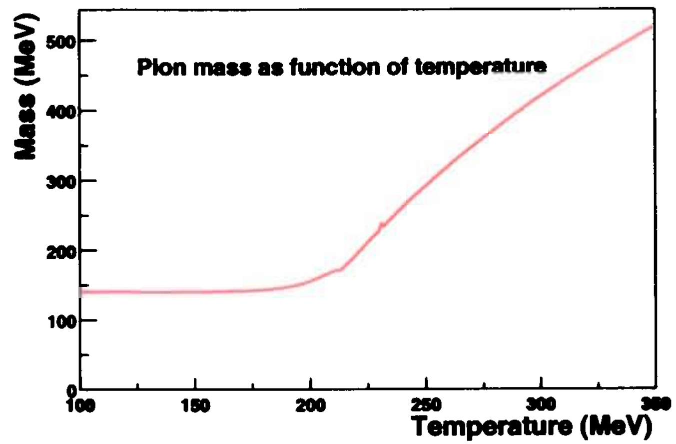
At high densities and/or temperatures

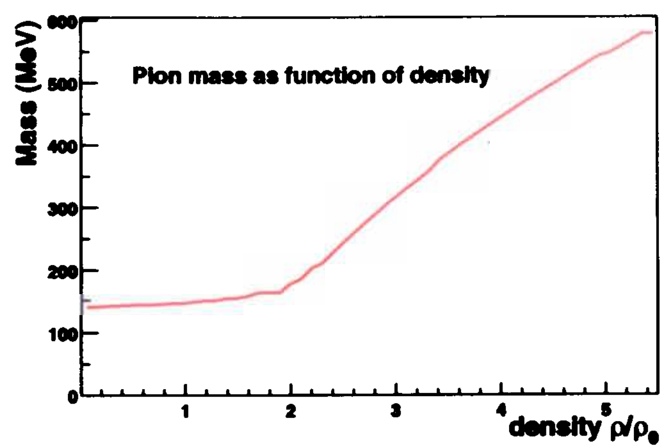
$$m_\pi \rightarrow \mathcal{R}e\ m_\pi + \frac{1}{2} i\Gamma_\pi$$

pion becomes unstable and decays into $q\bar{q}$
Mesons are bound $q\bar{q}$ states obtained as pole of the Bethe-Salpeter equation (which equals the RPA for point interactions)

In medium masses of hadrons

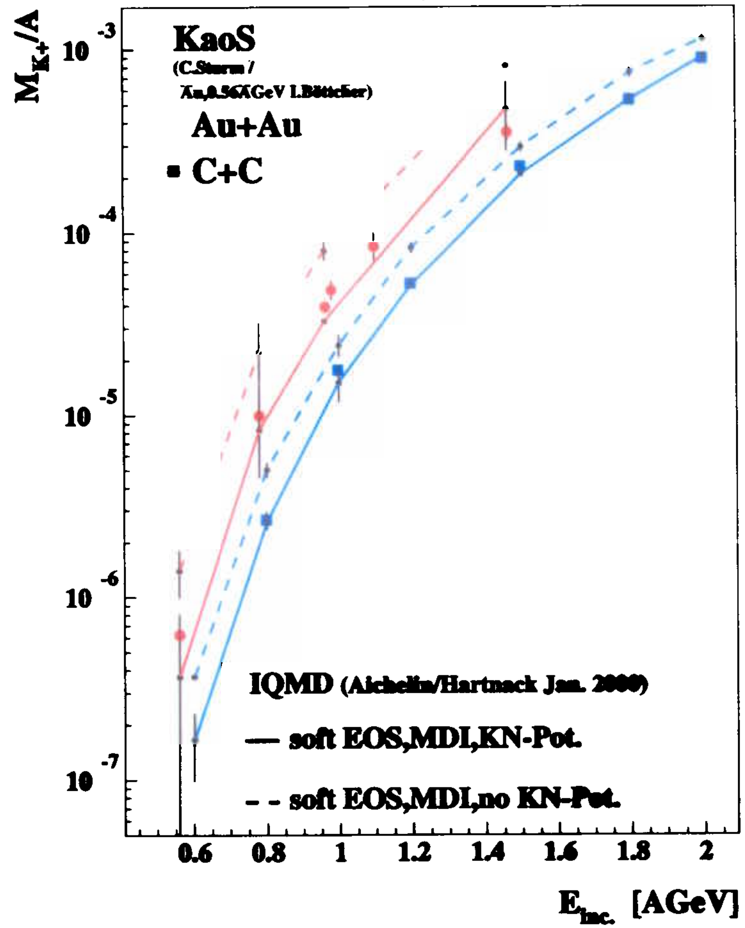
Meson masses depend on T and the baryonic density ($\doteq \mu_B$





As a Goldstone boson π is protected against any change of m_q as long as the chiral symmetry is broken.

