NZ - Aust Semiconductor Instrumentation Workshop

Signal Processing

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CHRISTCHURCH NEW ZEALAND

Signal Processing

- Sampling
- Degradation
- Image recovery problems
- Undersampling in MRI
- Recently introduced tools
- Implementation issues and trends

Sampling

Sampling—50 Years After Shannon

MICHAEL UNSER, FELLOW, IEEE

Proceedings of the IEEE, 88: 569-587, April 2000.

with reference to:

Shannon, C.E. “Communication in the presence of noise”, *Proc. IRE*, 37: 10-21, 1949.

→ Shannon-Whittaker-Kotel’nikov Theorem

“. . . *common knowledge in the communication art*”

Sampling

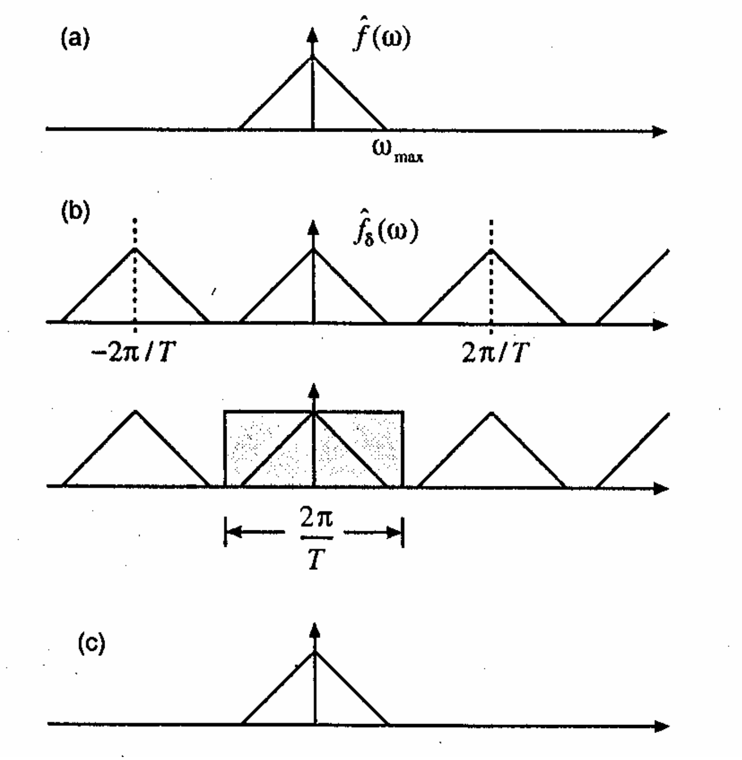
Shannon-Whittaker-Kotel'nikov Theorem:

If a function $f(x)$ contains no frequencies higher than ω_{max} (in radians per second), it is completely determined by giving its ordinates at a series of points spaced $T = \pi/\omega_{max}$ seconds apart.

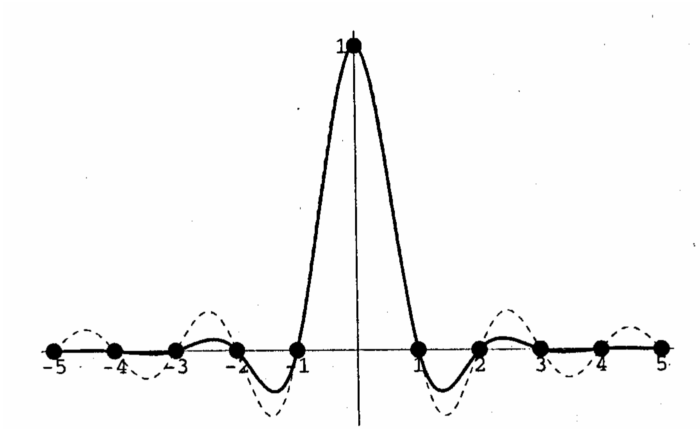
The reconstruction formula which complements the sampling theorem is:

$$f(x) = \sum_{k \in Z} f(kT)\,\mathrm{sinc}\left(\frac{x}{T} - k\right)$$

Reconstruction from samples



In the frequency domain



In the signal domain

Signal/image degradation

Our ability to use the data we measure is fundamentally limited by the errors in those measurements – the “noise”.

Noise has many causes; it is by its nature unpredictable and therefore best characterised statistically:

- a low flux of events may best be modelled by Poisson distribution
- at high fluxes, thermal effects tend to dominate $\rightarrow$ Gaussian distribution

Signal/image degradation

Data may also be "missing":

e.g.

- there may be no direct way of making a measurement
- the physics of the instrument may mean that information is lost
- we cannot wait long enough to make better measurements
- the medium may introduce gross distortions

Image recovery problems

Conference 5562 Monday-Tuesday 2-3 August 2004

Proceedings of SPIE Vol. 5562

Image Reconstruction from Incomplete Data III

Conference Chairs: **Philip J. Bones**, Univ. of Canterbury (New Zealand); **Michael A. Fiddy**, Univ. of North Carolina/Charlotte; **Rick P. Millane**, Univ. of Canterbury (New Zealand)

