



Lancaster University



A.G.Kozorezov

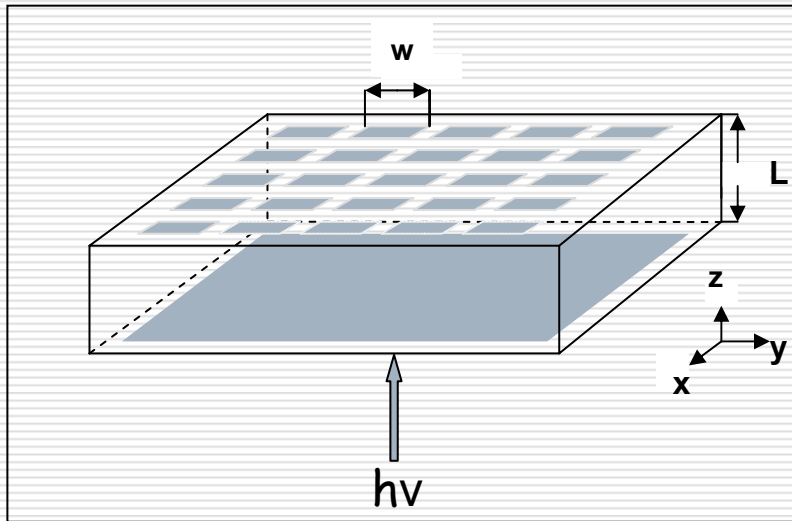
A.Owens,
R. den Hartog,
J.K.Wigmore,
A.Peacock
Hala Al Jawhari

**Analytical model of resolution
degradation of semiconductor
detectors due to carrier trapping**

Overview

- Detector resolution
- Analytical expression for carrier trapping contribution to signal broadening
- Asymptotic behaviour at small photon energy.
- Asymptotic behaviour at high photon energy
- Dependence on electron and hole transport. Optimisation
- TlBr data

Detector resolution



Square matrix on a
pixelated plane

$$z=0 : V=0$$

$$z=L : V=V_b$$

$$\Delta E = 2.355 \sqrt{\epsilon F E + \sigma_e^2 + \alpha_1 E^{\alpha_2}}$$

Analytical expression for carrier trapping contribution to signal broadening

$$\Delta E = 2.355 \sqrt{\varepsilon F E + \sigma_e^2 + G(E) E^2}$$

ε is e-h pair creation energy, F-Fano-factor, σ_e is electronic noise

$$G(E) = \frac{1 - \exp(-\Gamma(E)L)}{\Gamma(E)} \frac{\int_0^L dz \exp(-\Gamma(E)z) Q^2(\vec{r}_a, z)}{\left[\int_0^L dz \exp(-\Gamma(E)z) Q(\vec{r}_a, z) \right]^2} - 1$$

E is the photon energy, and $\Gamma(E)$ is the X-ray absorption coefficient

$$Q(\vec{r}_a, z_a) = \exp\left(-\frac{L - z_a}{l_e}\right) + \frac{1}{l_e} \left[L - z_a + \int_{z_a}^L dz \Phi_w(\vec{r}_a, z) \right] + \frac{1}{l_h} \int_0^{z_a} dz \Phi_w(\vec{r}_a, z)$$

$l_{e,h}$ are electron (hole) mean drift paths, Φ_w is the weighting potential

Asymptotic behaviour at small photon energy

$$E \rightarrow 0, \quad \Gamma L \gg 1$$

Absorption close to cathode

$$G(E) \rightarrow \frac{1}{\Gamma^2(E)l_e^2} (\ll 1)$$

At small photon energy trapping strength factor $G(E)$ shows universal behaviour. It does not depend on both pixel geometry and lateral co-ordinates of the photon absorption site. Its energy dependence is determined by that of the absorption coefficient

Asymptotic behaviour at high photon energy

$E \rightarrow \infty, \Gamma L \ll 1$ Uniform in-depth absorption

$$\Delta E = 2.355 \sqrt{\epsilon F E + \sigma_e^2 + \alpha(l_e, l_h, L, R_0, \vec{r}_a) E^2}$$

