High Energy Hadronic Scattering in the Color Glass Condensate

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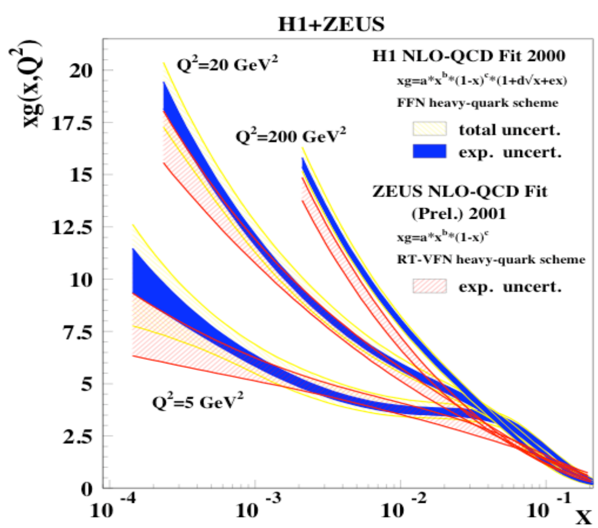
Recent Reviews on parton saturation & the CGC:

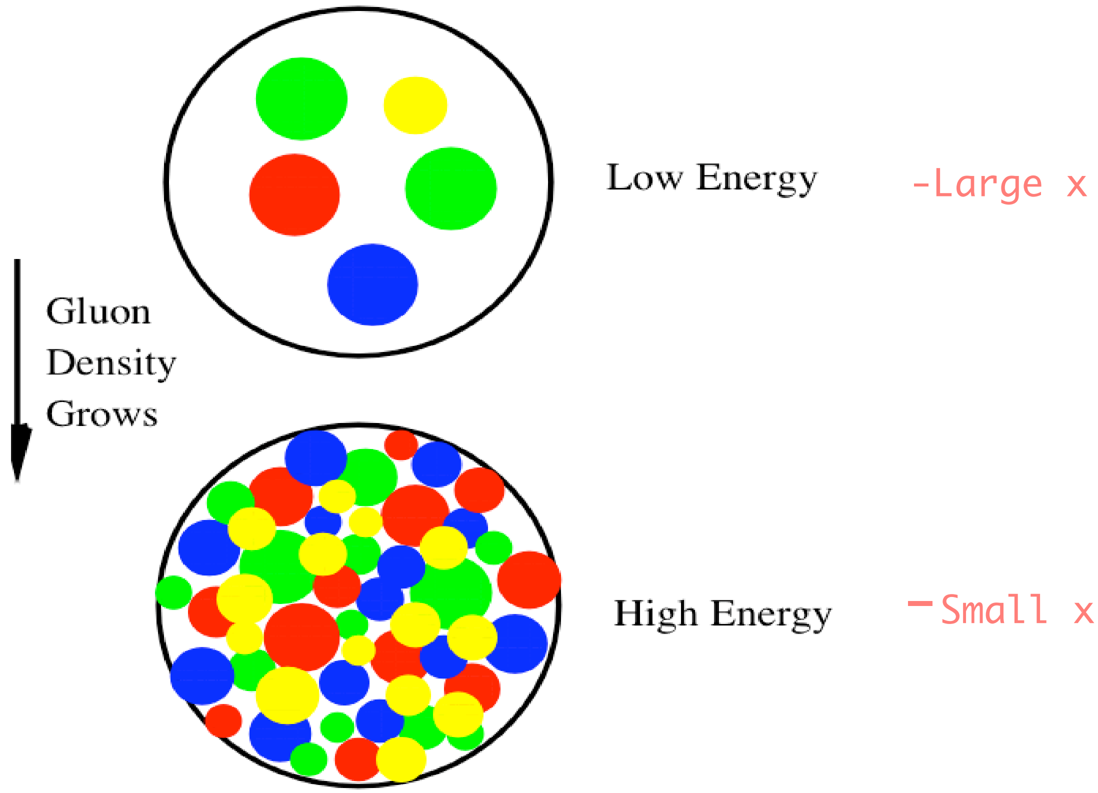
- L. McLerran, hep-ph/0311028
- E. Iancu & R. Venugopalan, hep-ph/0303204
- A. H. Mueller, hep-ph/9911289

Outline of talk:

- A classical effective theory for high energy QCD
- Quantum evolution *a la* JIMWLK and BK
- Hadronic scattering and k_t factorization in the Color Glass Condensate
- What the CGC tells us about the matter produced in AA collisions at RHIC.
- Thermalization and other open issues

Parton saturation at small x

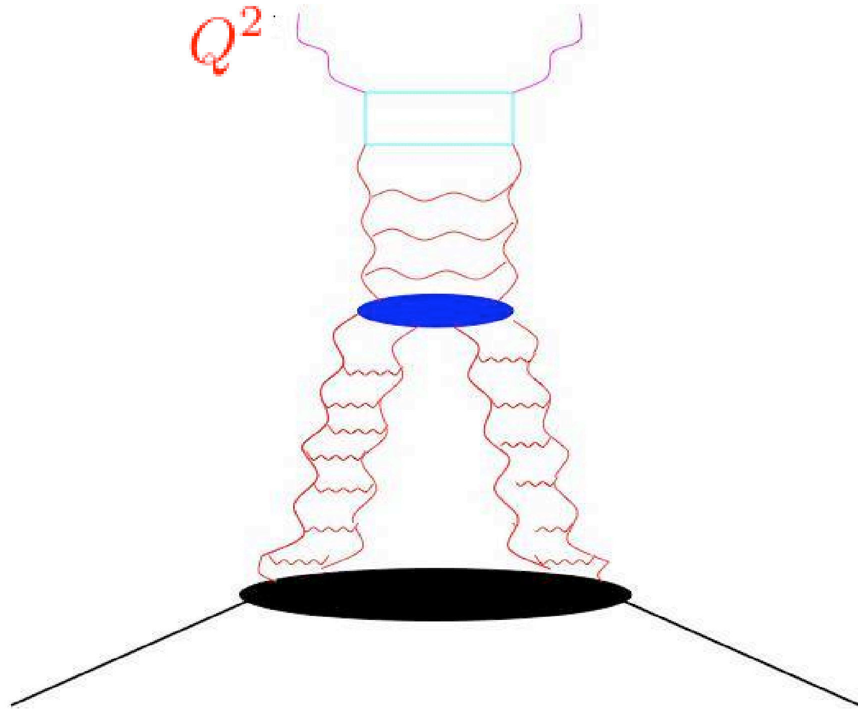




Phase space density grows rapidly-BFKL evolution breaks down when partons begin to overlap in transverse plane

Gluon density saturates at phase space density f= $1/\alpha_S$

Gluon recombination-higher twist effects



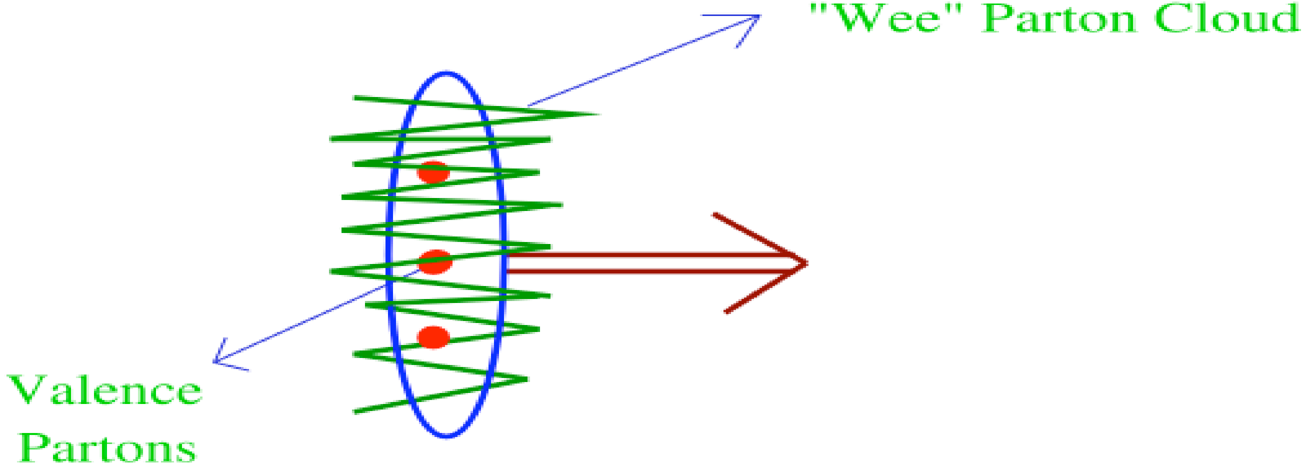
Gribov, Levin, Ryskin
Mueller, Qiu
Blaizot, Mueller

Recombination effects compete with DGLAP Bremsstrahlung effects when

$$\alpha_S x G(x, Q^2) \sim R^2 Q^2$$

Saturation of the gluon density for $Q \equiv Q_s(x)$

A hadron at high energies



$$|h> = |qqq> + |qqqg> + \cdots + |qqq \cdots gggq\bar{q}>$$

Each wee parton carries only a small fraction

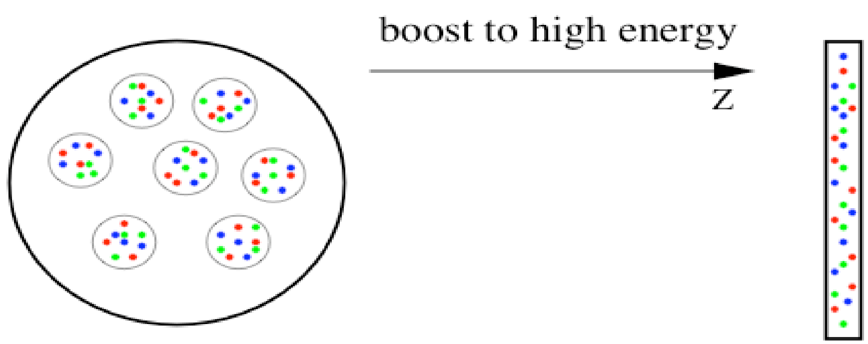
$x = k^+/P^+$ of the momentum P^+ of the hadron

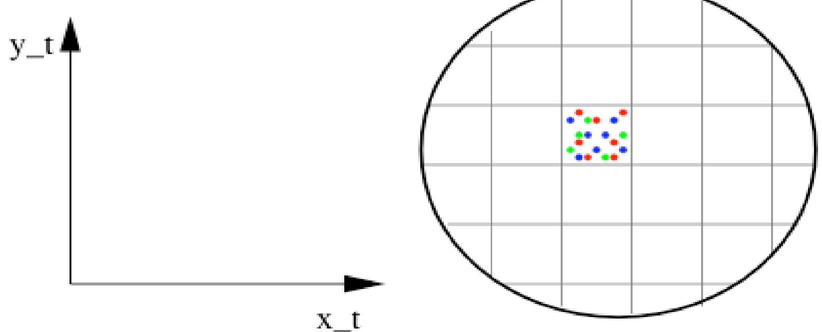
What is the behavior of wee partons in a high energy hadron?

