
Fermilab's
Primordial
SOUP

Cosmology for Particle Physicists

CERN Academic Lectures, May 2005

Sean Carroll, University of Chicago

Agenda (5 lectures):

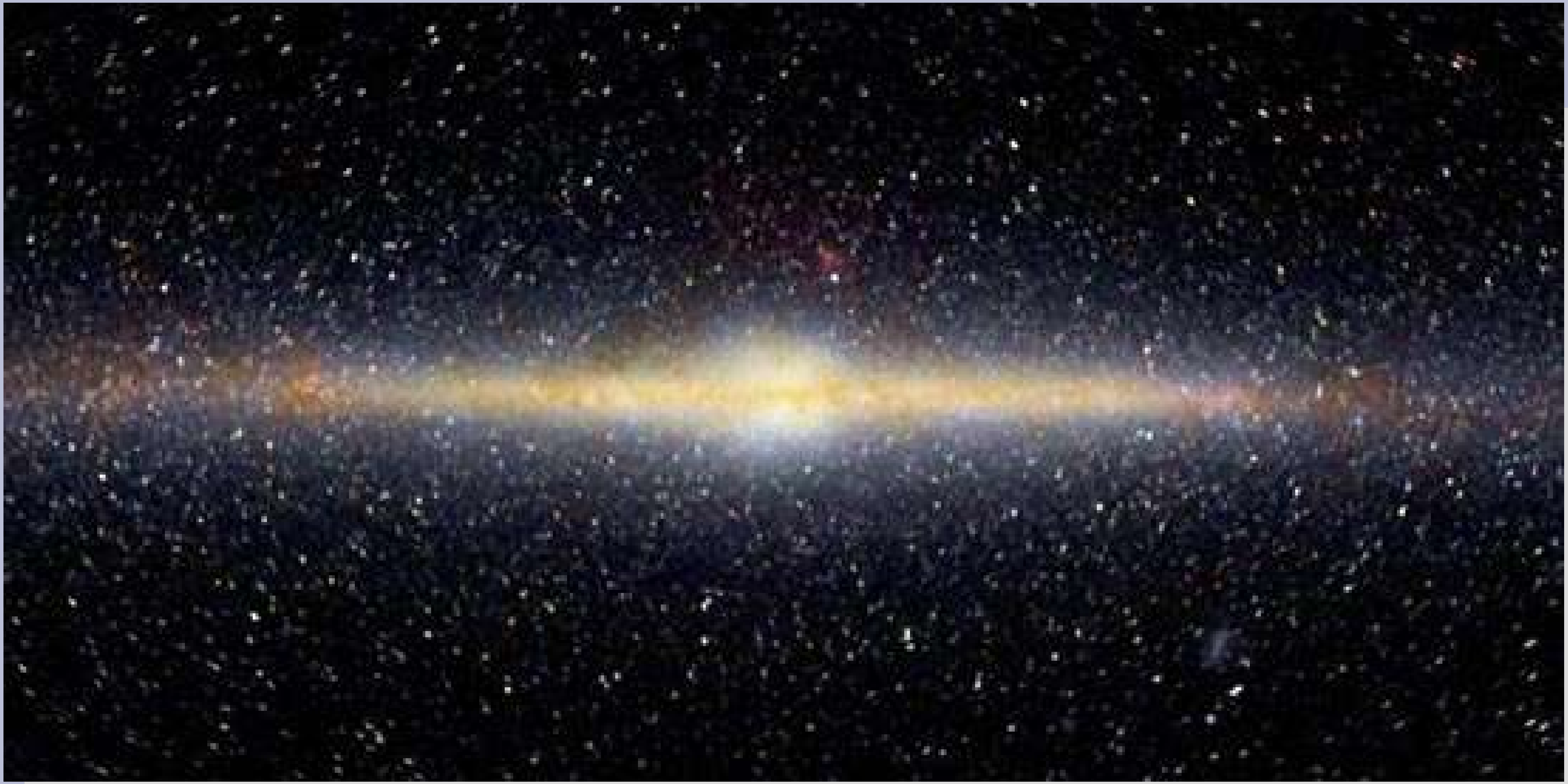
- Intro: Our Homogeneous and Isotropic Universe
- Ingredients: Dark Matter and Dark Energy
- Thermodynamics in the Early Universe
- Perturbations and Cosmic Structure
- Inflation and Beyond

Best way to think about the universe: look at it (or at least think about looking at it)

The Milky Way has fascinated astronomers since ancient times.

We now know that it's a collection of stars, similar to the Sun.

A better view (infrared, from space):



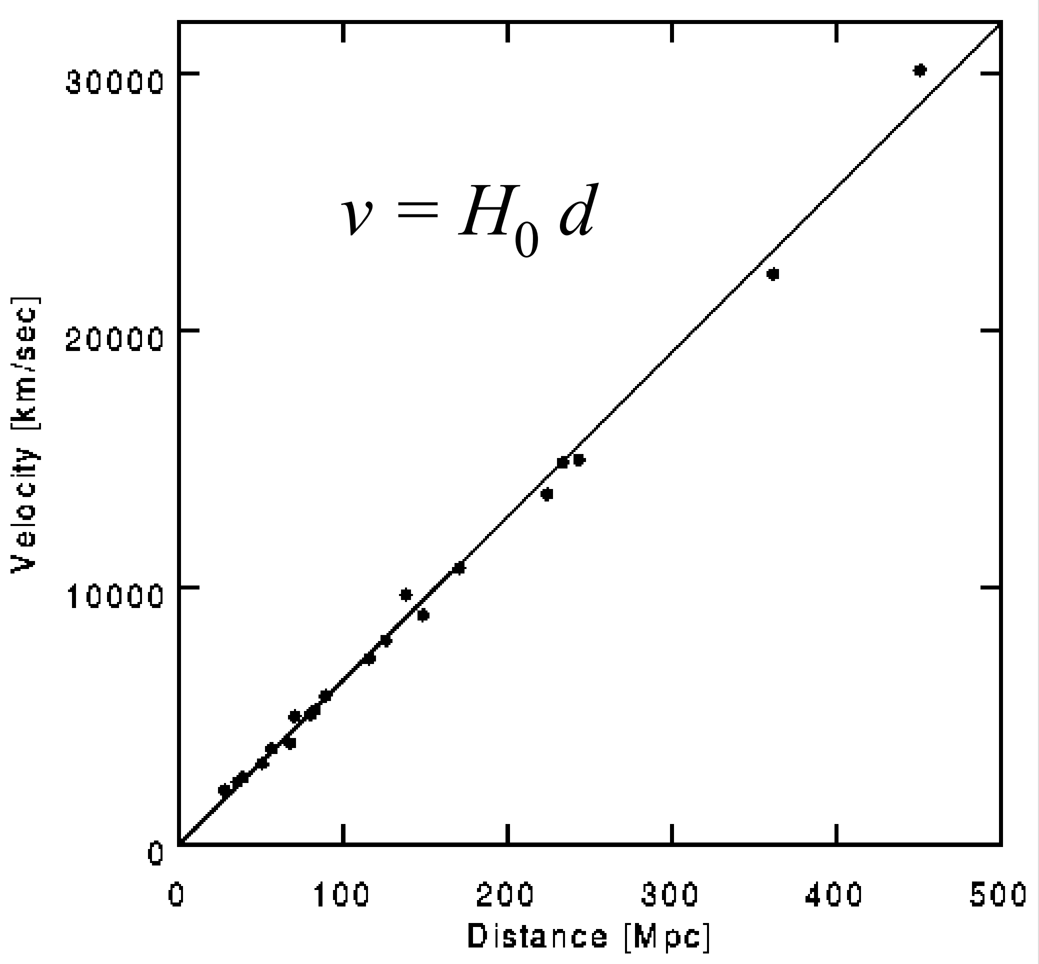
The Milky Way has about 100 billion stars (100 G★).

Looking more deeply, we see the universe is filled with galaxies.

1924: Edwin Hubble shows that each galaxy is a collection of stars, just like the Milky Way.

But don't get complacent: the universe is expanding.

1929: Hubble again, this time showing that the further away a galaxy is, the more rapidly it is moving away from us.



Modern version of Hubble's diagram.

After years of acrimonious debate, we've finally settled on a measured value of the Hubble constant:

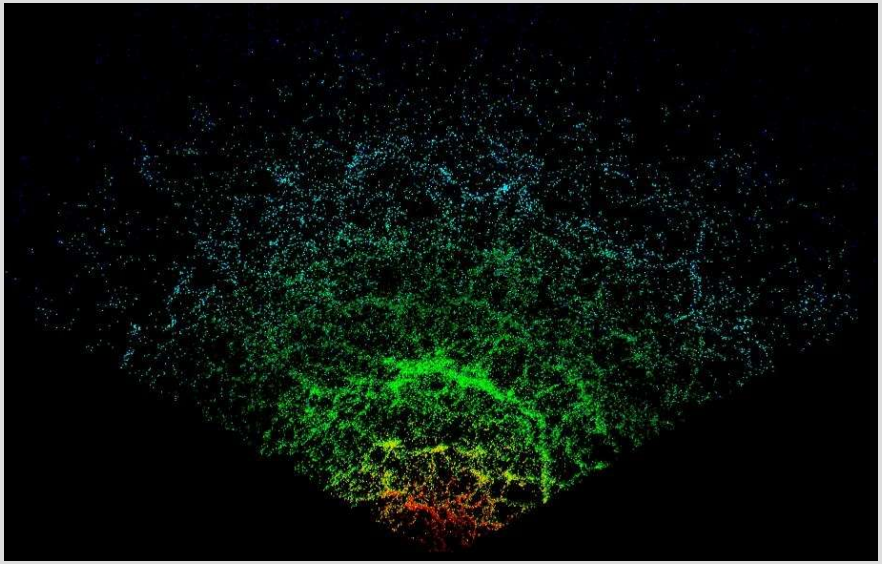
$$
\begin{aligned}
H_0 &= 72\ \text{km/sec/Mpc} \\
&\approx (10^{10}\ \text{years})^{-1} \\
&\approx (10^{18}\ \text{sec})^{-1} \\
&\approx (10^{28}\ \text{cm})^{-1} \qquad (c = 1) \\
&\approx 10^{-33}\ \text{eV}
\end{aligned}
$$

H_0^{-1} is the "Hubble time," and cH_0^{-1} is the "Hubble radius," characteristic scales for the universe.

