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DYSON-SCHWINGER APPROACH TO MESON SUBSTRUCTURE

EQUATIONS FOR GREEN'S FUNCTIONS (PROPAGATORS, VERTICES...) OF A THEORY:

DYSON - SCHWINGER EQUATIONS = MOST INSIGHTFUL APPROACH TO LIGHT PSEUDOSCALAR MESON SUBSTRUCTURE

* IN WEAK COUPLING EXPANSION - GENERATES FEYNMAN DIAGRAMS OF PERTURBATION THEORY

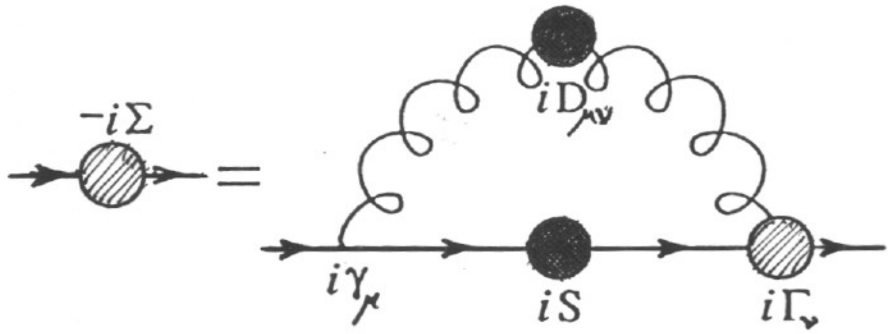
→ NO MODEL DEPENDENCE FOR LARGE SPACELIKE p^2, BUT → PERTURBATIVE QCD, MODEL DEPENDENCE LIMITED TO SMALL AND INTERMEDIATE p^2.

** NONPERTURBATIVE CHARACTER OF **DS** EQUATIONS = CRUCIAL FOR UNDERSTANDING LOW p^2 REGIME.

DRESSED QUARK PROPAGATOR:

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2) + m} = \frac{1}{i\gamma\cdot p + m + \Sigma(p)}$$

SELF-ENERGY $\quad \Sigma(p) = i\gamma\cdot p\left[A(p^2) - 1\right] + B(p^2)$



The Dyson-Schwinger equation for the quark self-energy.

WEAK COUPLING EXPANSION – PERTURBATIVE

RESULTS: $\quad A(p^2) \approx 1 \quad$ AND

$$B(p^2) = m\left[1 - \frac{3\alpha_s}{4\pi}\ln\left(p^2/m^2\right) + \dots\right]$$

NONLINEAR EQUATION ⇒ SELF-CONSISTENT SOLUTIONS WHICH ARE INACCESSIBLE TO PERTURBATION THEORY.

$\left.\begin{array}{l}\rightarrow \text{IN } \chi\text{-LIMIT } (m=0)\ \ B(p^2) \neq 0 \\ - \text{FAVORED IF EFFECTIVE COUPLING} \\ (\text{FOR LOW } p^2) \text{ IS STRONG ENOUGH.}\end{array}\right\}$ D χ S B

4

UNITY OF PHYSICS: SUCH "GAP" EQUATIONS PROVIDE NONPERTURBATIVE INFORMATIONS IN MANY-BODY PHYSICS, E.G., IN NAMBU-GORKOV FORMALISM OF SUPERCONDUCTIVITY.

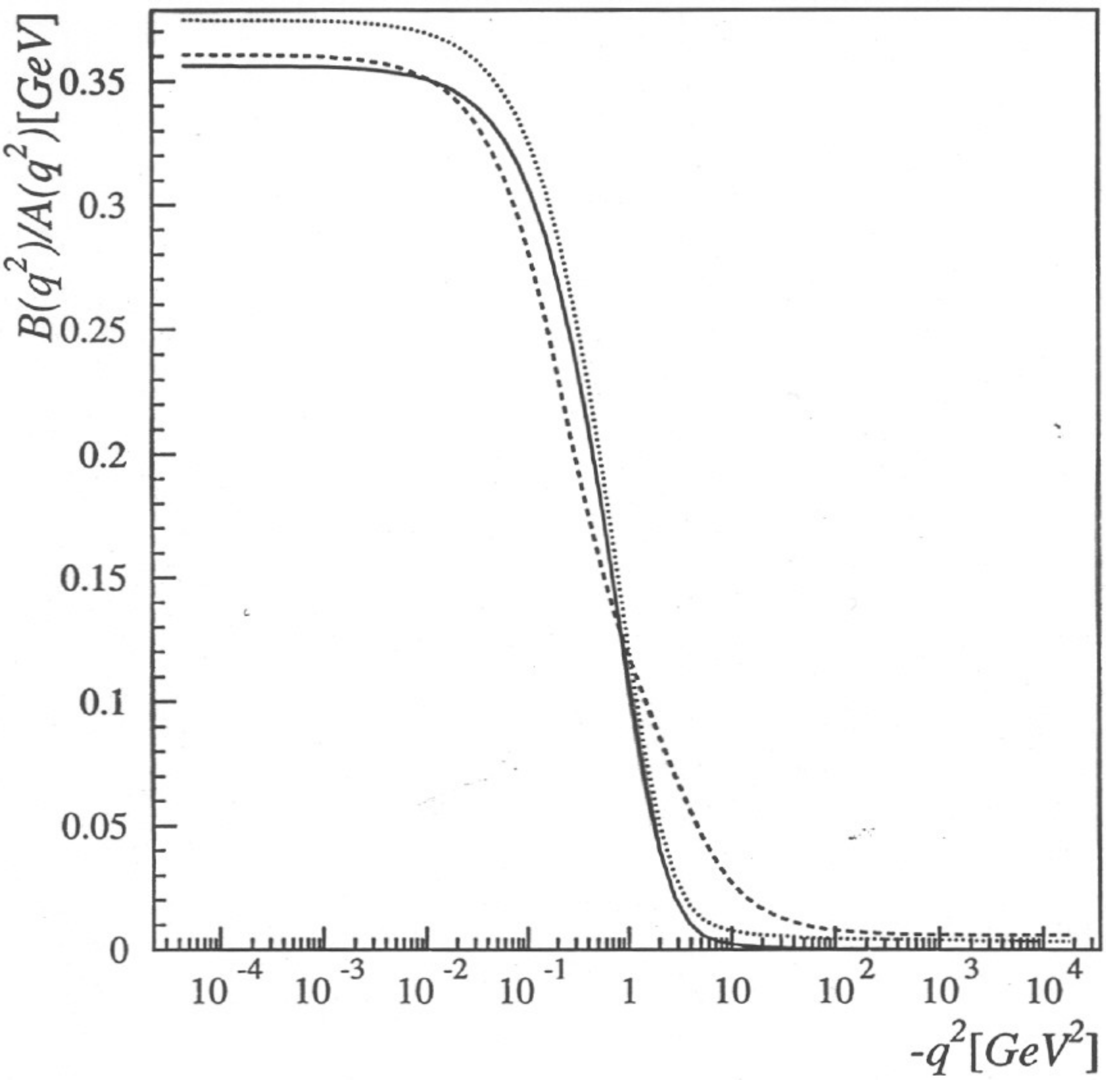
IN QCD CONTEXT, SUCH "STRONG PHYSICS" GIVES:

- CONFINEMENT (OR PROPAGATORS CONSISTENT WITH CONFINEMENT)
- DXSB → CONSTITUENT QUARKS
- NONPERTURBATIVE QCD VACUUM
- HADRON SPECTRA & BOUND STATES

5)

$$S(q) = \frac{1}{i\gamma\cdot q\, A(q^2) + B(q^2)} \equiv \frac{(1/A(q^2))}{i\gamma\cdot q + m(q^2)}$$

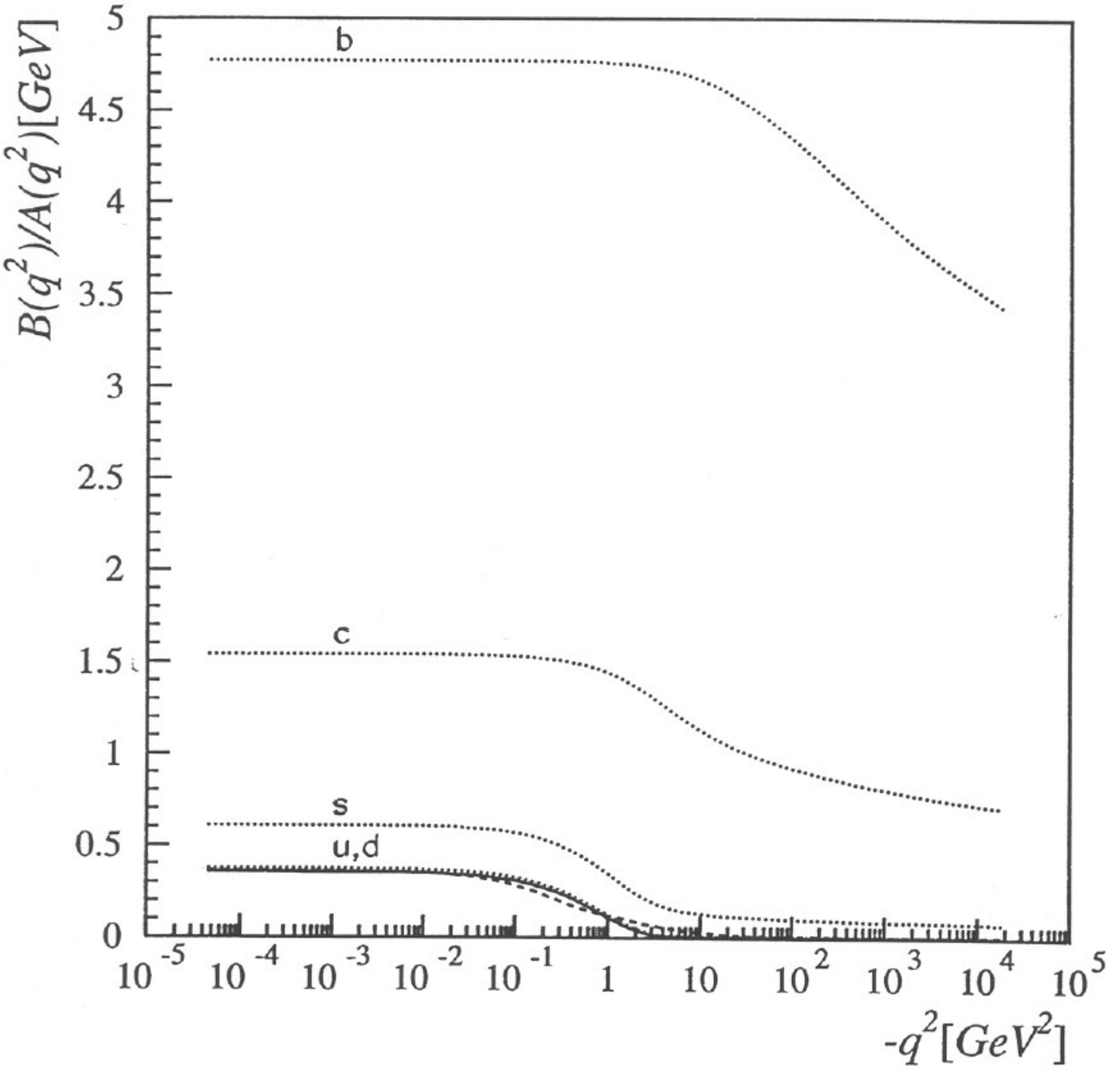
$$m(q^2) = \frac{B(q^2)}{A(q^2)}$$



$\tilde{m} = 0$: $-q^2 \to \infty$, $\quad m(q^2) = \dfrac{\frac{2\pi^2}{3} d \langle 0|\bar{q}q|0\rangle}{q^2 \left[\ln\left(\frac{-q^2}{\Lambda^2_{QCD}}\right)\right]^{1-d}}$