Classical Effective Theory

McLerran, RV; Kovchegov; Jalilian-Marian, Kovner,McLerran,Weigert

Consider large nucleus in the IMF frame: $P^+ \to \infty$

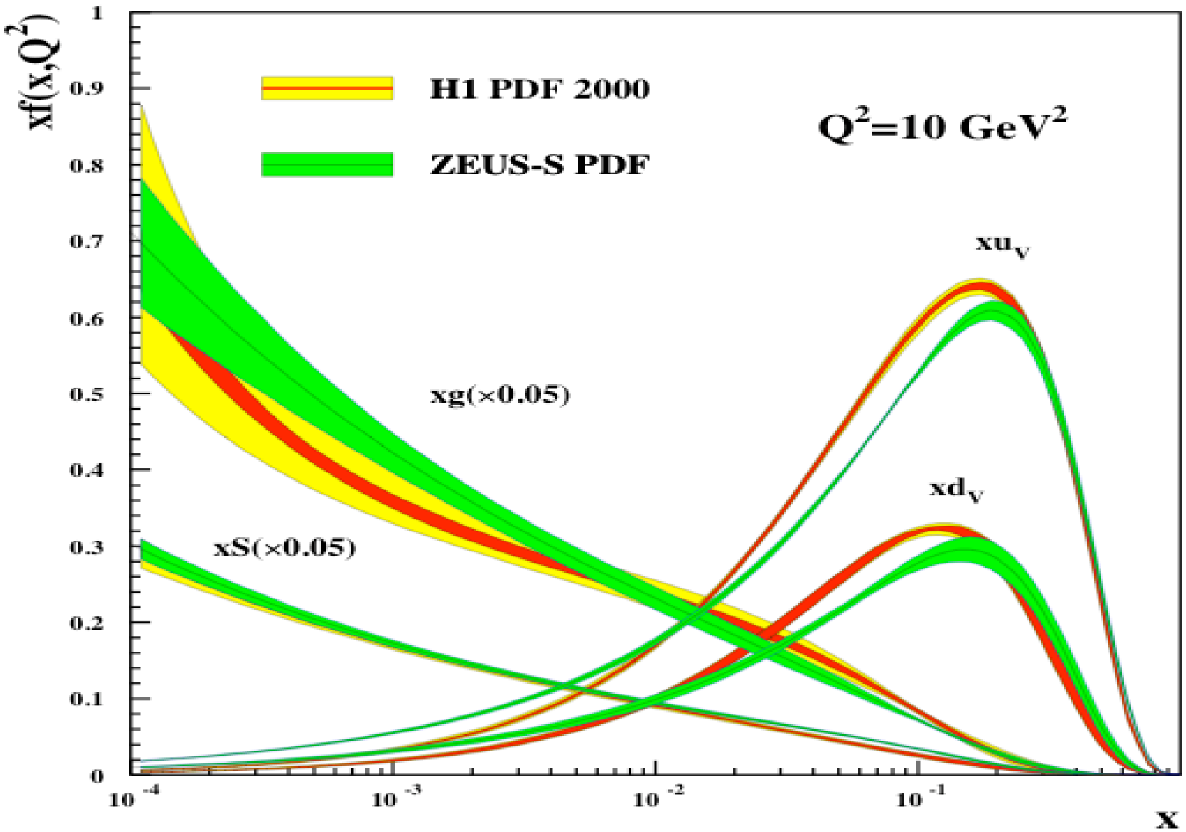




One large component of the current-others suppressed by $\frac{1}{P^+}$

Wee partons see a large density of valence color charges at small transverse resolutions.

Born-Oppenheimer: separation of large x and small x modes

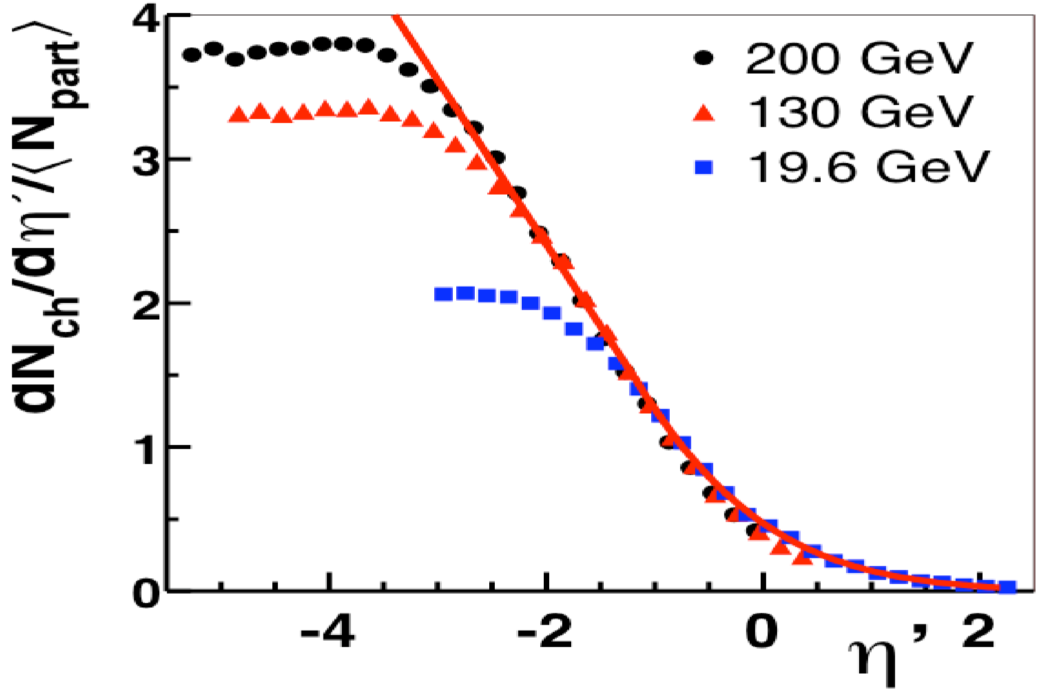


$$\tau_{\rm wee} \sim \frac{1}{k^-} = \frac{2k^+}{k_\perp^2} \equiv \frac{2x\,P^+}{k_\perp^2}$$

$$\tau_{\rm valence} = \frac{2P^+}{k_\perp^2} >> \tau_{\rm wee} \text{ for } x << 1$$

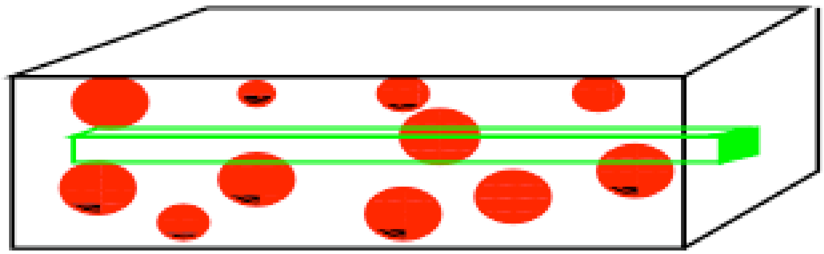
Valence partons are static over wee parton life times

Limiting fragmentation



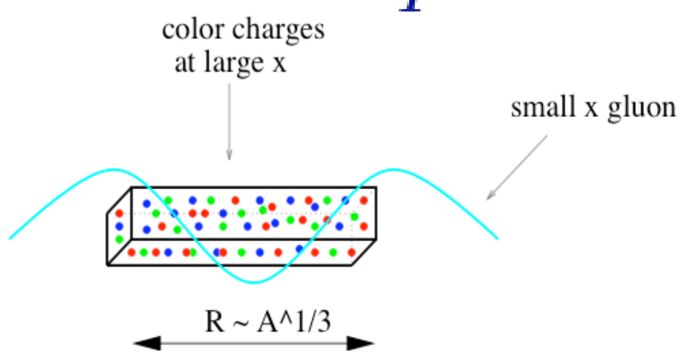
Suggestive that valence partons are recoil-less sources-unaffected by Bremsstrahlung of wee partons

Random walk in SU (N_c)



$$\lambda_{\rm wee} \sim \frac{1}{k^+} \equiv \frac{1}{xP^+} >> \lambda_{\rm valence} \equiv \frac{Rm_p}{P^+}$$

$$=> x << A^{-1/3}\,;$$



$$< \rho^a > = 0\,;\, < \rho^a(x_\perp)\rho^b(y_\perp) > = \mu_A^2 \delta^{ab} \delta^{(2)}(x_\perp - y_\perp)$$

Classical Gaussian random sources

- *Most likely representation -> "Higher dimensional" classical representation.*
- *Sum over distribution of representations => Classical path integral*

Jeon & RV

THE EFFECTIVE ACTION

Generating functional:

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho]}} \right\}$$

Scale separating sources and fields

Gauge invariant weight functional describing distribution of the sources

$$S[A,\rho] = \frac{-1}{4}\int d^4x\, F_{\mu\nu}^2 + \frac{i}{N_c}\int d^2x_\perp dx^- \delta(x^-) \mathrm{Tr}\left(\rho(x_\perp)\, U_{-\infty,\infty}[A^-]\right)$$

where $U_{-\infty,\infty}[A^-] = \mathcal{P}\exp\left(ig\int dx^+ A^{-,a}T^a\right)$

To lowest order, $= -J^+ A^-$ with $J^+ = g\,\rho(x_\perp)\,\delta(x^-)$

(*Note:* $\mathrm{Tr}\left(\rho(x_\perp)\ln(U_{-\infty,\infty})\right)$ *gives identical results*)

Jalilian-Marian,Jeon,RV

For a large nucleus,

$$W[\rho] = \exp\left(- \int d^2x_\perp \frac{\rho^a \rho^a}{2\,\mu_A^2} \right)$$

where, for valence quark sources, one has $\mu_A^2 = \dfrac{g^2 A}{2\pi R_A^2} \propto A^{1/3}\,\mathrm{fm}^{-2}$

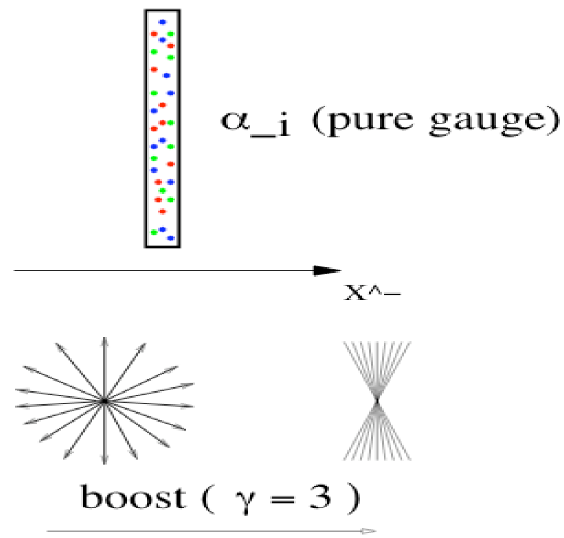
For A >>1, $\mu_A^2 >> \Lambda_{\rm QCD}^2$ and $\alpha_S(\mu_A^2) << 1$

Effective action describes a weakly coupled albeit non-perturbative system

The classical field of the nucleus at high energies

Saddle point of effective action-> Yang-Mills equations

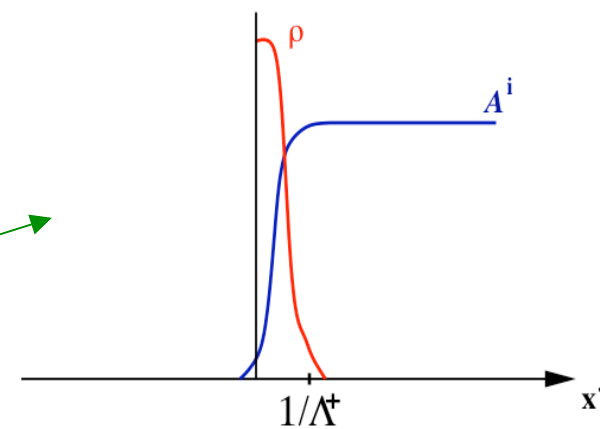
$$D_\mu F^{\mu\nu,a} = \delta^{\nu +}\delta(x^-)\rho^a(x_\perp)$$



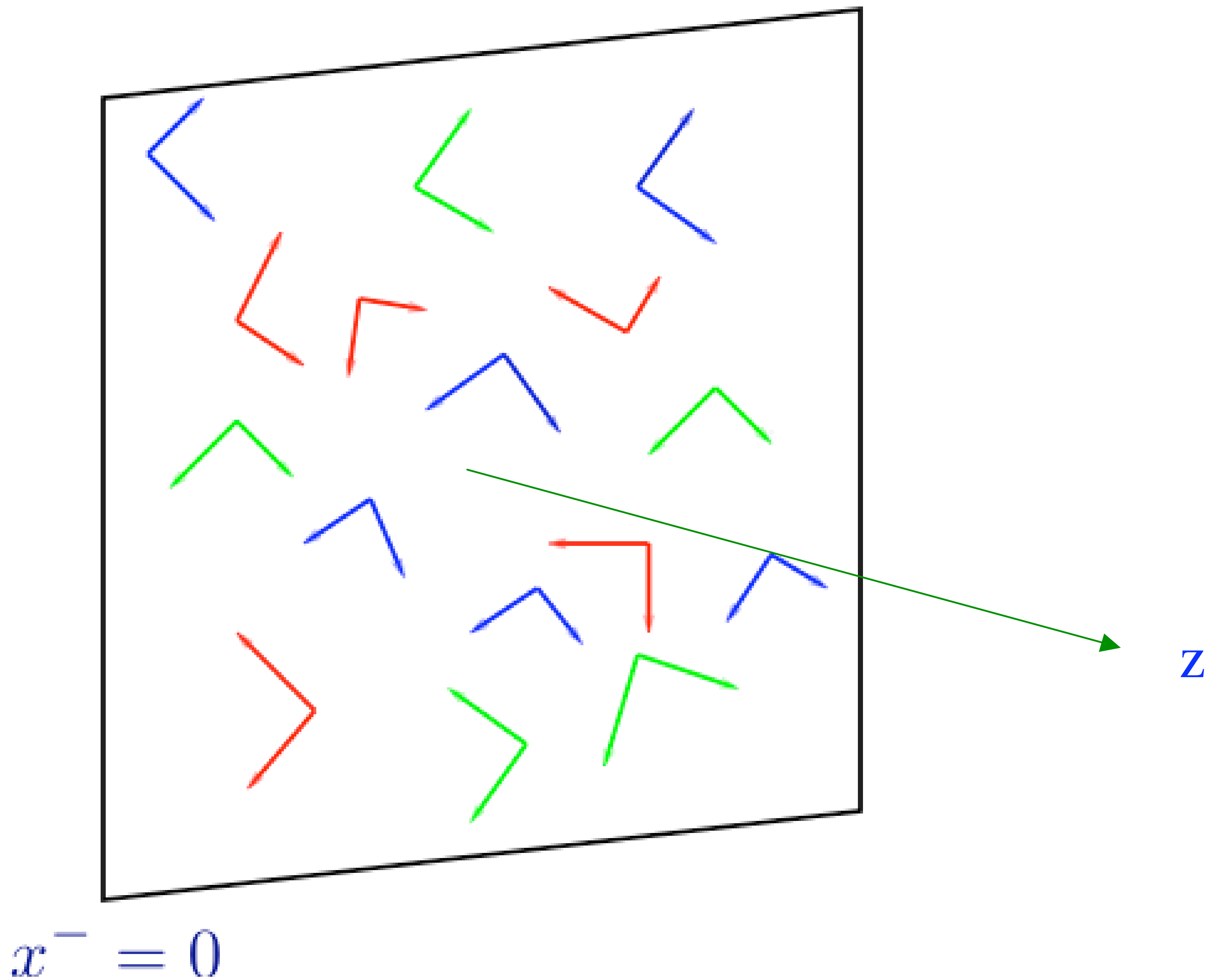
Solutions are non-Abelian Weizsäcker-Williams fields

$$A^+ = A^- = 0\ ;$$
$$F^{ij} = 0 => A^i = \theta(x^-)\alpha^i\ ,$$
$$\text{where } \alpha^i = \frac{-1}{ig} U\nabla^i U^\dagger$$
$$\text{and } \nabla\cdot\alpha = g\rho$$