SESSION 1: Optics and Phase

SESSION 2: Imaging Through Turbulence

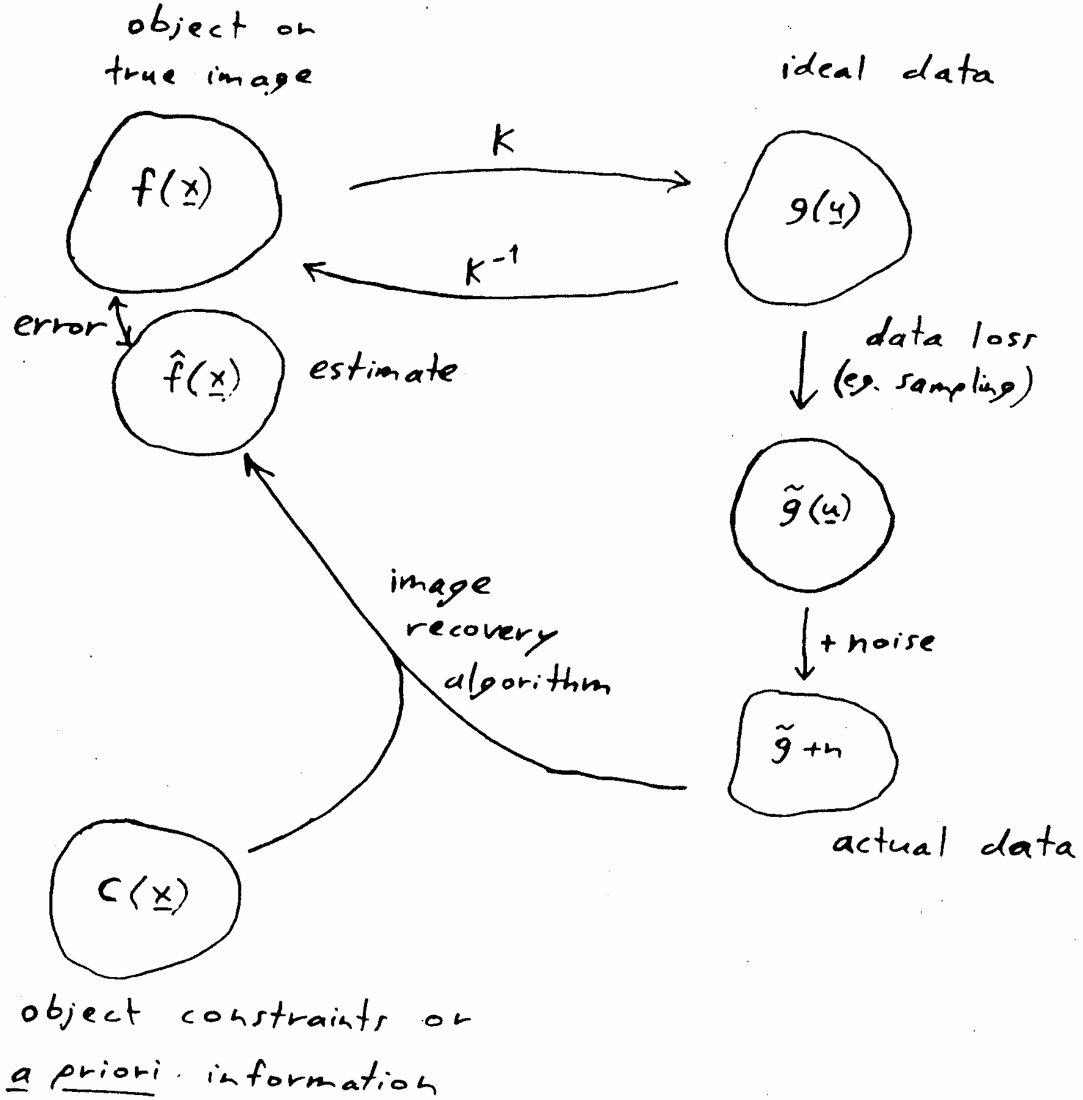
SESSION 3: Tomography

SESSION 5: Regularization and Numerical Methods

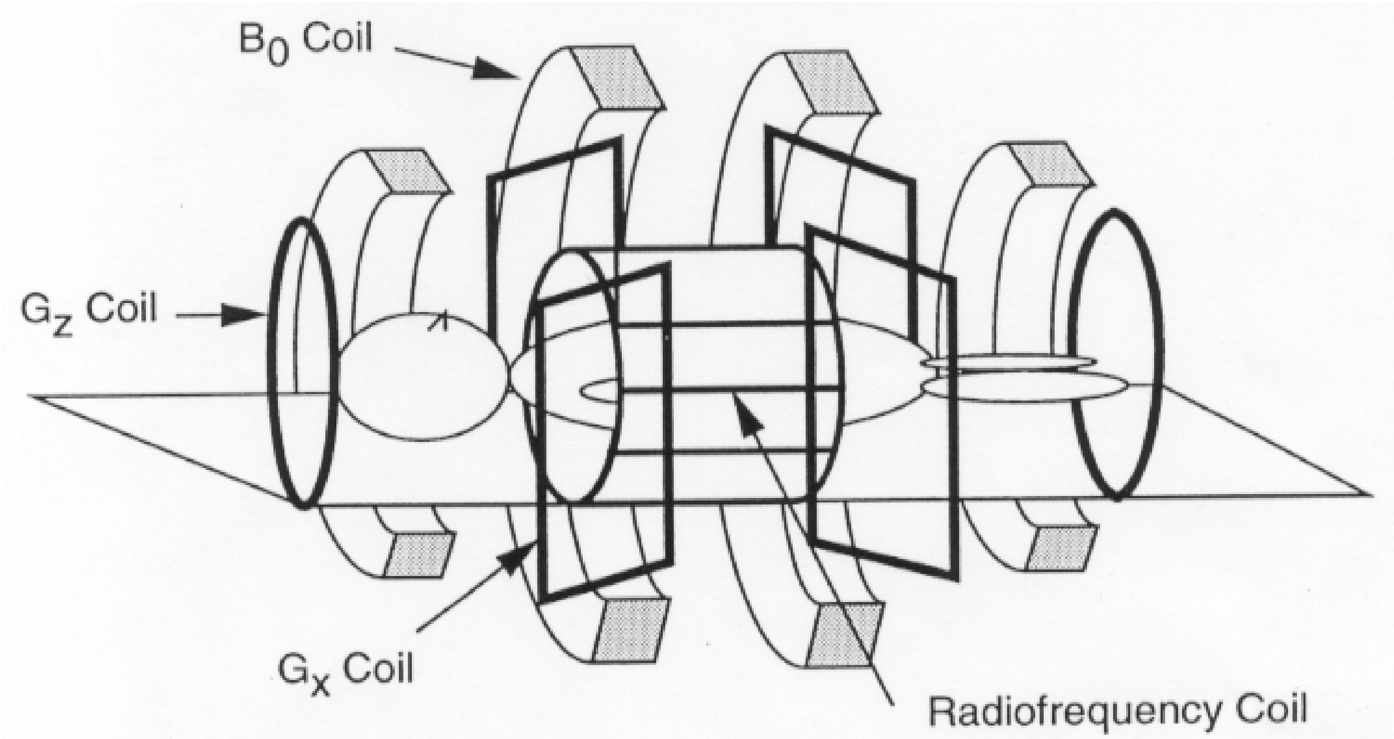
SESSION 6: Deconvolution

SESSION 7: Inverse Problems

Image recovery problems

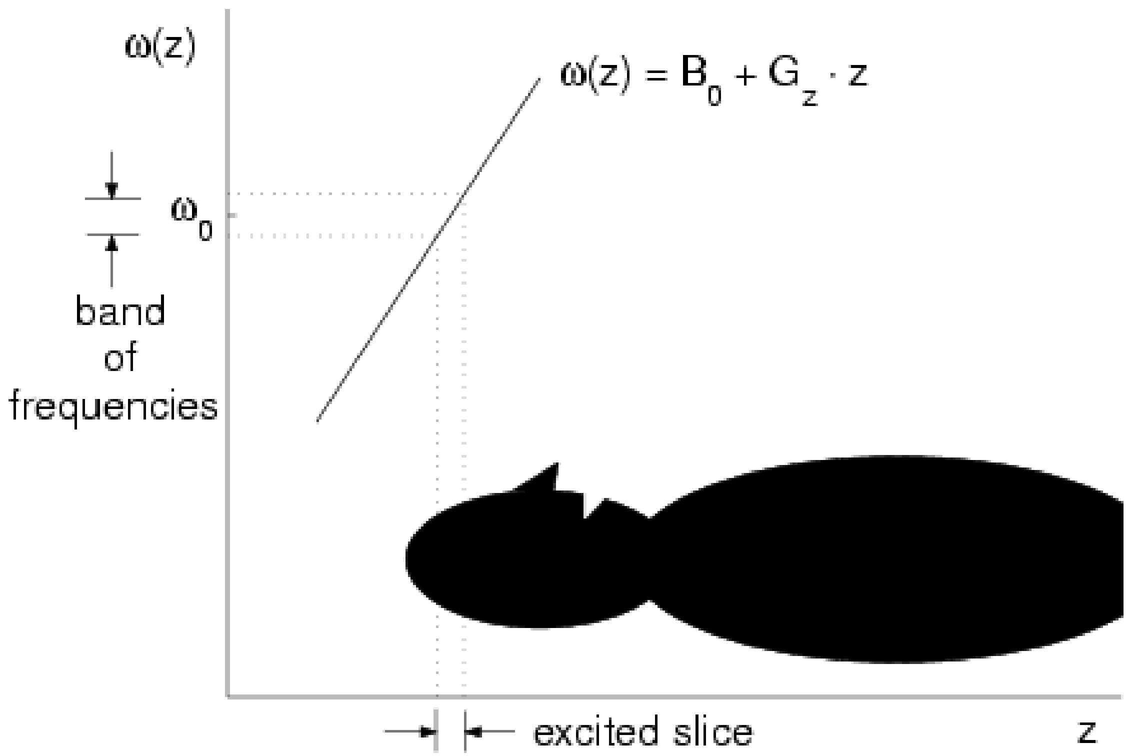


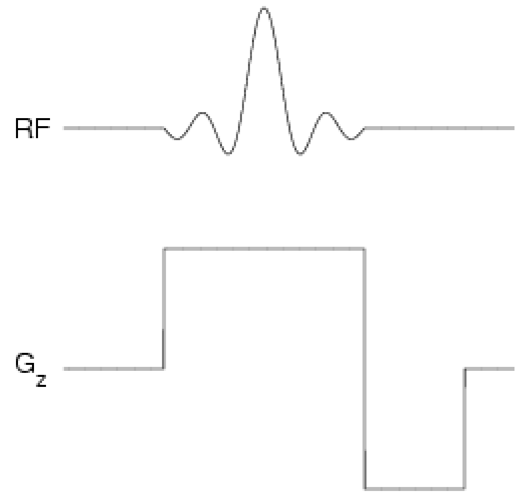
Example recovery problem: MRI scanner undersampling*



* Blakeley, Bones, & Millane, JOSA A, 20: 67-77, 2003.

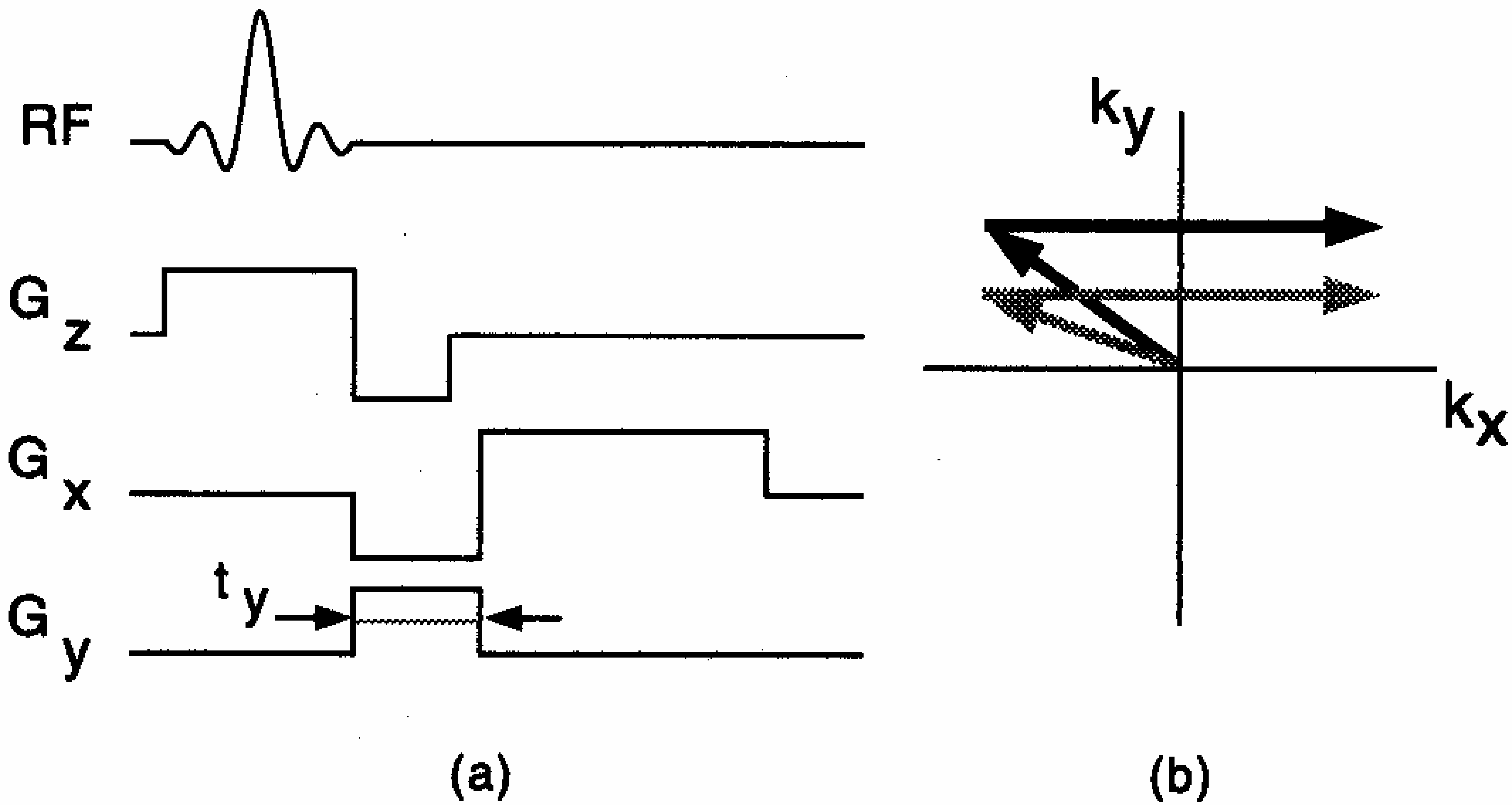
Slice selection . . .





. . . uses pulsed excitation
and z field gradient

Sampling over *k*-space



Motivation for undersampling

- Decreasing MR acquisition time allows throughput to be increased
- Alternatively, more resolution can be achieved in the same time

Sampling theorem

The Nyquist limit is well known (applied here in spatial frequency space):

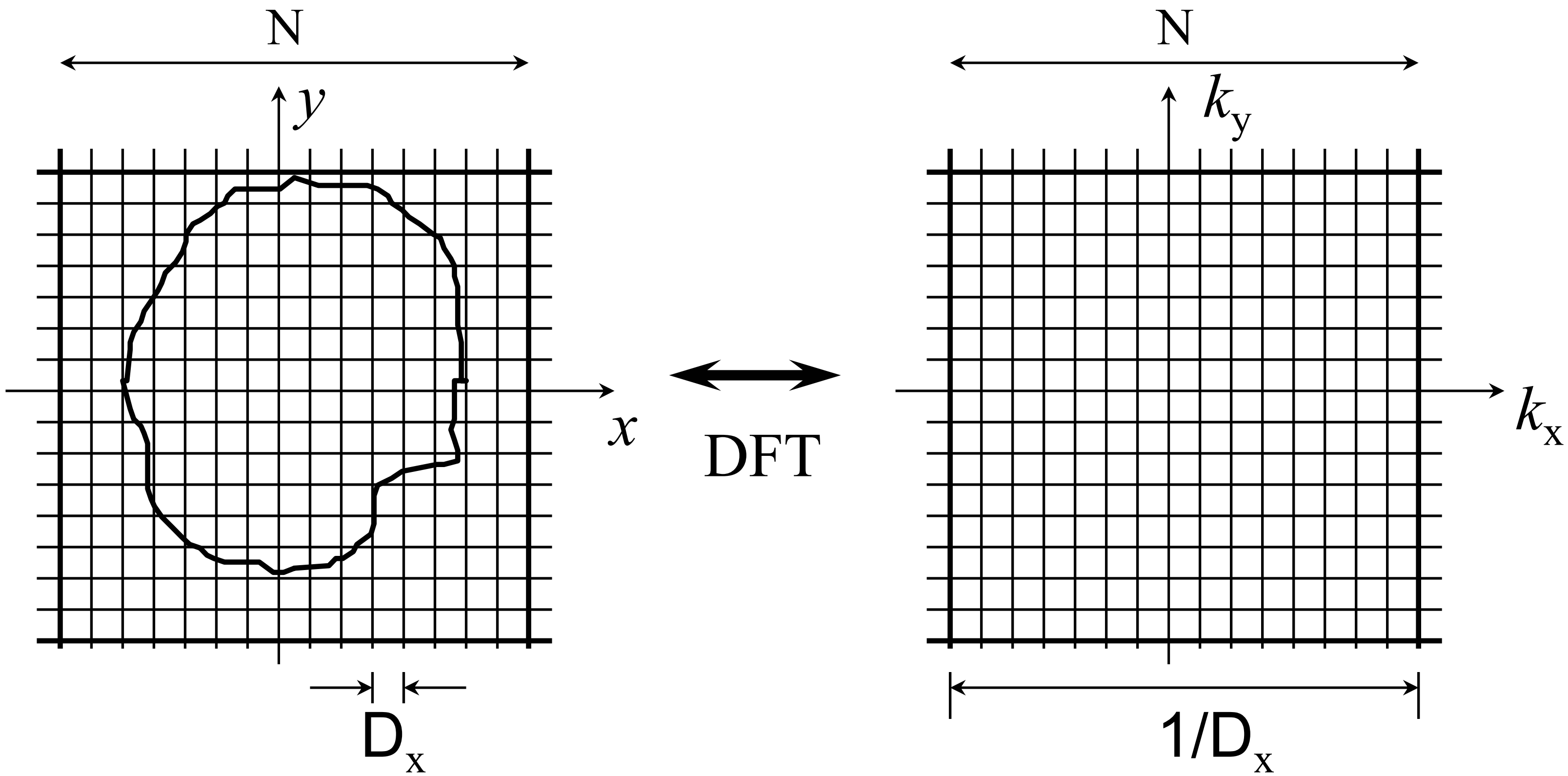
sample at the rate necessary to image the region of interest