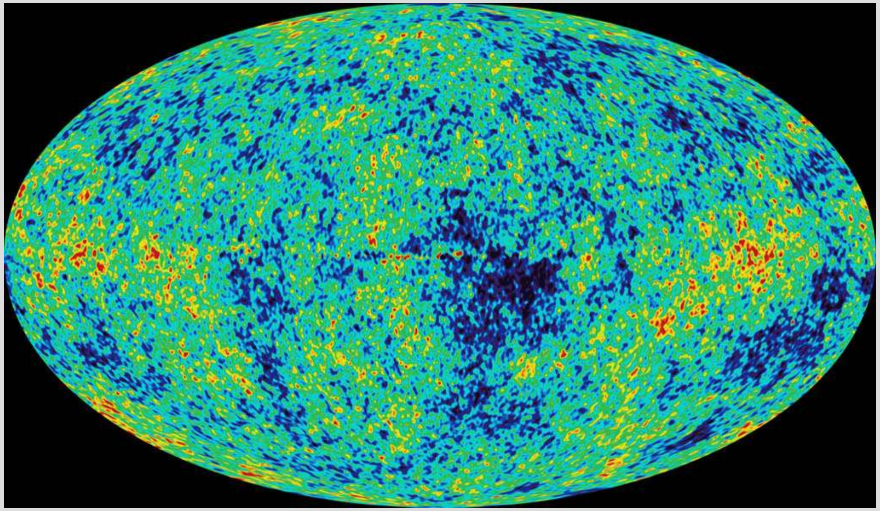
By the standards of particle physics, the universe is a very low-energy place.

Why cosmology is easy: the universe looks pretty much the same everywhere (homogeneous and isotropic)

Galaxy distribution: smooth on sufficiently large scales

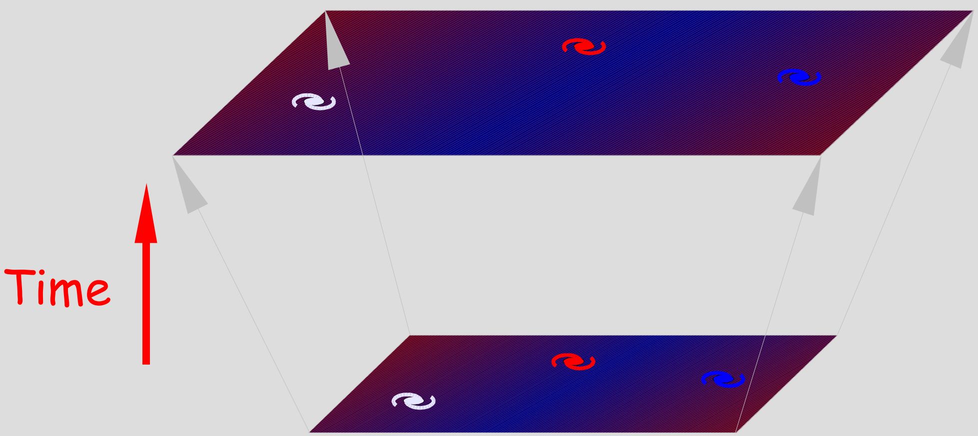


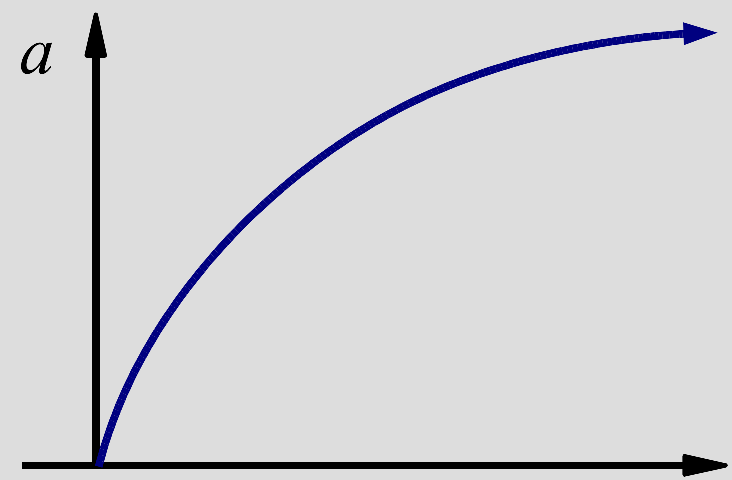
Background radiation: isotropic to one part in 100,000



Thus: a universe that is uniform (on large scales) over all of space, but expanding as a function of time.

Relative size of the universe is measured by the scale factor, $a(t)$. The Hubble parameter is $\dot{a}/a$, and the redshift of a distant object is $z = a^{-1} - 1$.

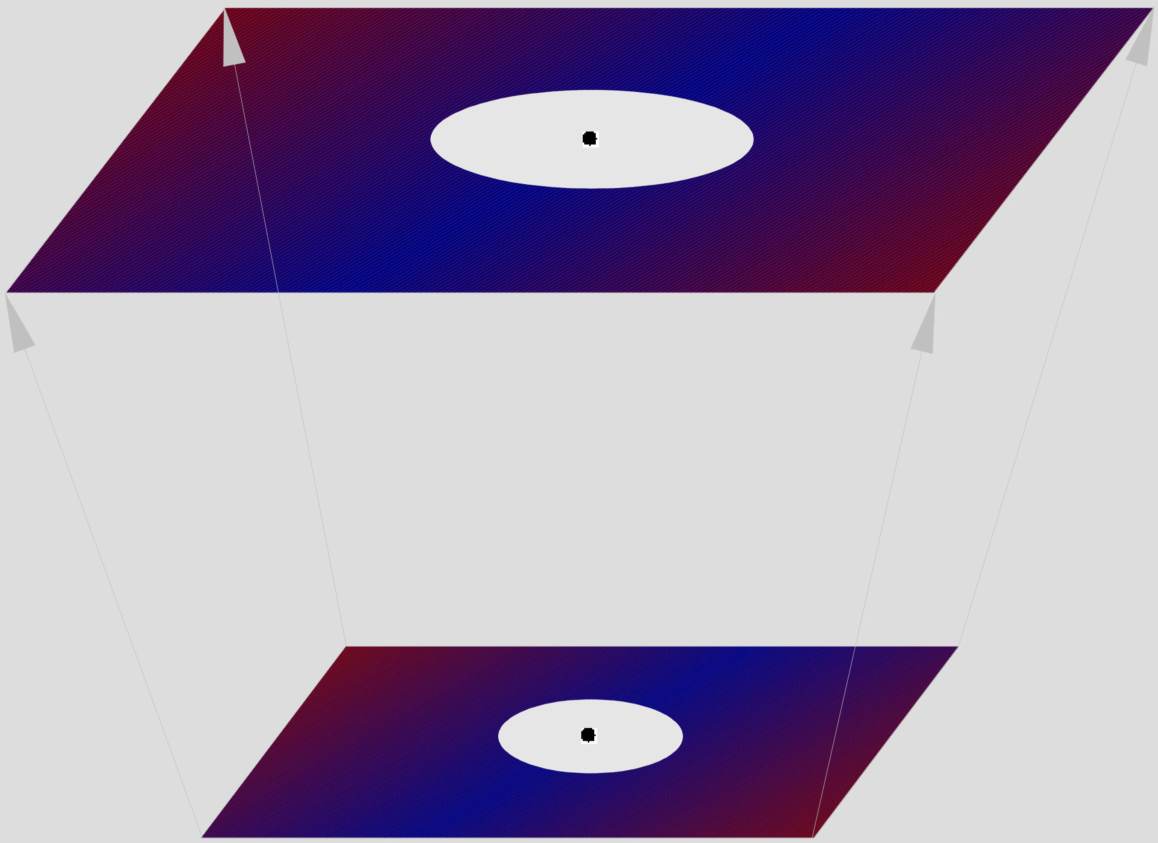




Note: since $v = H_0 d$, and d can be anything, we can get $v > c$ no problem.

Crucial fact about expansion: bound objects (or anything that has broken away from the background) do not expand along with the universe.

Consider taking a spherical region and concentrating all its mass in the center. The boundary will expand, but the metric inside will be exactly the Schwarzschild metric, without any trace of expansion. (Gauss's Law!)

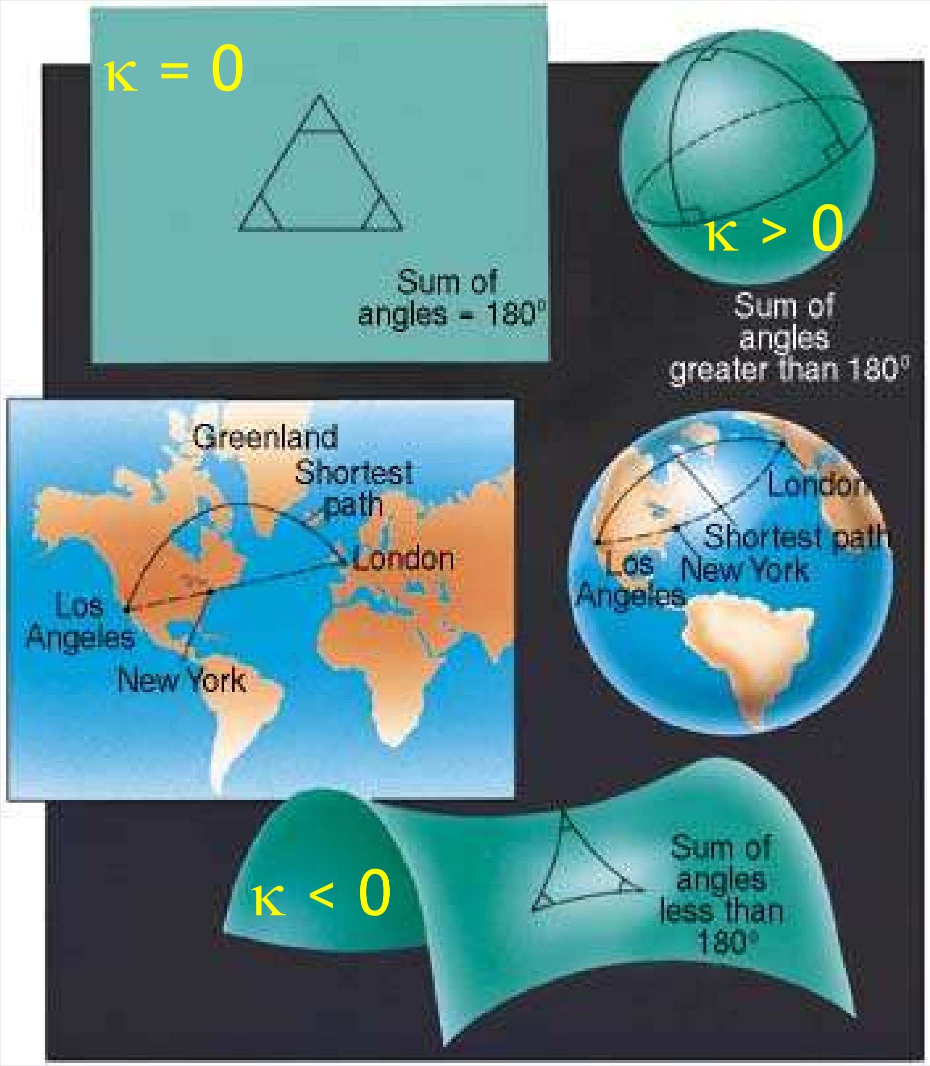


A smooth universe is described by a finite number of parameters (same at every point in space).