Careful solution requires smearing in x^-



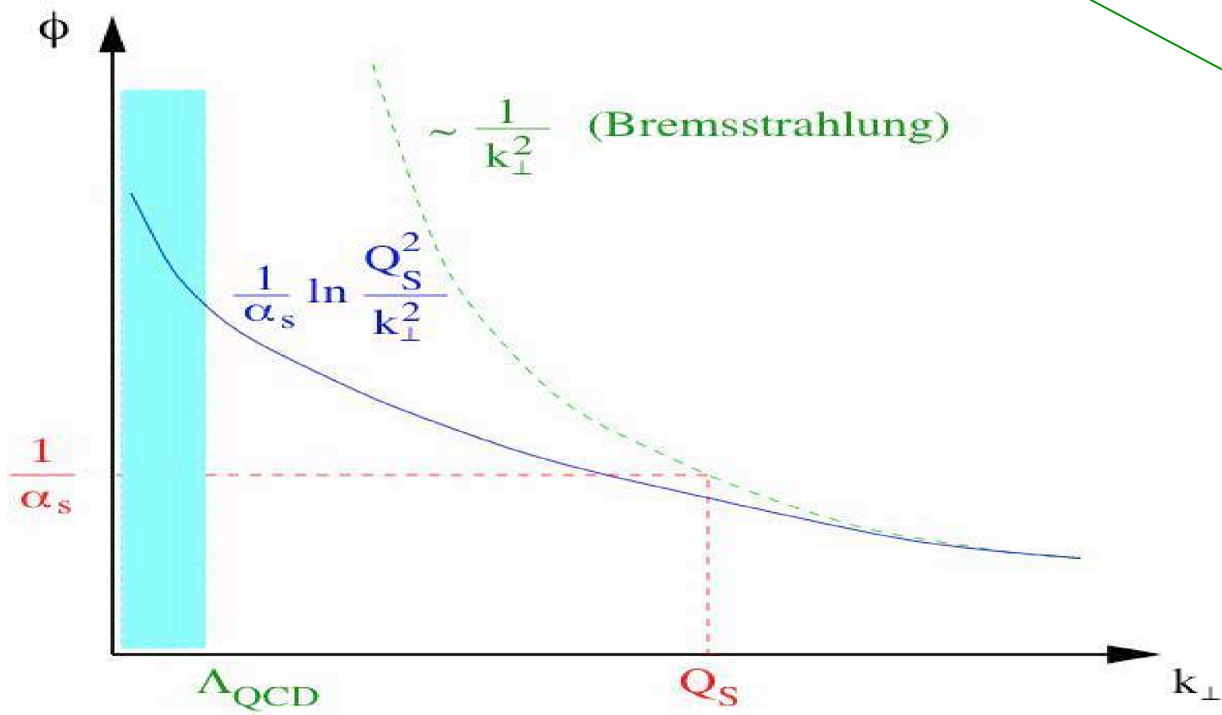
Random Electric & Magnetic fields in the plane of the fast moving nucleus



Average over ρ^a to compute gluon distribution $< AA >_\rho$

$$< AA >_\rho = \int [d\rho] A_{\rm cl.}(\rho) A_{\rm cl.}(\rho) W_{\Lambda^+}[\rho]$$

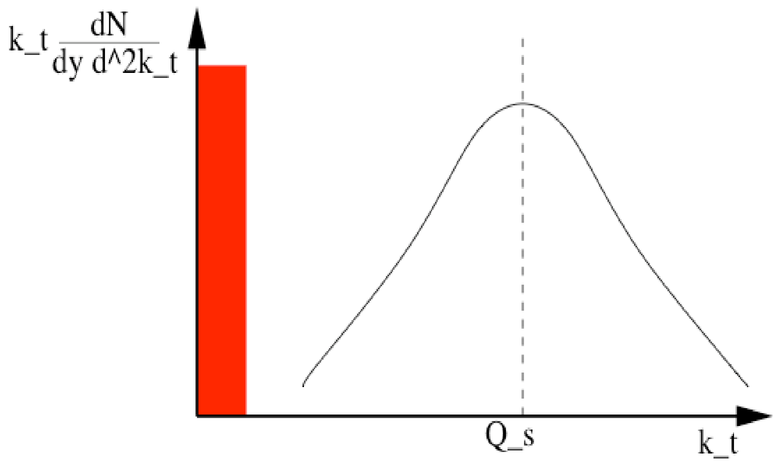
Gaussian in MV



ϕ = gluon phase space density = $\frac{(2\pi)^3}{2(N_c^2-1)} \frac{dN}{\pi R^2 d^2k_\perp dy}$

$Q_s^2 \approx \alpha_S N_c \mu_A^2 \ln\left(\frac{Q_s^2}{\Lambda^2}\right) \sim A^{1/3} \ln A \approx A^{1/3}$ for A >> 1

The Color Glass Condensate



✓ *Typical momentum of gluons is* Q_s

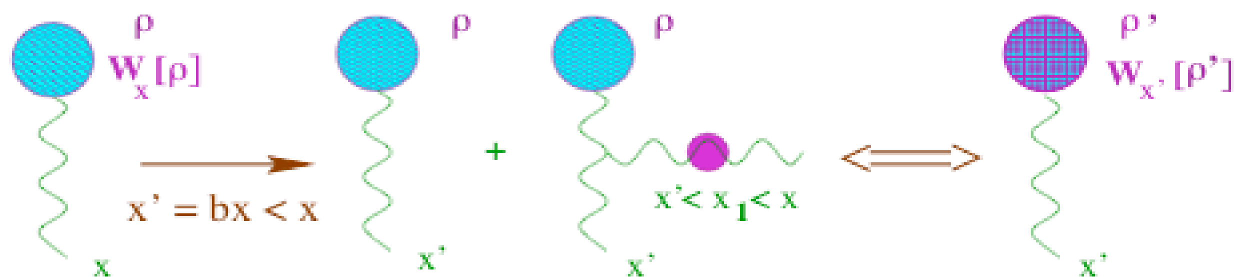
✓ *Bosons with large occupation #* $\sim \frac{1}{\alpha_S}$ *- form a condensate*

✓ *Gluons are colored*

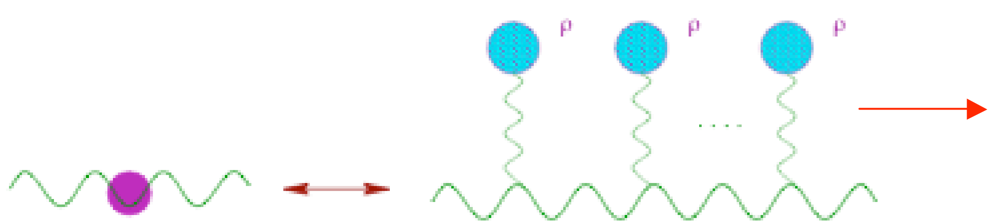
✓ *Random sources evolving on time scales much larger than natural time scales-very similar to spin glasses*

Hadron/nucleus at high energies is a Color Glass Condensate

Wilson RG at small x



Color charge grows due to inclusion of fields into hard source with decreasing x: $\rho' = \rho + \delta\rho => W_x[\rho] \rightarrow W_{x'}[\rho']$



Because of strong fields $A \sim 1/g$
All insertions are O(1)

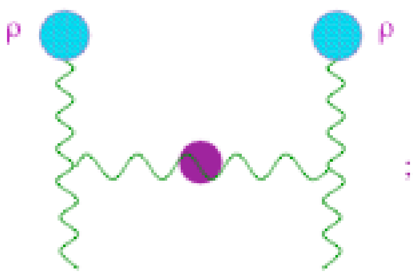
$W_x[\rho]$ *obeys a non-linear Wilson renormalization group equation-the JIMWLK equation*

(Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner)

At each step in the evolution, compute 1-point and 2-point functions in the background field

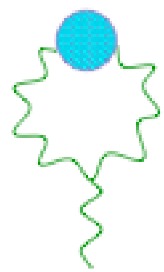
$$\sigma^a(x)[\rho] = <\delta\rho^a_Y(x)>_\rho \;;\; \chi^{ab}(x,y)[\rho] = <\delta\rho^a_Y(x)\delta\rho^b_Y(y)>_\rho$$

χ =



;

σ =



$$\sigma^a(x) = \frac{1}{2}\int d^2y \frac{\delta\chi^{ab}(x,y)}{\delta\rho^b_Y(y)}$$

The JIMWLK (functional RG) equation:

$$\frac{\partial W_x[\rho]}{\partial \ln(1/x)} = \frac{1}{2}\int_{x_\perp,y_\perp} \frac{\delta}{\delta\rho^a_x(x_\perp)}\chi^{ab}(x_\perp,y_\perp)[\rho]\frac{\delta}{\delta\rho^b_x(y_\perp)}W_x[\rho]$$

$\Rightarrow$ An infinite hierarchy of ordinary differential equations for the correlators $< A_1 A_2 \cdots A_n >_y$

Correlation Functions

Change of variables: $\rho^a \rightarrow \alpha^a \,;\, \nabla^2 \alpha = \rho$

$$< O[\alpha] >_Y = \int [d\alpha] O[\alpha] W_Y[\alpha]$$

Iancu, McLerran; Weigert

Brownian motion in functional space: Fokker-Planck equation!