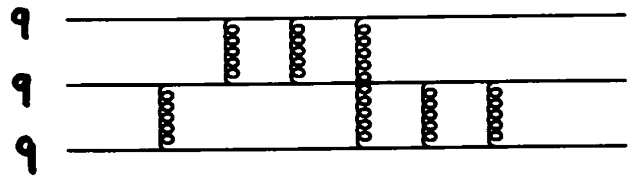
Observable in heavy ion reactions



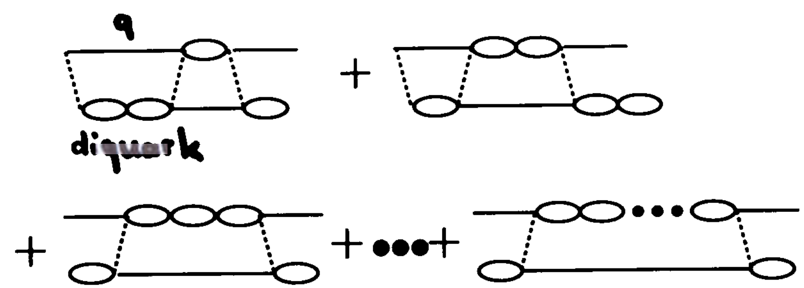
Due to the increase of m_{K^+} the K^+ yield decreases

Baryons as diquark-quark bound states

- Baryons are made of 3 quarks and appear as bound states of the quark - diquark interaction
- Starting from the basic picture (quarks interacting by gluon exchange) we introduce NJL point like interactions
  → Faddeev equation reduced to a quark-diquark Bethe-Salpeter equation
  For point like interactions it can be summed up
  Pole gives the mass of the baryon
- Previous work done by : Tjon[94] , Ishii[94], Reinhardt[92], Hanhart[95],...



Faddeev equation

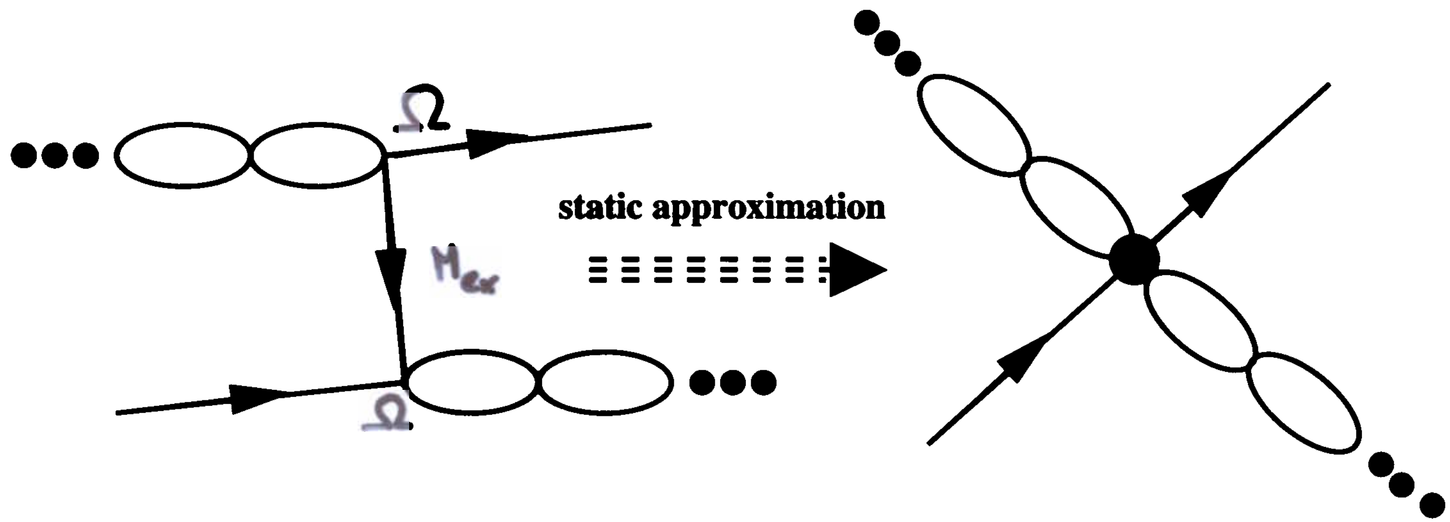


quark-diquark interaction

- Diquark-quark interaction vertex ?

Quark-Diquark vertex

- exchange of one quark



- STATIC APPROXIMATION (S.A.) : exchanged quark mass is heavy

$$Z_{bc}^{\beta\gamma} = g'_{Dqq'} \Omega_a^{\beta\delta} \left(-\frac{i}{M_{ex}} \right) \bar{\Omega}_b^{\gamma\alpha} g_{Dqq'}$$

- No momentum dependence → RPA summation possible
- Validity of the S.A.
  - Good for light quarks before the chiral restoration, for heavy quarks always.
  - Results more questionable when chir. Sym. is restored (but gluons have screening masses !)