Prior knowledge

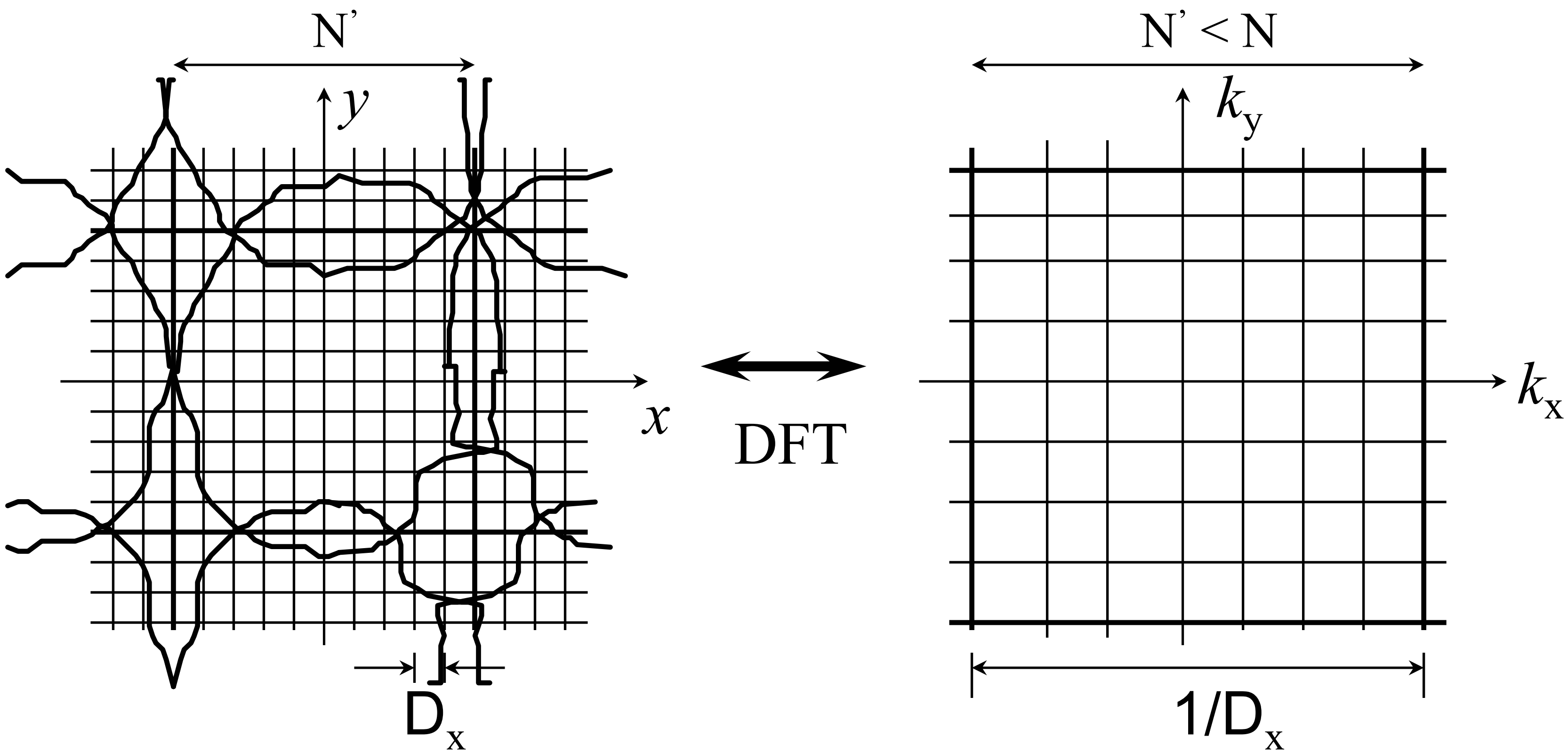
The proton density can only be non-zero *inside* the body

- the "support constraint"

Nyquist rate sampling



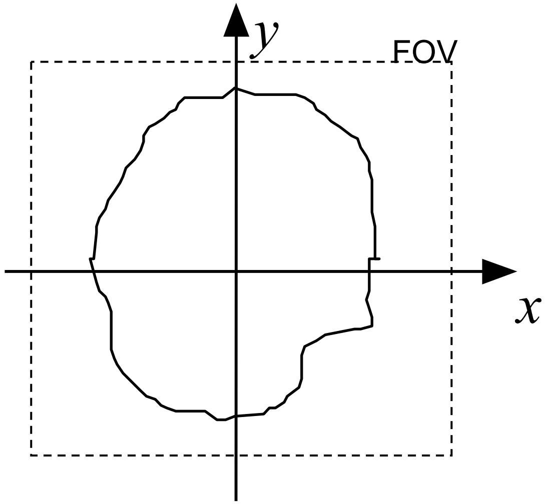
Undersampling in the frequency domain

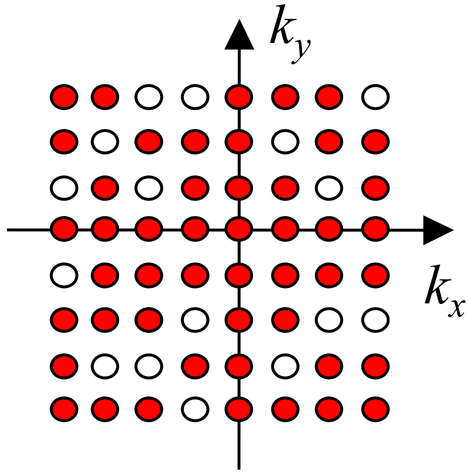


aliased image

The Problem

- Reconstruction of a limited support object sampled in the frequency domain

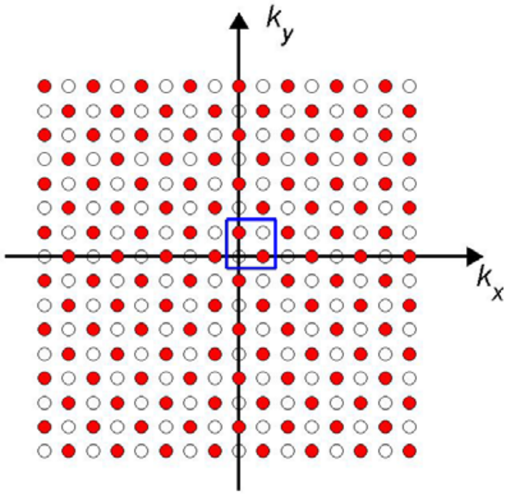


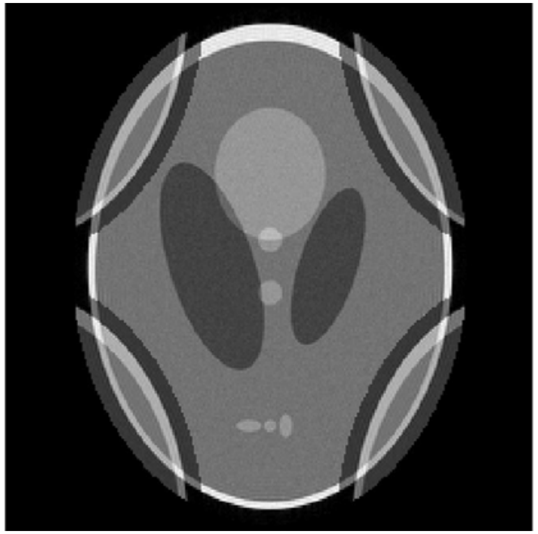


- Where should the samples be placed?
- What reconstruction algorithm should be used?

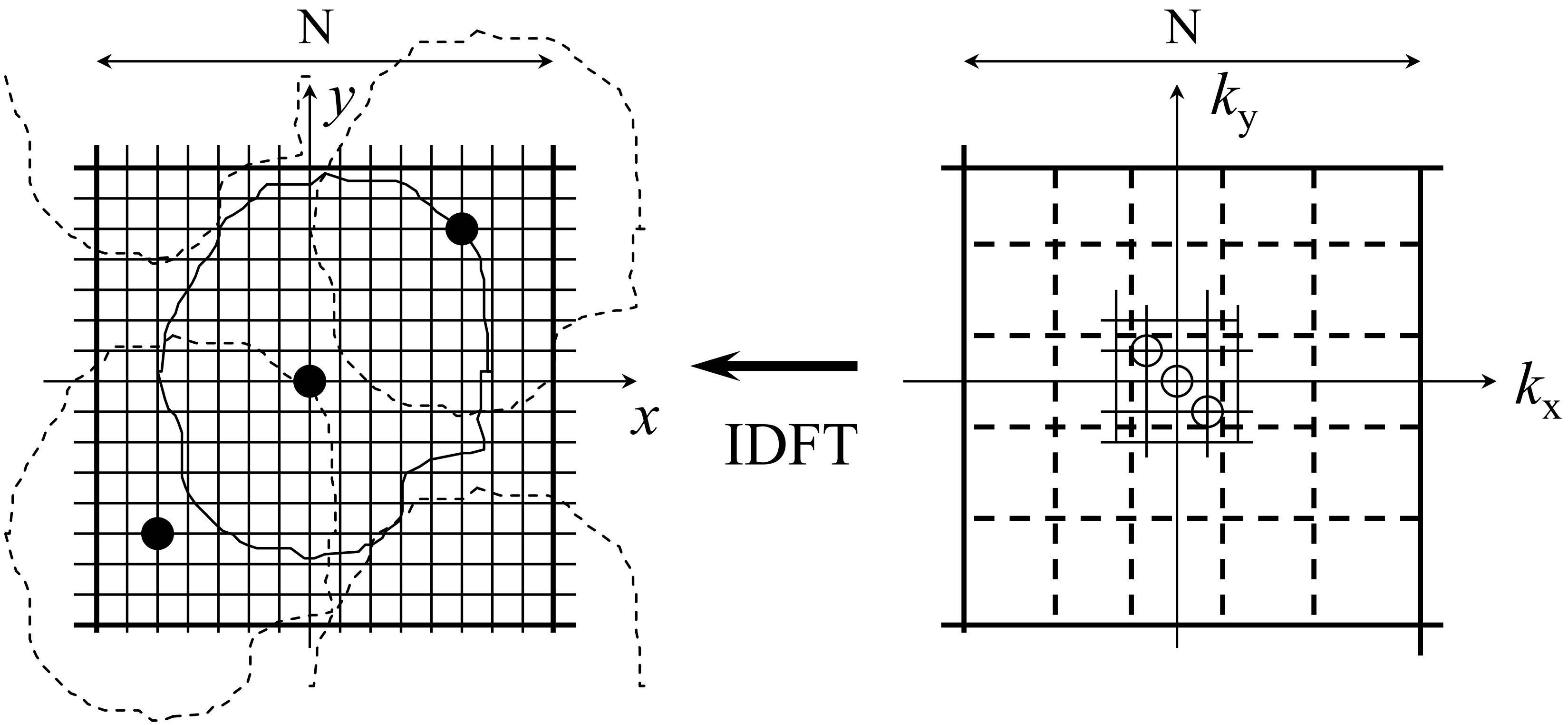
An observation

- A repeated sampling pattern and iterative algorithm results in perfect reconstruction in some regions and heavy aliasing in others