"diffusion coefficient"

$$=> \frac{\partial}{\partial Y} < O[\alpha] >_Y = < \frac{1}{2} \int_{x,y} \frac{\delta}{\delta \alpha_Y^a(x)} \chi_{x,y}^{ab} \frac{\delta}{\delta \alpha_Y^b(y)} O[\alpha] >_Y$$

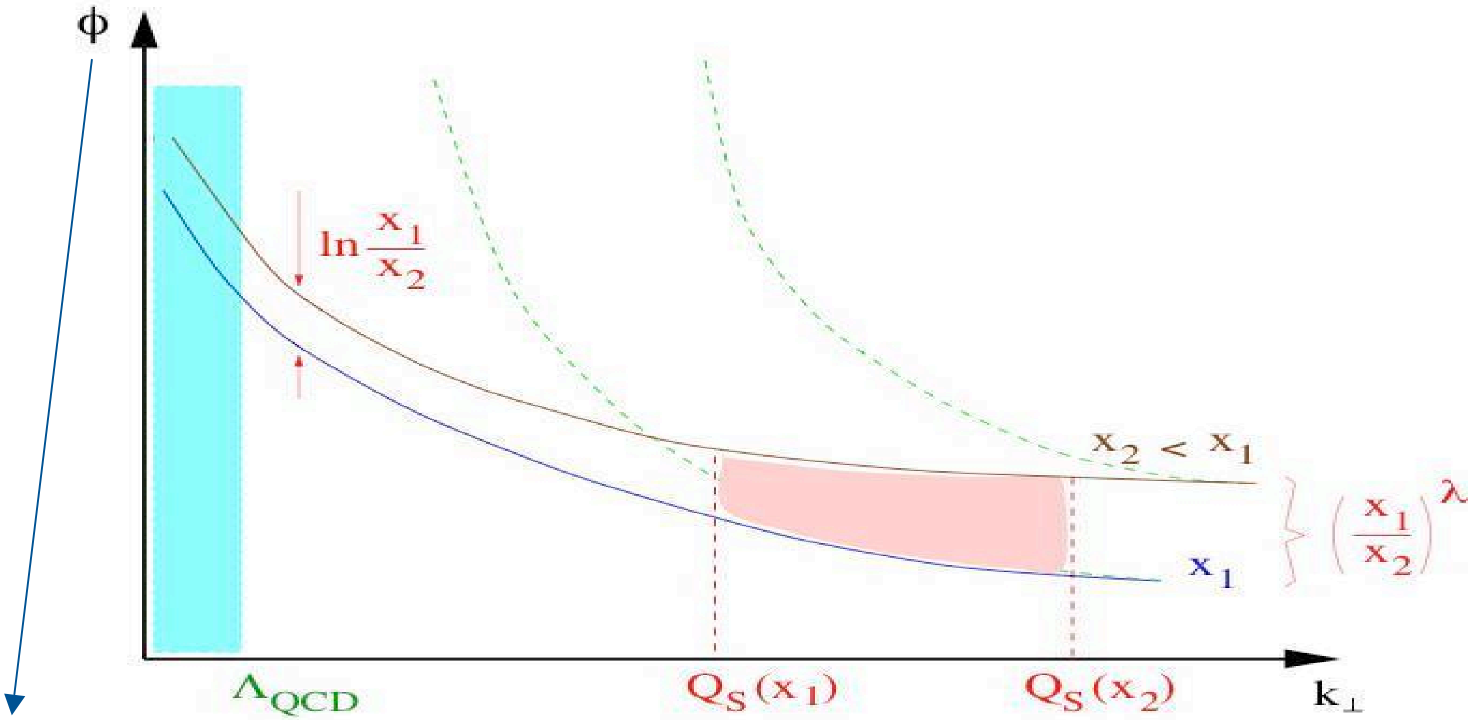
"time"

Consider the 2-point function: $< \alpha(x_\perp)\alpha(y_\perp) >_Y$

Can solve JIMWLK in the weak field limit: $g\,\alpha << 1$

Recover the BFKL equation in this low density limit

Can also solve JIMWLK in the strong field limit: $g\,\alpha \sim 1$ Iancu,McLerran



Gluon phase space density

$$\ln k^2 >> \alpha_s Y (\mathrm{MV, DGLAP}) : \phi \approx \frac{\mu_A^2}{k^2}$$

$$\ln k^2 \sim \alpha_s Y \text{ but } k^2 >> Q_s^2(Y)\,(\mathrm{BFKL}) : \phi \approx \left(\frac{\mu_A^2}{k^2}\right)^{1/2} e^{\omega \alpha_S Y}$$

$$k^2 << Q_s(Y) : \phi \approx \frac{1}{\alpha_S} \ln\left(\frac{Q_s^2(Y)}{k^2}\right)$$

How does one compute Q_s(Y) ?

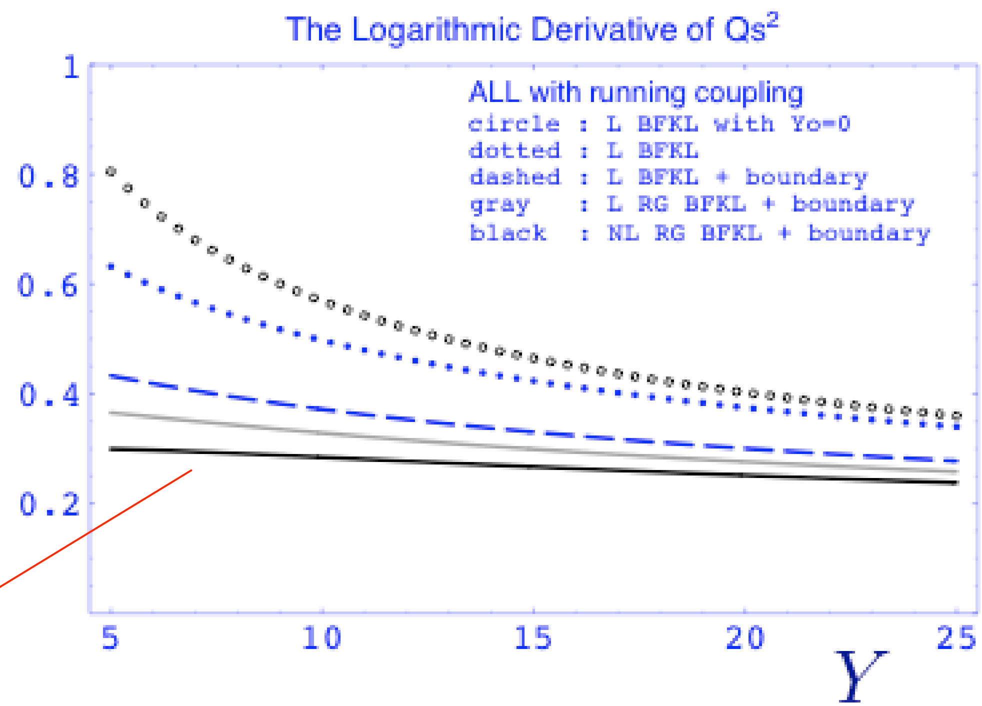
How does Q_s behave as function of Y?

Fixed coupling LO BFKL: $Q_s^2 = Q_0^2 e^{c\,\bar{\alpha}_s Y}$

LO BFKL+ running coupling: $Q_s^2 = \Lambda_{\rm QCD}^2 e^{\sqrt{2b_0 c(Y+Y_0)}}$

Re-summed NLO BFKL + CGC:

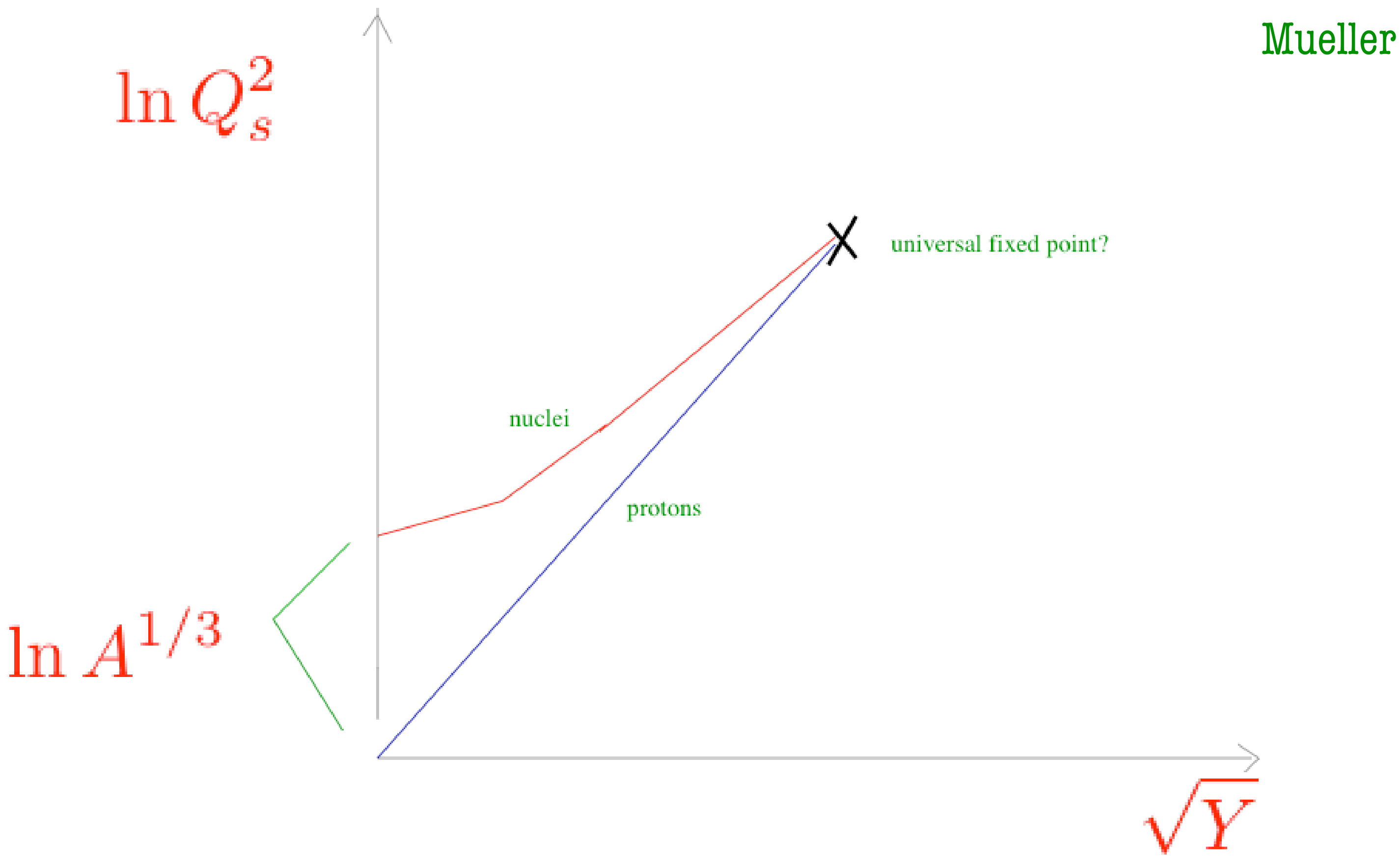
$$\lambda \equiv \frac{d \ln Q_s^2}{dY}$$



Triantafyllopolous

Very close to
HERA result!

A-DEPENDENCE OF SATURATION SCALE



Such interesting systematics may be tested at LHC & eRHIC

Geometrical Scaling

Iancu,Itakura, McLerran;
Mueller,Triantafyllopolous

Can write the solution of BFKL as:

$$\mathcal{N}_Y(r_\perp) \approx \exp\left(\omega\bar{\alpha}_s Y - \frac{\rho}{2} - \frac{\rho^2}{2\beta\bar{\alpha}_s Y}\right) \text{ with } \rho = \ln\frac{1}{r^2 Q_0^2}$$

ρ_S soln. where argument vanishes

$$=> Q_s^2 = Q_0^2\, e^{c\bar{\alpha}_s Y}, \text{with } c = 4.84$$

For $r_\perp < 1/Q_s$ (but close!), can write

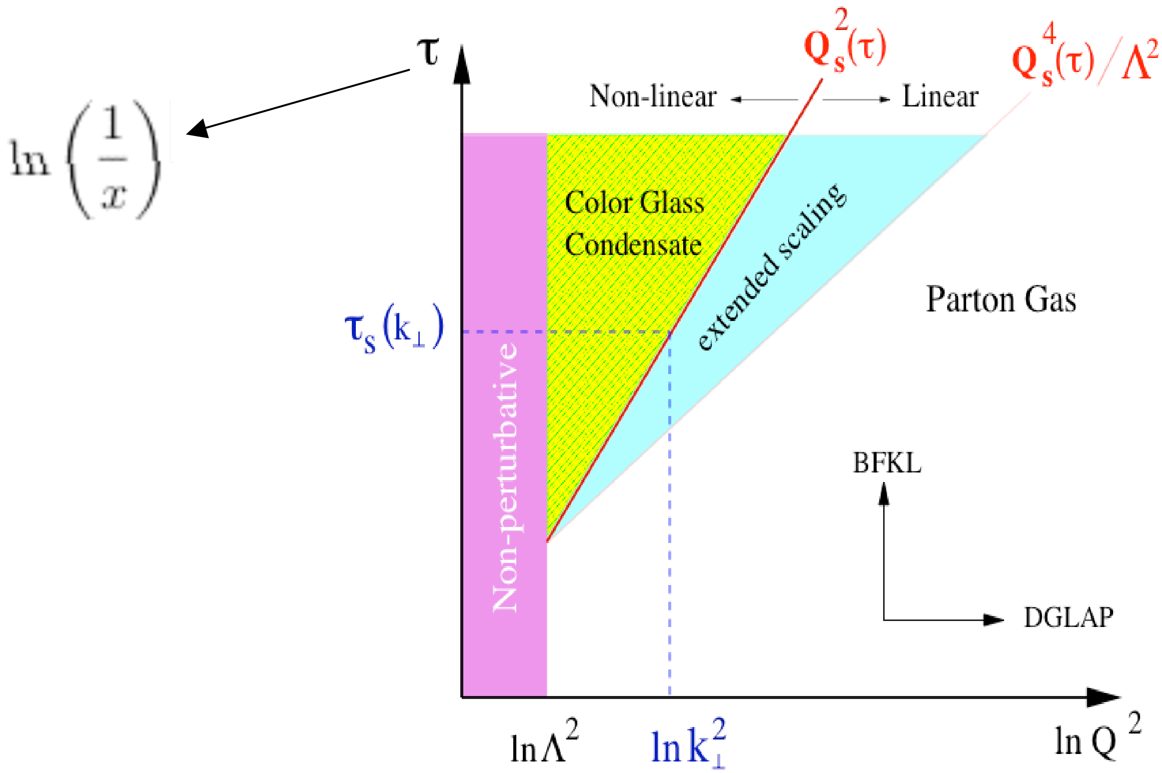
$$\rho = \rho_S(Y) + \ln\frac{1}{r_\perp^2 Q_s^2} \equiv \rho_S + \delta\rho$$