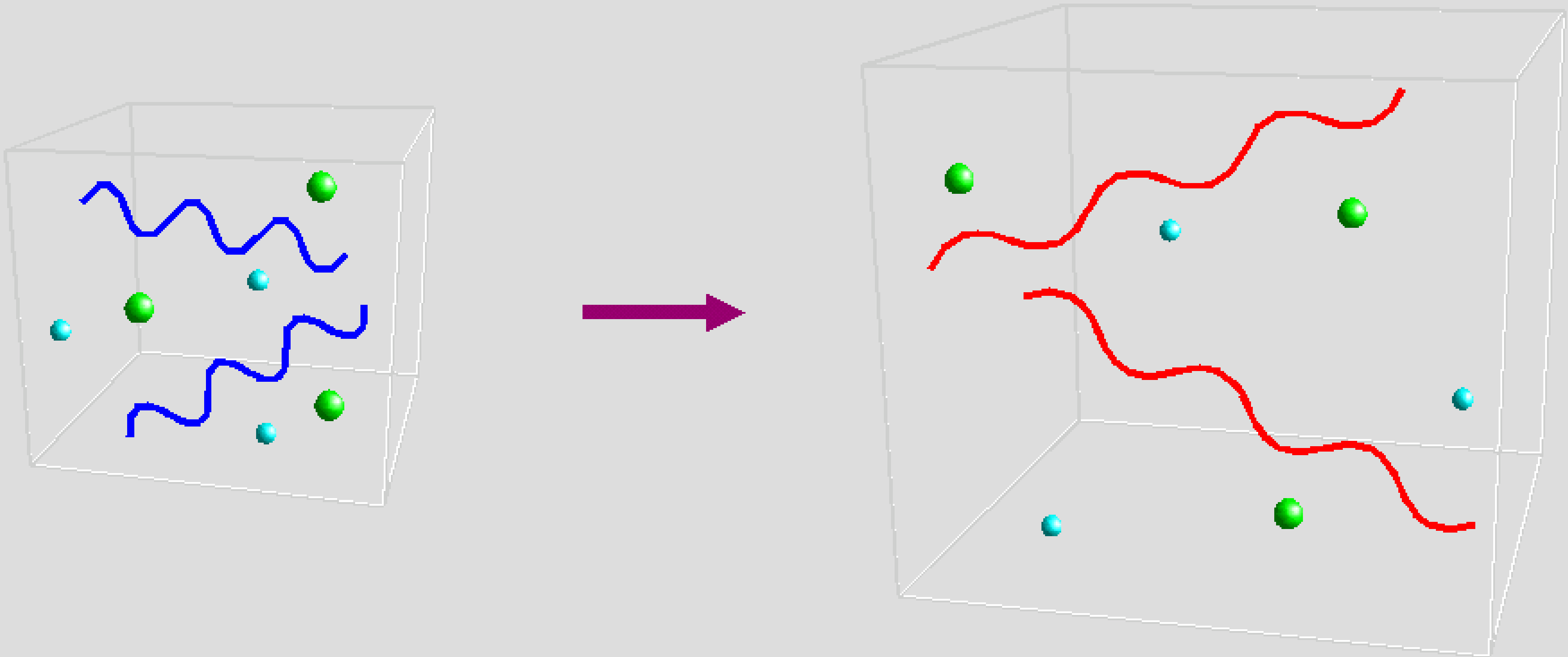
These are:

- Expansion rate: $H = \dot{a}/a$.
- Energy density in various forms, ρ_i. (e.g. matter, radiation, vacuum)
- Spatial curvature κ.

"Robertson-Walker universes"



Expansion dilutes matter (cold particles) and redshifts radiation.



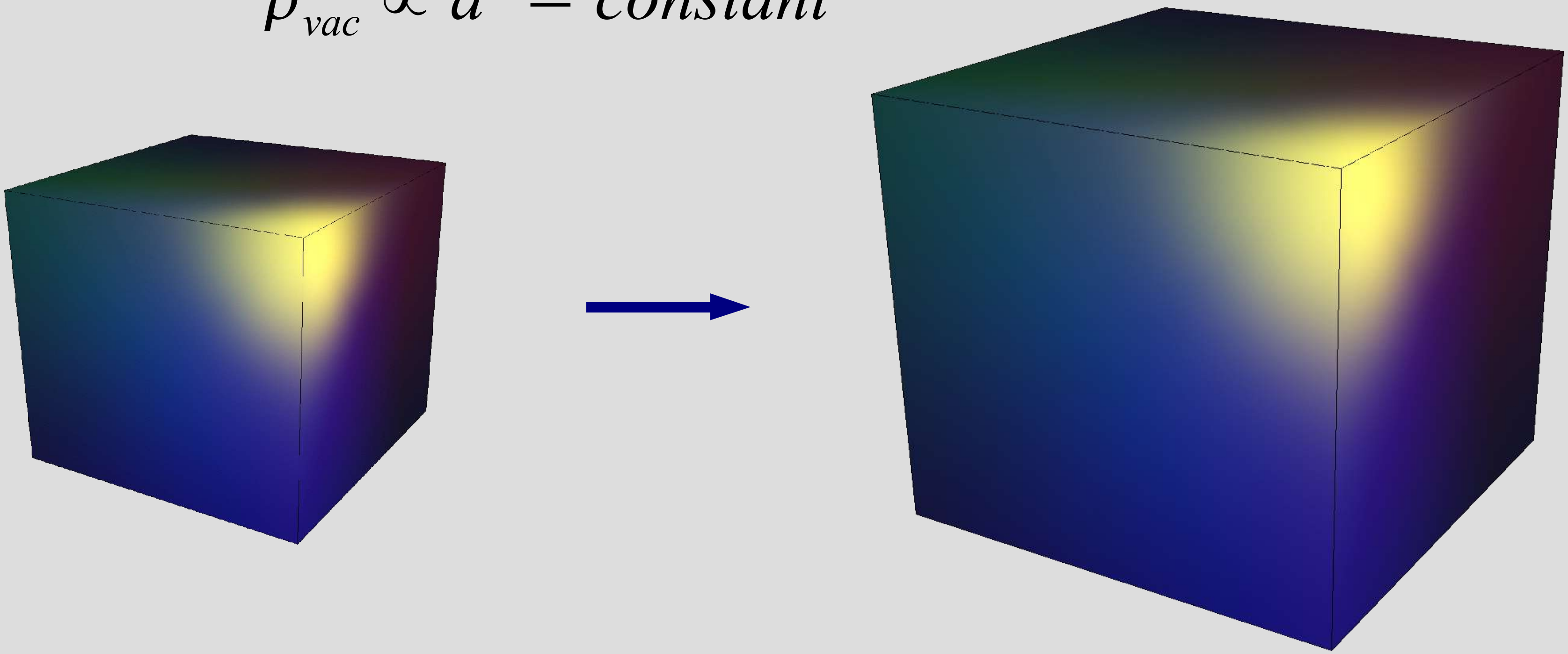
So the energy density in matter simply goes down inversely with the increase in volume: $\rho_M \propto a^{-3}$

And the energy density in radiation diminishes more quickly as each photon loses energy: $\rho_R \propto a^{-4}$

In contrast, vacuum energy simply remains at a constant density throughout space and time as the universe expands.

Equivalent to Einstein's cosmological constant.

$$\rho_{vac} \propto a^0 = constant$$



Note that, in an expanding universe, energy is not conserved.

These parameters are related by Einstein's equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

"curvature of spacetime" = $8\pi G$ "energy/momentum"

In cosmology, the curvature of spacetime nicely decomposes into expansion and spatial curvature. We end up with the Friedmann equation,

$$3H^2 + 3\frac{\kappa}{a^2} = 8\pi G \rho$$

or, more conventionally,

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}$$

The Friedmann equation gives us a relationship between the energy density and spatial curvature.

We can define the density parameter $\Omega = \frac{8\pi G}{3H^2}\rho$.

Then $\Omega = 1 + \kappa / \dot{a}^2$, so

$\Omega < 1$		$\kappa < 0$.
$\Omega = 1$	implies	$\kappa = 0$.
$\Omega > 1$		$\kappa > 0$.

Matter:	Ω_M	~ 0.3
baryons:	Ω_b	~ 0.05
dark matter:	Ω_{DM}	~ 0.25
neutrinos:	Ω_ν	~ 10^{-3}
Radiation:	Ω_R	~ 10^{-4}
Vacuum Energy:	Ω_{vac}	~ 0.7

← Values today, according to the best-fit model, as we will see.

Ω_{total} ~ 1.0 .

Solving the Friedmann equation, $H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}$

Note that $-\kappa / a^2$ can be thought of as an effective "curvature energy," $(8\pi G/3)\rho_\kappa \propto a^{-2}$. Then we have

$$\rho \propto a^{-n} \rightarrow a \propto t^{2/n}$$

Thus, when the universe is dominated by --

matter: $\rho = \rho_M \propto a^{-3} \rightarrow a \propto t^{2/3}$

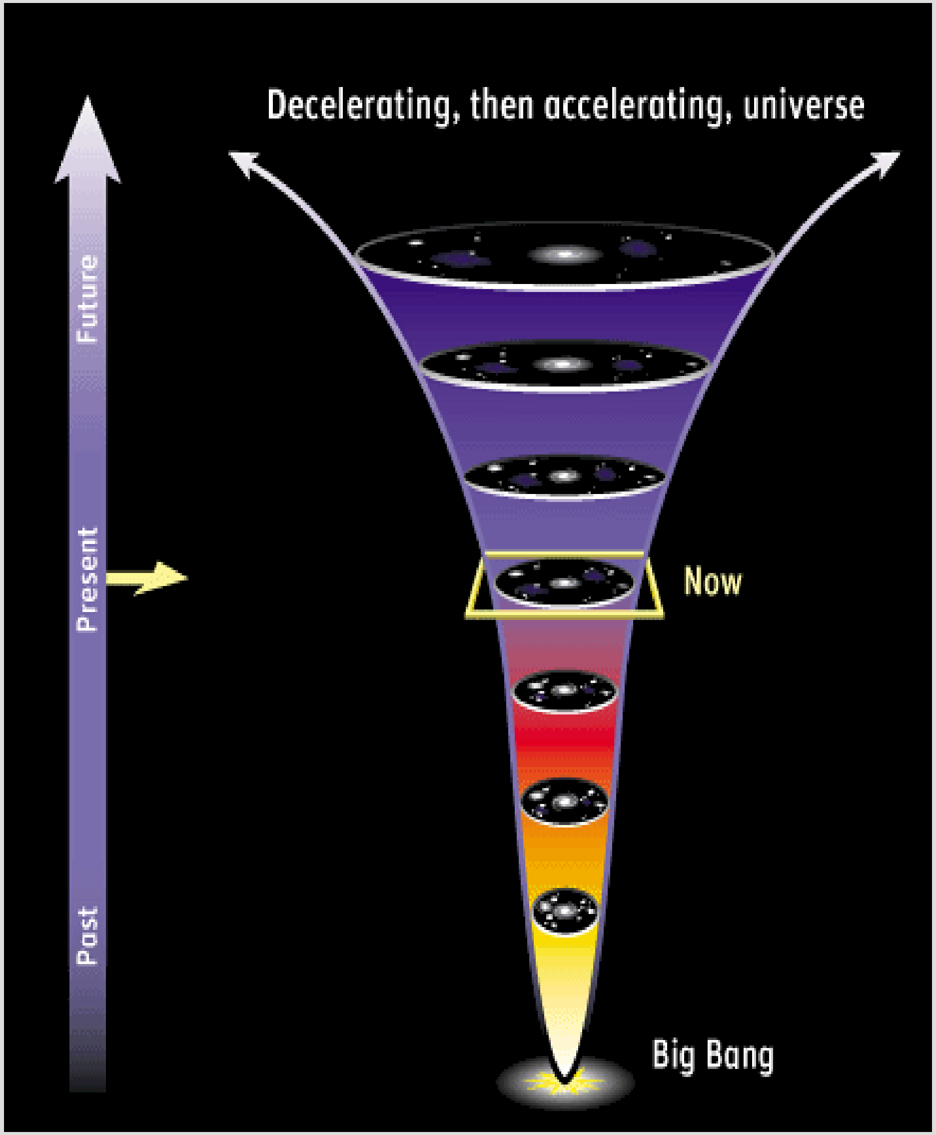
radiation: $\rho = \rho_R \propto a^{-4} \rightarrow a \propto t^{1/2}$