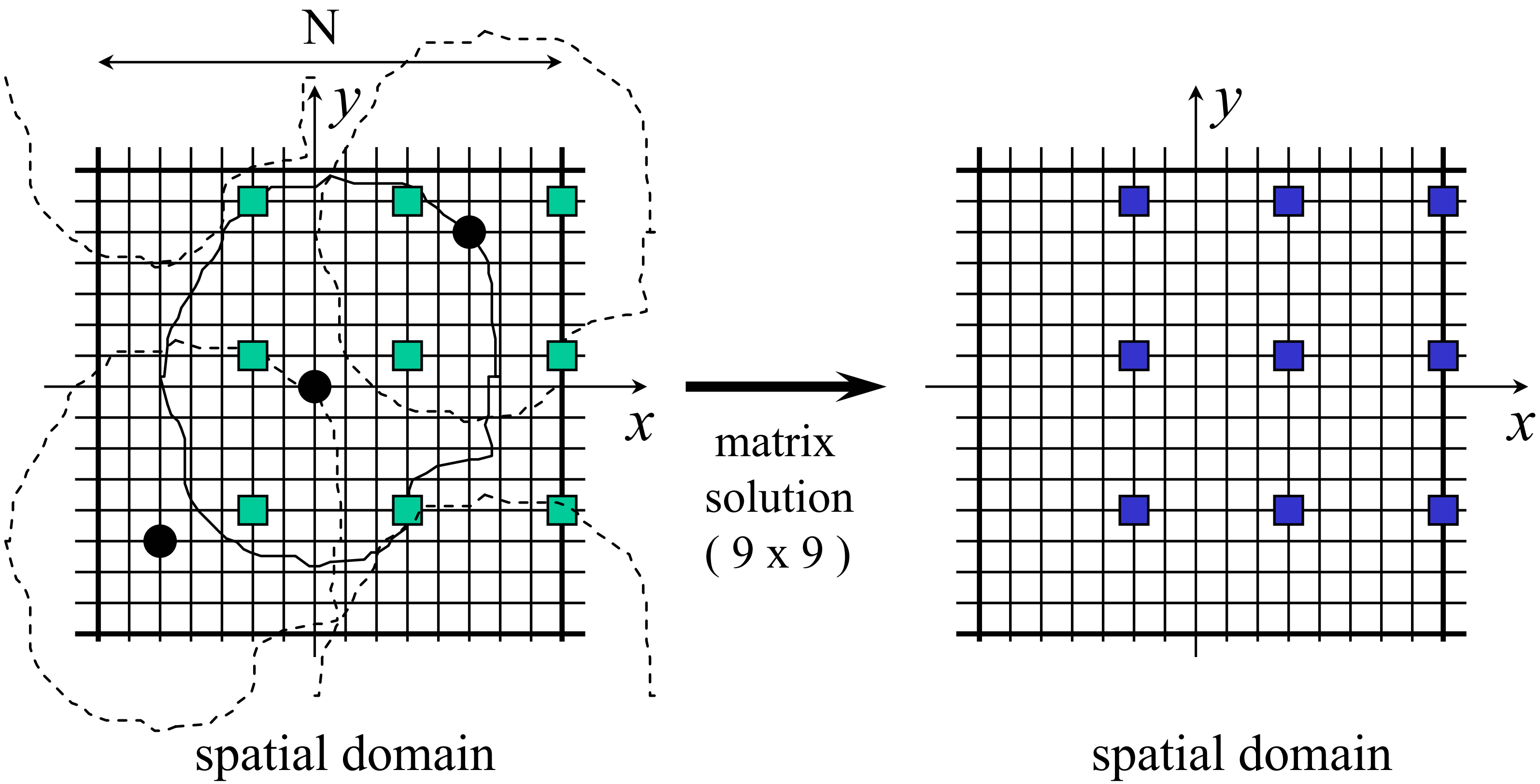




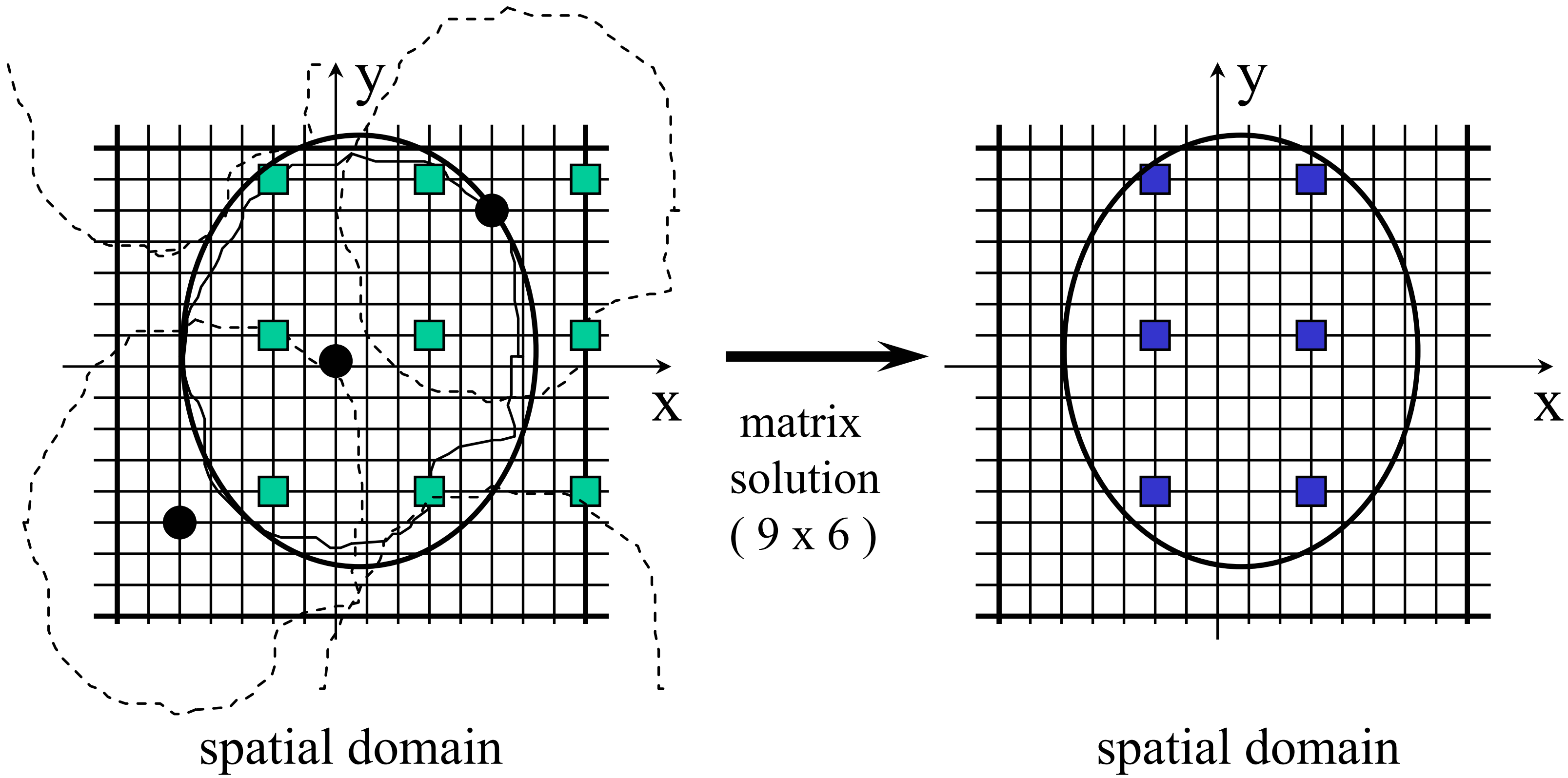
Regular undersampling ⇒ aliased image



Division into subproblems



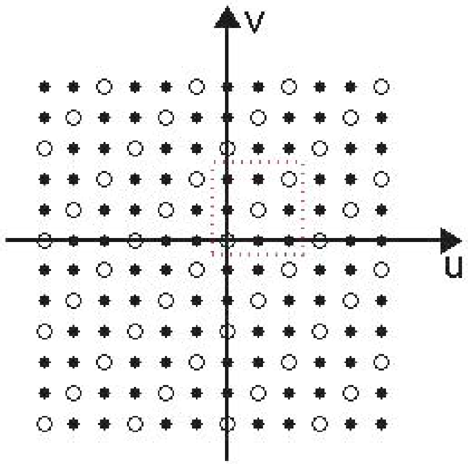
Imposing a support constraint

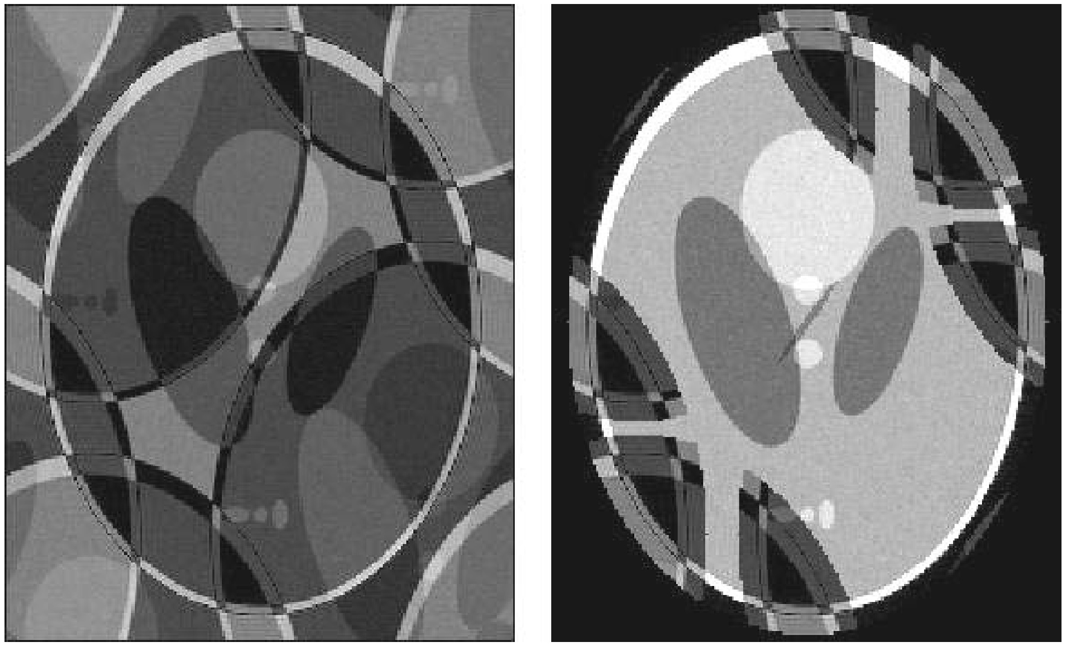


Reconstruction Algorithm

- Split the large overall problem into a number of much smaller subproblems
- Solve each subproblem independently using a matrix-based direct method
- Advantages:
  - Non-iterative
  - Conditioning information available
  - Prediction of unrecoverable regions *before* data acquisition