Plugging into N_Y, can show simply

$$\mathcal{N}_Y \approx \left(r_\perp^2 Q_s^2(Y)\right)^\gamma \text{ for } Q_s^2 << Q^2 << \frac{Q_s^4}{Q_0^2}$$

$\gamma \sim 0.64$ is large than BFKL anomalous dimension ~0.5

Novel regime of QCD evolution at high energies



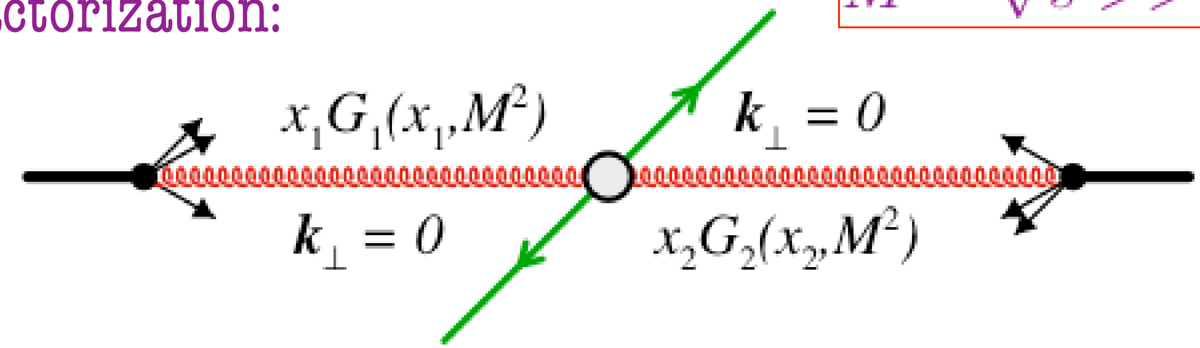
The Color Glass Condensate

Hadron & Nuclear Scattering at high energies

I: Universality: collinear versus k_t factorization

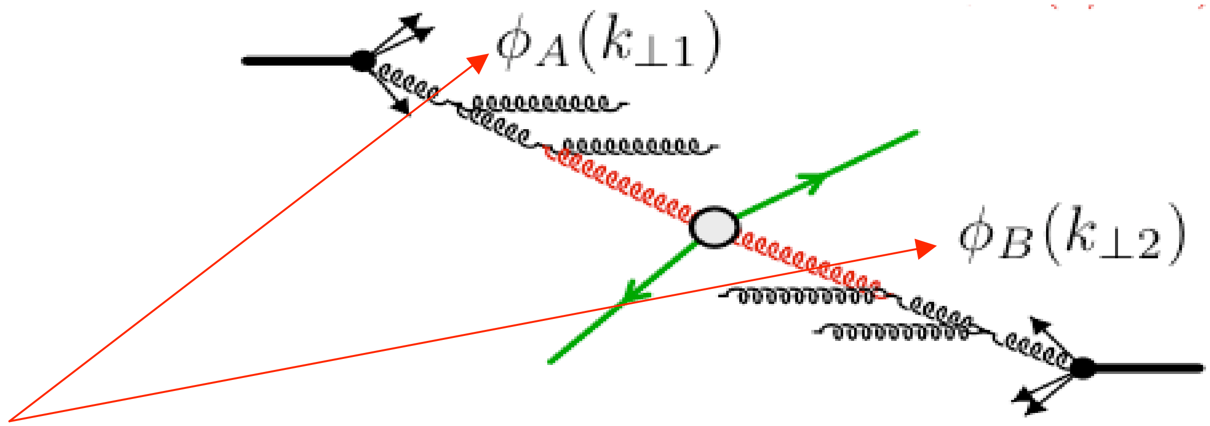
Collinear factorization:

$M \sim \sqrt{s} >> \Lambda_{QCD}$



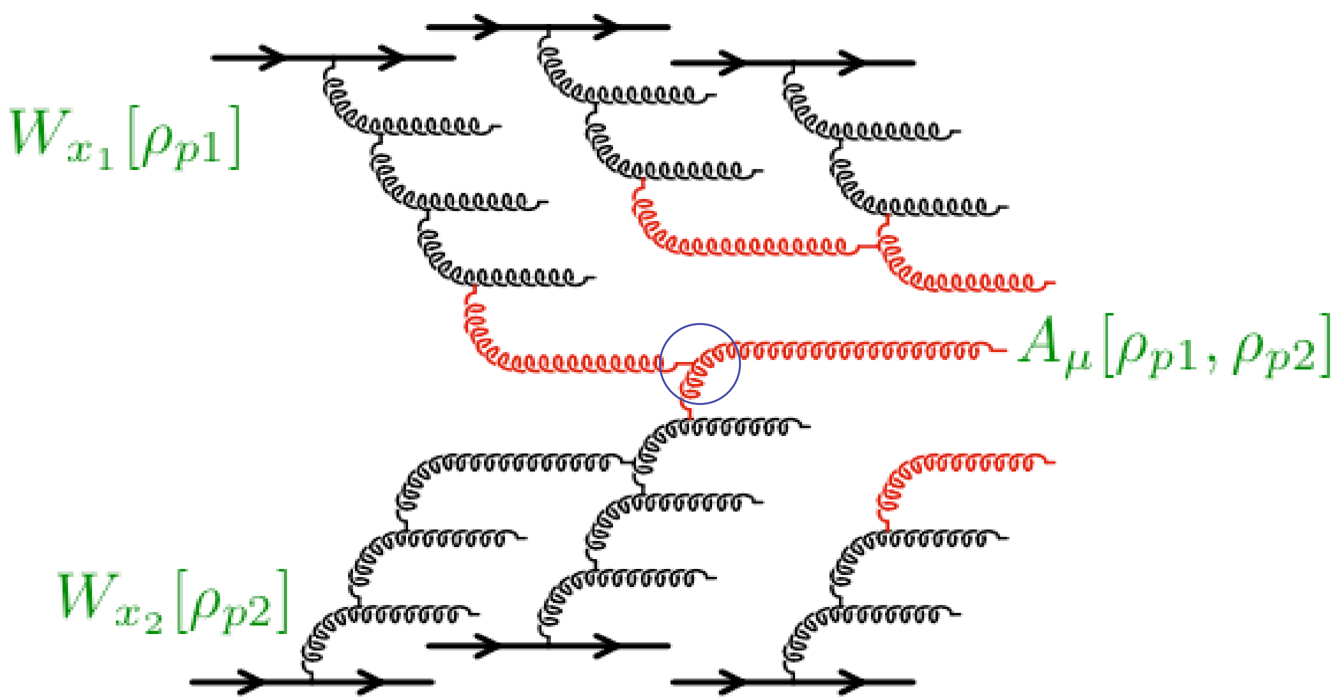
$\Lambda_{QCD} << M << \sqrt{s}$

k_t factorization:



Are these objects universal? Very important for extraction of "gluon" distributions.

Hadronic collisions in the CGC framework



Solve Yang-Mills equations for two light cone sources: ρ_{p1} & ρ_{p2}

For observables $O\big(A_\mu(\rho_{p1}, \rho_{p2})\big)$ average over $W_{x1}[\rho_{p1}]$ & $W[\rho_{p2}]$

Systematic power counting for scattering in the CGC

- Gluon & quark production to lowest order in sources (the dilute/pp case). $(\rho_{p1}/k_\perp^2,\, \rho_{p2}/k_\perp^2 << 1)$

- Gluon & quark production to lowest order in one source & all orders in the other (the semi-dense/pA case).
  $(\rho_p/k_\perp^2 << 1,\, \rho_A/k_\perp^2 \sim 1)$

- Gluon & quark production to all orders in both sources (the dense/AA case) $(\rho_{A1}/k_\perp^2,\, \rho_{A2}/k_\perp^2 \sim 1)$

- Dynamical evolution of soft & hard modes at late times in AA collisions

Gluon & quark production in the dilute/pp region

$(\rho_{p1}/k_\perp^2, \rho_{p2}/k_\perp^2) << 1$

Collinear Factorization:

Incoming partons have k_t=0. Applicable for $Q \sim \sqrt{s} >> \Lambda_{QCD}$

Gluon & quark distributions evaluated at the scale Q^2 *Are universal*

K_t factorization:

Collins & Ellis; Catani, Ciafaloni & Hautmann

Incoming partons have k_t-applicable when $\Lambda_{QCD} << Q << \sqrt{s}$

Described by unintegrated parton dists. $\phi_{p,A}(k_\perp)$

Is this k_t scale the saturation scale k_t ~ Q_s ? Levin, Ryskin, Shabelski, Shuvaev

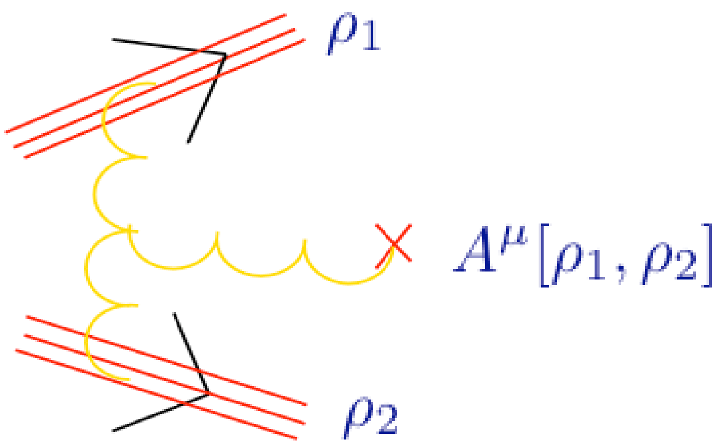
Several phenomenological studies by LRSS and Hagler et al studying spectra and correlations in pp-collisions

(Related approach by Raufeisen, Kopeliovich, Tarasov)

CGC is powerful formalism to study these issues at high energies. Collinear and k_t factorization arise as specific limits of the formalism

Inclusive gluon production in hadronic collisions to lowest order in ρ_1 & ρ_2 and α_S expressed in k_t factorized form.

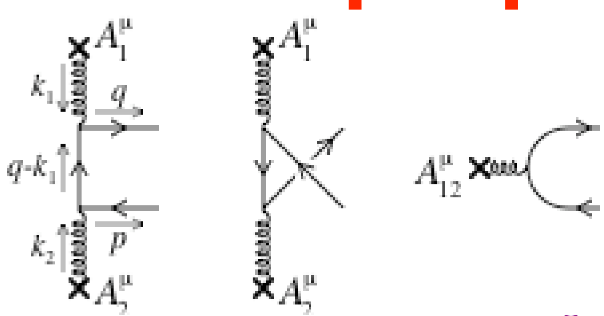
Kovner,Mclerran,Weigert
Kovchegov, Rischke
Gyulassy, McLerran



This diagram in $A^\tau = 0$ gauge is equivalent to sum of all Bremsstrahlung diagrams in covariant gauge.