$\tilde{m} \neq 0$: $-q^2 \to \infty$,

$$m(q^2) \simeq \frac{\hat{m}}{\left[\frac{1}{2}\ln\left(-q^2/\Lambda^2_{QCD}\right)\right]^{d}}$$



for $\hat{m} \neq 0$ $\qquad m_f(q^2) \equiv \frac{B_f(q^2)}{A_f(q^2)} \xrightarrow[-q^2 \to \infty]{} \frac{\hat{m}_f}{\left[\frac{1}{2}\ln\left(-q^2/\Lambda^2_{QCD}\right)\right]^{d}}$

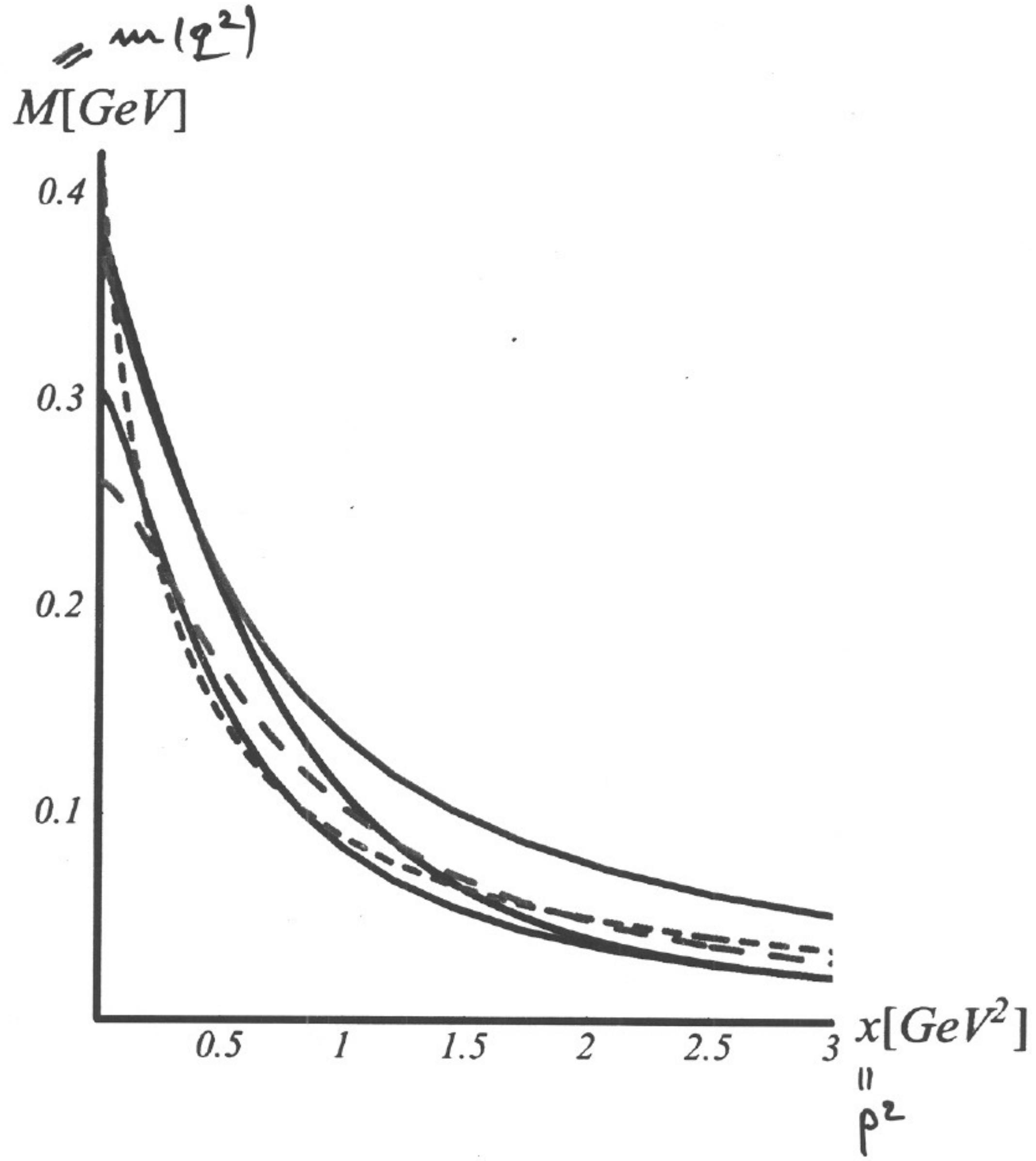
CURRENT MASSES $\hat{m}_f$:

$\hat{m}_{u,d} = 8.73$ MeV , $\hat{m}_s = 203$ MeV , ...

"CONSTITUENT MASSES" $m_f(0)$: $m_c(0) = 1,54$ GeV...

7)

FOR SUFFICIENT INTERACTION STRENGTH:



AT LOW MOMENTA, DYNAMICALLY GENERATED MASS $\approx \frac{1}{3} M_N \approx \frac{1}{2} M_\rho$ = TYPICAL CONSTITUENT QUARK MASS

SCHWINGER – DYSON APPROACH TO QUARK – HADRON PHYSICS *

= NONPERTURBATIVE, COVARIANT,

CHIRALLY WELL BEHAVED BOUND-STATE APPR.

* COUPLED INTEGRAL SD EQUATIONS

FOR GREEN FUNCTIONS OF QCD ($S, D_{\mu\nu}, \Gamma_\mu$ ...

... BUT WE CANNOT SOLVE THIS

EXACTLY, SINCE SD eq. FOR

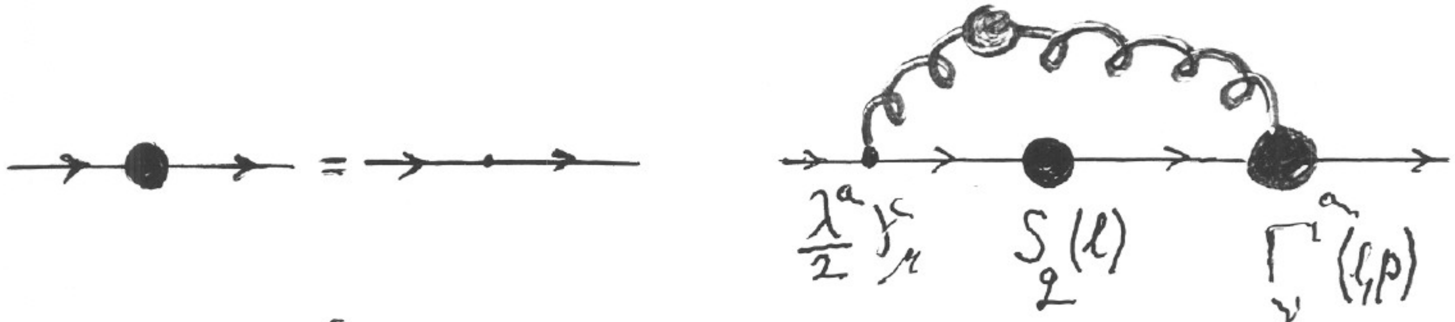
n-POINT FUNCTION CONTAINS (n+1) FUNCTION

ETC. UPWARDS.

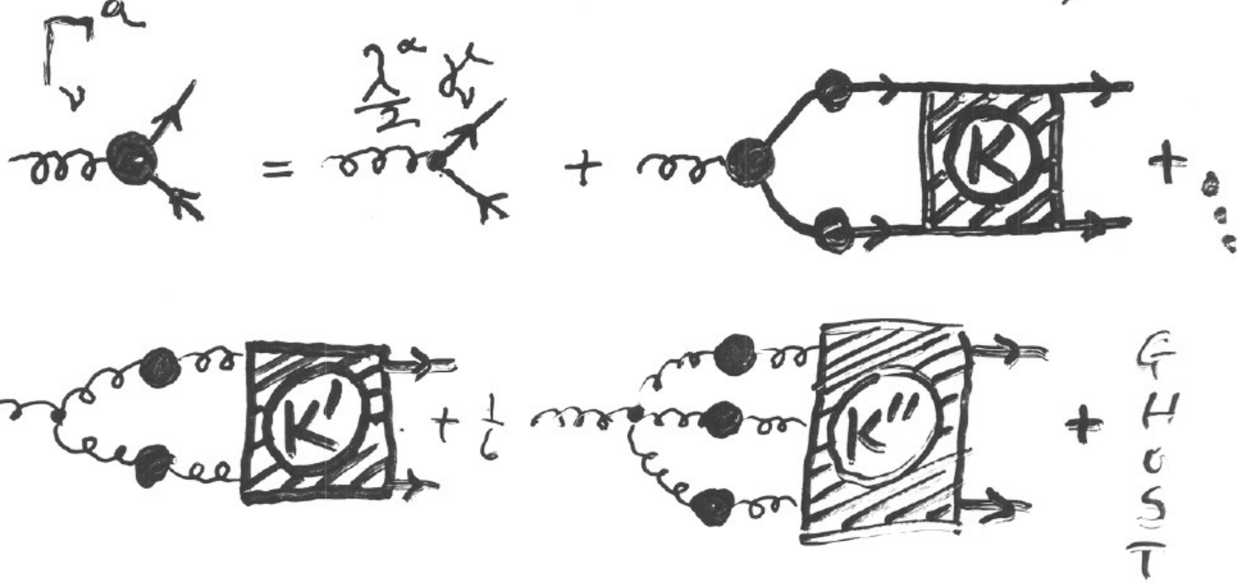
9) SCHWINGER-DYSON EQUATIONS FOR QUARK-HADRON PHYSICS:

EXAMPLE: "GAP" EQUATION FOR **DRESSED** S_q:

$$S_q^{-1}(p) = S_{q\text{-FREE}}^{-1}(p) + \int^{\Lambda} \frac{d^4\ell}{(2\pi)^4}\, g^2\, G^{\mu\nu}(p-\ell)\, \frac{\lambda^a}{2}\gamma_\mu\, S_q(\ell)\, \Gamma_\nu^a(\ell,p)$$



BUT

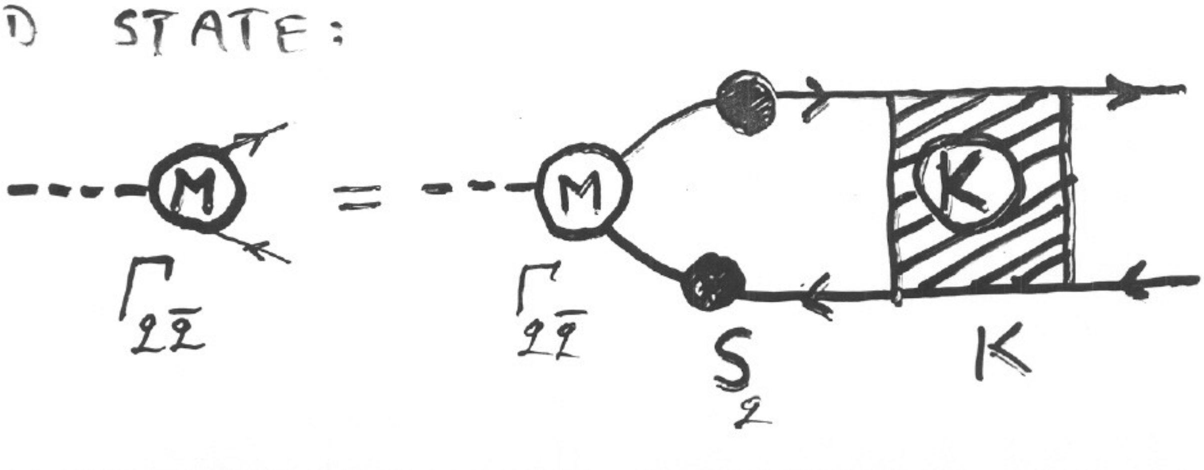


- THE SAME **UNKNOWN** INTERACTION

KERNEL K CREATING $q\bar{q}$ MESON (M)

BOUND STATE:

OMOGENEOUS
.S. EQUATION



→ MUST TRUNCATE ... MODEL ...