Results at $T = 0$ and $\mu_q = 0$

quarks and mesons

Particle	q	s	π	K
Theory (MeV)	422	627	137	550
Experience (MeV)	?	?	137	495

diquarks

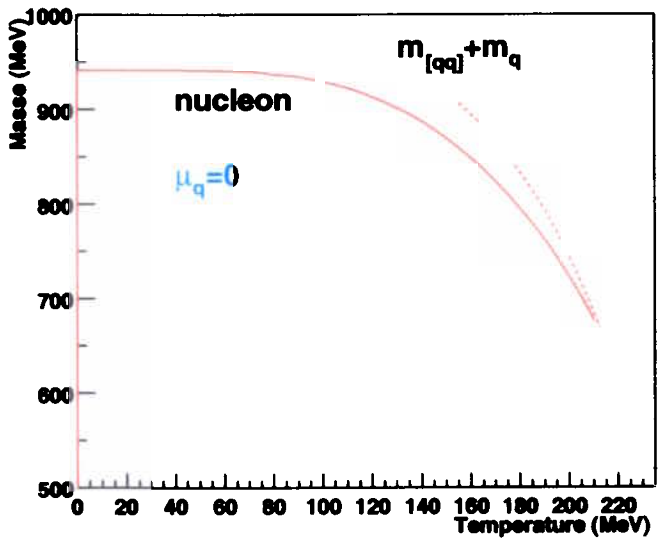
Particle	qq	qs
Theory (MeV)	557	759
Binding energy (MeV)	287	290

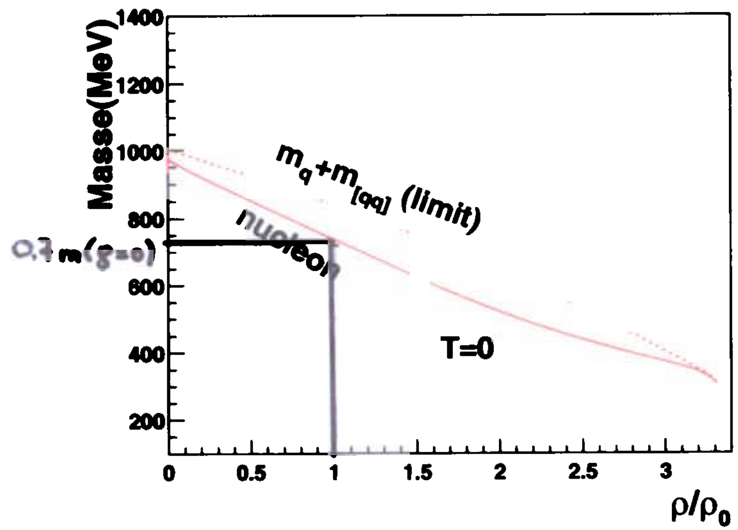
Baryons

Particle	N	Λ	Σ	Ξ
Theory (MeV)	938	[illegible]8	182	1276
Experience (MeV)	938	1116	1178	1315
Difference (%)	0	5	0.3	3
Binding energy (MeV)	38	13	0.	110

- ✔ Mass spectrum well described, all particles are stable in vacuum
- ✔ Need a high strange quark mass → **Kaons too heavy**

Nucleon





	$T_C(MeV)$	$M(T_C)(MeV)$	%
Nucleon	210	650	30

✔ Nucleons properties in good agreement with classical models

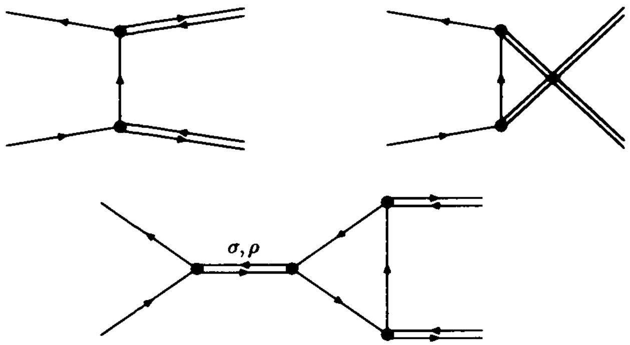
Parameters

This is all obtained by fixing 7 parameters to physical values.

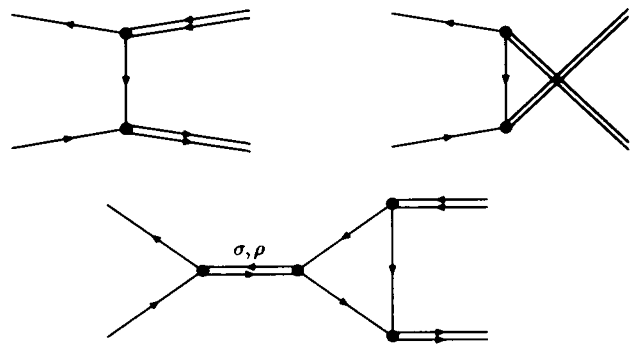
$m_0^{u,d}$	light quarks mass	m_π	pion mass
m_0^s	strange quark mass	m_K	kaon mass
G_D	U(1) breaking	m_η m_η	η, η' splitting
G_S	scalar interaction	$\langle \bar{q}q \rangle$	vacuum quark condensate
(G_V	vector interaction	m_ρ	ρ mass)
G_{DIQ}	quark-quark interaction	$m_{nucleon}$	nucleon mass
Λ	cut-off	f_π	pion decay constant