For $l_{e,h} \gg L$ the expression for α is simplified:

$$\alpha_{\text{single}} = \alpha_{\text{single}}(l_e, l_h, L) = \frac{L^2}{45} \left(\frac{1}{l_e} + \frac{1}{l_h} \right)^2 - \frac{L^2}{12l_e^2} \quad \text{for single pixel geometry}$$

$$\alpha_{\text{small}} = \frac{1}{12} \left(\frac{L}{l_e} \right)^2 \left(1 + O\left(\frac{R_0}{L} \right) \right) \approx \frac{1}{12} \left(\frac{L}{l_e} \right)^2 = \alpha_{\text{small}}(l_e, L)$$

for small pixel geometry $R_0 \ll L$

no dependence on pixel size and on absorption site location

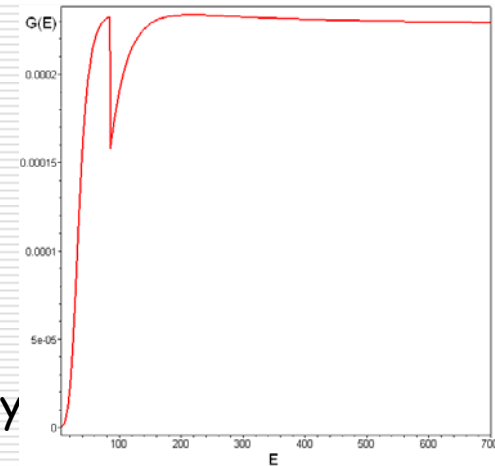
For $l_e = l_h = l \gg L$

$$\alpha_{\text{single}}(l, l, L) = \frac{1}{180} \left(\frac{L}{l} \right)^2 \ll \alpha_{\text{small}}(l, L)$$

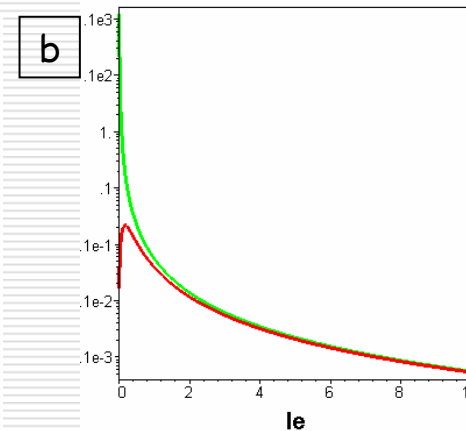
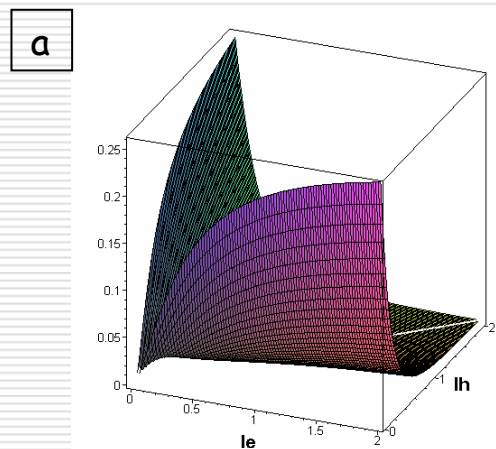
However for $l_e \geq (11/4)l_h \gg L$

$$\alpha_{\text{single}}(l_e, l_h, L) \geq \alpha_{\text{small}}(l_e, L)$$

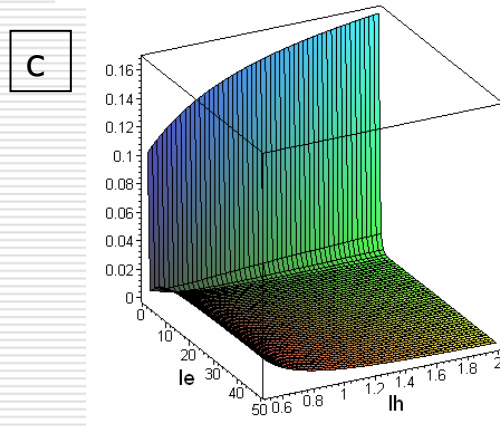
Small pixel geometry is worse than single pixel !!
No volume averaging, stronger fluctuations in near field region