curvature: $\rho = \rho_\kappa \propto a^{-2} \rightarrow a \propto t^{1}$

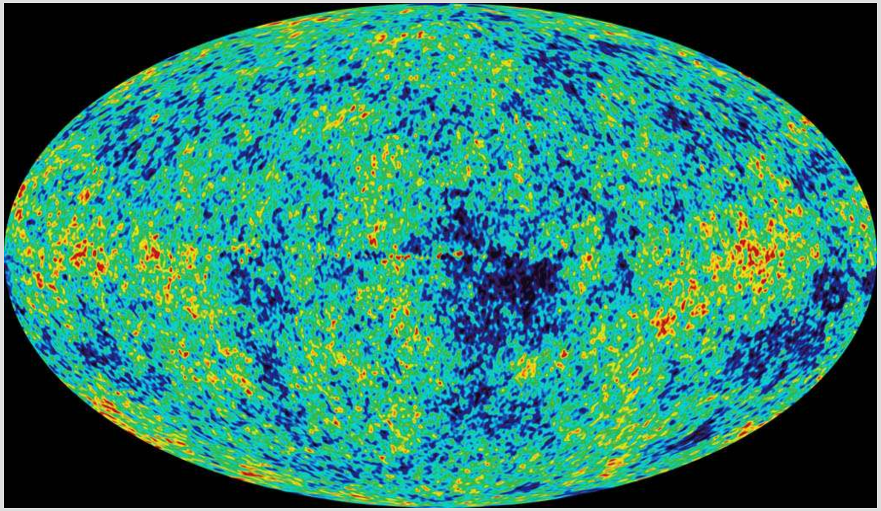
vacuum: $\rho = \rho_{vac} \propto a^{0} \rightarrow a \propto e^{H_0 t}$

(accelerating!)

An expanding universe was hotter and denser in the past, with temperature going roughly as $T \propto 1/a$.
We have a singularity as $T \to \infty$: the Big Bang.



The hot early stage of the universe gave off copious amounts of blackbody radiation, which we can observe today: the Cosmic Microwave Background.

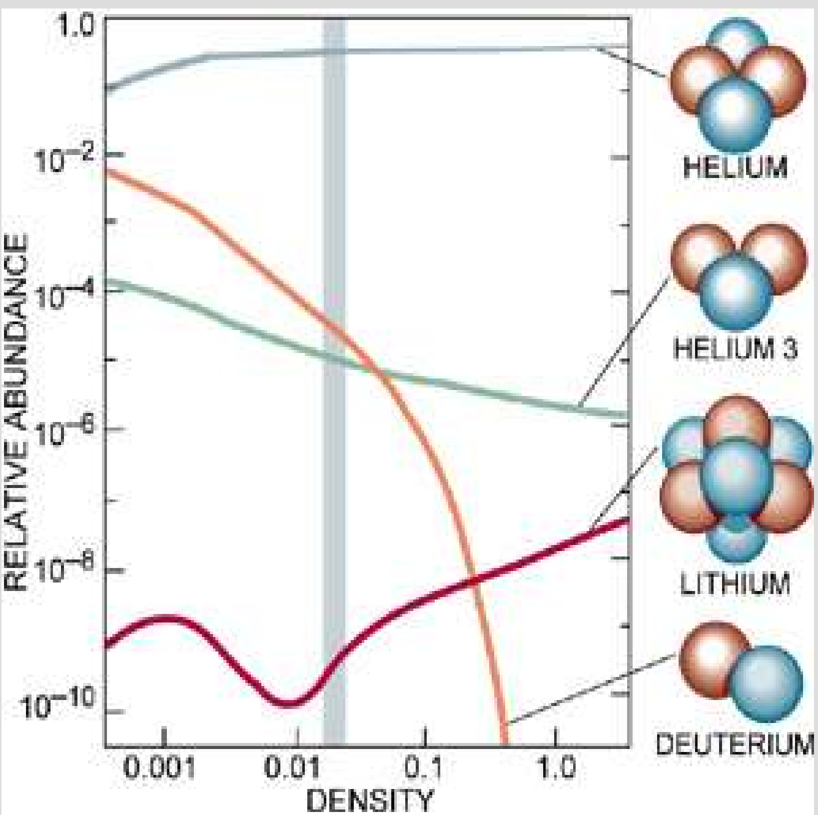


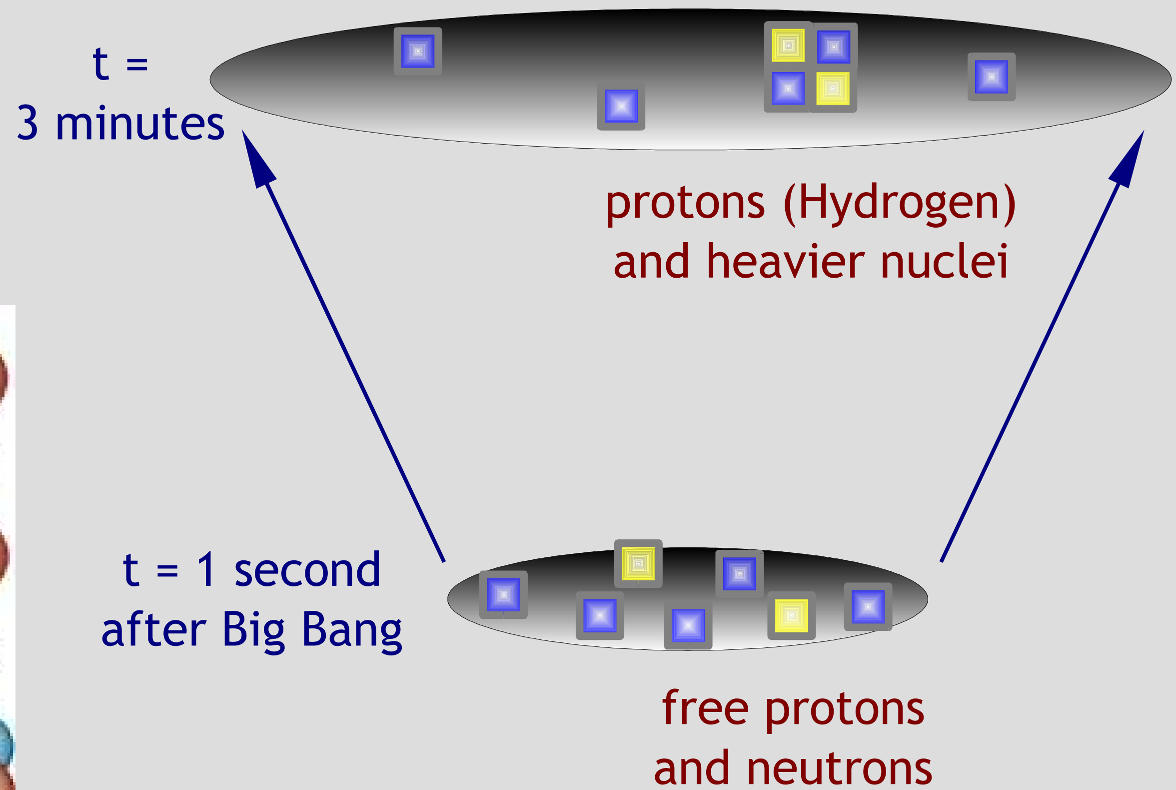
A Brief History

time	temperature	event
10^{-35} sec	10^{15} GeV	inflation (?)
10^{-9} sec	10^{2} GeV	electroweak SB
10^{-5} sec	0.3 GeV	QCD phase transition
10 sec	10^{2} keV	nucleosynthesis
370,000 yr	0.3 eV	recombination
13.7 Gyr	3×10^{-4} eV ~ 3K	today

The earliest era about which we have any empirical evidence is Big Bang Nucleosynthesis (BBN).