Inclusive pair production in CGC framework

Gelis, RV



$A^\mu_{1.2}[\rho_1, \rho_2] \propto O(\rho_1 \rho_2)$

Abelian

Non-Abelian vertex here is the Lipatov vertex

$$\frac{d\sigma}{dy_p dy_q d^2p_\perp d^2q_\perp} = \frac{1}{(2\pi)^6}\frac{1}{(N_c^2-1)^2}\int \frac{d^2k_{1\perp}}{(2\pi)^2}\frac{d^2k_{2\perp}^2}{(2\pi)^2}\delta(\vec{k}_{1\perp}+\vec{k}_{2\perp}-\vec{p}_\perp-\vec{q}_\perp)$$
$$\times\phi_1(k_{1\perp})\phi_2(k_{2\perp})\frac{\mathrm{Tr}\left(|m_{ab}^{-+}(k_1,k_2;q,p)|^2\right)}{k_{1\perp}^2 k_{2\perp}^2}$$

$|m_{ab}^{-+}(k_1,k_2;q,p)|^2$ *is identical to Collins & Ellis' k_t factorization result*

$$\frac{d\phi_1(k_{1\perp},x_\perp)}{d^2x_\perp} = \frac{\pi g^2}{k_\perp^2}\int d^2r_\perp e^{-i\vec{k}_\perp\cdot\vec{r}_\perp} < \rho_a(x_\perp+\frac{r_\perp}{2})\rho_a(x_\perp-\frac{r_\perp}{2}) >_\rho$$

is the un-integrated gluon distribution in the Gaussian MV-model

$$\frac{\mathrm{Tr}\left(|m_{ab}^{-+}(k_1,k_2;q,p)|^2\right)}{k_{1\perp}^2 k_{2\perp}^2}$$

is well defined in the collinear limit of $|k_{1\perp}|,|k_{2\perp}|\to 0$

↓

$|M|^2_{gg\to q\bar{q}}$ *after integration over azimuthal angles*

Recover lowest order collinear factorization result

Gluon & quark production in the semi-dense/pA region

$\left(\rho_p/k_\perp^2 << 1, \rho_A/k_\perp^2 \sim 1\right)$

Blaizot, Gelis, RV

- Solve classical Yang–Mills eqns.

$$[D_\mu, F^{\mu\nu}] = J^\nu \, ; \, [D_\nu, J^\nu] = 0$$

with two light cone sources

$$J^{\nu,a} = \underbrace{\delta^{\nu+}\delta(x^-)\rho_1^a(x_\perp)}_{\text{proton source}} + \underbrace{\delta^{\nu-}\delta(x^+)\rho_2^a(x_\perp)}_{\text{nuclear source}}$$

- $\boxed{\partial_\mu A^\mu = 0}$ => equations can be written as

$$(2\partial^+\partial^- - \nabla_\perp^2)A^\nu = J^\nu + ig[A_\mu, F^{\mu\nu} + \partial^\mu A^\nu]$$

need $A^\mu_{1\infty}$ =order $O(\rho_p)$ in proton & order $O(\rho_A^n)$; $n \to \infty$ in nucleus

$$(\partial^- + igA_{0\infty}\cdot T)J^+_{1\infty} = 0$$

$$(2\partial^+\partial^- - \nabla_\perp^2 + igA^-_{0\infty}\cdot T\partial^+)A^+_{1\infty} = J^+_{1\infty}$$

$$(2\partial^+\partial^- - \nabla_\perp^2 + 2igA^-_{0\infty}\cdot T\partial^+)A^+_{1\infty} = ig(A^-_{0\infty}\cdot T)\partial^i A^+_{1\infty} - ig(\partial^i A^-_{0\infty}\cdot T)A^+_{1\infty}$$

$$A^-_{1\infty} = \frac{1}{\partial+}(\partial^i A^i_{1\infty} + \partial^- A^+_{1\infty})$$

$$\boxed{A^-_{0\infty} = -\delta(x^+)\frac{1}{\nabla_\perp^2}\rho_A(x_\perp)}$$

$$J^+_{1\infty} \to A^+_{1\infty} \to A^i_{1\infty} \to A^-_{1\infty}$$

- The gluon field produced in pA collisions has the compact form:

$$q^2 \tilde{A}^{\mu}_{1\infty}(q) = i \int \frac{d^4k}{(2\pi)^4} \left(C^{\mu}_U U(k_2) + C^{\mu}_V V(k_2) + C^{\mu}_1 \mathbf{1}(k_2) \right) 2\pi\delta(k^-) \frac{\rho_1(k_\perp)}{k_\perp^2}$$

$$\mathrm{F.T.}\mathcal{P}_+ \exp\left[ig \int_{-\infty}^{\infty} dz^+ A^-_A(z^+, y_\perp) \cdot T \right] \qquad \mathrm{F.T.}\mathcal{P}_+ \exp\left[\frac{ig}{2} \int_{-\infty}^{\infty} dz^+ A^-_A(z^+, y_\perp) \cdot T \right]$$

- The well known Lipatov vertex is simply

$$C^{\mu}_L = C^{\mu}_U + \frac{1}{2} C^{\mu}_V$$

For on-shell gluons,

$$C^{\mu}_1 = 0\,;\, C_U \cdot C_V = C_V^2 = 0 \text{ and } C_U^2 = C_V^2 = -\frac{4k^2_{1\perp} k^2_{2\perp}}{q^2_\perp}$$

Thus only bi-linears of Wilson line U survive in the squared amplitude

☆ Final result for gluon multiplicity in pA

$$N_g = \frac{4g^2 N_c}{\pi^2 (N_c^2 - 1) q_\perp^2} \int \frac{d^3q}{(2\pi)^3 2E_q} \frac{d^2k_\perp}{(2\pi)^2} \int d^2x_\perp \frac{d\phi_p(k_\perp, x_\perp)}{d^2x_\perp} \frac{d\phi_A(q_\perp - k_\perp, x_\perp - b)}{d^2x_\perp}$$

Result is k_t factorized into product of proton * nuclear un-integrated distributions.

Kovchegov,Mueller
Kovchegov,Tuchin,
Kharzeev,Kovchegov,Tuchin

$$\phi_A(k_\perp, x_\perp) \propto < U_{ab}^\dagger U_{bc} >_{\rho_A}$$

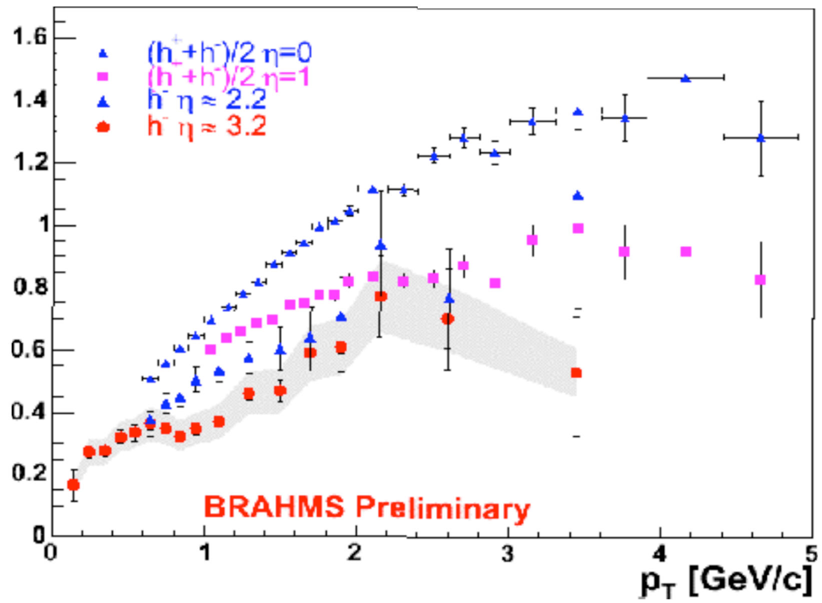
-Is non-linear, contains gluon density to all orders-
proportional to un-integrated gluon density at large k_t

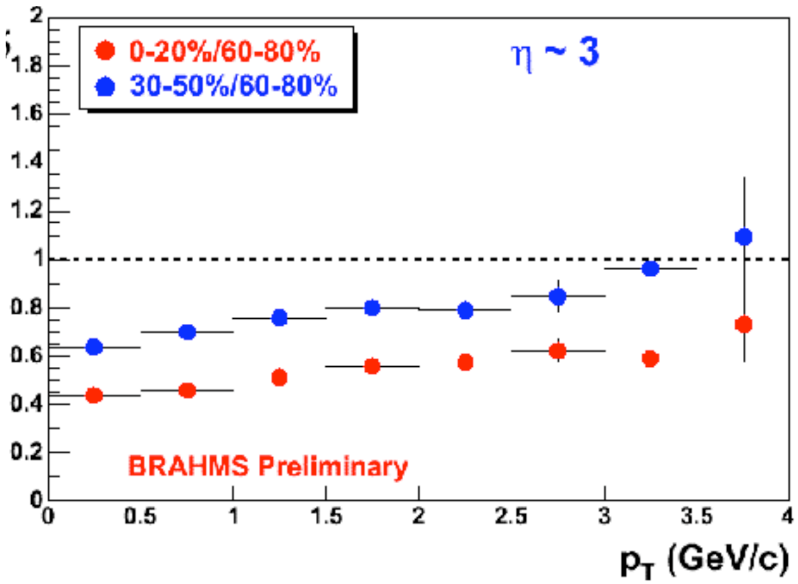
☆ Exactly equivalent to result of Dumitru+McLerran in $A^\tau = 0$ gauge

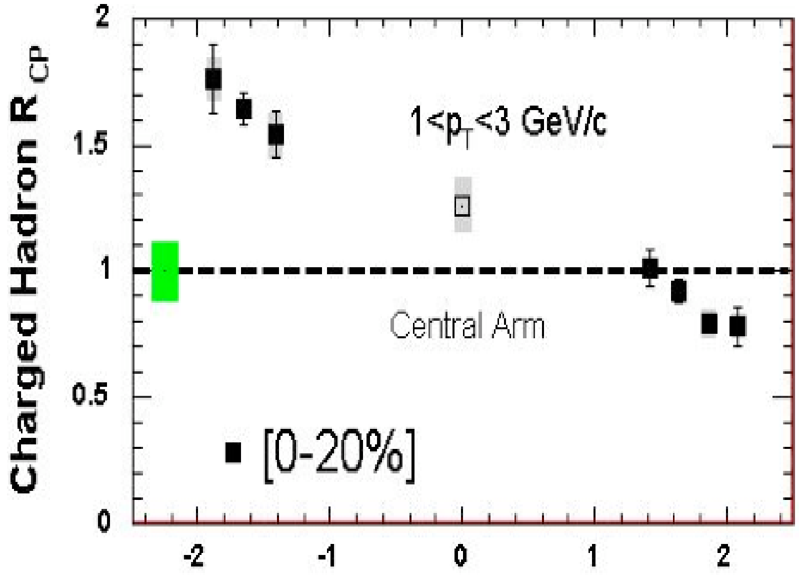
☆ Cronin effect ?