FORTUNATELY: a) INCREASED UNDERSTANDING HOW TO TRUNCATE AND PRESERVE: KNOWN BEHAVIORS (E.G. IN PERTURBATIVE REGIME), DχSB, ...

b) COMPUTER + SOFTWARE IMPROVEMENTS ⟹
⟹ BETTER AND BETTER APPROXIMATIONS

⟹ PROGRESS IN BOTH BRANCHES:

1.) „AB INITIO" S.D.-STUDIES OF Q.C.D. GREEN FUNCTIONS
(E.G. TÜBINGEN GROUP: ALKOFER + COLLAB., v. SMEKAL, BLOCH, ATKINSON, ...)

2.) PHENOMENOLOGICAL MODELING OF HADRONS AS QUARK BOUND STATES
(NOWADAYS, ROBERTS, MARIS, TANDY, ...COLLABORATORS)

- WILL NOT CONSIDER HERE SOME IMPORTANT DEVELOPING DIRECTIONS: *) $T \neq 0$ (... BLASCHKE, S. SCHMIDT...)
**) BARYONS AS qqq BOUND STATES

- BUT NARROW DOWN TO: I MESONS
II LIGHT SECTOR (u, d, s) PSEUDOSCALAR

CHIRAL (χ) SYMMETRY $\Leftrightarrow$ AXIAL W.T.I.

$$p_\mu \Gamma_{5\mu}(k,p) = S^{-1}\left(k+\frac{p}{2}\right) i\gamma_5 + i\gamma_5 S^{-1}\left(k-\frac{p}{2}\right)$$

AXIAL VERTEX DS EQUATION

$$\left[\Gamma_{5\mu}(k,p)\right]_{tu} = \left[\gamma_5\gamma_\mu\right]_{tu} + \int \left[S\left(q+\frac{p}{2}\right)\Gamma_{5\mu}(q,p)S\left(q-\frac{p}{2}\right)\right]_{sr} \left[K(q,k;p)\right]_{tu}^{rs} d^4q$$

CONTAINS $q\bar{q}$ INTERACTION KERNEL K FROM BSE

AND S FROM GAP EQUATION $\longrightarrow$ GOOD

CHIRAL BEHAVIOR = SENSITIVE TO

TRUNCATIONS OF BOTH K AND GAP EQ KERNEL

- FROM RENORMALIZABILITY & ASYMPTOTIC FREEDOM, FOR LARGE SPACELIKE MOMENTA $Q^2 \equiv (k-q)^2 \sim k^2 \sim q^2$,

$$K_{tu}^{rs}(q,k;p) = 4\pi\alpha_S(Q^2) G_{\rho\sigma}^{free}(Q) \left[\frac{\lambda^a}{2}\gamma_\rho\right]_{ts} \left[\frac{\lambda^a}{2}\gamma_\sigma\right]_{ru} \qquad (*)$$

(+ CONTRIBUTIONS SUPPRESSED BY ADDITIONAL POWERS OF $\frac{1}{Q^2}$)

AXIAL WTI IS THEN AUTOMATICALLY SATISFIED IF THE SAME TRUNCATION IS IN THE GAP EQUATION :

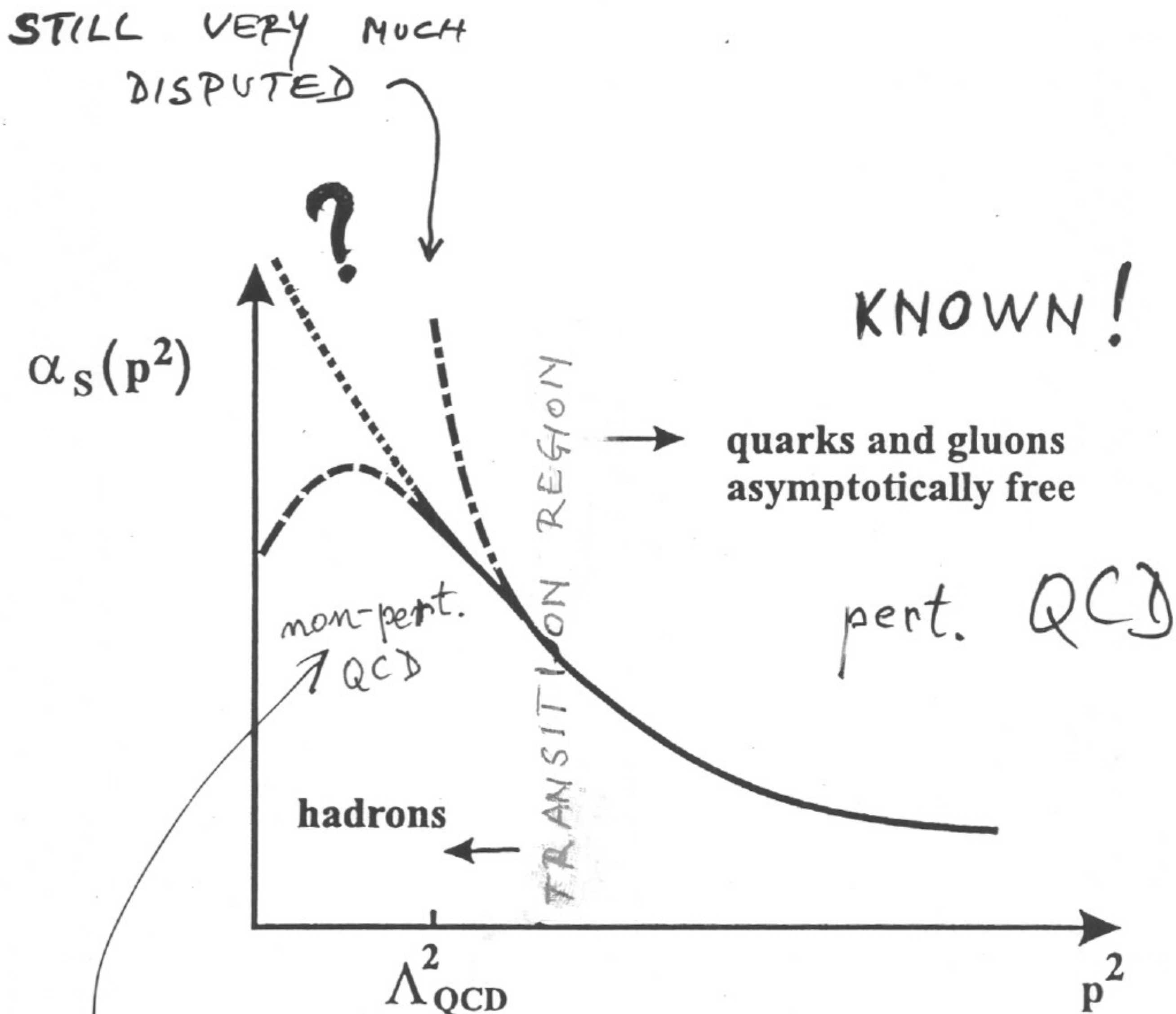
$$S^{-1}(p) = S^{-1}_{FREE}(p) + \int \frac{d^4q}{(2\pi)^4} \, 4\pi \alpha_S([p-q]^2) \, G^{FREE}_{\mu\nu}(p-q) \, \frac{\lambda^a}{2} \gamma_\mu \, S(q) \, \frac{\lambda^a}{2} \gamma_\nu .$$

THIS RENORM-GROUP-IMPROVED RAINBOW-LADDER TRUNCATION SUPPOSE THAT (*) IS ALSO A GOOD APPROXIMATION AT LOW MOMENTA ($\lesssim 1 \, GeV^2$)

- RECENTLY, ATTEMPTS BEYOND THIS TRUNCATION, BUT THIS ASSUMPTION IS STILL THE BASIS OF EXISTING DS PHENOMENOLOGY OF MESONS,

ALTHOUGH $\alpha_S(p^2) \neq$ KNOWN AT LOW !

⇒ ? WHAT TO DO AT LOW p^2 ?



ATTEMPTS TO GET $\alpha(p^2)$ AT LOW p^2 FROM
„AB INITIO" D.S. CALCULATIONS AND LATTICE (ALKOFER ET AL, BLOCH, ...)