Hadronization Cross Sections

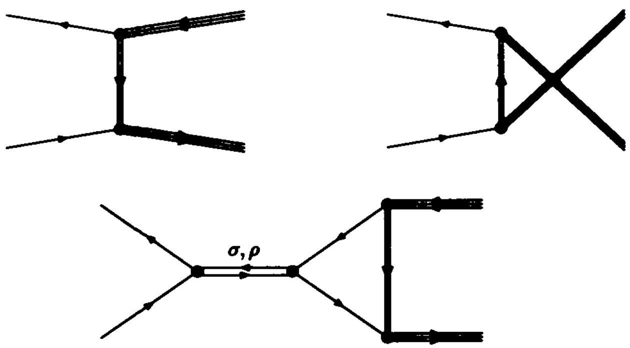
obtained n the NJL model by $N_{\ldots}$ xpansion of the Lagrangian (Heidelberg group)



$q\bar{q} \to MM$ channel



$q\bar{q} \to D\bar{D}$ channel



$q\bar{q} \to B\bar{B}$ channel

NJL Lagrangian contains

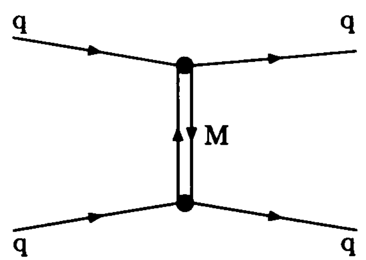
- ❒ Meson, Diquark, Baryon production

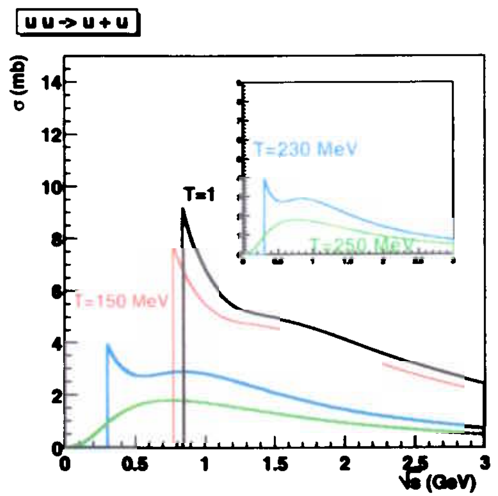
most of the σ's on the order c mb

in addition: diquark+q $\to$ baryon+ meson .....