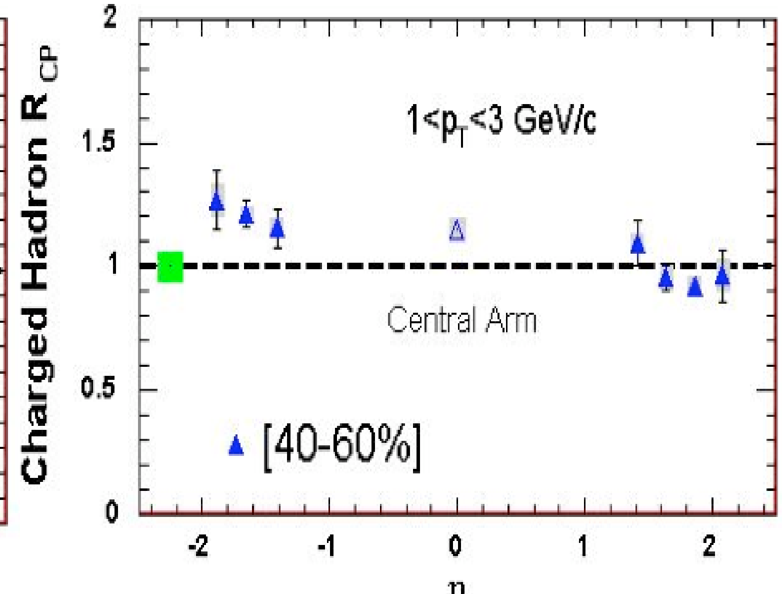
Dumitru,Gelis,Jalilian-Marian

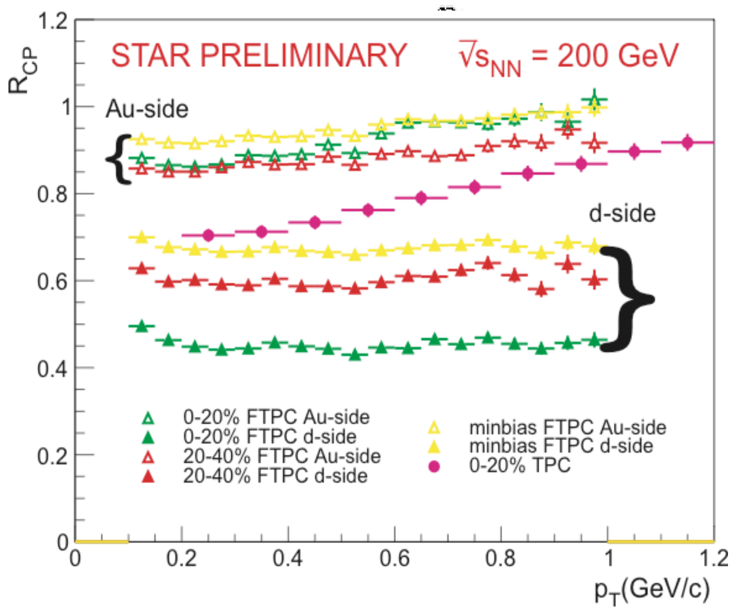
RHIC DATA ON THE CRONIN EFFECT

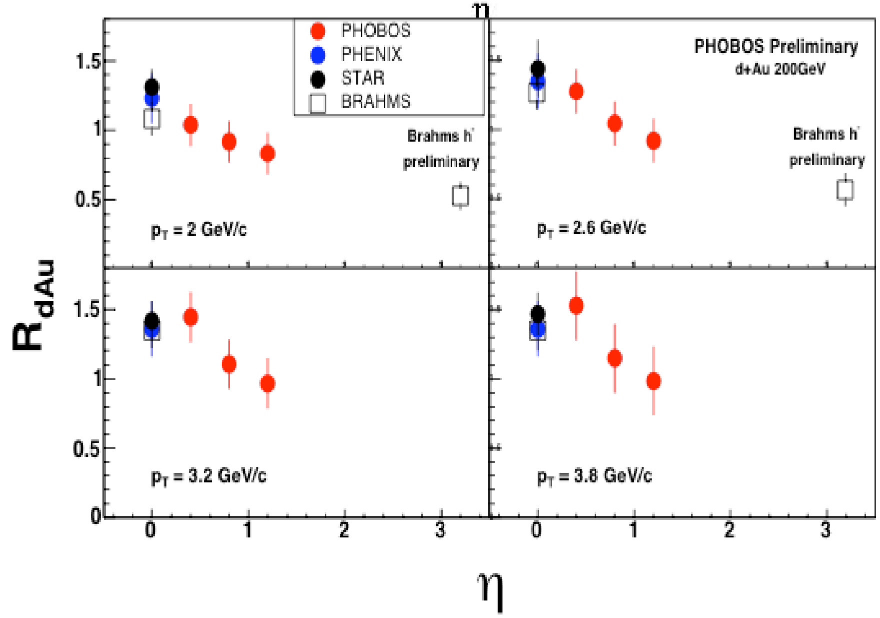




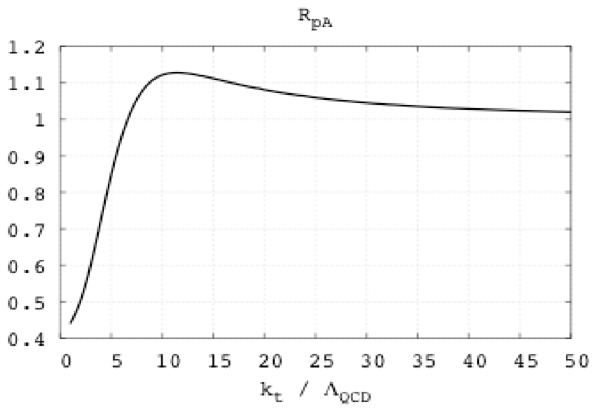

Charged Hadron R CP
1<p_T<3 GeV/c
Central Arm
[0-20%]
2
1.5
1
0.5
0
-2
-1
0
1
2


Charged Hadron R CP
1<p_T<3 GeV/c
Central Arm
[40-60%]
2
1.5
1
0.5
0
-2
-1
0
1
2
η

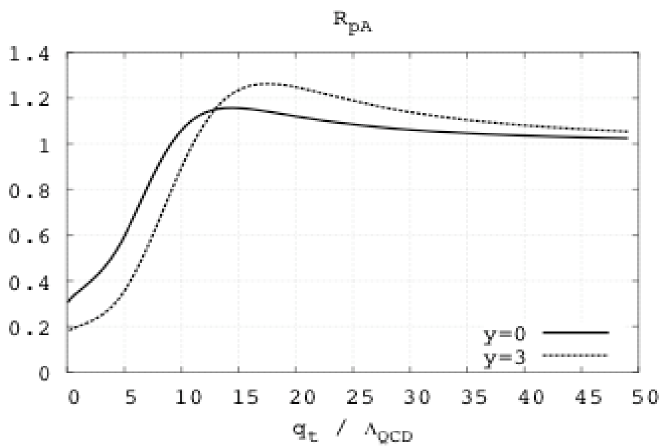

STAR PRELIMINARY
√s_NN = 200 GeV
R_CP
Au-side
d-side
0-20% FTPC Au-side
0-20% FTPC d-side
20-40% FTPC Au-side
20-40% FTPC d-side
minbias FTPC Au-side
minbias FTPC d-side
0-20% TPC
p_T(GeV/c)
0
0.2
0.4
0.6
0.8
1
1.2


PHOBOS
PHENIX
STAR
BRAHMS
PHOBOS Preliminary
d+Au 200GeV
Brahms h⁻
preliminary
p_T = 2 GeV/c
p_T = 2.6 GeV/c
p_T = 3.2 GeV/c
p_T = 3.8 GeV/c
R_dAu
η
1.5
1
0.5
0
0
1
2
3

Compute R_pA

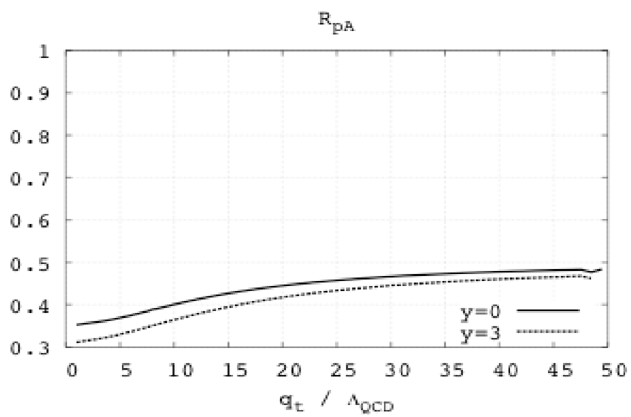


MV model with fixed Q_s



MV-with naïve quantum evolution a la Golec-Biernat-Wusthoff

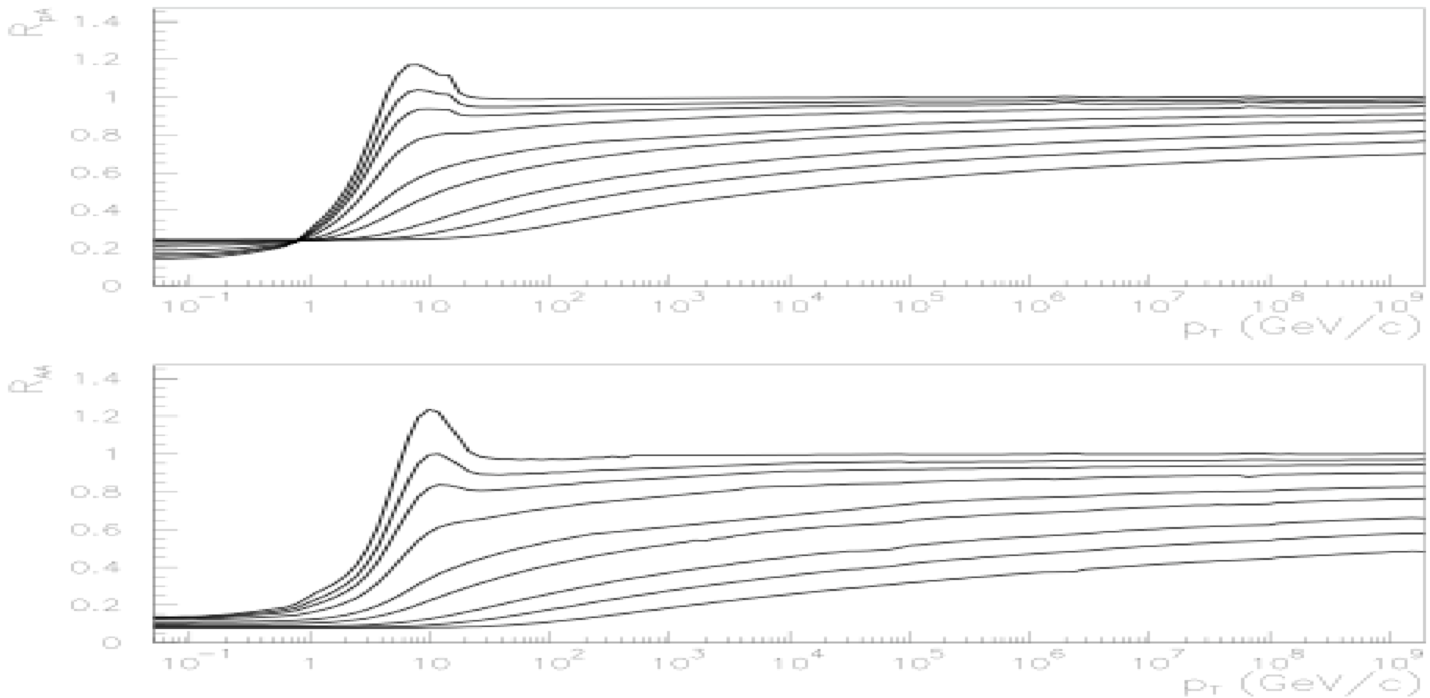
$$Q_s^2 = A^{1/3} \left(\frac{x_0}{x} \right)^\lambda$$



"Super saturated" quantum evolution a la Iancu,Itakura,McLerran

$$\mu_A^2 = \frac{4\pi}{g^2 N_c} k_\perp^2 \ln \left(1 + \left(\frac{Q_s}{k_\perp} \right)^{2\gamma} \right)$$

Numerical solutions of B-K equation with MV initial conditions



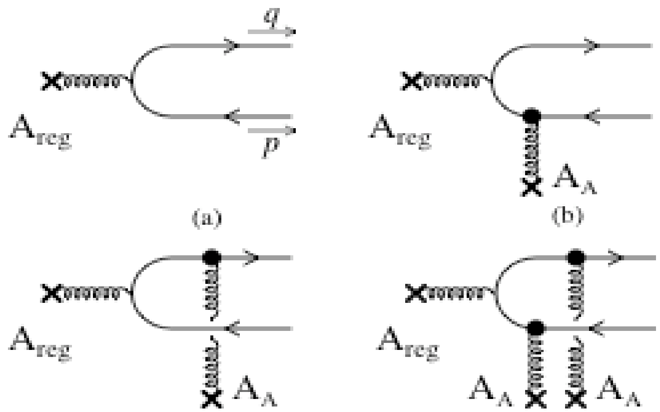
Albacete,Armesto,Kovner,Salgado,Wiedemann
see also, Kharzeev,Kovchegov,Tuchin

Very Important to understand running coupling BFKL effects!

Rummukainen & Weigert
Mueller & Triantafyllopolous

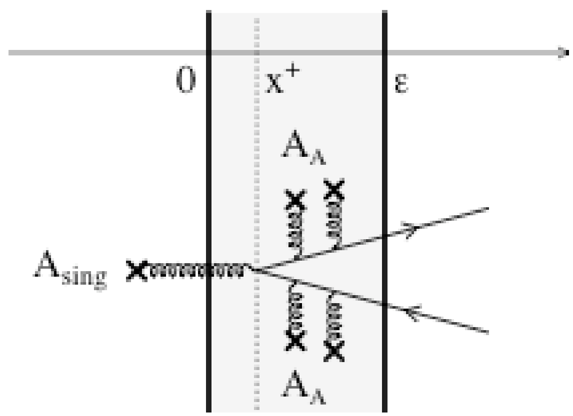
See very recent work by Iancu, Itakura, Triantafyllopolous ; Albacete et al

Quark production to all orders in pA



Blaizot, Gelis, RV

$A_{\rm reg}$ is the gluon field to $O(\rho_p\, \rho_A^n)\,;\, n \to \infty$



The "V"-Wilson lines disappear- need contribution from pair scattering in nucleus

☆ *Result for neither pair-production nor single quark production is k_t factorizable.*