Dependence on electron and hole transport. Optimisation



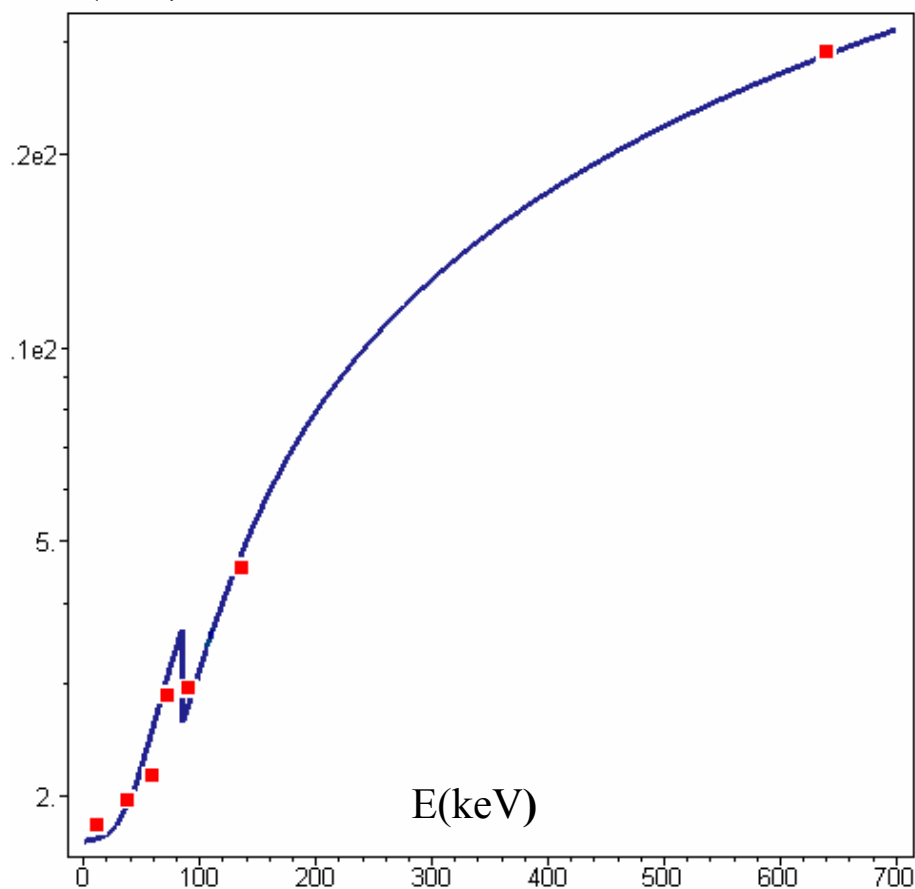
- a. $G_{\infty}(I_e, I_h, L)$ as a function of electron and hole transport parameters for a single pixel geometry (I_e and I_h are in units of L);
- b. $G_{\infty}(I_e, I_e, L)$ vs I_e .
Upper curve: $1/180(L/I_e)^2$



- c. $G_{\infty}(I_e, I_h, L, R_0)$ as a function of electron and hole transport parameters for a small pixel geometry (I_e and I_h are in units of L), $R_0=0.05L$;

TlBr: Experiment

$\Delta E(\text{keV})$



TlBr detector resolution
 3x3 square matrix array
 $L=1000\mu\text{m}$, $R_0=350\mu\text{m}$, $V_b=300\text{V}$
 $\mu_e T_e = 5 \cdot 10^{-4} \text{cm}^2 \text{V}^{-1}$
 $\mu_h T_h = 3 \cdot 10^{-6} \text{cm}^2 \text{V}^{-1}$
 Solid curve - theory,
 Squares - experiment.

Onodera et al (2004) TlBr
 3x3 square matrix array
 $L=1.8\text{mm}$, $R_0=570\mu\text{m}$, $V_b=1800\text{V}$
 $\mu_e T_e = 1.7 \cdot 10^{-4} \text{cm}^2 \text{V}^{-1}$
 $\mu_h T_h = 6.4 \cdot 10^{-5} \text{cm}^2 \text{V}^{-1}$
 $\Delta E_{\text{experiment}} = 9.7 \text{keV}$ at $E=660 \text{keV}$
 $\Delta E_{\text{theory}} = 34.4 \text{keV}$. However, for
 $\mu_e T_e = 5 \cdot 10^{-4} \text{cm}^2 \text{V}^{-1}$ $\Delta E_{\text{theory}} = 9.7 \text{keV}$

Summary

- ❑ The noise due to incomplete charge collection because of carrier trapping results in the resolution degradation described by the $G(E)E^2$ variance.
- ❑ $G(E)$ has universal low energy asymptotic with no dependence on pixel geometry
- ❑ In the opposite limit of high photon energies G is a constant depending on both pixel geometry and carrier transport parameters
- ❑ Theoretical resolution curve shows an excellent agreement with experiment for TlBr detector in the range 5-660 keV without any fitting parameters.