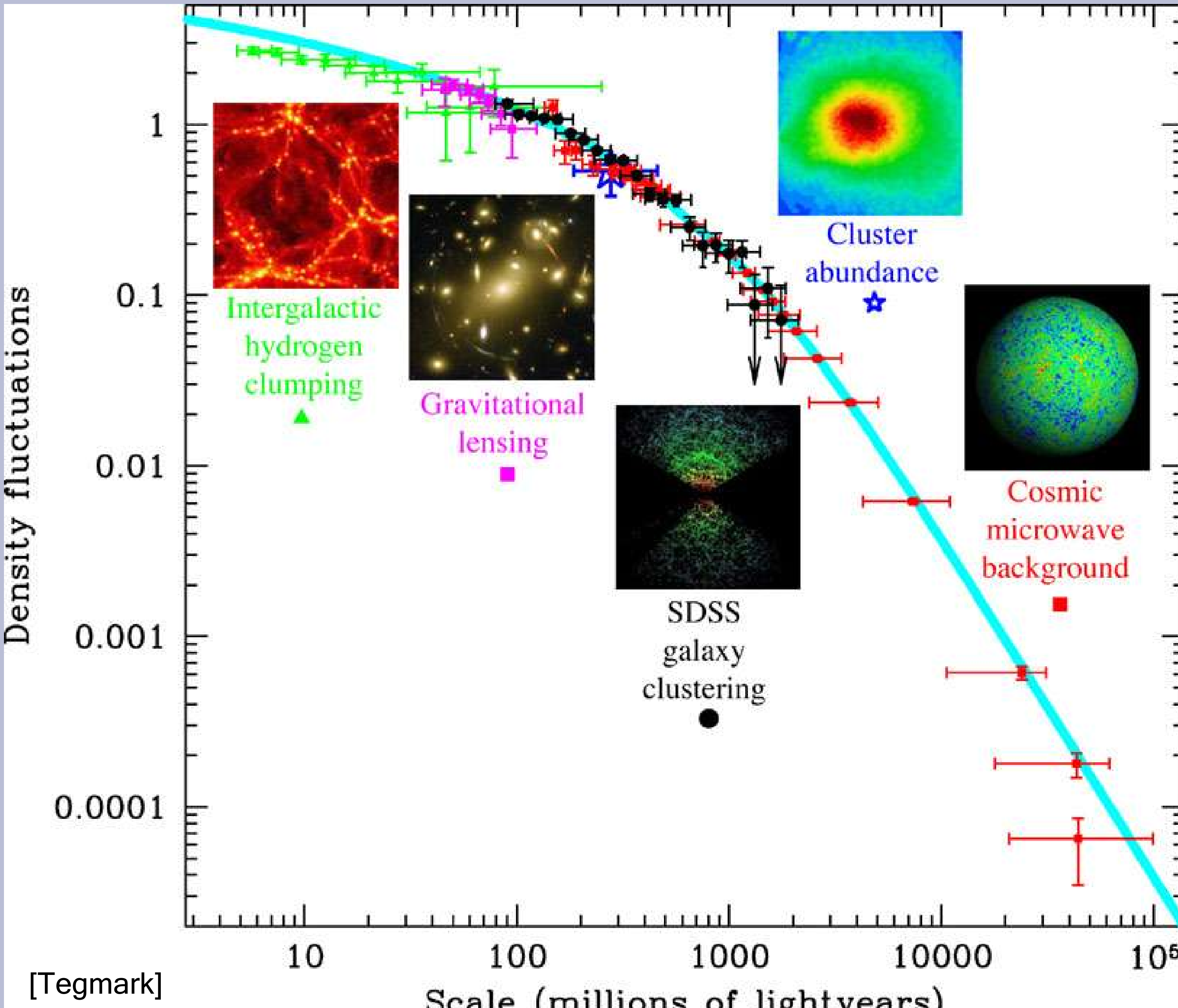
Prediction: 24% Helium, 10^{-5} D and ^{3}He per proton, 10^{-10} ^{7}Li.





Matching primordial light-element abundances requires $\Omega_{baryon} \sim 0.05$.

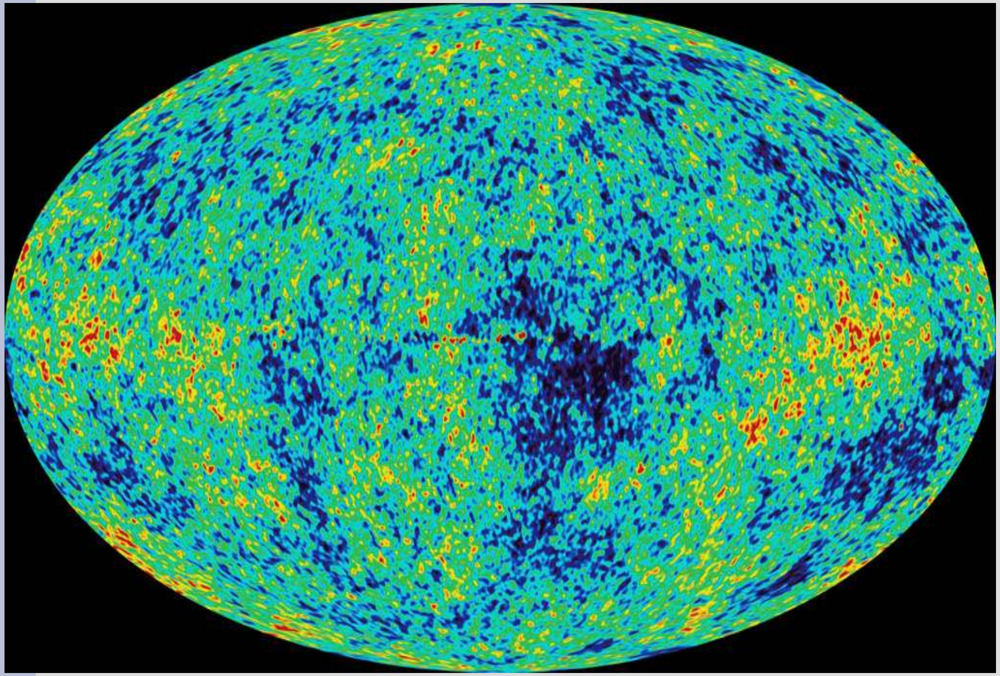
Our universe isn't truly smooth: there are structures on all scales, from small clouds up to superclusters.

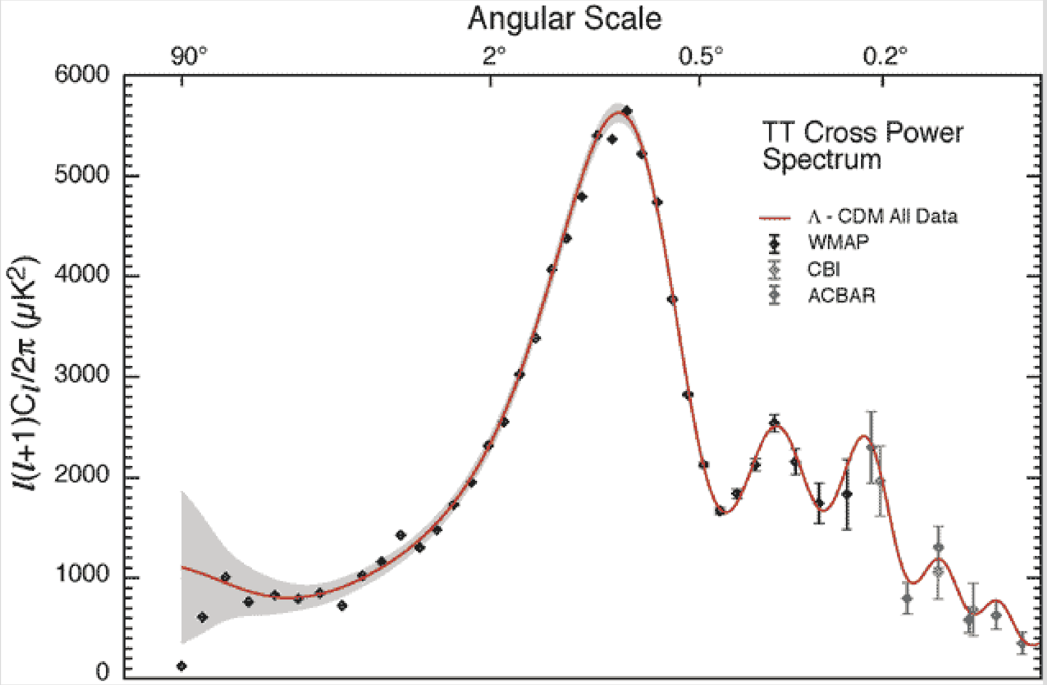


[Tegmark]

The power spectrum of perturbations is consistent with <u>equal power on all scales</u> at very early times.

Temperature anisotropies in the CMB tell us what the perturbations were doing at a redshift $z \sim 1200$.





[WMAP]

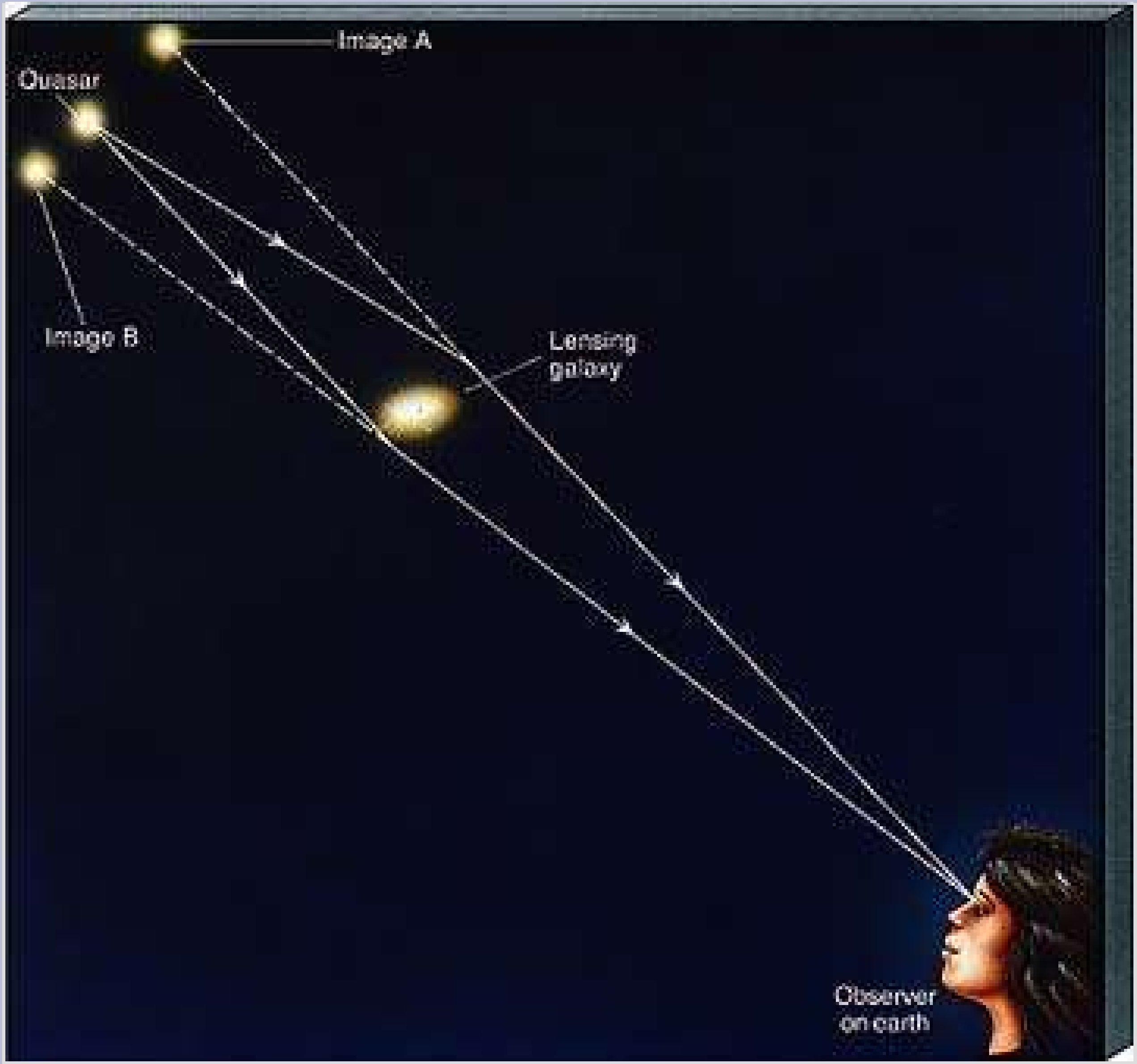
Wiggles in the CMB power spectrum are sensitive to nearly all cosmological parameters: total density, dark matter, baryon density, Hubble constant, etc.