Result can however still be factorized

$$\frac{d\sigma^{pA\to q\bar{q}X}}{dy_p dy_A d^2p_\perp q_\perp} \propto \phi_p \times [A\phi_{g,g} + (B\phi_{g;q\bar{q}} + c.c) + C\phi_{q\bar{q};q\bar{q}}]$$

$$< U_A(x_\perp)U_A^\dagger(y_\perp) >$$

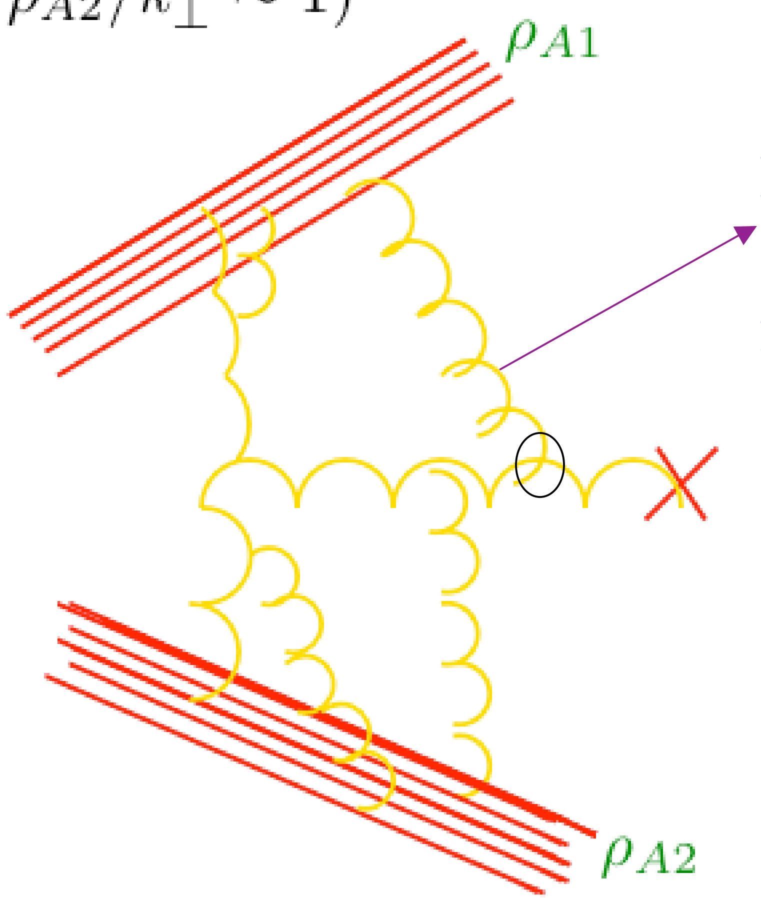
$$< U_F(x_\perp)\tau^a U_F^\dagger(y_\perp)U_F(y'_\perp)\tau^b U_F(x'_\perp) >$$

$$< U_F(x_\perp)\tau^a U_F^\dagger(y_\perp)\tau^{b'}(U_A^{a'b'})^\dagger(y'_\perp) >$$

Can be computed in the Gaussian approximation

Gluon & Quark production in the dense/AA region

$(\rho_{A1}/k_\perp^2,\ \rho_{A2}/k_\perp^2 \sim 1)$

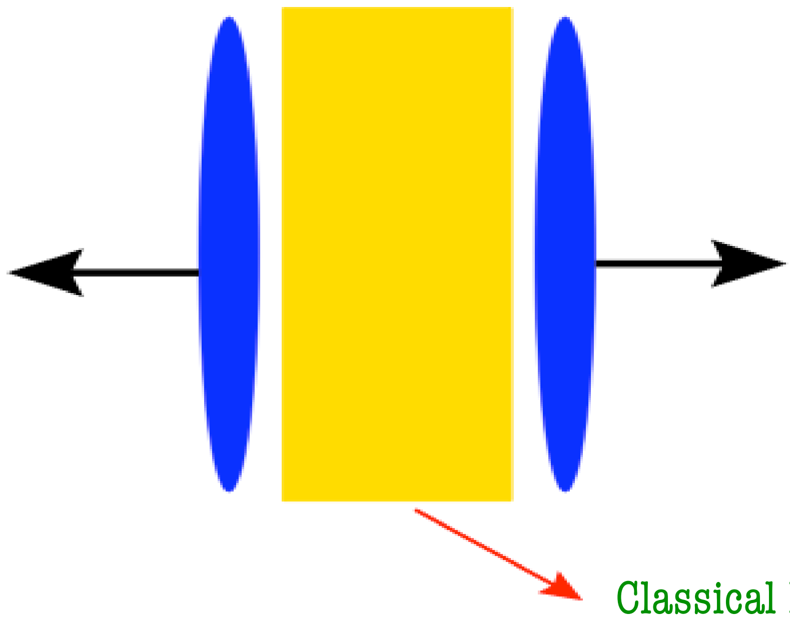


Breaks k_t factorization?
Contribution is same order in coupling

- Wave-fn evolution effects (beyond MV) difficult to include-work of Rummukainen & Weigert is promising.
- Classical evolution shows re-scattering-hence energy loss at eta=0 must be especially strong in AA!

Colliding Sheets of Colored Glass at High Energies

Krasnitz,Nara,RV;
Lappi



Classical Fields with occupation # f= $\frac{1}{\alpha_S}$

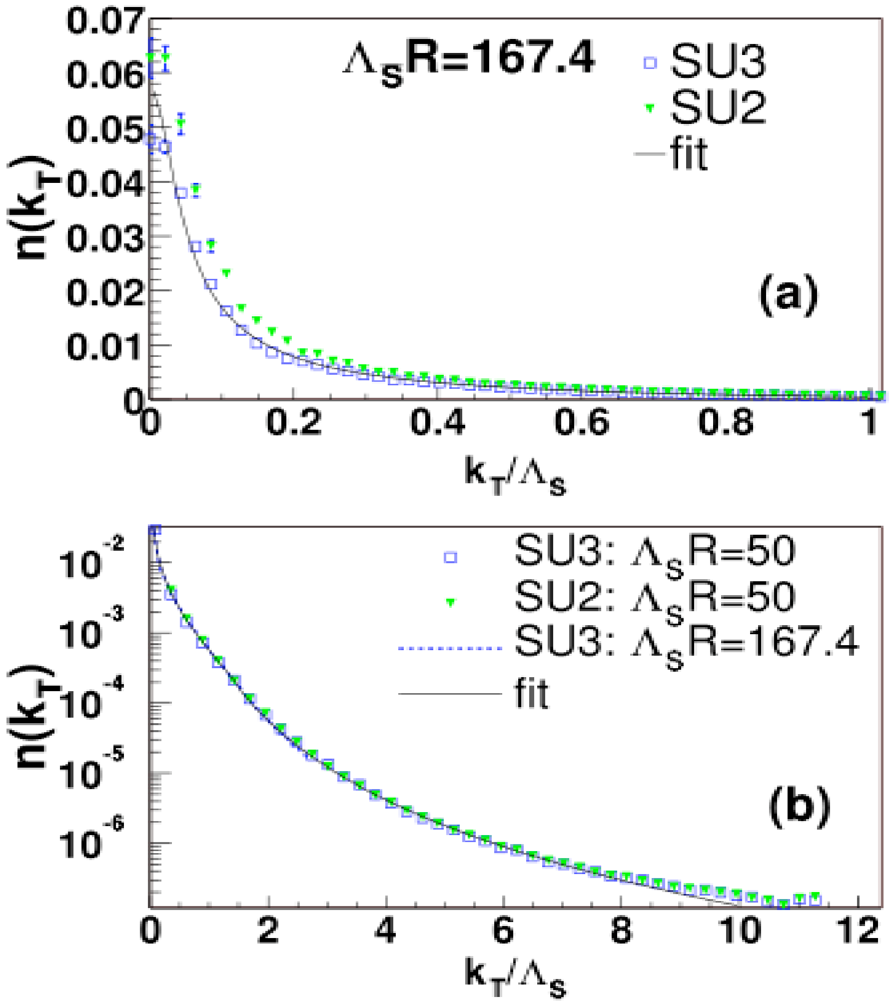
Initial energy and multiplicity of produced gluons depends on Q_s

$$\frac{1}{\pi R^2}\frac{dE_\perp}{d\eta} = \frac{0.25}{g^2} Q_s^3 \qquad \frac{1}{\pi R^2}\frac{dN}{d\eta} = \frac{0.3}{g^2} Q_s^2$$

Classical approach breaks down at late times when f <<1

$$\tau >> \frac{1}{Q_s} \text{ but } \tau << R$$

Transverse momentum distributions of gluons



$$n(k_\perp) = \tilde{f}_N/(N_c^2 - 1)$$

The SU(3) gluon distribution is fitted by the form

$$\frac{1}{\pi R^2}\frac{dN}{d\eta d^2k_\perp} = \frac{\tilde{f}_N}{g^2}$$

where

$$\tilde{f}_N = \frac{a_1}{\exp\left(\frac{\sqrt{k_\perp^2+m^2}}{T_{\rm eff}}\right) - 1}$$

for $k_\perp/\Lambda_s < 1.5$

and

$$\tilde{f}_N = a_2\,\Lambda_s^4 \ln(4\pi k_\perp/\Lambda_s)\, k_\perp^{-4}$$

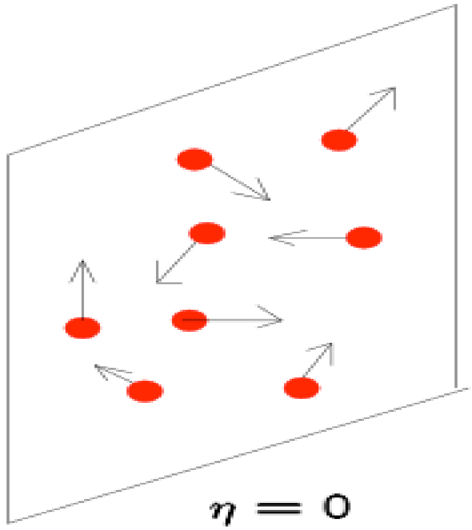
for $k_\perp/\Lambda_s > 1.5$

$$T_{\rm eff} = 0.47\Lambda_s$$

$a_1 = 0.137$; $a_2 = 0.009$; $m = 0.04\Lambda_s$

- *The transverse momentum dist. is infrared finite...*

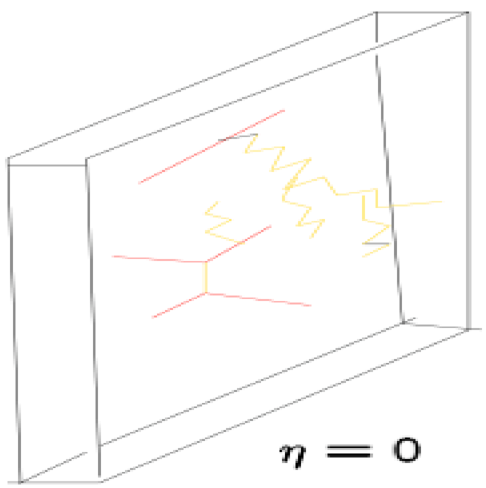
- *The CGC describes only the initial state—produced gluons may re—scatter and thermalize...*



$\tau \sim 1/\Lambda_s$

$p_\perp \sim \Lambda_s$

$p_z \sim 0$

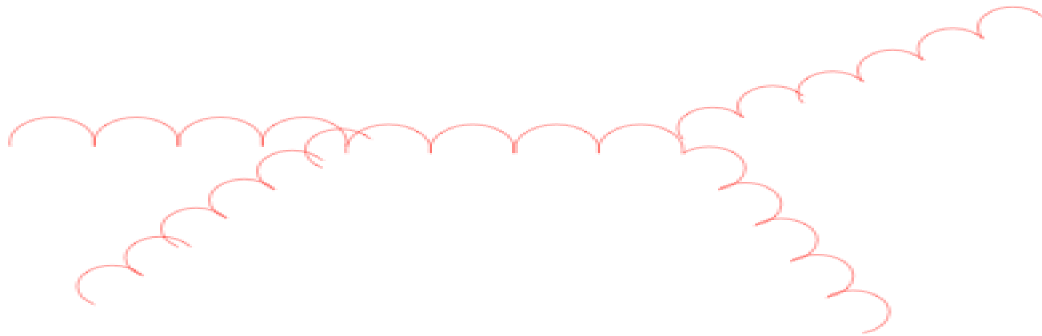


$1/\Lambda_s << \tau << R$

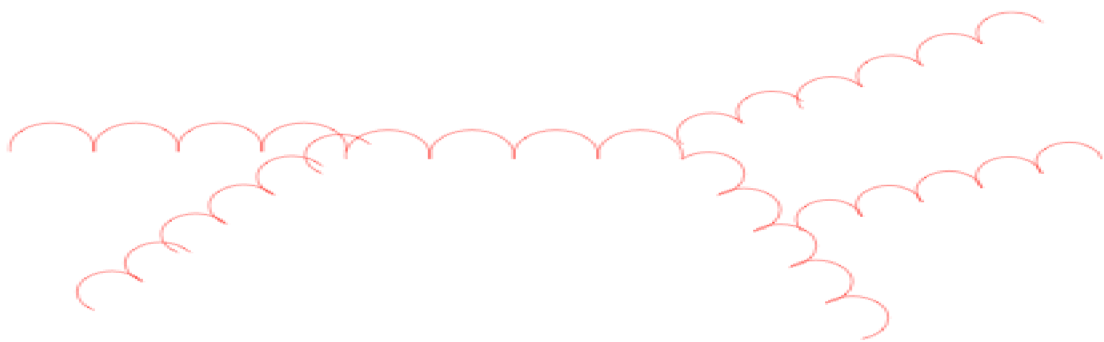
$p_\perp \sim p_z \sim T$

A. H. Mueller
Bjoraker , R.V

- *Small angle scattering drives the system only slowly towards equilibrium...*



- *2 −−> 3 processes may be more efficient...*



Baier, Mueller, Schiff, Son

Role of collective instabilities in thermalization? Arnold,Lenaghan,Moore

Space-time history of a heavy ion collision