... BUT TO REPRODUCE MESON PHENOMENOLOGY THROUGH B-S EQUATION, $\alpha_s(p^2)$ MUST BE:

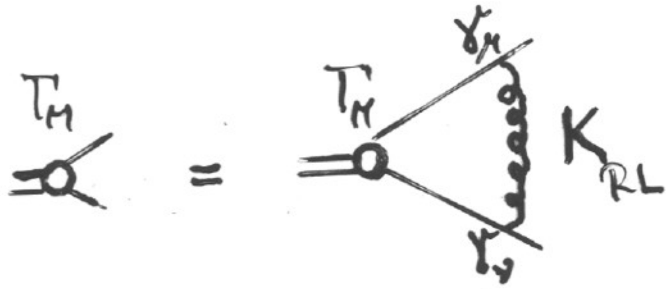
- STILL MODELED AT LOW p^2 (JAIN-MUNCZEK, ROBERTS + MARIS, TANDY...)
- OR, OBTAINED FROM THE INTERPLAY OF DIM-2 AND DIM-4 GLUON CONDENSATES (KEKEZ, KLABUČAR,

USED IN BS EQUATION FOR BOUND STATES

$$[\Gamma_M(k,p)]_{tu} = \int \frac{d^4q}{(2\pi)^4} [S(q+\tfrac{p}{2})\Gamma_M(q,p)S(q-\tfrac{p}{2})]_{sr} K(k,q;p)^{rs}_{tu}$$

RAINBOW-LADDER KERNEL (*) $\Rightarrow$

$$\Gamma_M(k,p) = \int \frac{d^4q}{(2\pi)^4} \frac{\lambda^a}{2}\gamma_\mu S(q+\tfrac{p}{2})\Gamma_M(q,p)S(q-\tfrac{p}{2})\frac{\lambda^a}{2}\gamma_\nu \times$$
$$4\pi\alpha_s([k-q]^2)\, G^{FREE}_{\mu\nu}(k-q)$$



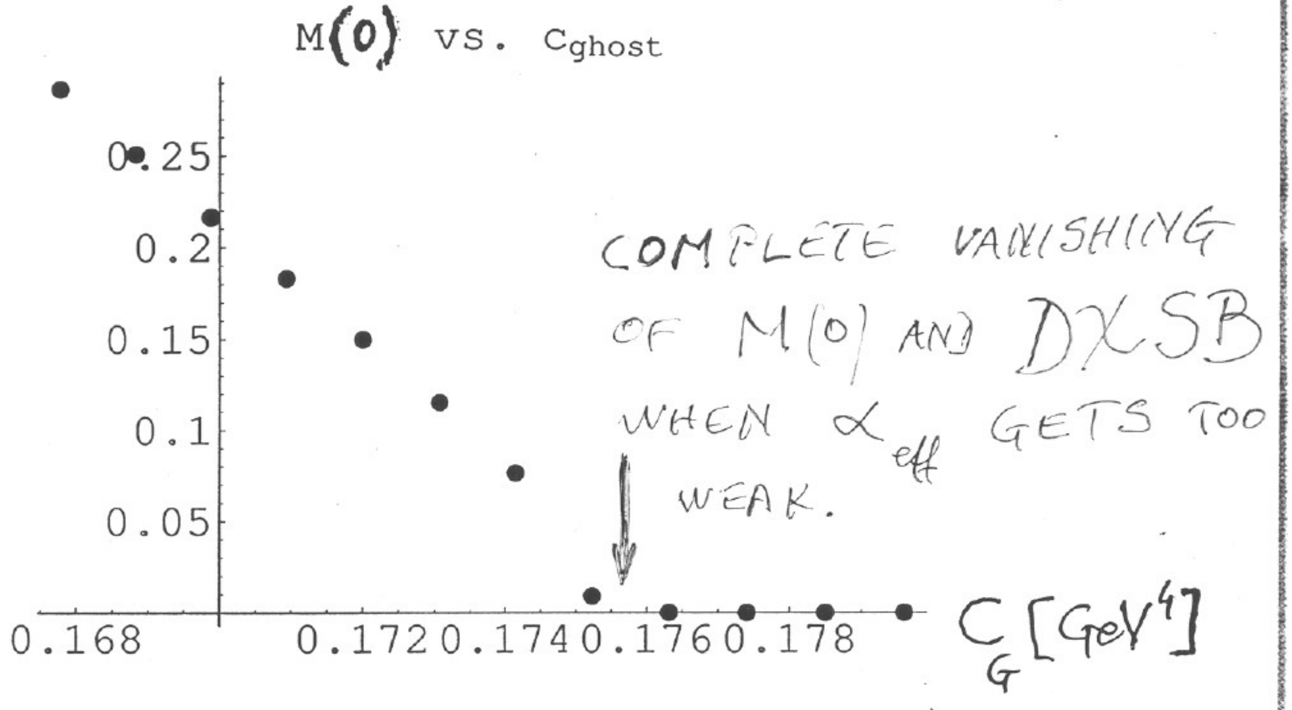
FOR A GOOD MODEL α_s^{IR}, GOOD MESON MASSES AND DECAY CONSTANTS FROM LIGHT TO HEAVY - ESP. $\mathbb{P}$ - AND $\mathbb{V}$ MESONS.

MOST NOTABLY: MODEL-INDEPENDENTLY, LIGHT PSEUDOSCALAR MESONS = (ALMOST) MASSLESS GOLDSTONES OF DχSB

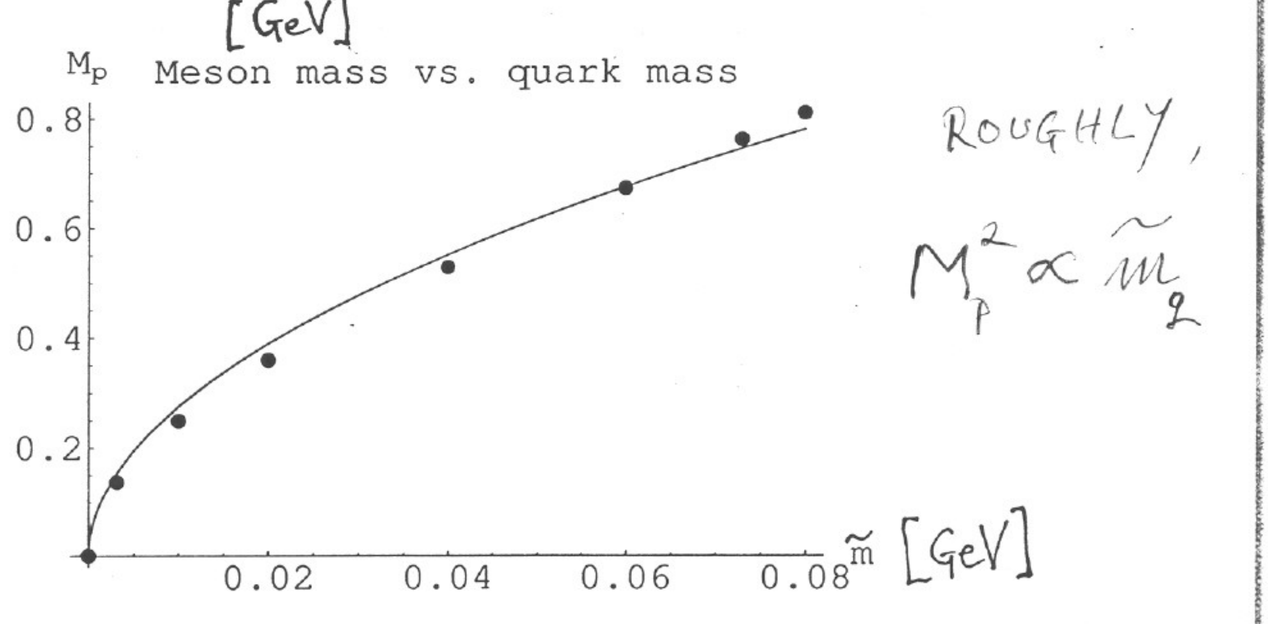
CORRECT QCD CHIRAL BEHAVIOR:

GMOR: $f_\pi^2 M_\pi^2 = -2m\langle 0|\bar{q}q|0\rangle$

Chiral symmetry



The constituent quark mass $M(0) = B(0)/A(0)$ dependence on the parameter $C_G = C_A$.

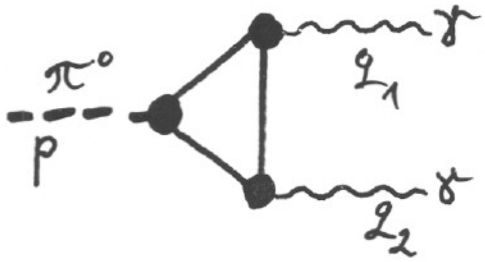


Pseudoscalar meson mass M_H vs. bare quark mass $\tilde{m}$.

COUPLED SD-BS APPROACH

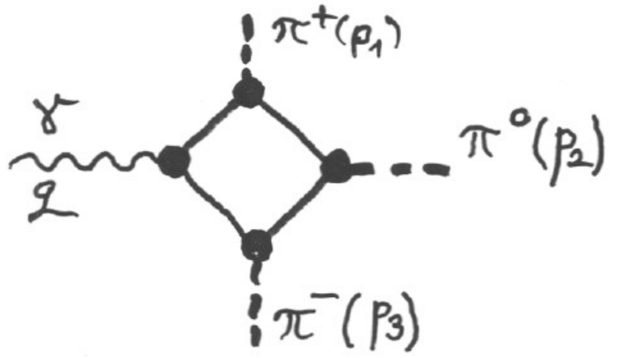
= THE BOUND-STATE APPROACH SUITABLE FOR ANOMALOUS PROCESSES: IT HAS CORRECT CHIRAL SYMM. BEHAVIOR, & LIGHT PSEUDOSCALARS ARE BOTH $q\bar{q}$ AND (QUASI)GOLDSTONES OF DχSB

→ INDEPENDENTLY OF INTERNAL $q\bar{q}$ STRUCTURE, EXACTLY REPRODUCES △ & ◇ AMPLITUDES FIXED BY AXIAL ANOMALY:



$$T_{\pi}^{2\gamma}(m_\pi = 0) = \frac{e^2 N_c}{12\pi^2 f_\pi}$$

(ROBERTS, MARIS, TANDY)



$$F_\gamma^{3\pi}(p_{1,2,3}=0)_{M_\pi=0} = \frac{T_\pi^{2\gamma}(0)}{e f_\pi^2} = \frac{e N_c}{12\pi^2 f_\pi^3}$$

(ALKOFER, ROBERTS, BISTROVIĆ, KLABUČAR)

CHIRAL & SOFT LIMIT RESULTS!

* SD-BS APPROACH SUITABLE ALSO BEYOND THESE LIMIT.

KLABUČAR, KEKEZ, BISTROVIĆ

1.) $\eta, \eta' \to \gamma\gamma^{(*)}$ $(m_\eta, m_{\eta'} \gg m_\pi \neq 0)$

2.) $F_\gamma^{3\pi}(p_1, p_2, p_3) = ?$ FOR $p_{1,2,3} \neq 0$, $M_\pi \neq 0$

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GLUON $\langle A^2 \rangle$ CONDENSATE AND SCHWINGER-DYSON APPROACH TO π, K,...

(BASED ON HEP-PH/0307110, TO APPEAR IN PHYS. REV. D)

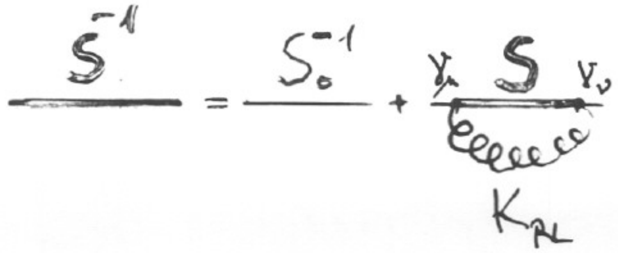
IN PHENOMENOLOGICAL S-D APPROACH,

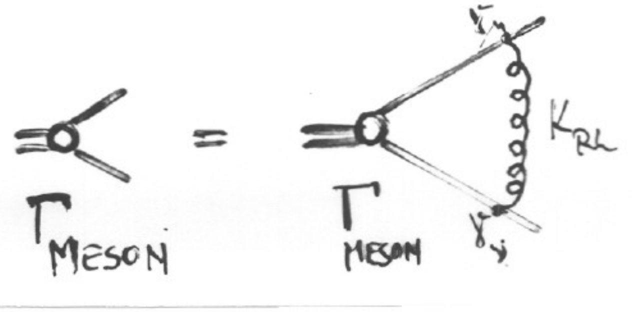
→ DχSB → LIGHT PSEUDOSCALAR MESONS AS (QUASI-) GOLDSTONE BOSONS FOR THE CONSISTENT APPROXIMATION - E.G., RAINBOW-LADDER (RL) IN **LANDAU** GAUGE

HAS

$$K_{RL}(p) = 4\pi \frac{\alpha_{eff}(p^2)}{p^2}\,\delta_{ab}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\frac{\lambda^a}{2}\gamma_\mu \otimes \gamma_\nu \frac{\lambda^b}{2}$$

BOTH IN "GAP" AND IN B.-S. EQS





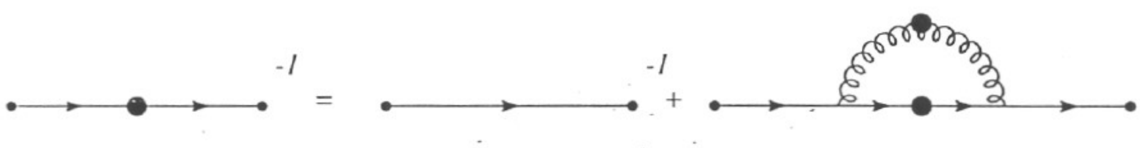
18.