Tiny perturbations in the CMB ($\Delta T/T \sim 10^{-5}$) have grown into huge galaxies and clusters we see today.

Gravitational dynamics of galaxies and clusters provide a direct handle on the total mass density of the universe.

[Virgo consortium]

Clusters are sufficiently large that they are "fair samples" of the universe. They can be weighed by various different techniques.

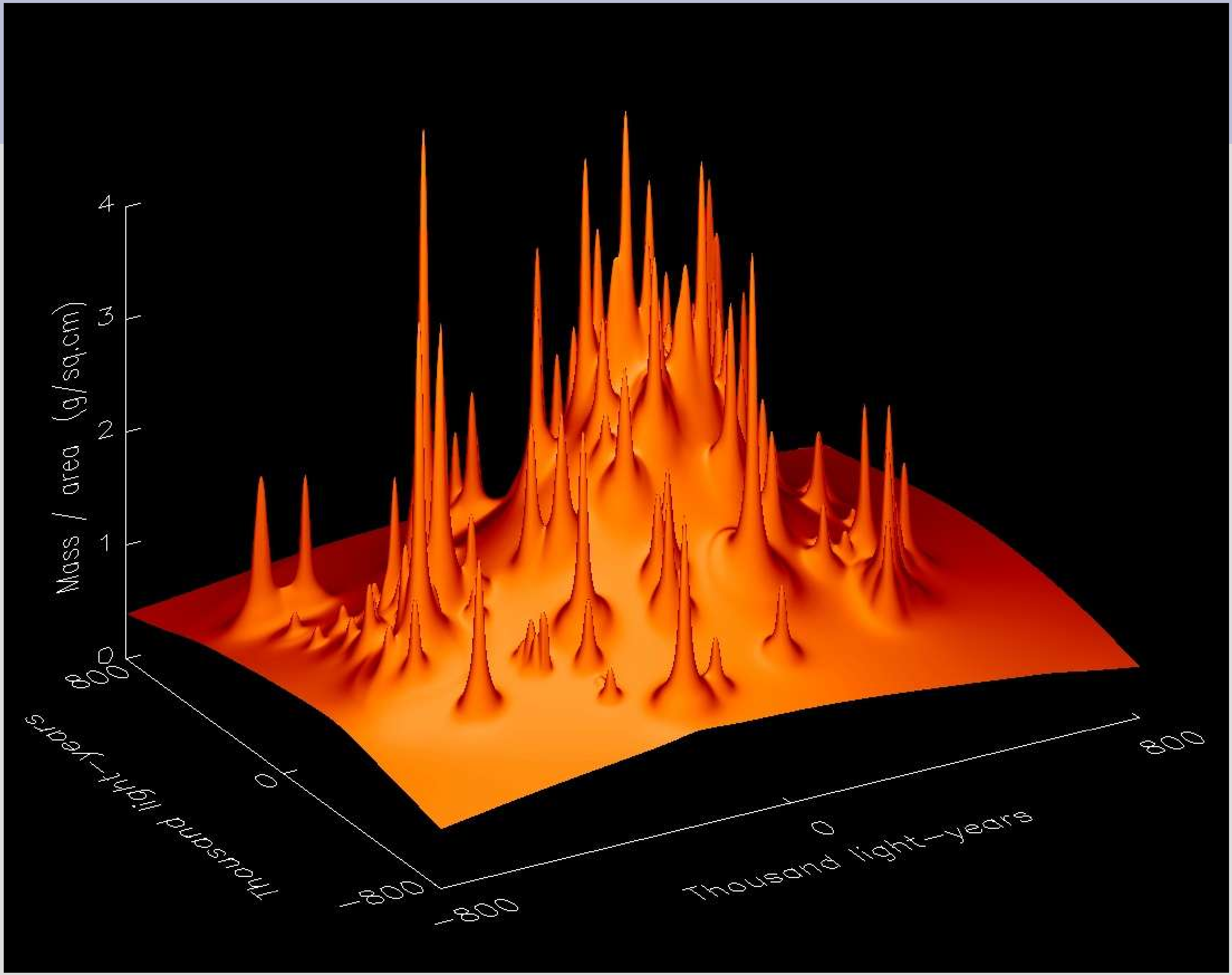


One aesthetically satisfying method is gravitational lensing of objects in the background.

Lensing angle is proportional to overall mass of the system.

Hubble Space Telescope Image of a galaxy cluster.

Mass reconstruction from gravitational lensing.



[Tyson et al.]

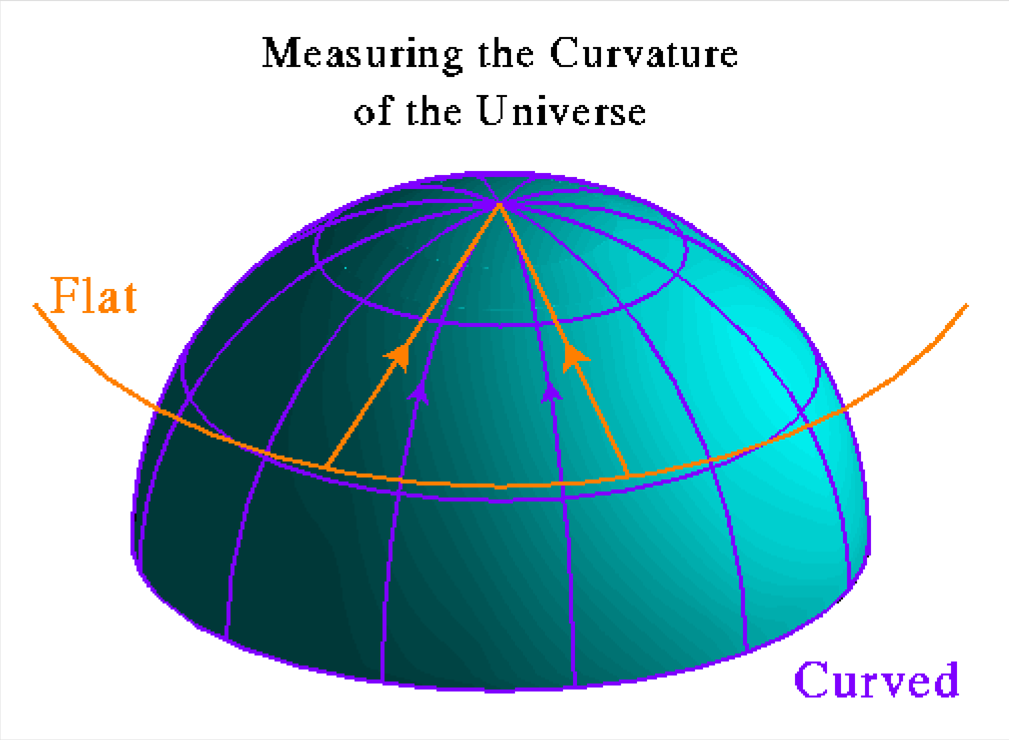
There is much more matter in the universe than can be accounted for by "ordinary" matter (the Standard Model).

We conclude that there must be something else: Dark Matter.

The dark matter density parameter is $\Omega_{DM} \sim 0.25$, while that in baryons is only $\Omega_b \sim 0.05$. The dark matter must be a completely new kind of particle. (Neutralinos? Axions?)

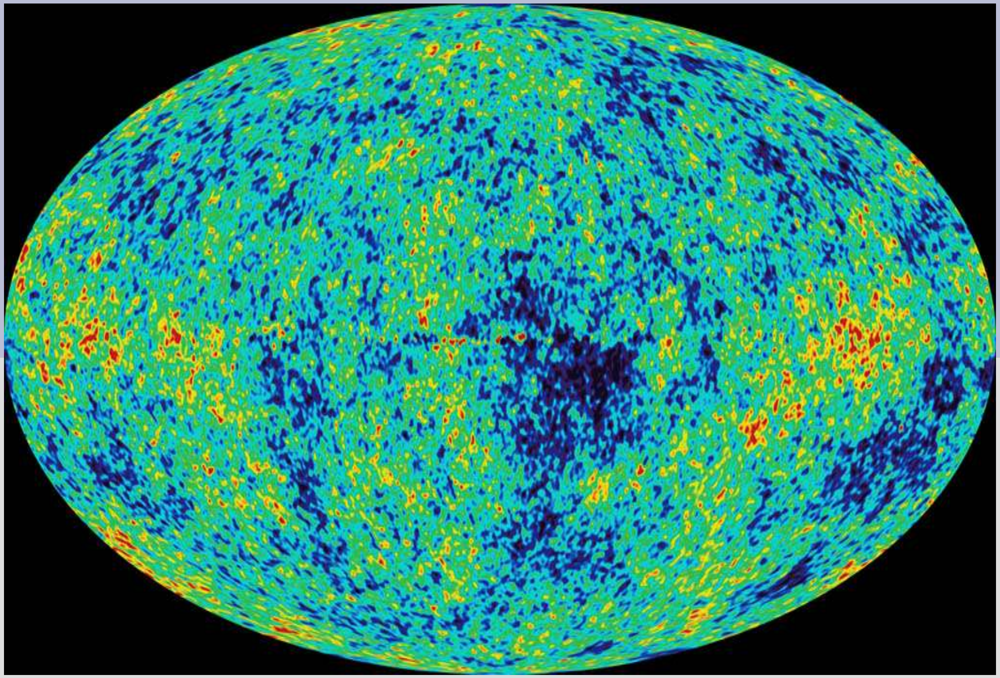
All matter (ordinary + dark) adds up to $\Omega_M \sim 0.3$, intriguingly less than the critical density.

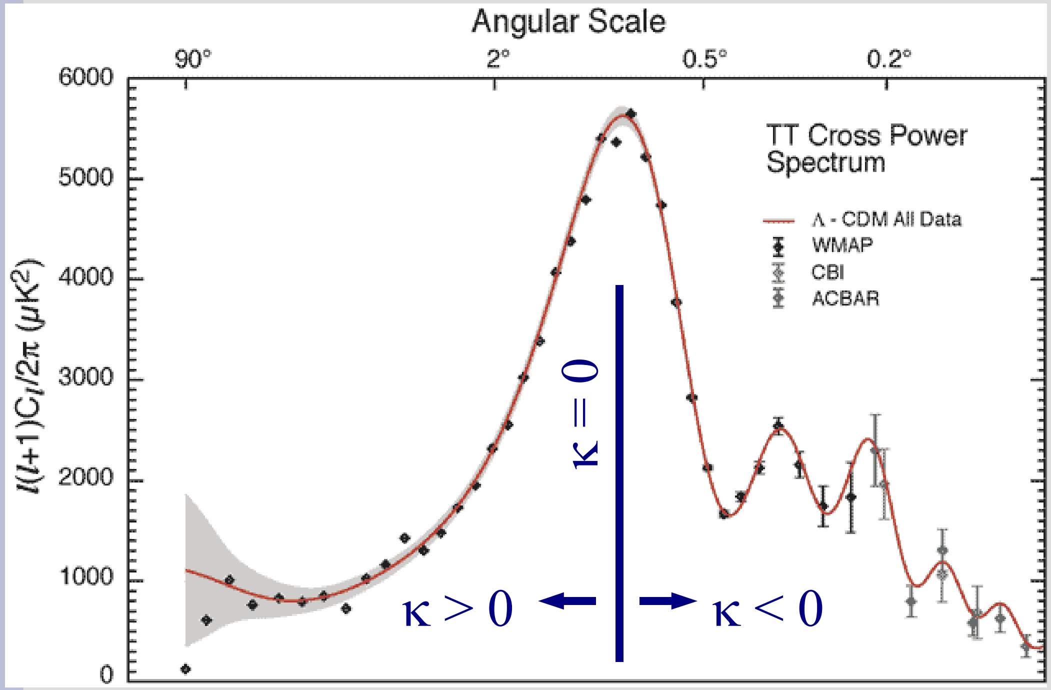
We can check the total energy of the universe by looking at spatial curvature: $\Omega = 1 + \kappa / \dot{a}^2$.