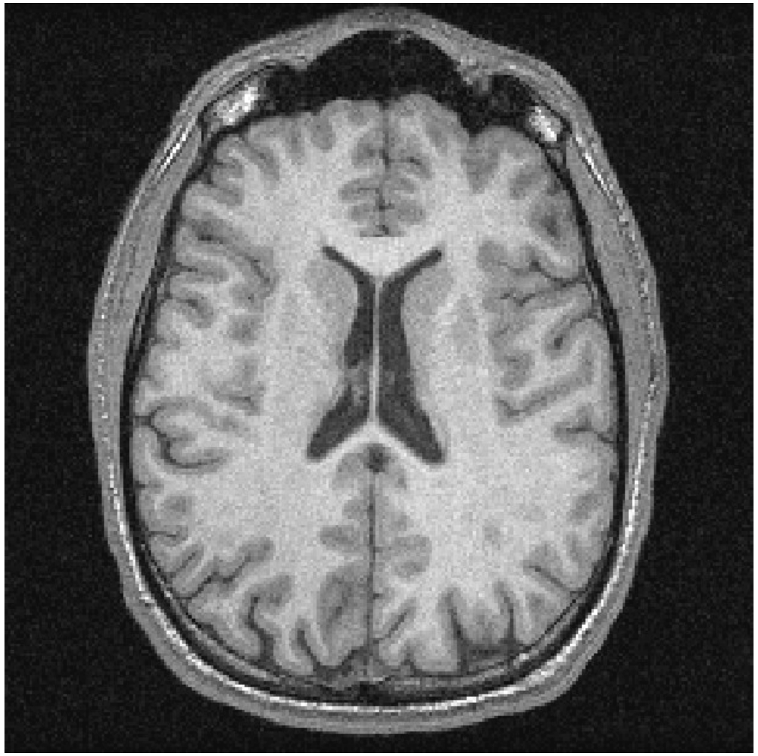
Results: direct partial recovery



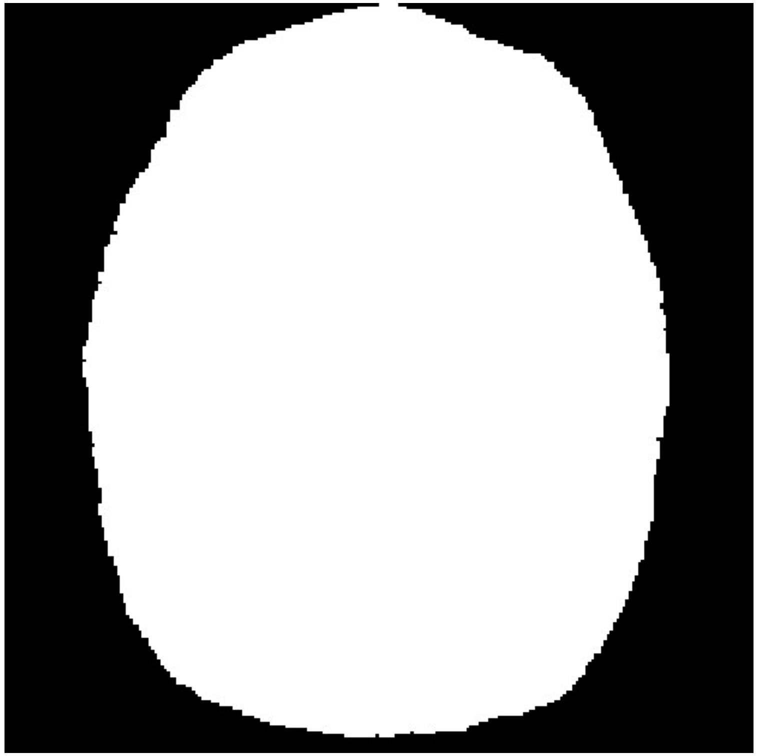


Results: direct partial recovery

original

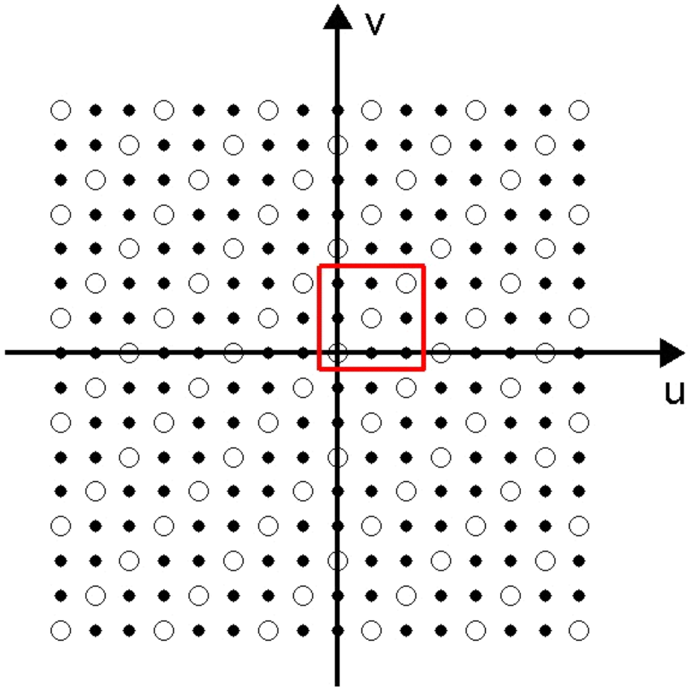


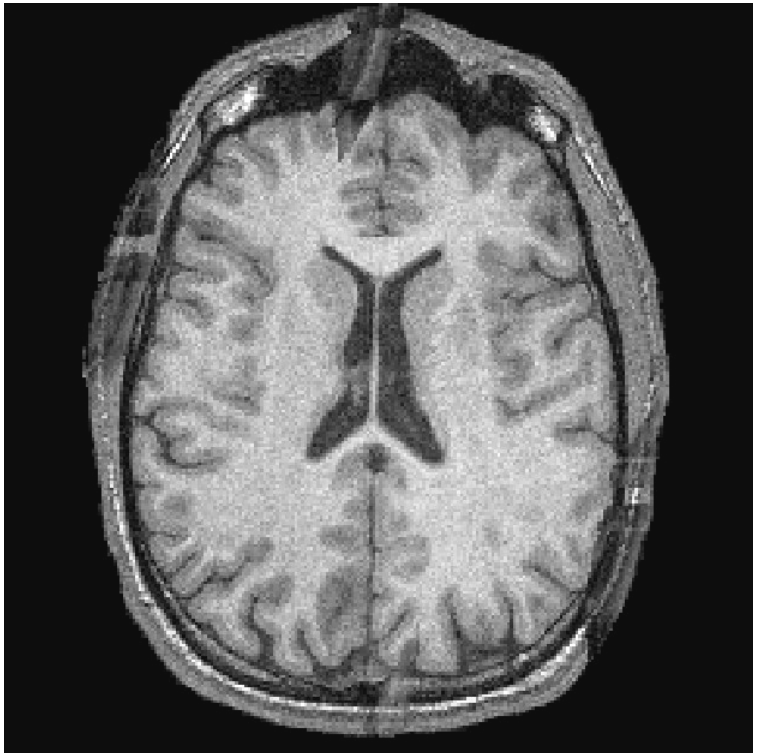
support



Results: direct partial recovery

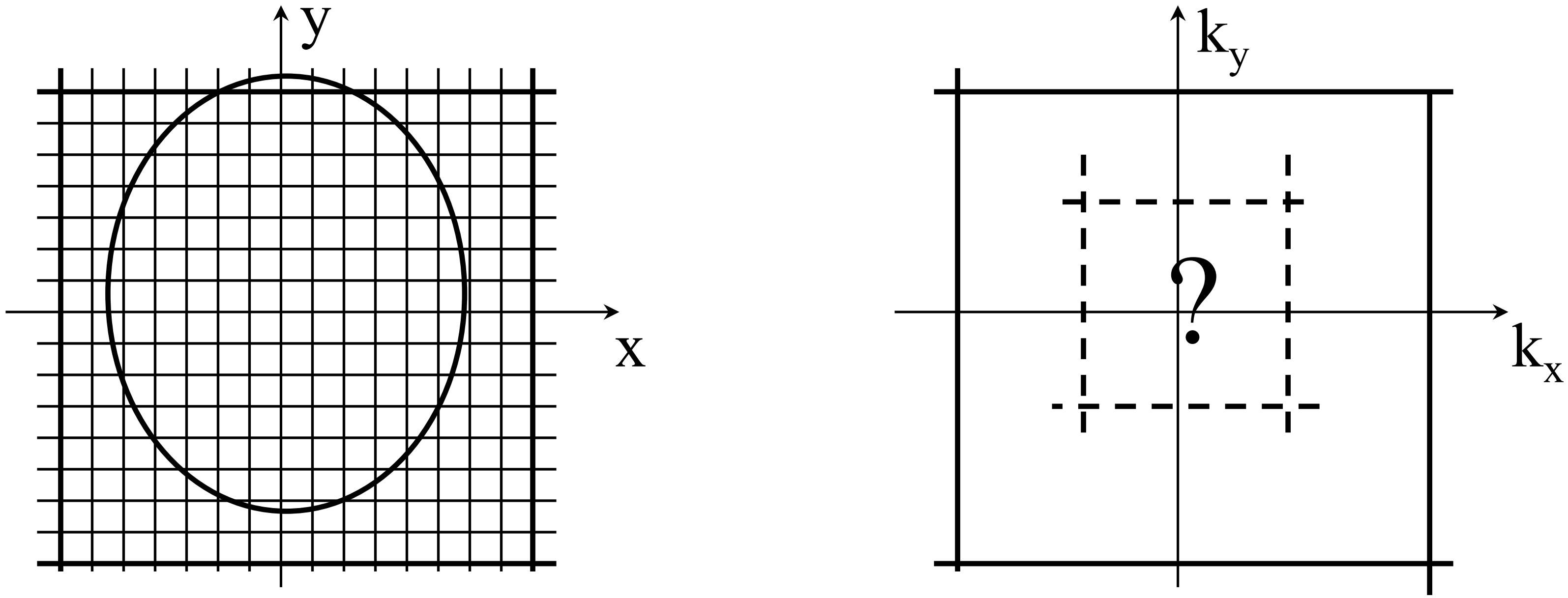
k-space sampling





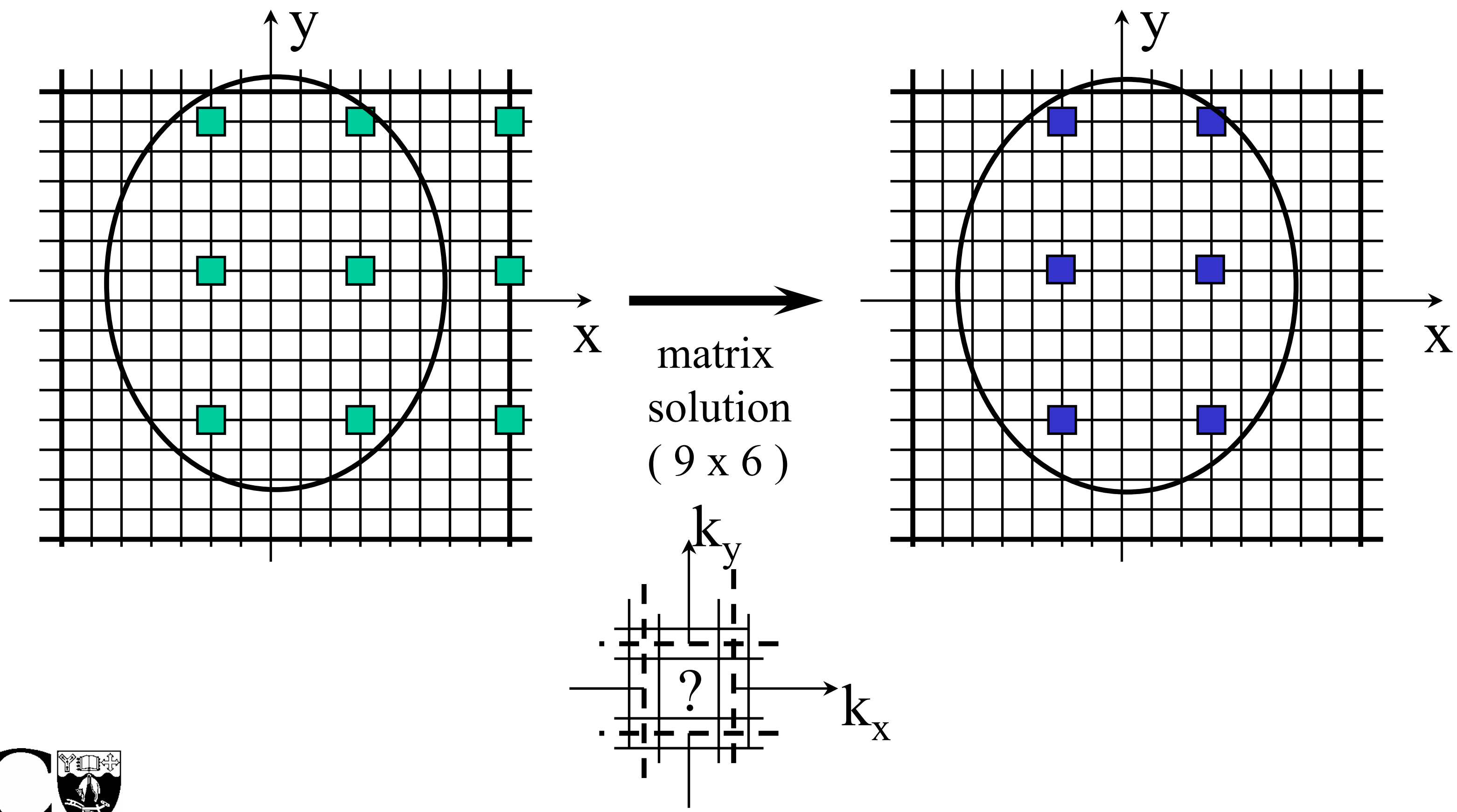
“Universal” sampling patterns

Given a support . . .

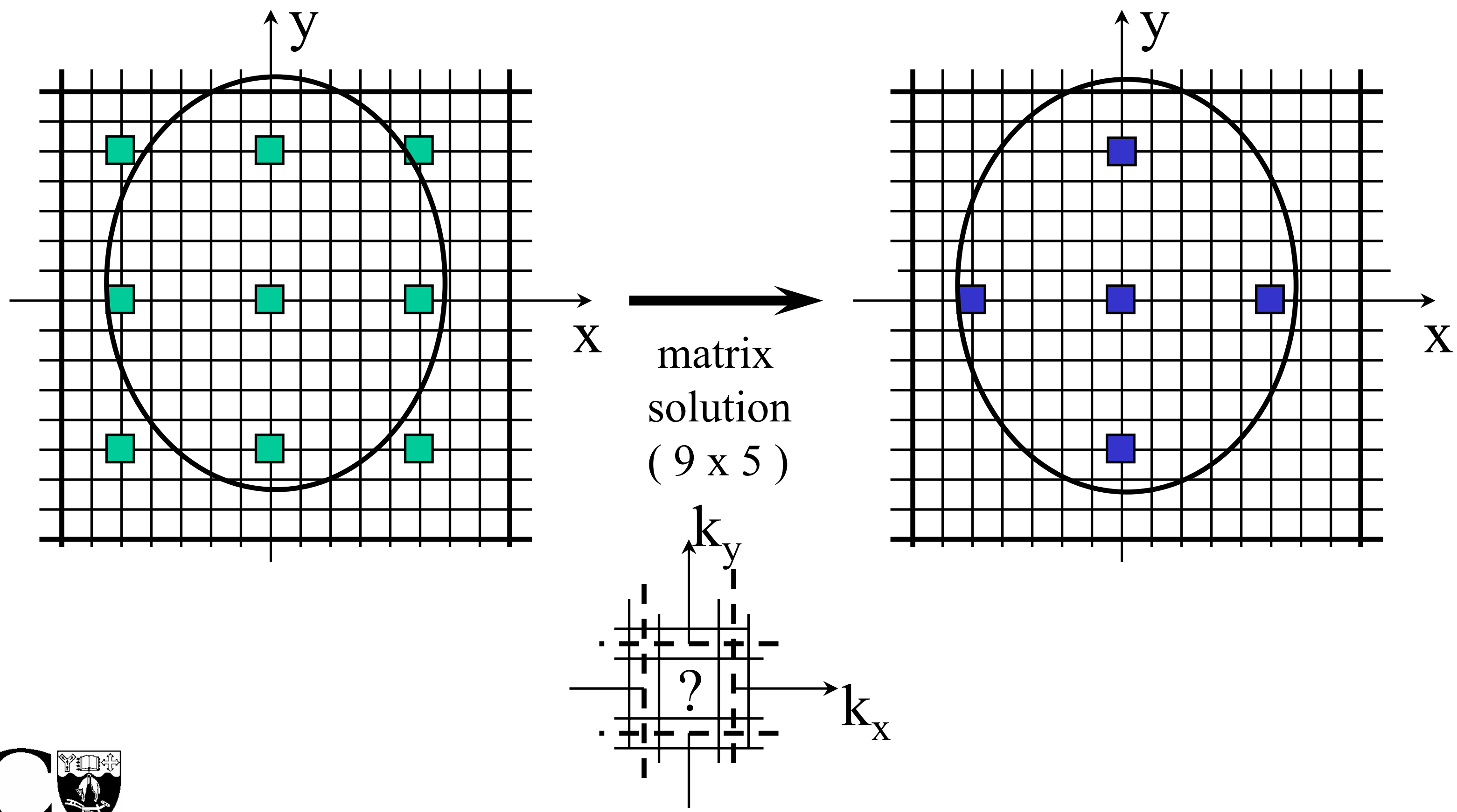


. . . which pattern gives a completely recoverable image?