Elastic Cross Sections

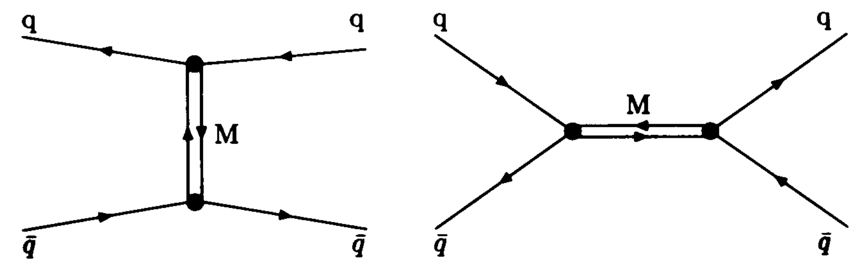
$qq \to qq$: has t and u channel

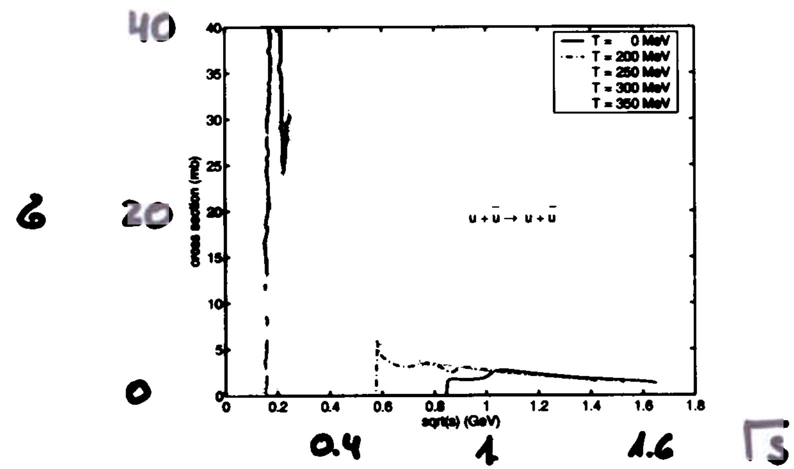




$\sigma(qq \to qq) \approx 5mb$ close to T_c

$q\bar{q} \to q\bar{q}$: has s and t channel





$\sigma(q\bar{q} \to q\bar{q}) \approx 50mb$ close to T_c

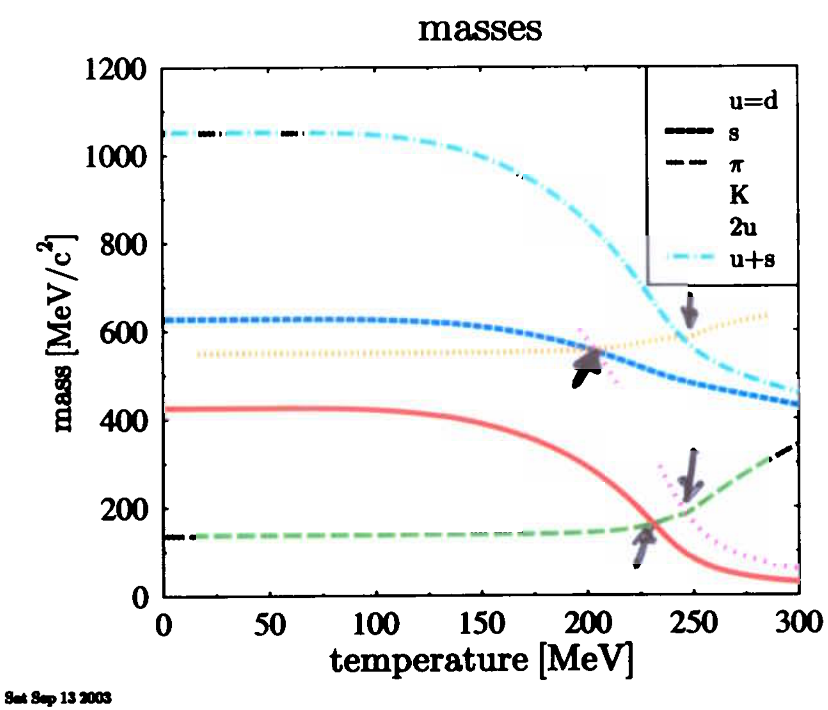
due to the resonan hannel

close to T_c the system is more liquid han plasma

ast equilibration and radial flow

Large Cross Sections I

Two relevant temperatures



T_{Mott} : phase transition

mass of meson = mass of constituents

- $m_K = m_s + m_q$
- $m_\pi = 2m_q$

$T_{exo/endo}$:

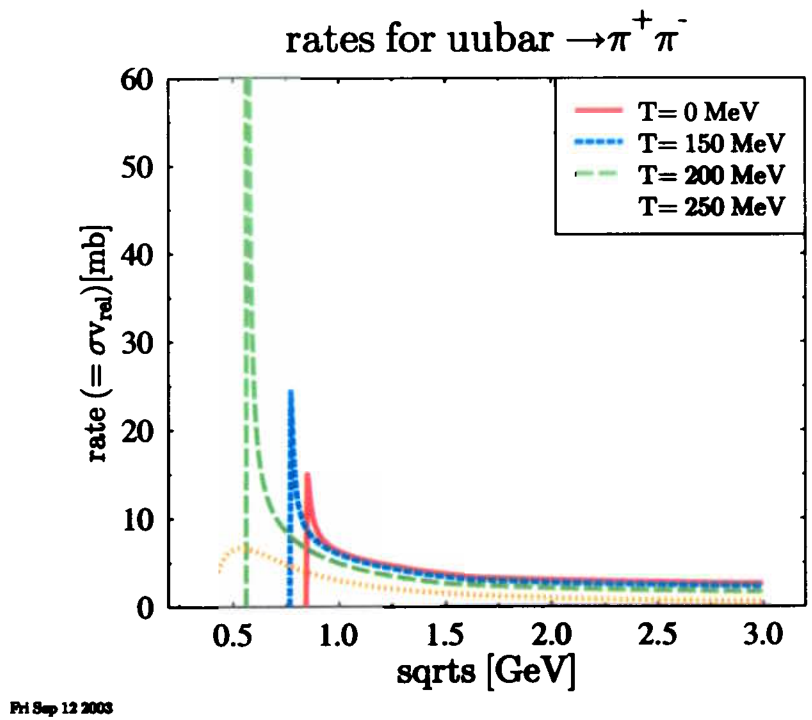
$T < T_{exo/endo}$ meson production exothermic

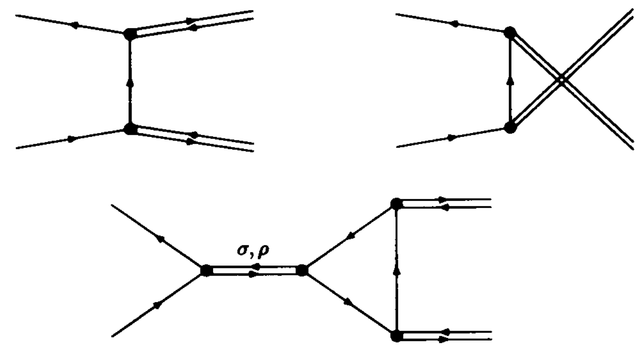
- $m_K = m_s$
  production: $(s\bar{s} \to KK)$
- $m_\pi = m_q$
  production: $(u\bar{u} \to \pi\pi)$

$T_{exo/endo} < T_{Mott}$

Large Cross Sections II

channe rate fo $q\bar{q} \to MM$ becomes large close to :inemati threshold





$q\bar{q} \to MM$ channel

$\sqrt{s} >> \sqrt{s_{thres}}$:

- t+u channel dominate
- s channel $\propto m_q$

$\sqrt{s} \approx \sqrt{s_{thres}}$:

s-channel becomes resonan

$T^s(\sqrt{s}, T) \propto \frac{1}{1-2G\Pi_\sigma(\sqrt{s},T)}$

shows strong enhancement around

$T = T_{Mott}$

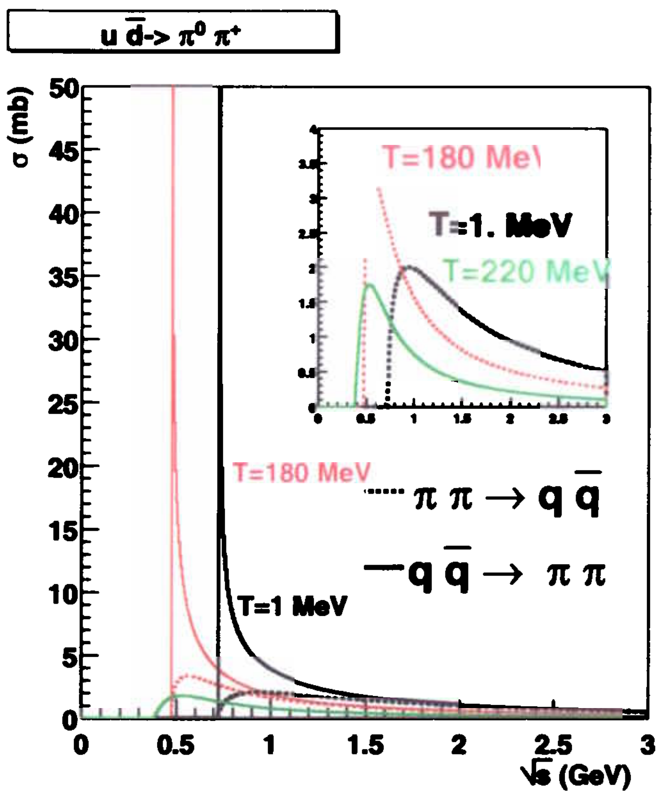
Large Cross Sections III

consequences of detailed balance

$$\sigma(\pi\pi \to q\bar{q})(\sqrt{s},T) = \frac{s-4m_q^2}{s-4m_\pi^2}\sigma(q\bar{q} \to \pi\pi)(\sqrt{s},T)$$

favors

- for $T > T_{exo/endo}$ $q\bar{q}$ production,
- for $T < T_{exo/endo}$ $\pi\pi$ production,



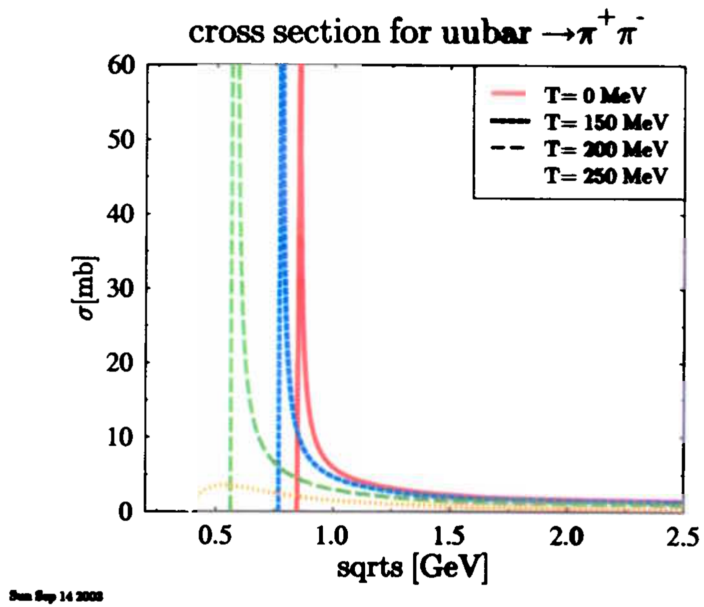
Resonant matrix element is large for a wide kinematic range

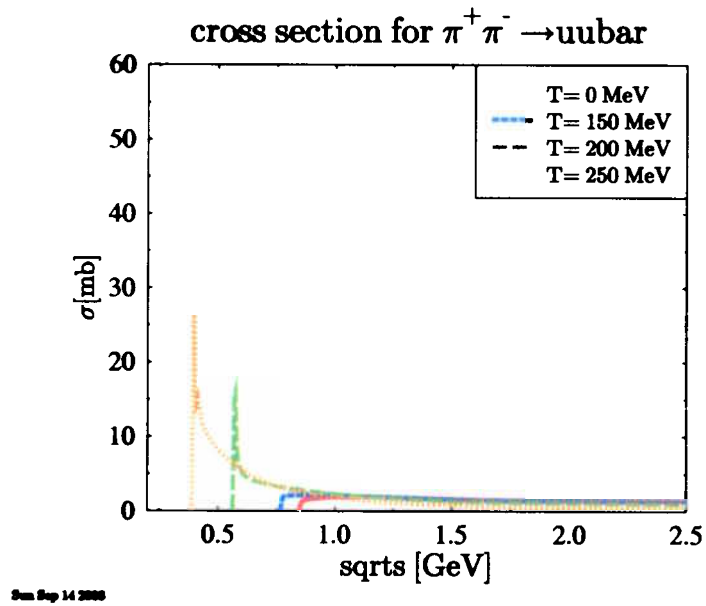
→ σ's large at threshold even for $T < T_{exo/endo}$

→ strong pion production → quarks disappear into hadrons

Expansion of the plasma I

goes from high to low temperatures !!