To do this we need a standard triangle, happily provided by temperature fluctuations of the CMB.

CMB fluctuations peak at a <u>characteristic length scale</u> of 370,000 light years; observing the corresponding <u>angular scale</u> measures the geometry of space.





Take angular power spectrum of temperature fluctuations; the position of the peak is a curvature-meter.

Observation: $\theta_{peak} = 1^o$.

Consistent with a <u>flat universe</u>:

$\Omega = 1.0$.

A brief mathematical interlude:

If $\Omega_M \sim 0.3$, but $\Omega_{Total} \sim 1.0$, then $\Omega_{something\ else} \sim 0.7$.

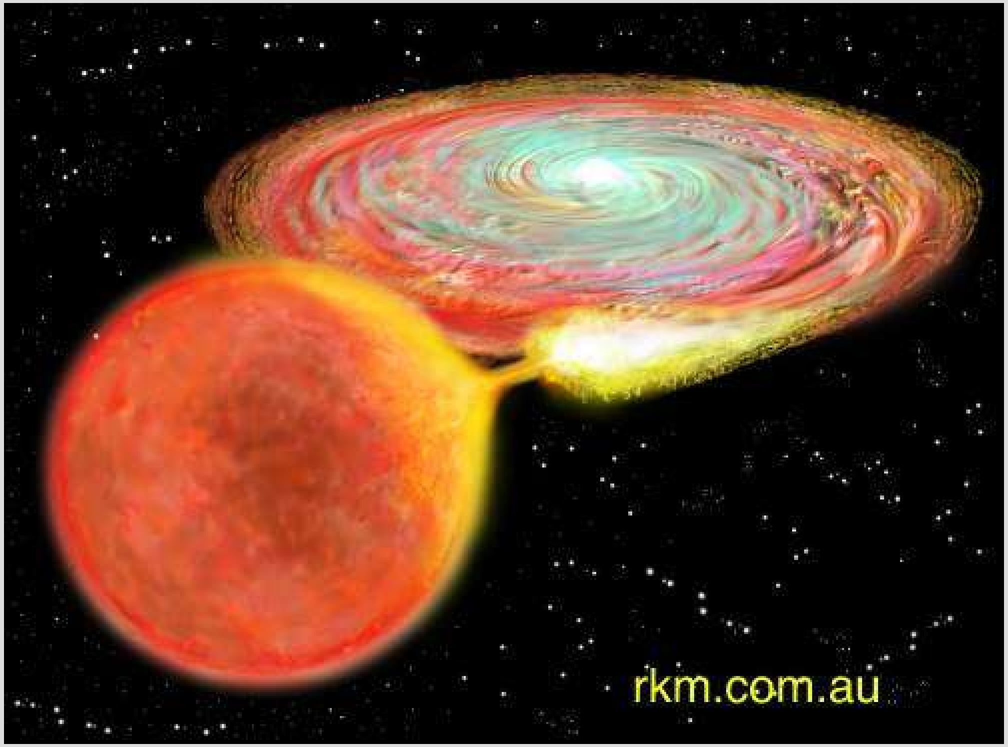
What could this something else be? Let's call it Dark Energy.

It must be smoothly distributed through space, or we would have noticed it via lensing and other experiments. But it can't be fast-moving particles (hot dark matter), or it would have ruined the formation of structure on small scales.

Hmm.

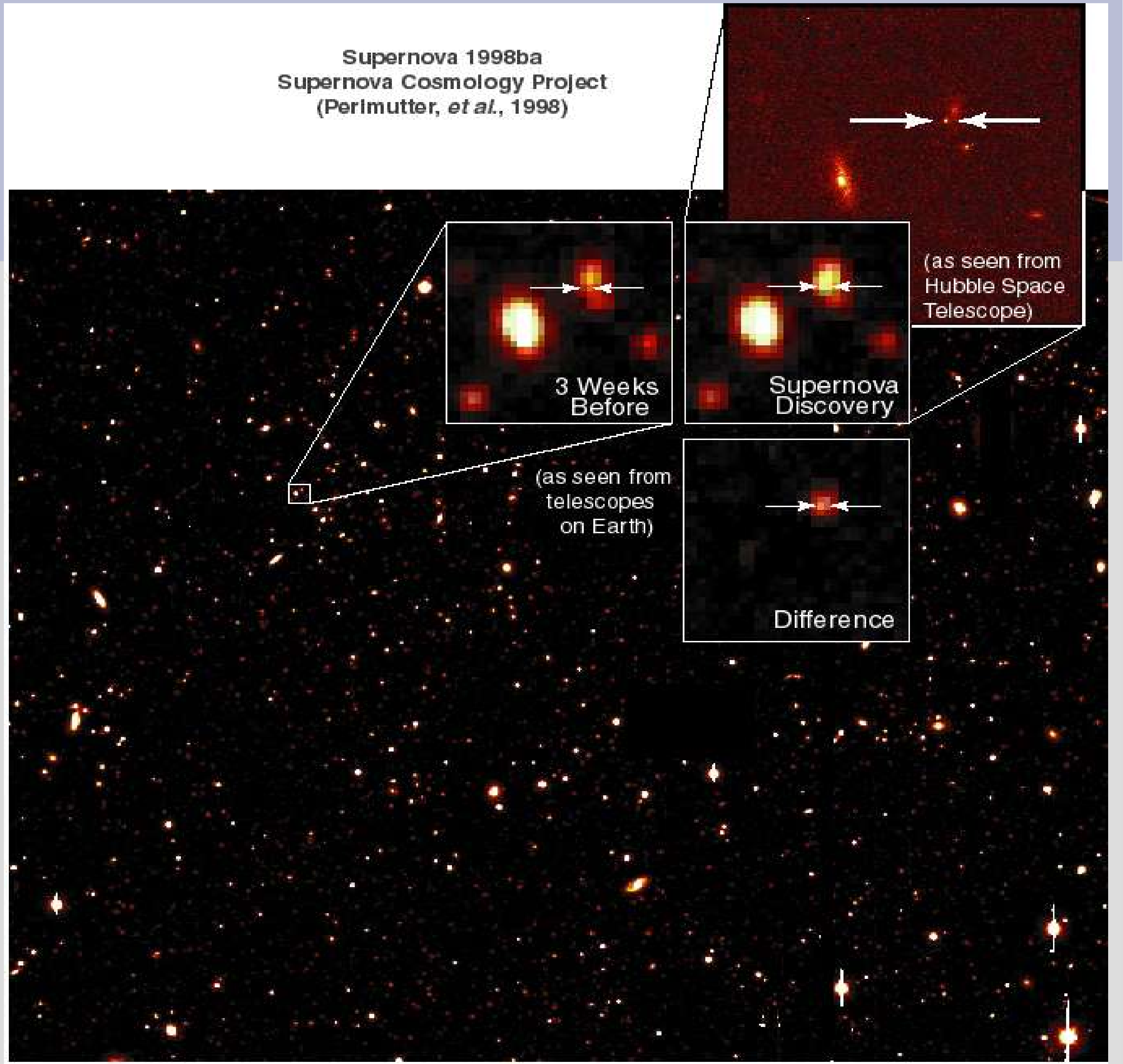
An independent characteristic of the total energy density of the universe is the change in the expansion rate over time -- acceleration/deceleration.

We need a standard (or at least standardizable) candle, visible very far away. Best examples are Type Ia Supernovae.