Schwinger–Dyson equation

- rainbow approximation

$$S_a^{-1}(p) = \not{p} - \tilde{m}_a - iC_F\, g^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S_a(k) \gamma^\nu G_{\mu\nu}(p-k)$$

$$S_a^{-1}(p) = A_a(p^2)\not{p} - B_a(p^2) = A_a(p^2)[\not{p} - M_a(p^2)]$$

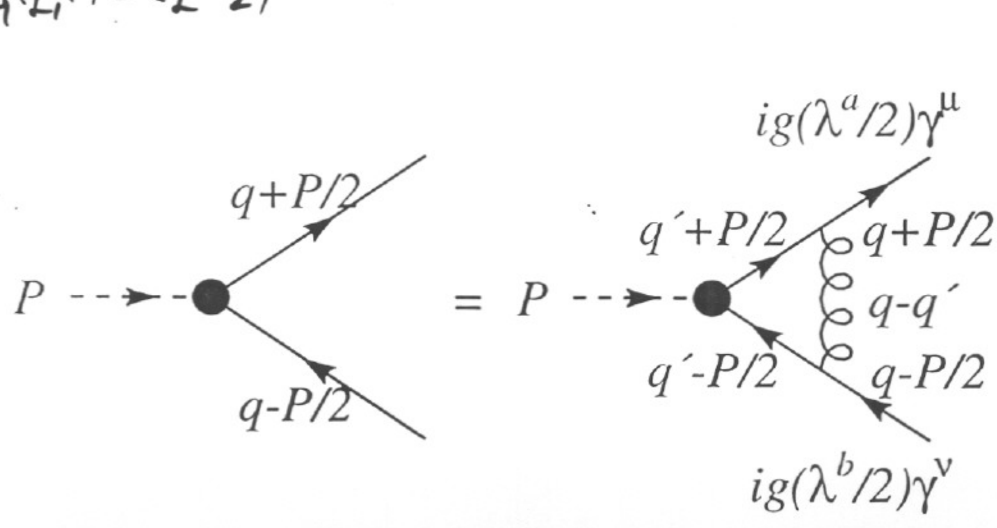


Bethe–Salpeter equation

- improved ladder approximation

$$S_a^{-1}(q+\frac{P}{2})\chi_{ab}(q,P)S_b^{-1}(q-\frac{P}{2}) = ig^2 C_F \int \frac{d^4q'}{(2\pi)^4}\gamma^\mu \chi_{ab}(q',P)\gamma^\nu G_{\mu\nu}(q-q')$$

$$\underbrace{\chi(q,P)}_{S(q+\frac{P}{2})\Gamma_M(q,P)S(q-\frac{P}{2})} = \gamma_5[\chi_0(q,P) + \not{P}\chi_1(q,P) + \not{q}\chi_2(q,P) + [\not{P},\not{q}]\chi_3(q,P)]$$



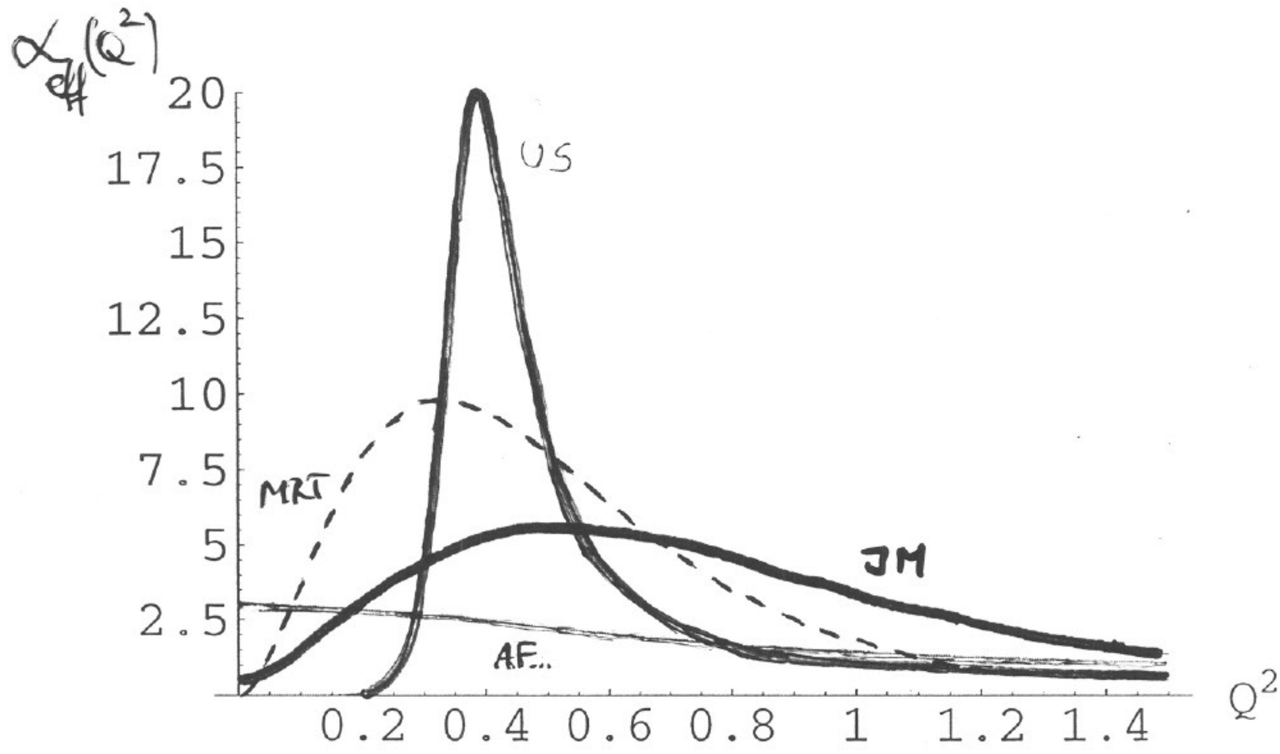


Fig. 1: The momentum dependence of various strong running couplings mentioned in the text. JM [9] and MRT [11,6,7] $\alpha_{eff}(p^2)$ are depicted by, respectively, blue and dashed curves. The effective coupling (15) proposed and analyzed in the present paper is depicted by the red curve, and $\alpha_s(p^2)$ (2) of Fischer and Alkofer [3] (fit A) by the green curve.

ENHANCEMENT OF α_{eff}

REQUIRED AT INTERMEDIATE

MOMENTA (SQUARED) ~ 0.3-0.6

AT LARGE SPACELIKE p, $\alpha_{eff}(p^2) \longrightarrow \alpha_{pert}(p^2)$ OF pQCD

BUT $\alpha_{eff}(p^2)$ = UNKNOWN IN NONPERTURBATIVE DOMAIN

... BUT PHENOMENOLOGICALLY SUCCESSFUL K_{RL} HAVE $\alpha_{eff}(p)$ ENHANCED AROUND

$p^2 \sim 0.5$ GeV

WHY?

EXPLANATION OF THAT ENHANCEMENT = ?

WE PROPOSE: DIMENSION 2 GLON CONDENSATE $\langle A^a_\mu A^{a\mu}\rangle \equiv \langle A^2\rangle$ MAY ENHANCE THE FORM FOR COUPLING USED IN "AB INITIO" S-D STUDIES IN LANDAU GAUGE, TÜBINGEN LATTICE, ...

$$\alpha_s(p^2) = \alpha_s(\mu^2)\, Z(p^2)\, [G(p^2)]^2$$

$$\alpha_s(\mu^2) = \frac{g^2}{4\pi}, \quad Z(\mu^2)\, G(\mu^2)^2 = 1$$

AT THE RENORMALIZATION POINT $p^2 = \mu^2$

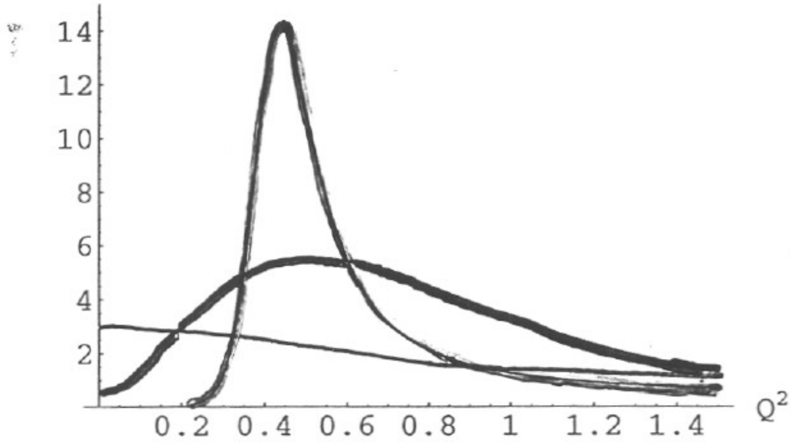


Figure 1: The effective strong coupling constant $\alpha_s(Q^2)$. Blue = Munczek and Jain model [3]. Red = our model, "old standard set" of input parameters. Green = Alkofer, fit A, Refs. [11, ?].

while Eqs. (31) and (32) relate c_{gluon} and c_{ghost} with the $\langle G^2 \rangle$ condensate... The effective strong coupling constant α_s is shown on Figs. 1 (and 2) as a function of Q^2. It has the four poles in the complex Q^2 plane given by

$$(Q^2)_1 = \frac{1}{2}\left(M_{\text{gluon}}^2 - i\sqrt{4c_{\text{ghost}} - M_{\text{gluon}}^4}\right) , \tag{43}$$

$$(Q^2)_2 = \frac{1}{2}\left(M_{\text{gluon}}^2 + i\sqrt{4c_{\text{ghost}} - M_{\text{gluon}}^4}\right) , \tag{44}$$

$$(Q^2)_3 = \frac{1}{2}\left(-M_{\text{gluon}}^2 - i\sqrt{4c_{\text{gluon}} - M_{\text{gluon}}^4}\right) , \tag{45}$$

$$(Q^2)_4 = \frac{1}{2}\left(-M_{\text{gluon}}^2 + i\sqrt{4c_{\text{gluon}} - M_{\text{gluon}}^4}\right) . \tag{46}$$

For $\min\{c_{\text{ghost}}, c_{\text{gluon}}\} > M_{\text{gluon}}^4/4$ there is no pole on the real axis.