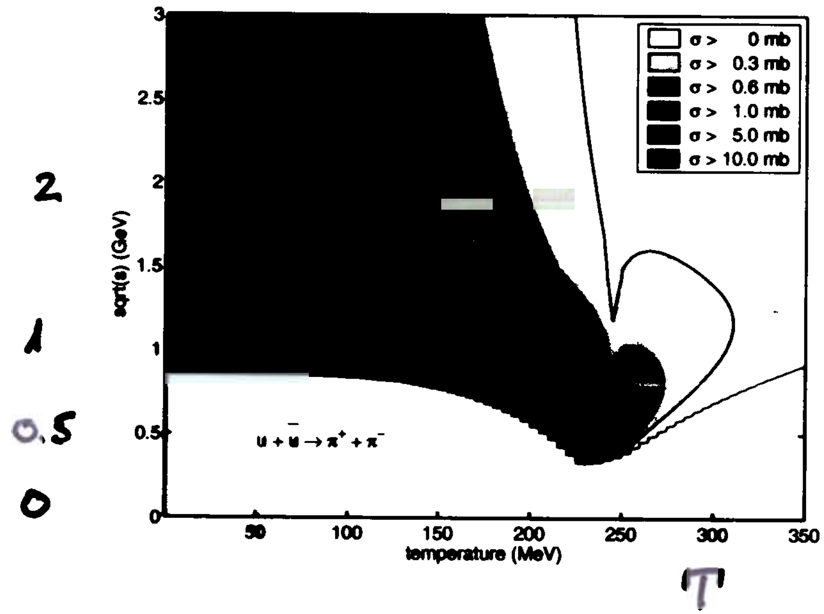
$T > T_{Mott}$: π's rarely created and immediately destroyed

$T \approx T_{Mott}$: $\sigma(q\bar{q} \to \pi\pi) > \sigma(\pi\pi \to q\bar{q})$, π stable

$T < T_{Mott}$: π production almost divergent at threshold but threshold changes

Expansion of the plasma II

The cross sectio is larg over a wide kinematic range



This allows almost all quarks to finc a hadronizatioı partne:

↓

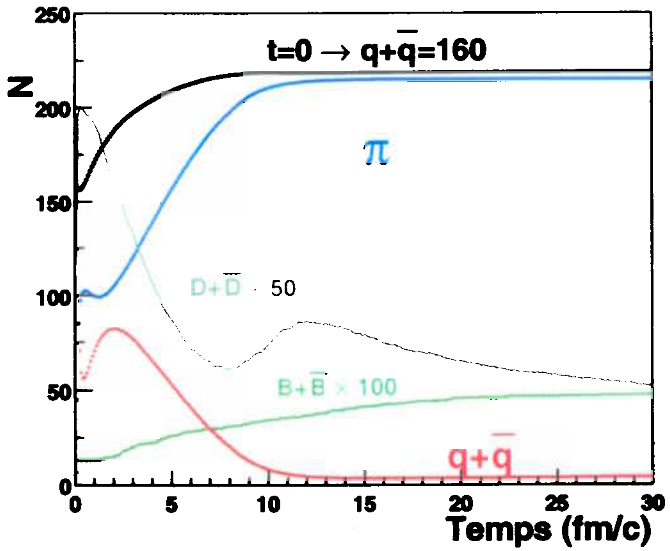
disappearance of the quarks during the expansion

"kinemati confinement possible

Expansion of a $q\bar{q}$ plasma III

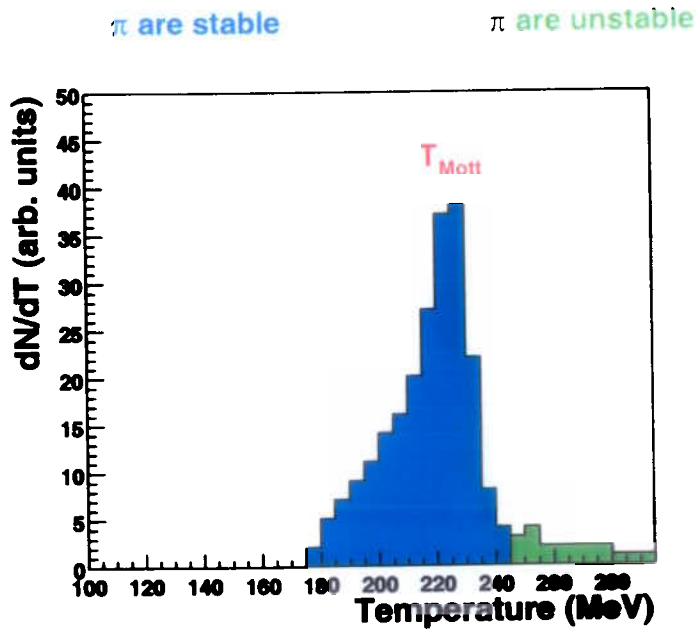
- Define the initial condition:
  - a number of quarks, anti-quarks and (unstable) π
  - a size $\rightarrow$ scalar density ($r \simeq 2.5 fm$)
  - a temperature $\approx 260 MeV$
  - a chemical potential (baryonic density)
- Let the system expand.