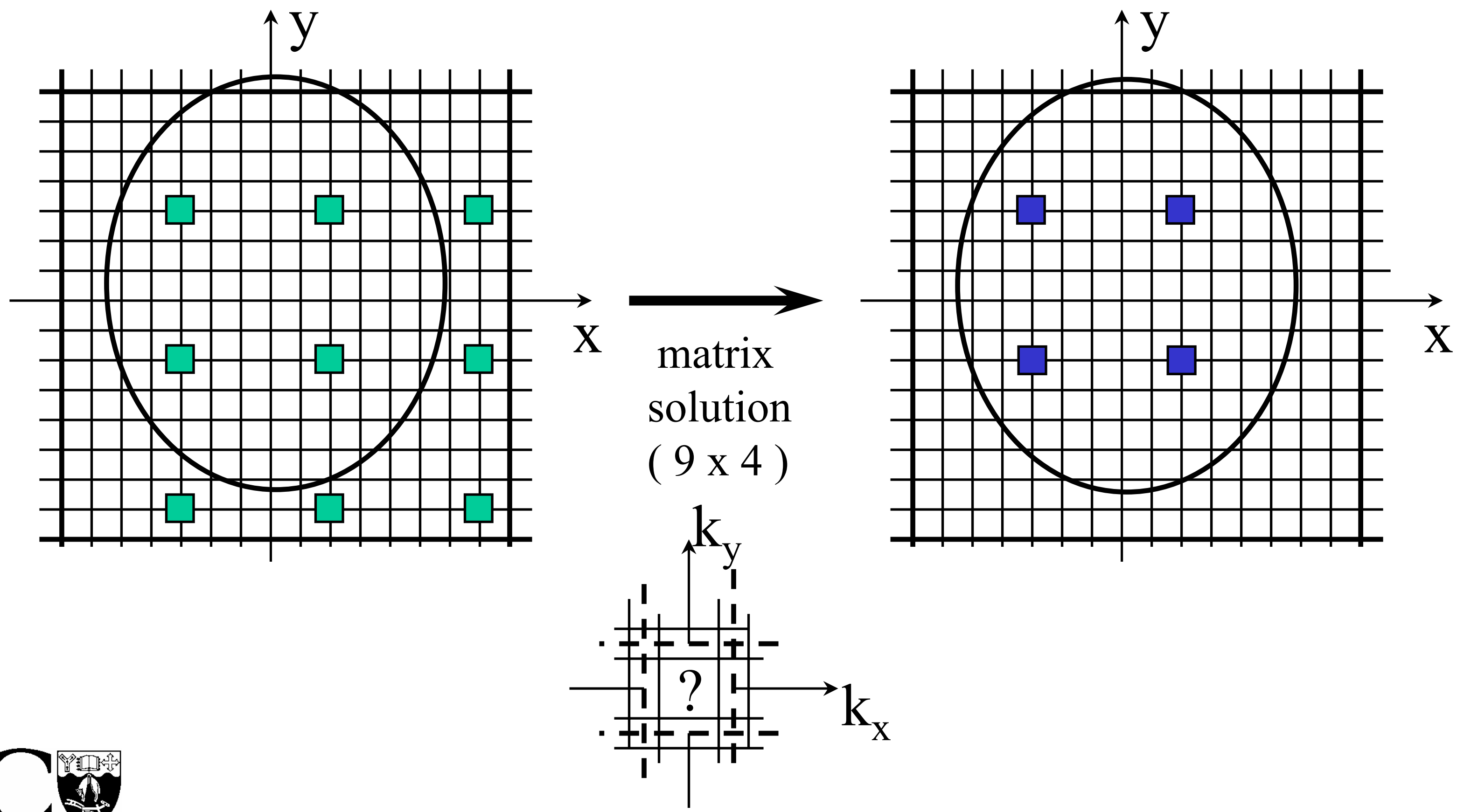
Universal sampling pattern



Universal sampling pattern

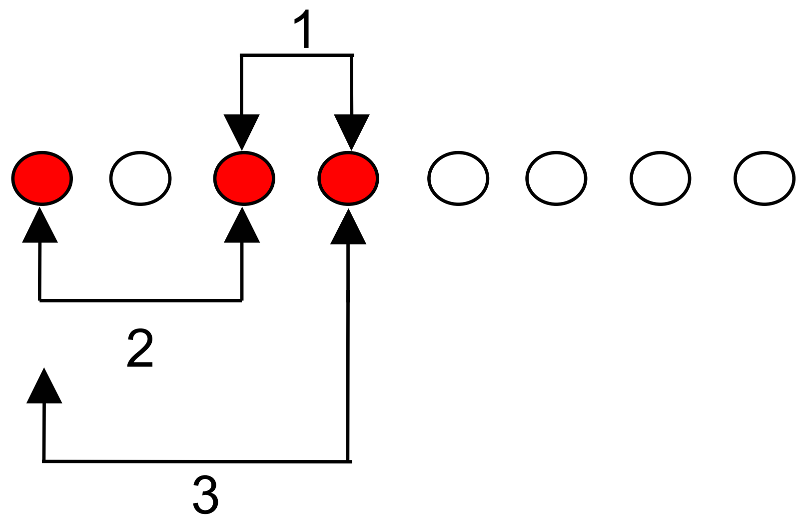


Universal sampling pattern

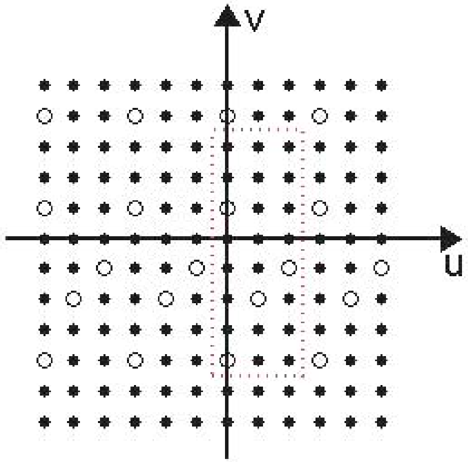


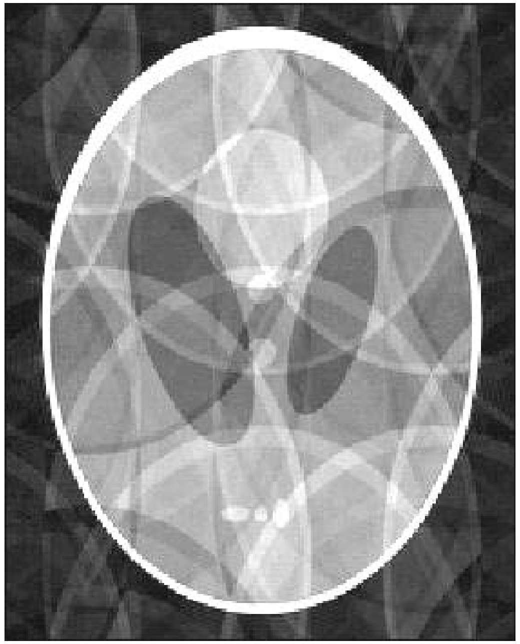
Finding universal patterns

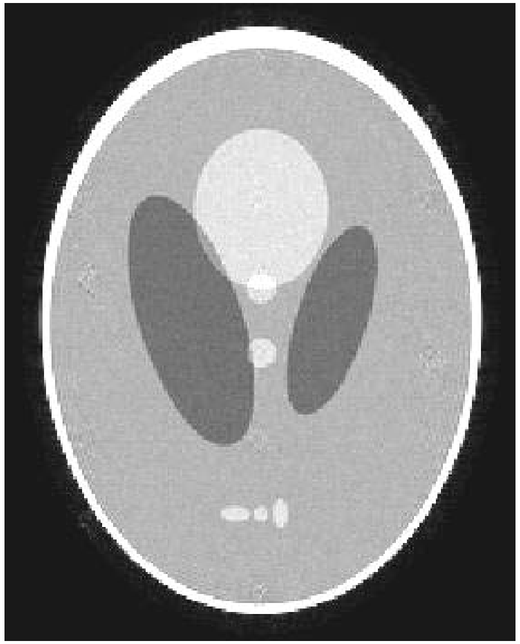
- 1-D problem related to higher dimensions in certain circumstances
- There are ${}^{N}C_{p}$ possible sampling patterns
  - Which are universal?
  - Which are 'better'?
- Use heuristic metrics to ensure a fast algorithm
  - Based on distances between sample locations



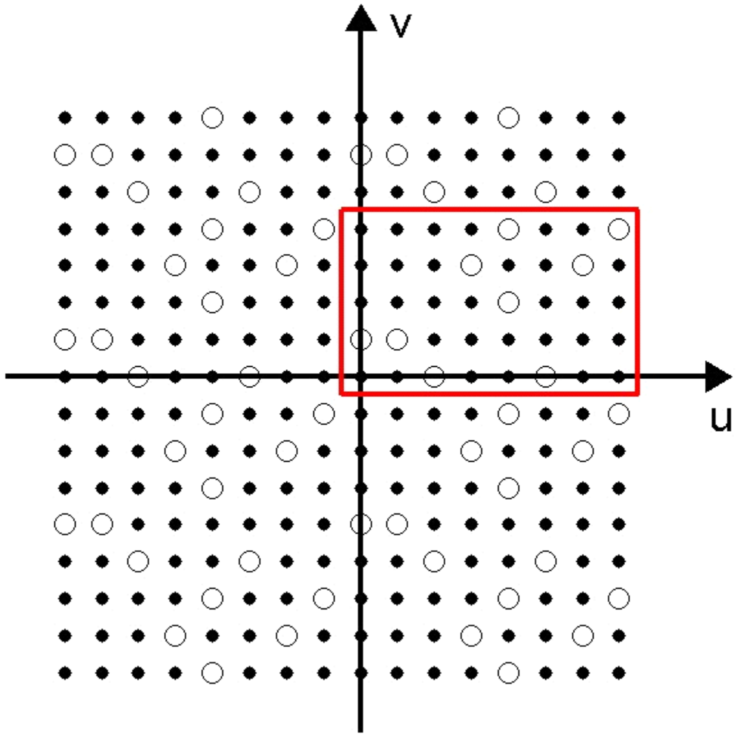
Result: recovery from a universal sampling pattern

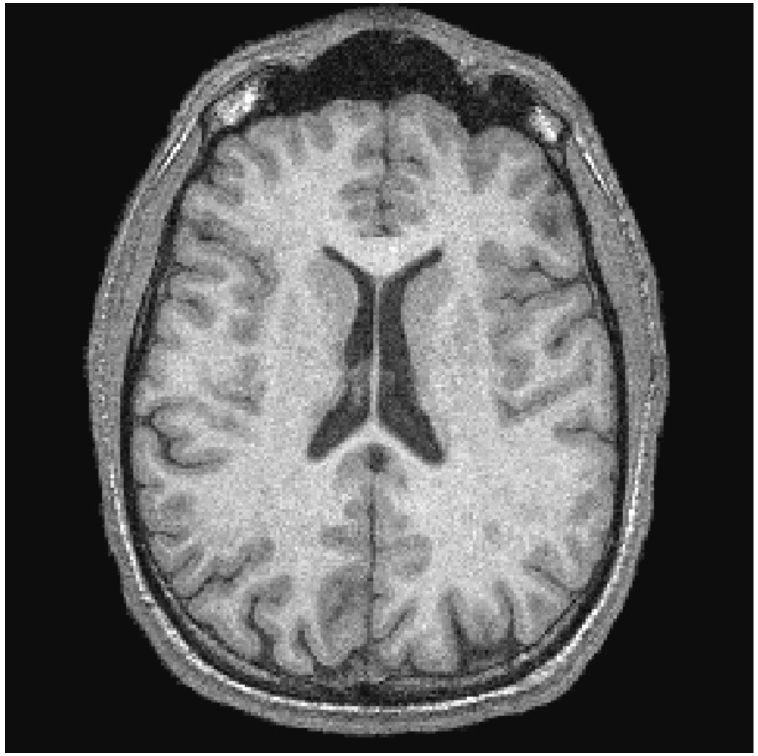






Result: recovery from a universal sampling pattern





Speed of metrics-based algorithm

A – a sequential search method based on linear algebraic properties
B – our algorithm employing the metrics in a sequential search

Time to find a pattern based on a 15 x 8 block:

A	1590 sec
B	0.06 sec

Note that an exhaustive search becomes impractical for N >> 20

Conclude that prior information can allow the Nyquist limit to be relaxed and useful sampling patterns can be found with a fast algorithm

Recently introduced tools

- Wavelets
- Neural networks
- Genetic algorithms

Valuable toolbox items or mainly fashion?