9 Results...

The constituent quark mass $M(0) = B(0)/A(0)$ dependence on the parameter $c_{\text{ghost}} = c_{\text{gluon}}$ is shown on Fig. 7 for $m = 0$ and $M_{\text{gluon}} = 0.884$ GeV.

$$D_G^{ab}(p) = -\delta^a_b \frac{G(p^2)}{p^2}, \quad G(p^2) = \frac{1}{1 + \frac{\Pi_G(p^2)}{p^2}},$$

$$Z(p^2) = \frac{1}{1 + \frac{\Pi_A(p^2)}{p^2}}$$

$$D^{ab}_{\mu\nu}(p) = \frac{Z(p^2)}{p^2}\,\delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

INSTEAD OF "AB INITIO" SOLUTIONS,

USE O.P.E. RESULTS OF LAVELLE, SCHADEN, ..., KONDO

$$\Pi_i(p^2) = m_i^2 + \mathcal{O}_i(1/p^2)\,, \qquad (i = A, G)\,,$$

$$\left.\begin{matrix} D = 4 \\ N_c = 3 \\ \text{LANDAU} \\ \text{GAUGE} \end{matrix}\right\} \quad m_A^2 = \frac{3}{32}\, \underbrace{g^2\langle A^2\rangle}_{\simeq (2.79\,\text{GeV})^2} \overset{!}{=} -m_G^2 \quad \simeq (0.845\,\text{GeV})^2$$

(BOUCAUD & AL, LATTICE)

THESE CONTRIBUTIONS TO GLUON & GHOST PROPAGATORS IMPLY $\langle A^2 \rangle$ CONDENSATE = IMPORTANT FOR DS APPROACH TO HADRONS.

...?... BUT UNLIKE DIMENSION-4 $\langle F^2 \rangle$ CONDENSATE, $\langle A^2 \rangle$ IS <u>NOT</u> GAUGE INVARIANT ... THEN HOW IT CAN INFLUENCE OBSERVABLE QUANTITIES? (MASSES, f_H ...)

GUBAREV, STODOLSKY, ZAKHAROV, ... MORE RECENTLY, DUDAL & VERSCHELDE... KONDO...:

- IT TURNS OUT, LANDAU-GAUGE $\langle A^2 \rangle$ EQUALS A NON-LOCAL, BUT GAUGE-INVARIANT QUANTITY: MINIMAL VALUE (WITH RESPECT TO GAUGE CHOICE) OF INTEGRATED $A^a_\mu A^a_\mu$, NAMELY

$$\min_U \left[\frac{1}{2} \int d^4x \, A^{a\,U}_\mu(x) \, A^{a\,U}_\mu(x) \right] \Big/ VT$$

$$[A^U] = \{ A^U = UAU^\dagger + U dU : U(x) \in SU(N_c) \}$$

→ $\langle A^2 \rangle$ IN LANDAU GAUGE MAY HAVE A PHYSICAL MEANING.

– OUTSIDE L.G., BESIDES $\langle A^2 \rangle$ OTHER CONDENSATES OF DIMENSION 2 APPEAR. THESE GHOST CONDENSATES LIKELY CANCEL VARIATIONS WHICH $\langle A^2 \rangle$ SUFFERS WITH GAUGE CHANGES, SINCE THE PHYSICS BEHIND ALL THESE DIFFERENT DIM.-2 CONDENSATES IN DIFFERENT GAUGES IS THE SAME:

GLUON-GHOST CONDENSATION LOWERS THE QCD VACUUM ENERGY E, WHICH IS A PHYSICAL, GAUGE-INVARIANT QUANTITY, TO A STABLE ("$E < 0$") VACUUM.

J.H. FIELD, PHYSICAL REVIEW D 66, 013013 (2002)

"PHENOMENOLOGICAL ANALYSIS OF GLUON MASS EFFECTS..

Reference	Estimation method	Gluon mass
[12] PARISI & PETRONZIO	$J/\psi \to \gamma X$	800 MeV
[8] CORNWALL	Various	500 ± 200 MeV
[59] DONNACHIE & LANDSHOFF	Pomeron parameters	687–985 MeV
[61] HANCOCK & ROSS	Pomeron slope	800 MeV
[62] NIKOLAEV + al.	Pomeron parameters	750 MeV
[63] SPIRIDONOV & CHETYRKIN	$\Pi^{\mathrm{c.m.}}_{\mu\nu}$, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	750 MeV
[64] LAVELLE '91	$qq \to qq$, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	640 $\mathrm{MeV}^2/Q(\mathrm{MeV})$
[67] KOGAN & KOVNER	QCD vacuum energy, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	1.46 GeV
[68] FIELD	PQCD at low scales (various)	$1.5^{+1.2}_{-0.6}$ GeV
[39] LIU & WETZEL	$\Pi^{\mathrm{c.m.}}_{\mu\nu}$, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	570 MeV
	Glueball current, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	470 MeV
[66] YNDURÁIN	QCD potential	10^{-10}-20 MeV
[69] LEINWEBER & AL. '99	Lattice gauge	1.02 ± 0.10 GeV
This paper FIELD	$J/\psi \to \gamma X$	$0.721^{+0.016}_{-0.068}$ GeV
	$\Upsilon \to \gamma X$	$1.18^{+0.09}_{-0.29}$ GeV

ALEXANDROU, DE FORCRAND, FOLLANA: (2002) LATTICE 600^{+150}_{-30} MeV

$\Rightarrow m_A = 0.845$ GeV = REASONABLE ESTIMATE FOR GLUON DYNAMICAL MASS

WE CAN ONLY PARAMETERIZE $\mathcal{O}_i\left(\frac{1}{p^2}\right) \simeq \frac{C_i}{p^2}$

BUT RECALL LAVELLE '91 :

$$\mathcal{O}_A\left(\frac{1}{p^2}\right) \sim \Pi_A^{\langle F^2 \rangle} = \frac{34 N_c \pi \alpha_s \langle F^2 \rangle}{9(N_c^2-1)\, p^2} \equiv \frac{C_A}{p^2} = \frac{(0.640\ \mathrm{GeV})^3}{p^2}$$

= JUST A TENTATIVE ESTIMATE FOR C_A

A HEURISTIC DERIVATION

SUPPOSE ONE CAN DISENTANGLE pert FROM Npert PARTS:

$$\Pi_i = \Pi_i^{pert} + \Pi_i^{Npert} \qquad i = A, G$$

$$Z(p^2) \equiv \frac{1}{1+\frac{\Pi_A(p^2)}{p^2}} \approx \frac{1}{1+\frac{\Pi_A^{pert}(p^2)}{p^2}}\,\frac{1}{1+\frac{\Pi_A^{Npert}(p^2)}{p^2}} \equiv Z^{pert}(p^2)\,Z^{Npert}(p^2)\,,$$

(NEGLECT $\frac{\Pi_A^{pert}(p^2)\,\Pi_A^{Npert}(p^2)}{p^4}$ AT HIGH p^2)

$$\frac{g^2}{4\pi}\,Z^{pert}(p^2)\,G^{pert}(p^2)^2 \equiv \frac{g^2}{4\pi}\,\frac{1}{1+\frac{\Pi_A^{pert}(p^2)}{p^2}}\left(\frac{1}{1+\frac{\Pi_G^{pert}(p^2)}{p^2}}\right)^2 = \alpha_{pert}(p^2)$$

["AB INITIO" S-D : $\alpha_s(p^2) = \frac{g^2}{4\pi}\,Z(p^2)\,G(p^2)^2$]

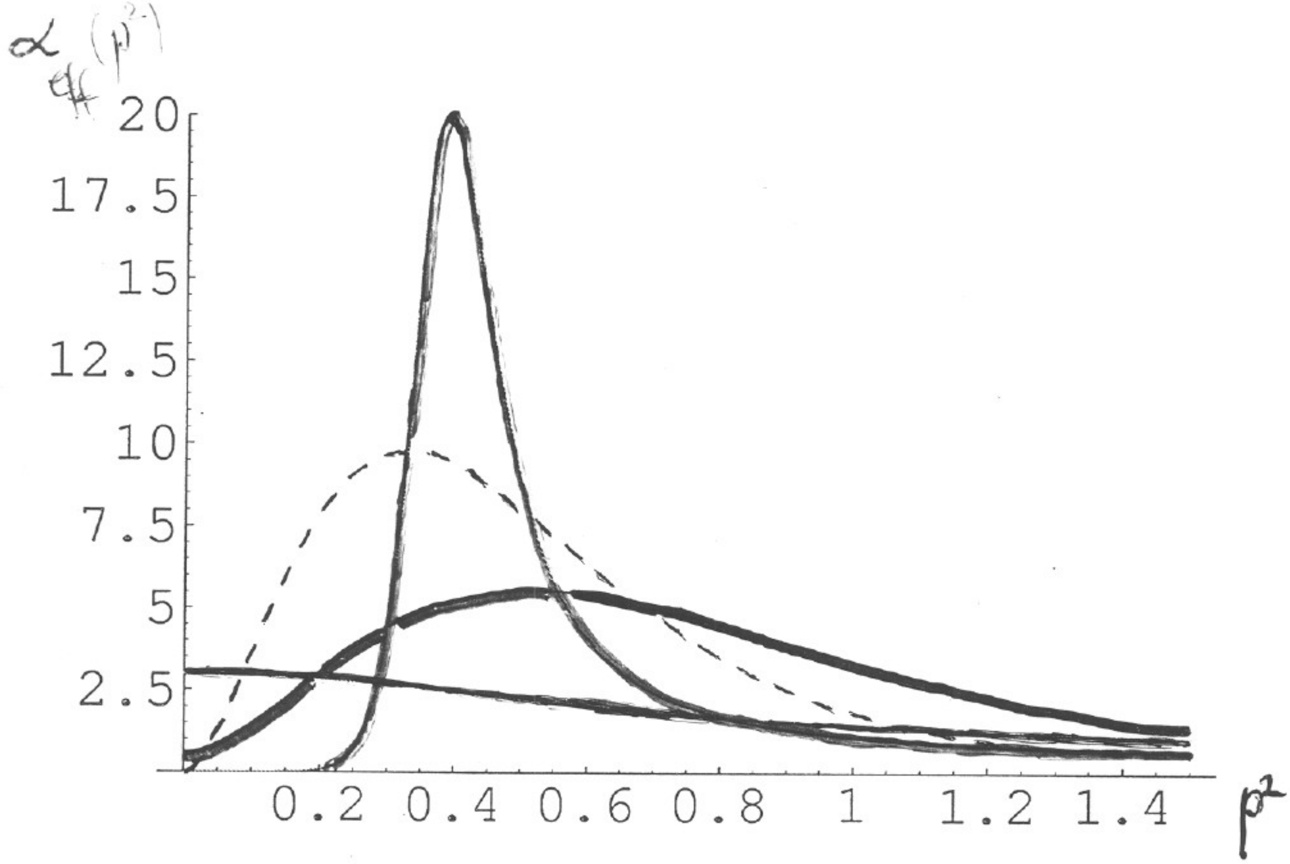
$$Z^{Npert}(p^2) = \frac{1}{1+\frac{m_A^2}{p^2}+\frac{C_A}{p^4}}\,,$$

$$G^{Npert}(p^2) = \frac{1}{1-\frac{m_A^2}{p^2}+\frac{C_G}{p^4}}\,,$$

(BREAKS DOWN FOR $p^2 \to 0$, BUT SMALL-p^2 CONTRIBUTIC IN PHENOMENOLOGICAL APPLICATIONS = NOT VERY IMPORTANT.)