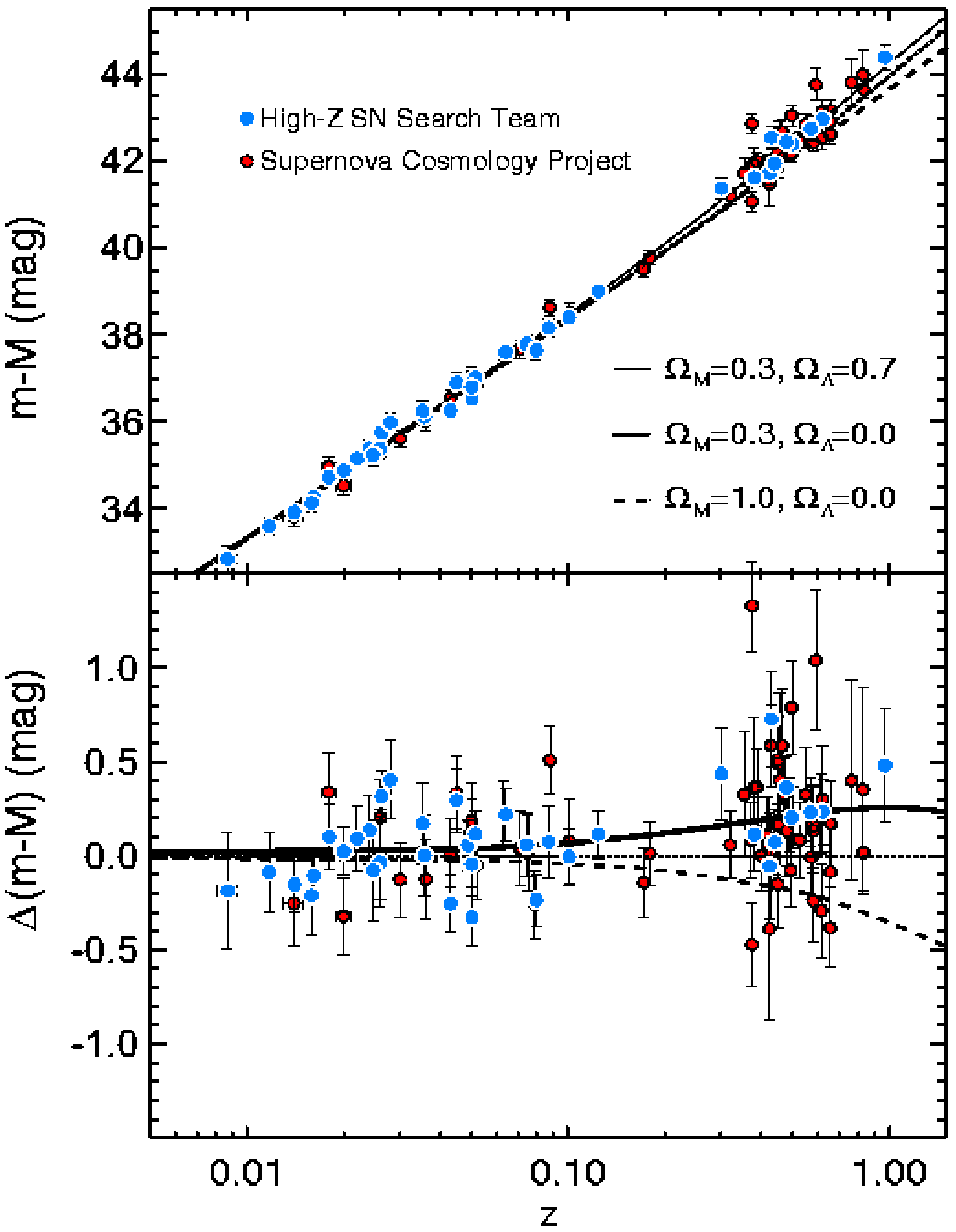


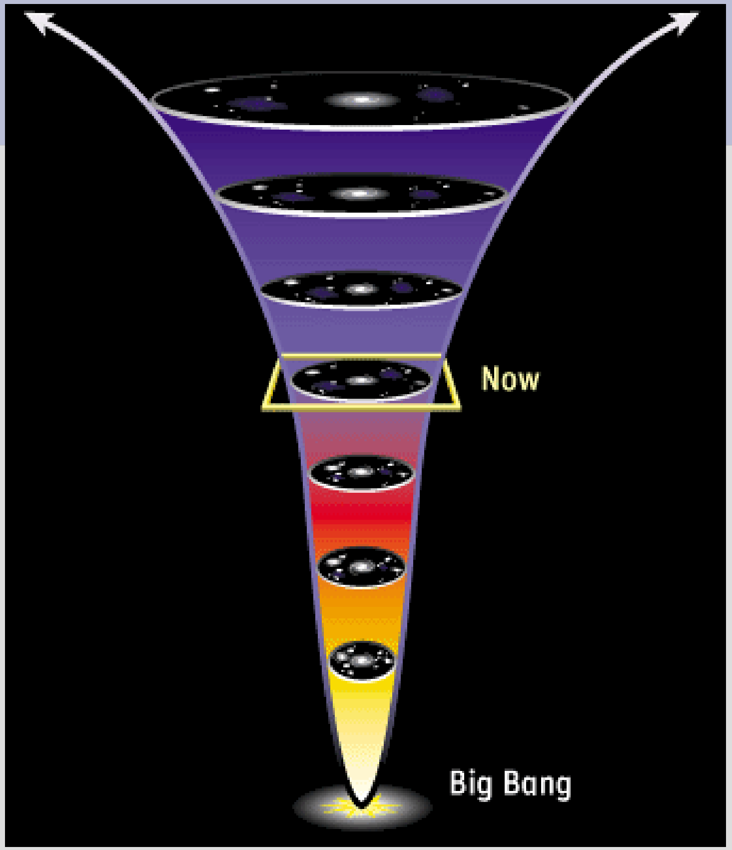
Matter gradually accretes onto a compact white dwarf star, until the gravitational pull becomes too great and the white dwarf collapses and explodes.

Strategy: stare at one tiny patch of the sky for a long time, waiting for a supernova to appear.



Astonishing result: the universe is accelerating!





accelerating

decelerating

How do we make the universe accelerate?
Some energy density that <u>doesn't go away</u>.

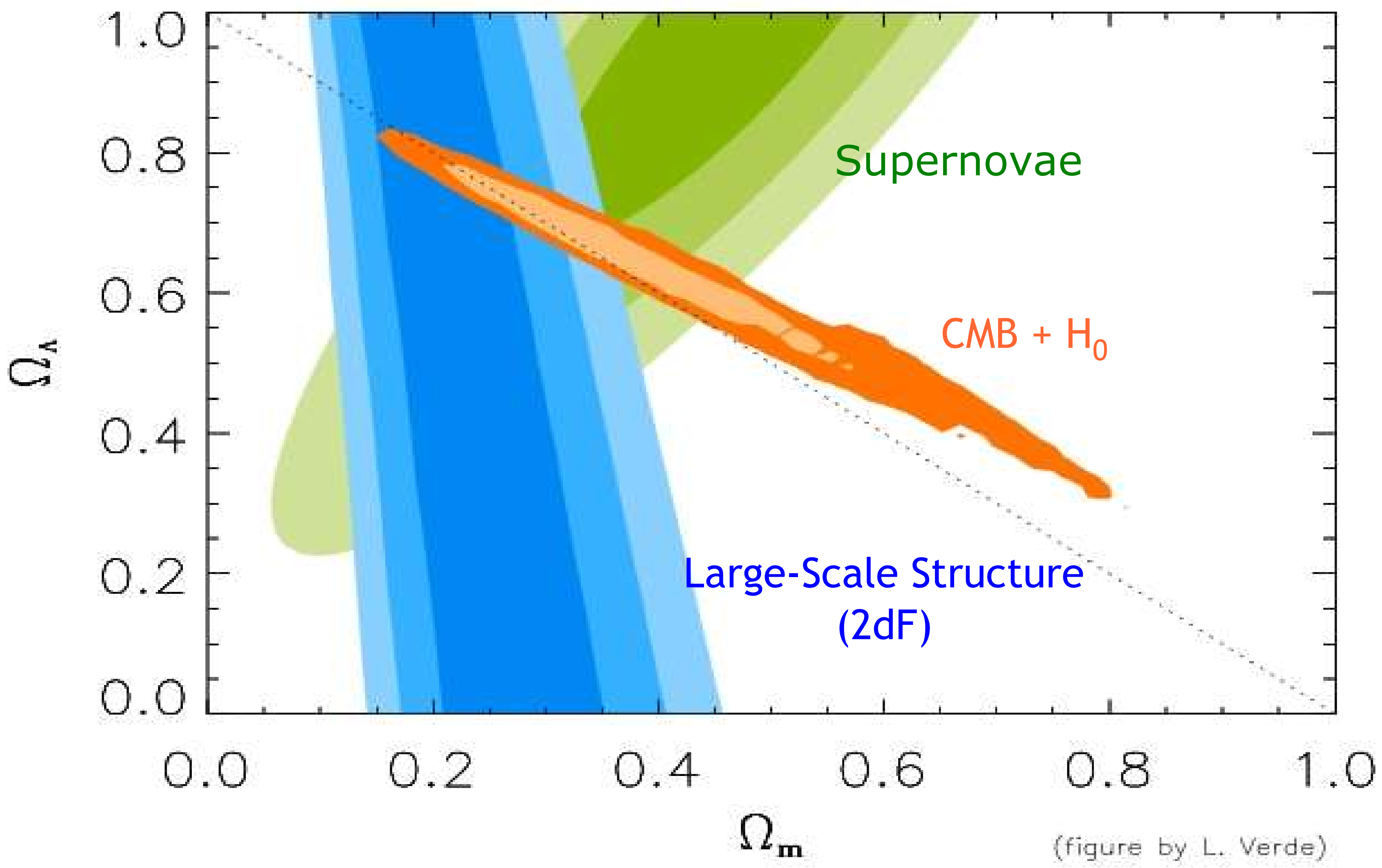
Friedmann (Ω = 1.0): $$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

Multiply by a^2 to get: $$\dot{a}^2 \propto a^2 \rho$$

So, to get acceleration ($\dot{a}$ increasing), we need ρ to be persistent – going away more slowly than a^{-2}.

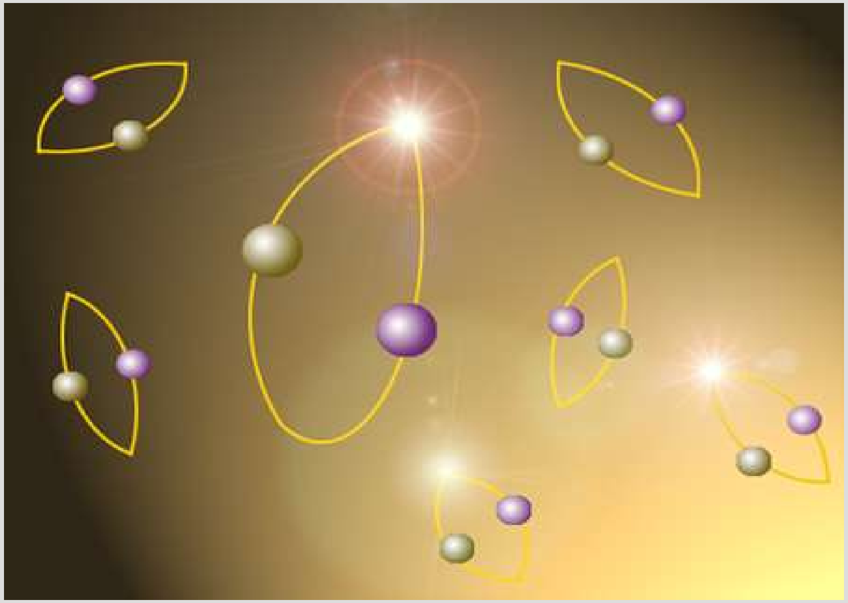
All consistent with the existence of a dark energy component that is nearly constant through both space and time, with Ω_{DE} ~ 0.7.

Concordance: $\Omega_M \sim 0.3$, $\Omega_\Lambda \sim 0.7$.



If dark energy is smoothly distributed through space, and persistent through time, the simplest candidate is vacuum energy – perfectly constant in spacetime.

Vacuum energy is easy to get; there are certainly vacuum fluctuations, and every reason for them to contain energy.



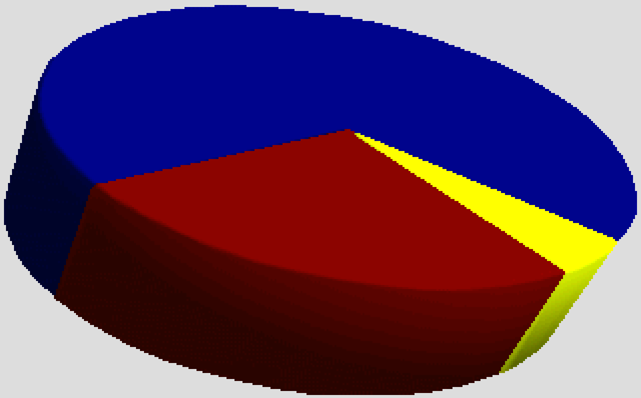
Only one problem...

Naively: $\rho_{vac} = \infty$, or at least $\rho_{vac} = E_{Pl}/L_{Pl}^3 = 10^{120}\rho_{vac}^{(obs)}$.

This is the infamous "cosmological constant problem."

So the final accounting seems to be:

5% Ordinary Matter
25% Dark Matter
70% Dark Energy



It fits all the data, it just makes no sense.

Our job: figure it all out!