Wavelets

Basis functions are compact in both signal and frequency spaces

Extent in signal space is measured in wavelengths

Both impulse-like and wave-like properties of the signal can be represented and located

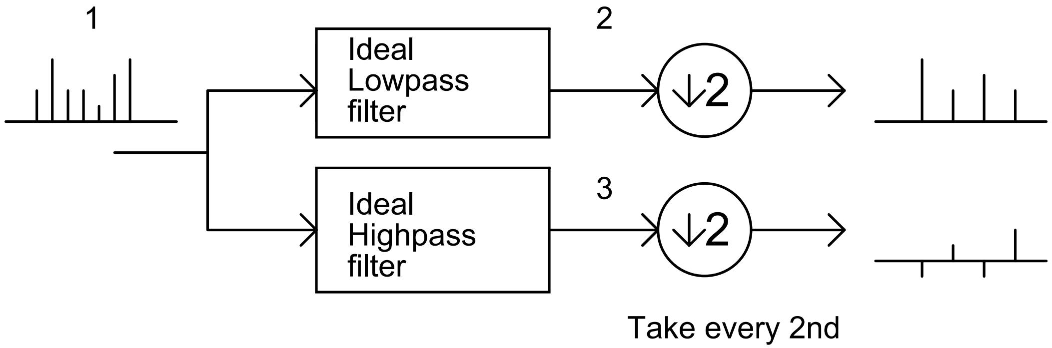
Both continuous (complex) and discrete forms of transform

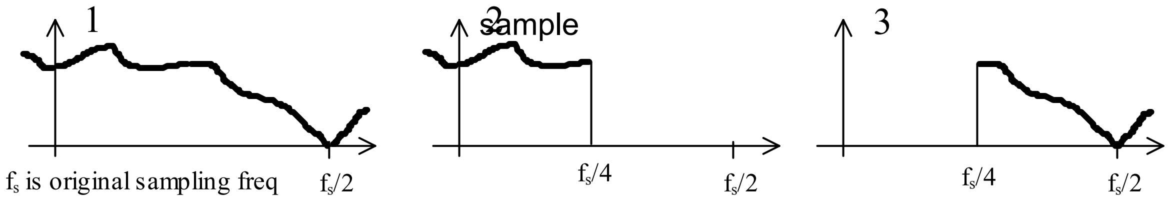
Discrete wavelet transform (DWT) is useful at isolating and locating features in an image

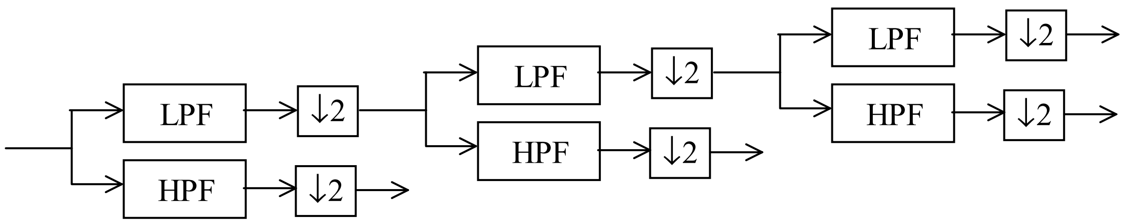
DWT is O(N) - compare: FFT is O(N logN)

2-D DWT has been incorporated into JPEG2000

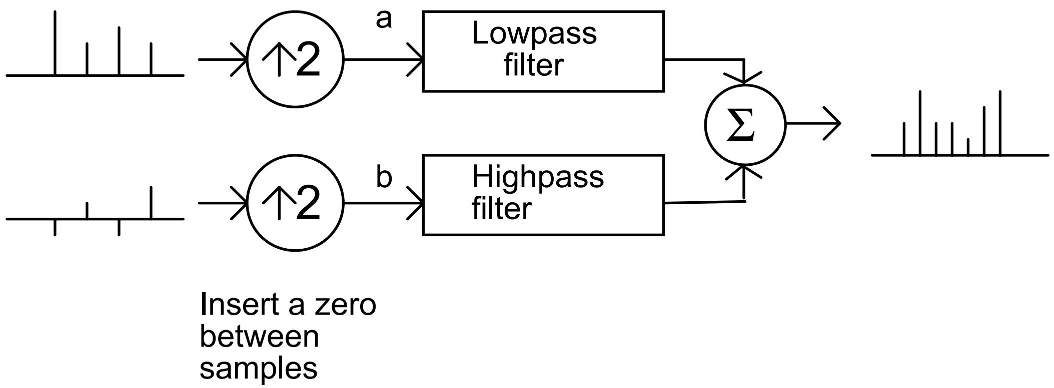
Wavelets - DWT



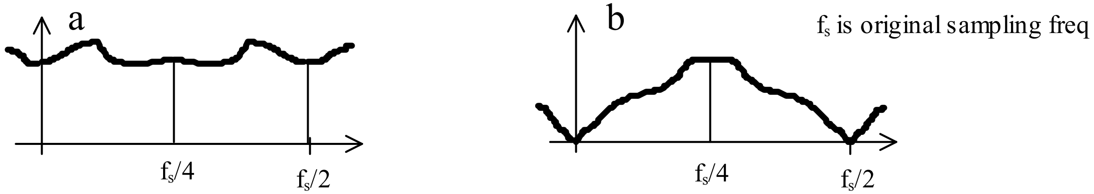




Wavelets

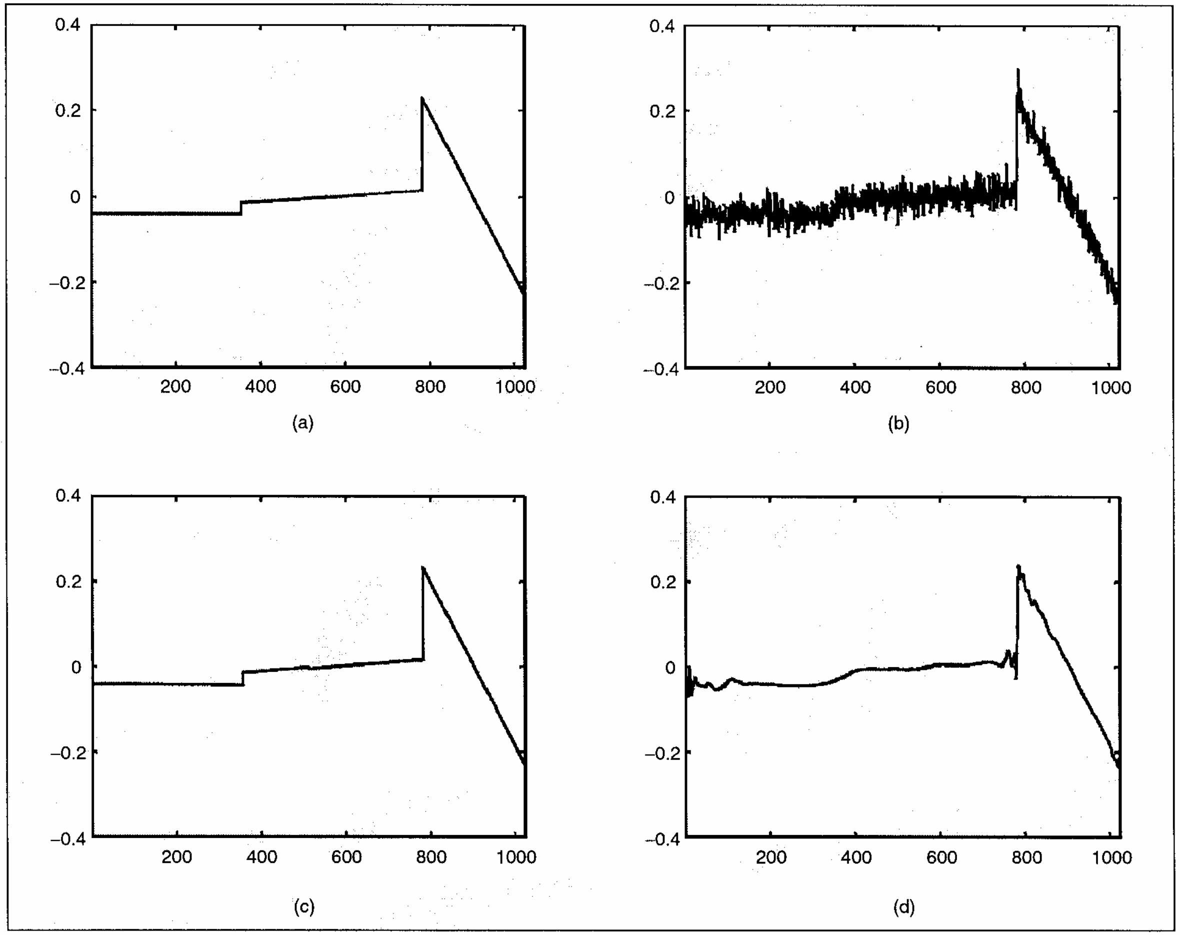


a
Lowpass filter
b
Highpass filter
Σ
↑2
↑2
Insert a zero between samples



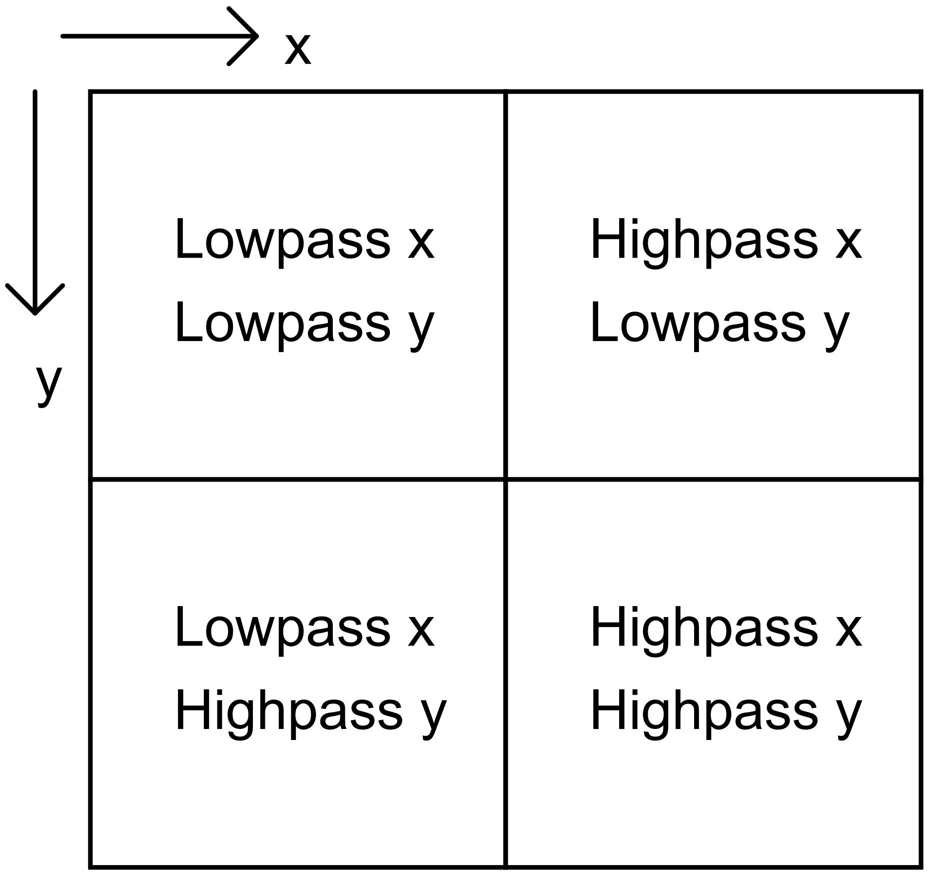
a
$f_s/4$
$f_s/2$
b
$f_s/4$
$f_s/2$
f_s is original sampling freq

Wavelets – "denoising"



▲ 11. Denoising in wavelet domain using thresholding and footprints. (a) Original. (b) Noisy version (SNR = 11.1 dB). (c) Denoising using footprints (SNR = 31.4 dB). (d) Denoising using standard wavelet thresholding (SNR = 20.1 dB).