$$\underline{\alpha_{eff}(p^2) = \alpha_{\rm pert}(p^2)\,Z^{Npert}(p^2)\,G^{Npert}(p^2)^2}$$

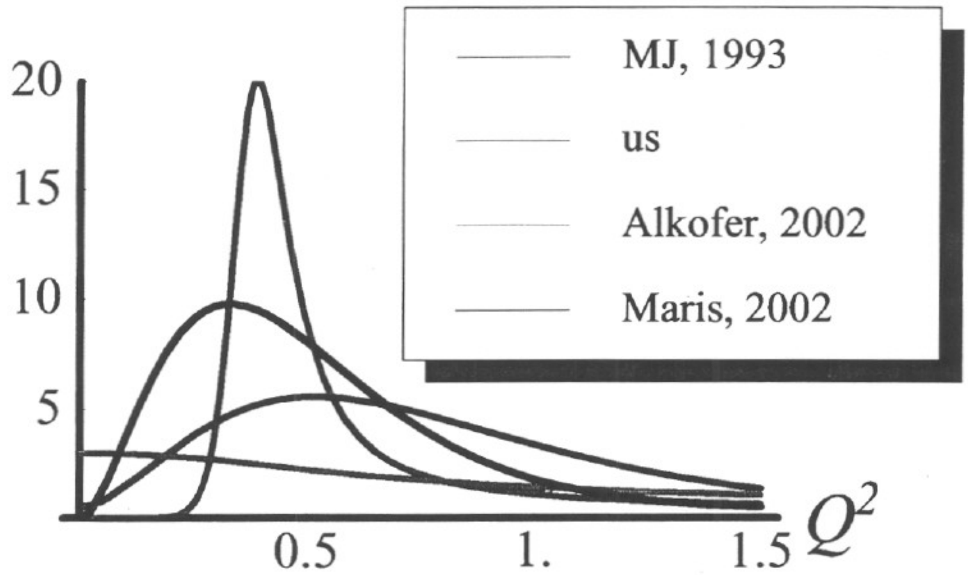
ONLY "A POSTERIORI" JUSTIFICATION BELOW $p^2 \sim 1$ GeV



COMPLEX

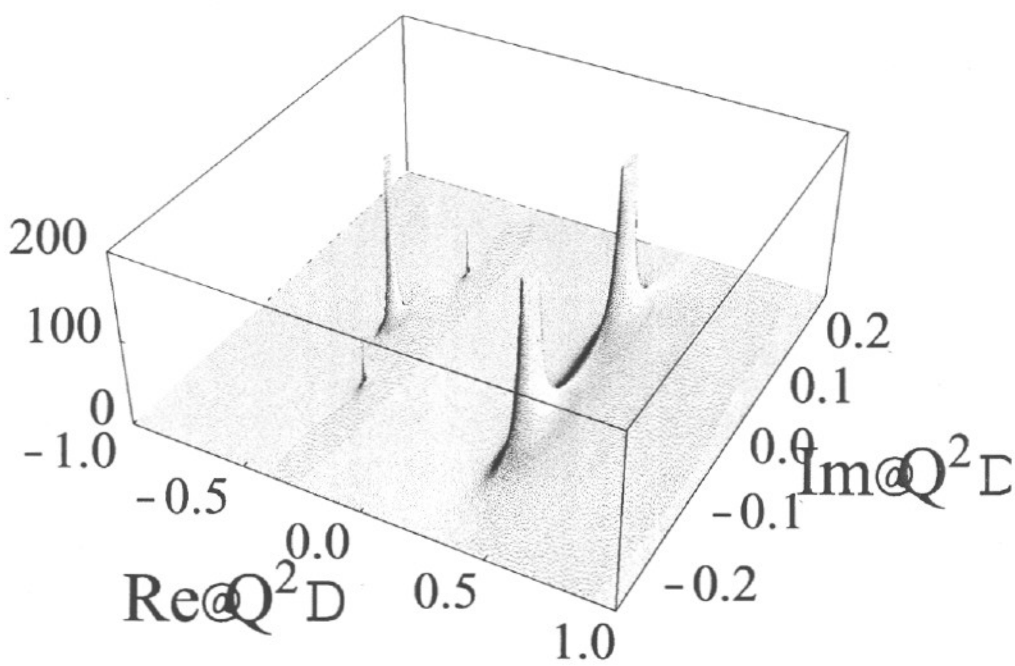
$$(p^2)_{1,2} = \frac{1}{2}\left(m_A^2 \mp i\sqrt{4C_G - m_A^4} \right) \qquad \text{[poles of } G^{Npert}(p^2)\text{]} \; ,$$

$$(p^2)_{3,4} = \frac{1}{2}\left(-m_A^2 \mp i\sqrt{4C_A - m_A^4} \right) \qquad \text{[poles of } Z^{Npert}(p^2)\text{]}$$



- the poles of $\alpha_{\rm eff}(Q^2)$

$$
\begin{aligned}
(Q^2)_1 &= \frac{1}{2}\left(m_A^2 - i\sqrt{4C_G - m_A^4}\right)\\
(Q^2)_2 &= \frac{1}{2}\left(m_A^2 + i\sqrt{4C_G - m_A^4}\right)\\
(Q^2)_3 &= \frac{1}{2}\left(-m_A^2 - i\sqrt{4C_A - m_A^4}\right)\\
(Q^2)_4 &= \frac{1}{2}\left(-m_A^2 + i\sqrt{4C_A - m_A^4}\right)
\end{aligned}
$$



A BETTER DERIVATION OF $\alpha_{eff}(p^2)$:

CONSTANTS AT TREE LEVEL DEVELOP $\log - p^2$-DEPENDENC:

$$m_A^2 = \frac{N_c g^2 \langle A^2 \rangle}{4(N_c^2-1)} = -m_G^2 \longrightarrow m_A^2(p^2) = -m_G^2(p^2)$$

DUE TO PERTURBATIVE QCD CORRECTIONS

E.G., $Z(p^2) = \dfrac{1}{1 + \dfrac{N_c g^2 \langle A^2\rangle}{4(N_c^2-1)}\dfrac{1}{p^2} + \dfrac{O(1/p^2)}{p^2}} \xrightarrow[\text{KONDO + AL. IN L.G.}]{\text{1-LOOP}}$

$$\rightarrow Z(p^2) = \frac{1}{\left[\dfrac{\ln(p^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\right]^{\frac{13N_c}{6\beta_0}} + \dfrac{N_c g^2\langle A^2\rangle}{4(N_c^2-1)p^2}\left[\dfrac{\ln(p^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\right]^{\frac{61 N_c}{12\beta_0}} + \dfrac{O\left(\frac{\ln p^2}{\Lambda^2 p^2}\right)}{p^2}}$$

GENERIC FORMS OF GLUON & GHOST RENORMALIZATION FUNCTIONS FROM O.P.E. WITH pQCD CORRECTIONS

$$Z(p^2) = \frac{1}{d_0^A(p^2) + \dfrac{d_2^A(p^2)}{p^2} + \dfrac{d_4^A(p^2)}{p^4} + \cdots}$$

$$G(p^2) = \frac{1}{d_0^G(p^2) + \dfrac{d_2^G(p^2)}{p^2} + \dfrac{d_4^G(p^2)}{p^4} + \cdots}$$

$d_n^{A,G}(p^2)$ = TERM OF MASS-DIMENSION n FOR A OR G CASE.

AT ONE-LOOP LEVEL: $\alpha_{pert} = \frac{g^2}{4\pi} = \frac{4\pi}{\beta_0 \ln(p^2/\Lambda^2)}$

ANOMALOUS DIMENSIONS $\Bigg\}$ - GLUON: $\gamma = \frac{-13N_c + 4N_f}{22N_c - 4N_f} = \frac{-13N_c + 4N_f}{6\beta_0}$

- GHOST: $\delta = \frac{-9N_c}{44N_c - 8N_f} = \frac{-3N_c}{4\beta_0}$

→ RESULTS OF KONDO + AL FOR PURE Y.M. ($N_f = 0$) GIVE

$$d_0^A(p^2) = \left[\frac{\alpha_{pert}(p^2)}{\alpha_{pert}(\mu^2)}\right]^{\gamma}$$

$$d_0^G(p^2) = \left[\frac{\alpha_{pert}(p^2)}{\alpha_{pert}(\mu^2)}\right]^{\delta}$$

- FROM KONDO + AL, BOUCAUD + AL, FOR PURE Y.M.

$$\frac{d_2^A(p^2)}{d_0^A(p^2)} = -\frac{d_2^G(p^2)}{d_0^G(p^2)} \equiv m_A^2(p^2) = -m_G^2(p^2)$$

THIS GIVES

$$Z(p^2) = \left[\frac{\alpha_{pert}(p^2)}{\alpha_{pert}(\mu^2)}\right]^{-\gamma} \frac{1}{1 + \frac{m_A^2(p^2)}{p^2} + \frac{C_A(p^2)}{p^4} + \cdots}$$

$$G(p^2) = \left[\frac{\alpha_{pert}(p^2)}{\alpha_{pert}(\mu^2)}\right]^{-\delta} \frac{1}{1 - \frac{m_A^2(p^2)}{p^2} + \frac{C_G(p^2)}{p^4} + \cdots}$$

OF COURSE, TERMS OF DIM. 4 (AND HIGHER) HAVE <u>NOT</u> BEEN CALCULATED — JUST SUGGESTIVE NOTATION

$$C_{A,G}(p^2) = \frac{d_4^{A,G}(p^2)}{d_0^{A,G}(p^2)}$$

TO POINT OUT THE CORRESPONDENCE WITH EARLIER TREE-LEVEL RELATIONS WHICH DIFFER JUST BY THE ABSENCE OF THE

SINCE OBVIOUSLY

$$\gamma + 2\delta = -1,$$

$$\alpha_s(p^2) = \alpha_s(\mu^2)\, Z(p^2) \left[G(p^2)\right]^2 \quad \text{GIVES}$$

$$\alpha_s(p^2) = \alpha_{pert}(p^2)\, \frac{1}{1 + \frac{m_A^2(p^2)}{p^2} + \frac{C_A(p^2)}{p^4} + \cdots} \cdot \left(\frac{1}{1 - \frac{m_A^2(p^2)}{p^2} + \frac{C_G(p^2)}{p^4} + \cdots} \right)^2$$

p^2-DEPENDENCE OF $C_{A,G}$ AND HIGHER TERMS = UNKNOWN, EXCEPT THAT IT IS LOGARITHMIC.