Evolution

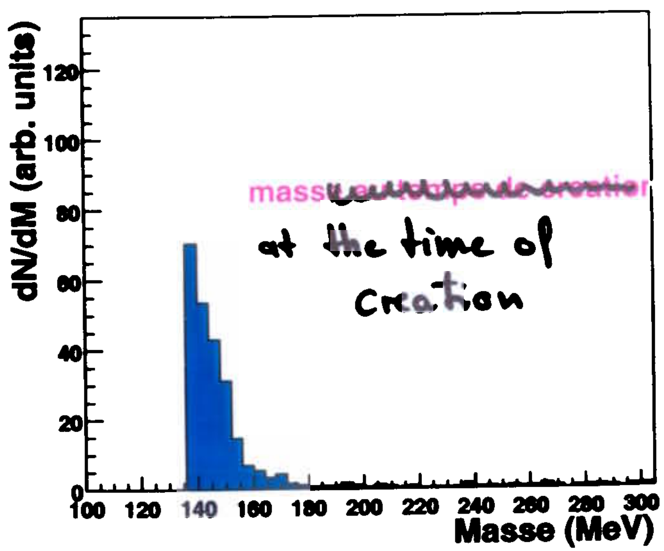


- ✔ Number of free quarks first goes up due to pion decay ($\pi \rightarrow q\bar{q}$)
- ✔ Later number of pions increases, number of quarks decreases
- ✔ Number of free quarks should be zero at the end
  This is (almost) the case due to kinematic confinement.
- ✔ Number of baryons goes up, because no suppression is implemented!

Creation of pions

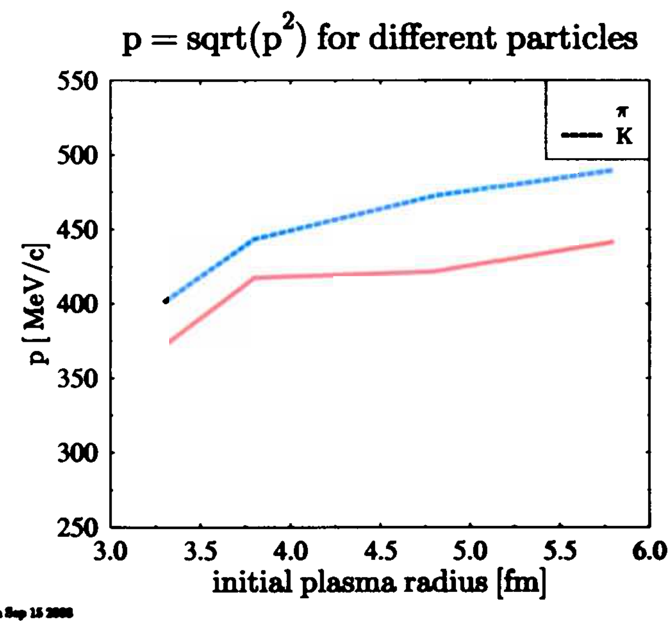


- ✔ Initial Temperature ≈ 260 MeV
- ✔ System in equilibrium : ll π s createc ıt T_{Mott}(≈ 240 MeV
- ✔ System in expansion: Most of π's are created below T Mot
- ✔ Same is true for Kaons



- ✔ Most of the pions are created with a mass 139 MeV
- ✔ At the end : all pions have their vacuum mass (139 MeV)

Observables



plasma with initially 40 u,d,s,$\bar{u}$, $\bar{d}$, $\bar{s}$ each

same total energy

- Due to their different production kinematics: different hadrons have a different $< p >= \sqrt{p^2}$
- The momentum increases with the mass of the particle species -as seen in experiment

Conclusions

- NJL provides a framework to udy the chiral phas transition
- For $T > T_{Mott}$ quarks are the degrees of freedom, for $T < T_{Mott}$ mesons and baryons are the effective degrees of freedom
- Meson and baryon properties at T= 0 =ρ well reproduced → allows for connection with experiment. Meson and baryon properties at finite T and ρ agree with the results of other approaches (ChPT, Walecka...)
- Description of dynamical chiral phase transition possible
- Effective confinement (although not in the Lagrangian) due to s- channel resonance of $\sigma(q\bar{q} \to q\bar{q}, \pi\pi, KK, p\bar{p}) \to$ large cross section close to T_c.
- Results show that the transition from the plasma to the hadrons happens by surface emission of the hadrons
- In this model pions, kaons and baryons carry information on the plasma
- Approach shows how mall pQCD cross sections (dE/dx $\approx$ 1GeV/fm and the observed thermalisatio (RHIC) can be reconciled