Wavelets - 2-D DWT



Neural networks

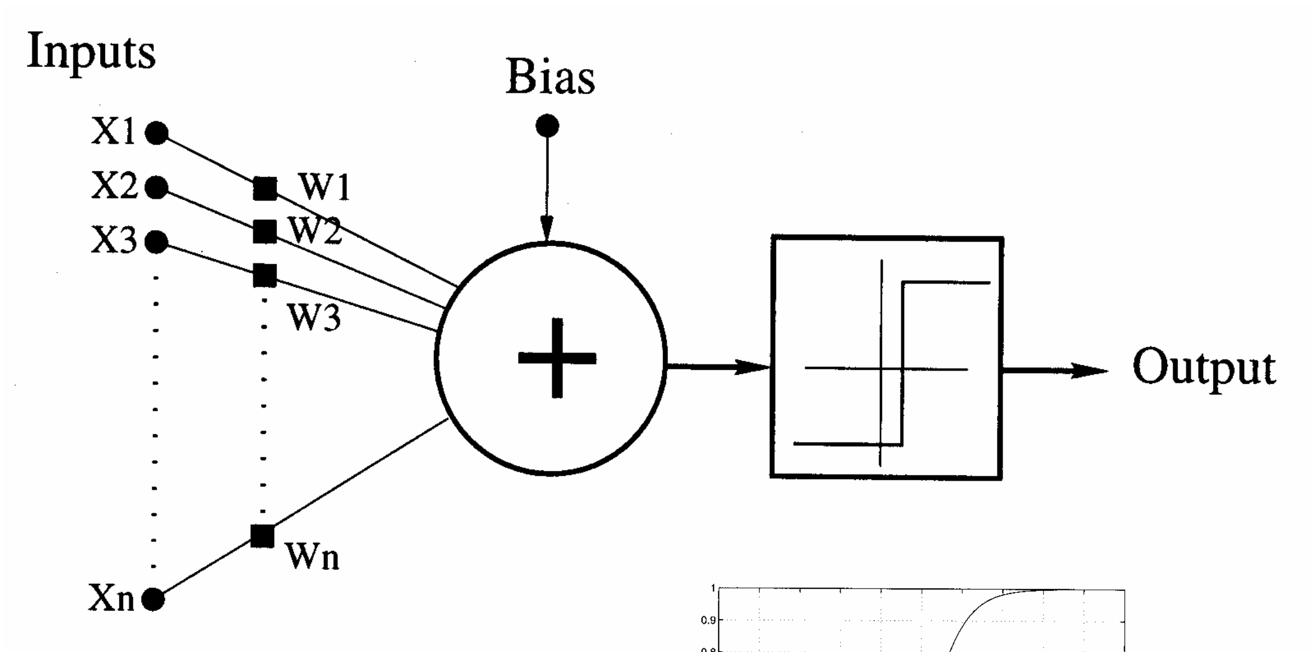
Based on ideas formulated by McCulloch and Pits in the 1940s

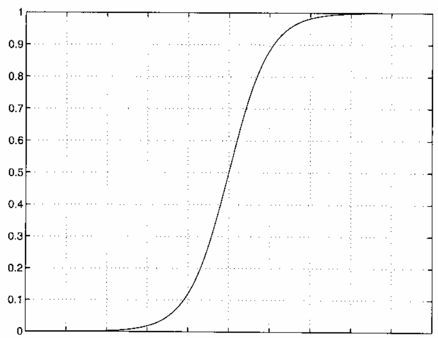
Blossomed with the back propagation algorithm in the 1980s

Radial basis function networks and Kohonen self organising networks have since been added

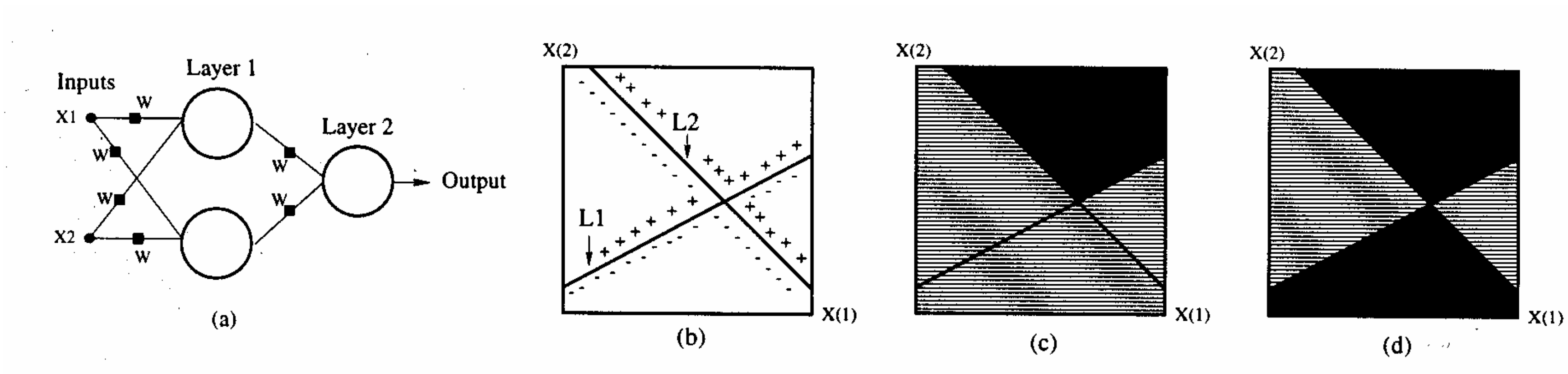
Useful for providing increased performance where signals are not generated by linear, stationary and Gaussian systems

Neural networks

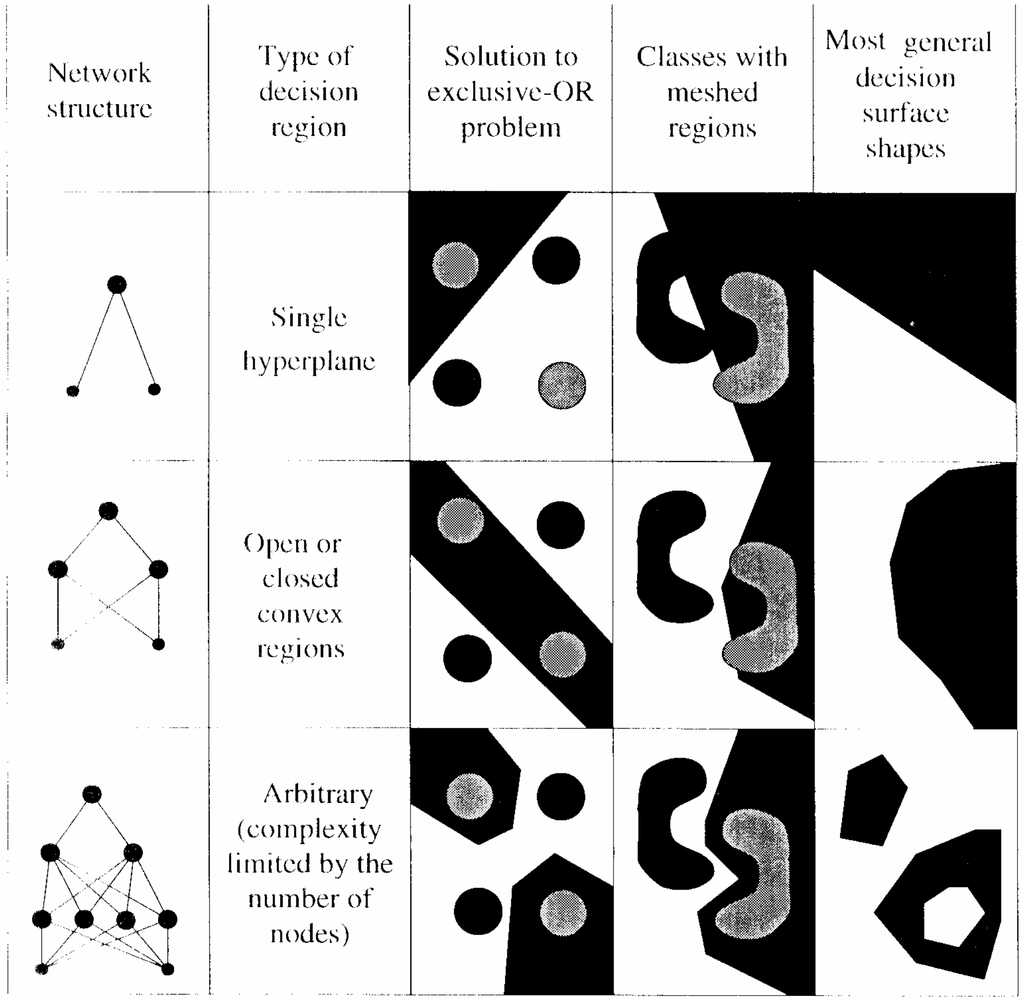




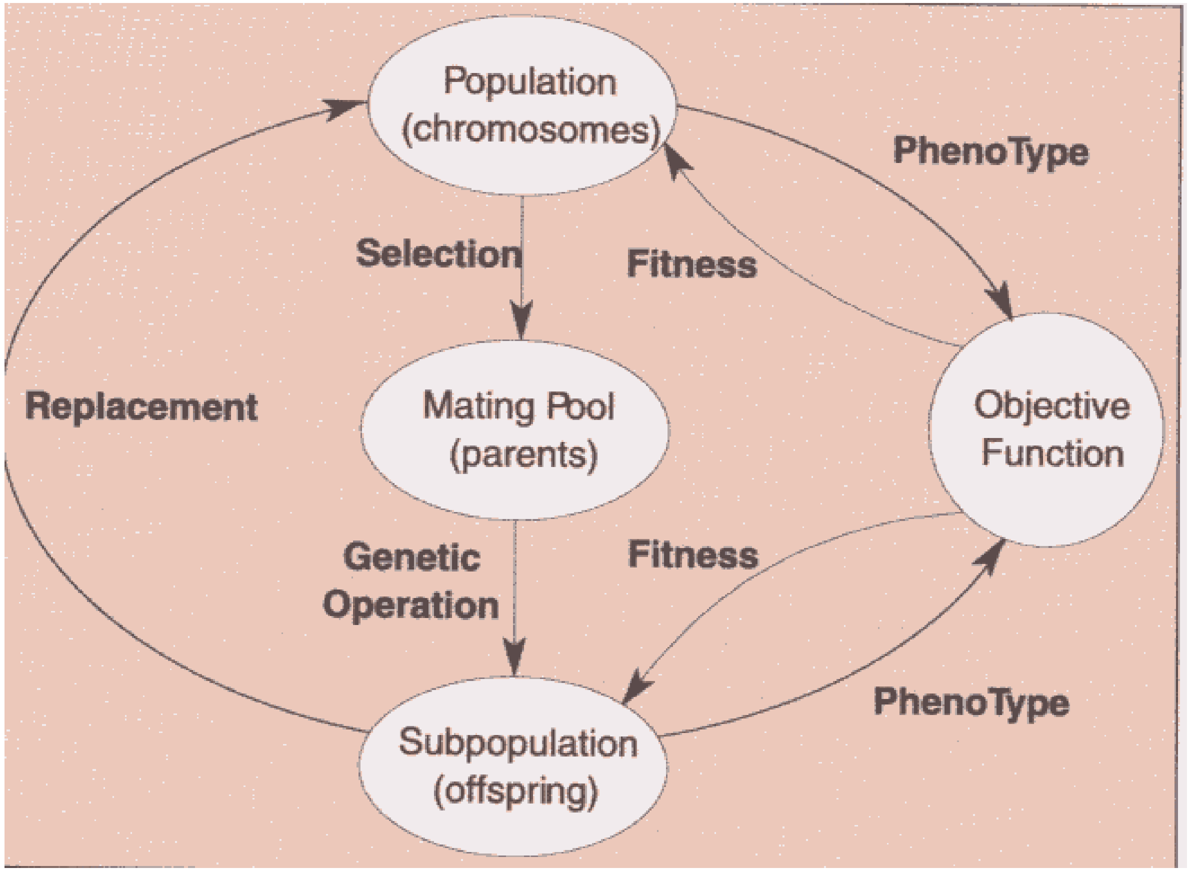
Neural networks



Neural networks



Genetic algorithms



Genetic algorithms

1) Randomly generate an initial population $\mathbf{X}(0):=(\mathbf{x}_1,\mathbf{x}_2,...\mathbf{x}_N)$;
2) Compute the fitness $F(\mathbf{x}_i)$ of each chromosome $\mathbf{x}_i$ in the current population $\mathbf{X}(t)$;
3) Create new chromosomes $\mathbf{X}_r(t)$ by mating current chromosomes, applying mutation and recombination as the parent chromosomes mate;
4) Delete numbers of the population to make room for the new chromosomes;
5) Compute the fitness of $\mathbf{X}_r(t)$, and insert these into population;
6) $t:=t+1$, if not (end-test) go to step 3, or else stop and return the best chromosome.

Genetic algorithms

Strengths	- complement the conventional optimisation methods
	- can be made to be adaptive
Weaknesses	- difficult to predict performance for a GA
	- slow
	- very wide range of choices for the designer

Implementation issues and trends

Industry imperatives are driven by:

Multimedia	- compression of image, sound, video
Communications	- error detection/correction coding
	- encryption
	- low power
Pattern recognition	- “homeland” security and personal identification
	- watermarking
	- database mining
Data networks	- packet processing
	- smart routing

Implementation issues and trends

Hardware development is dominated by:

DSP chips	- more and more pipelining
	- wider and wider instructions
	- still based on the MAC instruction
Gate arrays	- DSP cores
	- general-purpose RISC cores

The top of Mt Aspiring (3082 m), New Zealand

just minutes from Christchurch