NEGLECTING COMPLETELY UNKNOWN HIGHER TERMS ("...") AND SLOWLY VARYING LOG DEPENDENCE IN $C_{A,G}$ AND THEN ALSO IN m_A,

$$\alpha_s(p^2) \approx \alpha_{pert}(p^2)\, \frac{1}{1 + \frac{m_A^2}{p^2} + \frac{C_A}{p^4}} \left(\frac{1}{1 - \frac{m_A^2}{p^2} + \frac{C_G}{p^4}} \right)^2$$

$$\equiv \alpha_{eff}(p^2) = \alpha_{pert}(p^2)\, Z^{N_{pert}}(p^2) \left[G^{N_{pert}}(p^2) \right]^2$$

AS FROM THE QUICK HEURISTIC DERIVATION.

IN THE CHIRAL LIMIT

$\tilde{m}_q = 0 \longrightarrow$ ONLY PARAMETERS

ARE m_A, C_A, C_G

ON THE BASIS OF LAVELLE, SCHADEN, ..., KONDO, ...

$\rightarrow$ EXPECT $C_G \sim C_A \Rightarrow$ TRY $C_G = C_A$

LAVELLE '91 $\rightarrow$ INITIAL ESTIMATE $C_A = (0.64\ \mathrm{GeV})^4$

$\rightarrow$ WITH $m_A^2 = \frac{3}{32} g^2 \langle A^2 \rangle = (0.845\ \mathrm{GeV})^2$ <u>ALMOST</u>

FITTED CHIRAL LIMIT

JUST 5% CHANGE: $m_A \longrightarrow 0.884$ GeV

"OLD PARAMETER SET": $m_A = 0.884$ GeV, $C_G = C_A = (0.64\ \mathrm{GeV})^4$

	$\langle \bar{q}q \rangle$ [GeV^3]	f_π [GeV]	$-\frac{\langle \bar{q}q \rangle}{f_\pi^2}$ [GeV]	$\lim_{m\to 0} \frac{M_\pi^2}{2m}$ [GeV]
MJ	**-0.0110365**	**0.0898237**	**1.36788**	**1.40093**
this work	**-0.00981022**	**0.0882591**	**1.24073**	**1.29289**

$(-214\ \mathrm{MeV})^3$

Test of the Gell-Mann-Oaks-Renner (GMOR) relation:

$$\lim_{m \to 0} \frac{M_\pi^2}{2m} = -\frac{\langle \bar{q}q \rangle}{f_\pi^2} .$$

$$T^{\chi\,\mathrm{lim}}_{\pi^0 \to \gamma\gamma} \equiv T^{\chi\,\mathrm{lim}}_{\pi^0}(0,0) = \frac{1}{4\pi^2 f_\pi} = 0.282\ \mathrm{GeV}^{-1}, \quad \mathbf{O.K.}$$

AWAY FROM THE CHIRAL LIMIT

- FIRST, WITHOUT ANY REFITTING:
- TAKE "STANDARD" (LONG USED) $\tilde{m}_q \neq 0$

FIXED BY MUNCZEK-JAIN FIT '93:

$$\boxed{\begin{aligned} \tilde{m}_u = \tilde{m}_d &= 3.1 \text{ MeV} \\ \tilde{m}_s &= 73.0 \text{ MeV} \end{aligned}}$$

(WITH α^{MJ}_{eff})

$$\boxed{\begin{aligned} m_A &= 0.884 \text{ GeV} \\ C_G = C_A &= (0.640 \text{ GeV})^4 \end{aligned}}$$

OLD SET OF PARAMETERS, AGAIN THE SAME $\alpha_{PERT}(p^2)$

H	M_H [GeV]	f_H [GeV]	$T_H(0,0)$ [GeV^{-1}]
π	0.1367	0.09124	0.27170
K^+	0.5207	0.11213	
$s\bar{s}$	0.7621	0.13293	0.08828

RE-FIT $y = \{M_{\pi^0}, M_{K^0}, f_{\pi^\pm}, f_{K^\pm}\}$

- MINIMIZE $F[x] = \sum_y \left(\frac{y_{EXP} - y_{TH}}{y_{EXP}} \right)^2$

A SET OF the NEW input parameters $x = \{\tilde{m}_u, \tilde{m}_s, m_A, C_G\}$

$\tilde{m}_u(L) = \tilde{m}_d(L)$	$3.046 \cdot 10^{-3}$ GeV
$\tilde{m}_s(L)$	$67.70 \cdot 10^{-3}$ GeV
$\Lambda_{\rm QCD}$	0.228 GeV
N_f	5
x_0	10 $\sim \frac{m_A^2}{\Lambda_{QCD}^2}$
m_A	0.8402 GeV
C_G	0.6060^4 GeV^4
C_A	0.6060^4 GeV^4

NEW ≈ OLD M-J

ALWAYS "STANDARD" IN $\alpha(p^2)_{PERT}$

IN $\alpha(p^2)_{PERT}$

$\ln\left(\frac{p^2}{\Lambda^2_{QCD}}\right) \to \ln\left(x_0 + \frac{p^2}{\Lambda^2_{QCD}}\right)$

NEW

- Calculated pseudoscalar meson masses and decay constants, all in units of GeV

H	M_H	f_H	$T_H(0,0)$	$\Gamma(H \to \gamma\gamma)$
π	0.134957	0.0929336	0.256045	$0.673973 \cdot 10^{-8}$
K^+	0.494925	0.111454		
$s\bar{s}$	0.722104	0.132934	0.0859868	$0.116435 \cdot 10^{-6}$

- Experimental values, all in units of MeV

	M_H	f_H
π^0	134.9766 ± 0.0006	91.9 ± 3.5
$\pi^\pm$	139.57018 ± 0.00035	92.42 ± 0.26
K^0	497.672 ± 0.031	
$K^\pm$	493.677 ± 0.016	113.0 ± 1.0

$T^{EXP}_{\pi^0}(0,0) = 0.274 \pm 0{,}010$ GeV^{-1}

FIXED: $C_A = (0.606\ \text{GeV})^4 = C_G$, $m_A = 0.8402$ GeV

χ LIMIT RESULTS: $f_{\pi^+} = f_{\pi^0} = 90.5$ MeV

$T_{\pi^0}^{\gamma\gamma}(\chi\ \text{LIM}) \equiv \dfrac{1}{4\pi^2 f_\pi} = 0.280\ \text{GeV}^{-1}$, $\langle \bar{q}q \rangle = (-217\ \text{MeV})^3$

	M_{π^0}	f_{π^+}	M_{K^+}	f_{K^+}	$T_{\pi^0}^{\gamma\gamma}$ [GeV^{-1}]
MJ: $\tilde{m}_{u,d} = 3.1$ MeV, $\tilde{m}_s = 73$ MeV	136.17	93.0	516.28	112.5	0.256
NEW: $\tilde{m}_{u,d} = 3.046$ MeV, $\tilde{m}_s = 67.70$ MeV	134.96	92.9	494.92	111.5	0.256
EXP. VALUES	134.98	92.4 ± 0.3	493.68	113.0 ± 1.0	0.274 ± 0.010

VARY $\tilde{m}_u, \tilde{m}_s$; $C_G = C_A$ FIXED

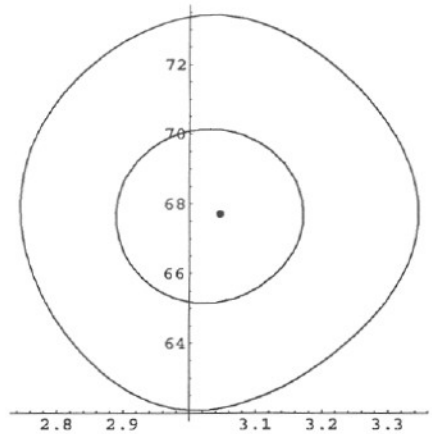


Figure 14: The solutions of the equations $F = 2.5\%$ and $F = 5.0\%$ in (m_u, m_s) plane. The red point is position of the minimum.

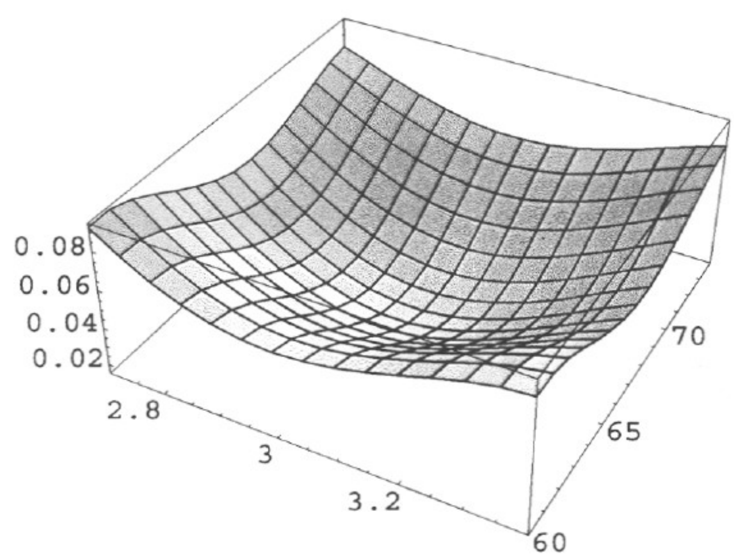


Figure 15: F vs. (m_u, m_s).

CONTRARY TO <u>HIGH</u> SENSITIVITY ON $\tilde{C}_G$

AND <u>HIGH</u> SENSITIVITY ON m_A.

- IN CONTRAST TO $(\tilde{m}_u, \tilde{m}_d)$, MANY PAIRS OF (m_A, C_G) GIVE GOOD FIT ($F \sim 1{,}5\%$) AS LONG AS

$$[C_G]^{1/4} = 0.774\, m_A - 0.044 \text{ GeV}$$

- PRESERVES "INTEGRATED STRENGTH"
- "ONE-PARAMETER MODEL" $(0.6 \text{ GeV} \lesssim C_G^{1/4} \lesssim 0.9 \text{ GeV})$

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$$[C_G]^{1/4} = 0.774\, m_A - 0.044 \text{ GeV}$$

$$C_G^{1/4} \in (0.6,\ 0.9)\,\text{Ge}$$

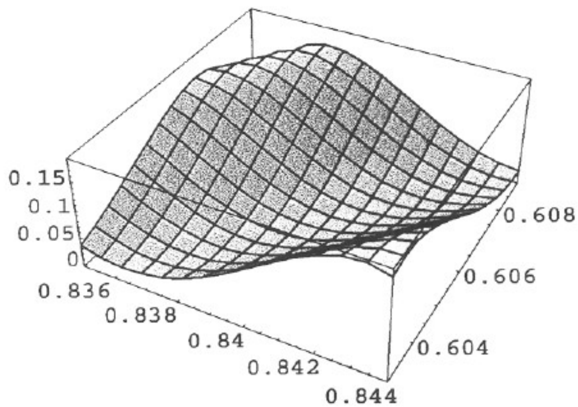

Figure 16: F vs. $(M_{\text{gluon}}, c_{\text{ghost}}^{1/4})$ 3D plot.