Parton Evolution and Saturation in Deep Inelastic Scattering

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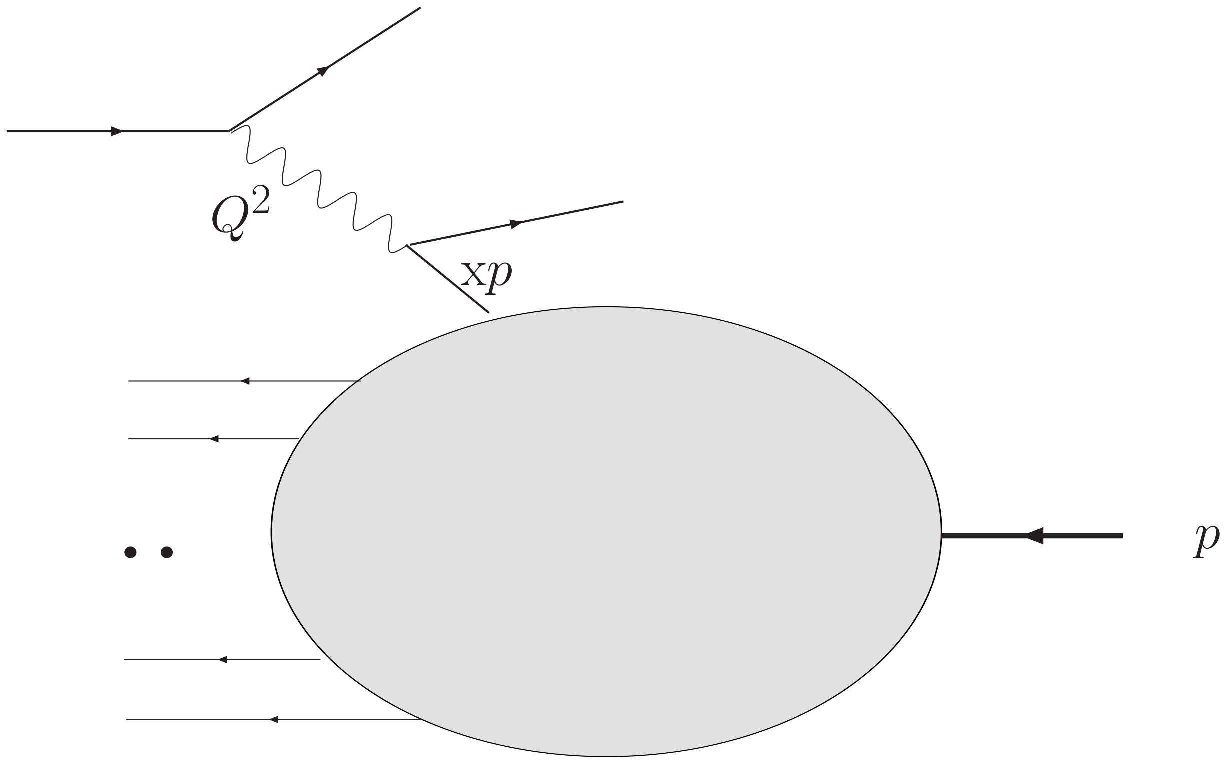
Ericeira, November 5, 2004

- Introduction: Saturation
- (Experimental) evidence at HERA
- Theory: evolution equations
- Conclusions

Deep Inelastic Electron Proton Scattering



Introduction

Saturation in deep inelastic scattering: basic idea

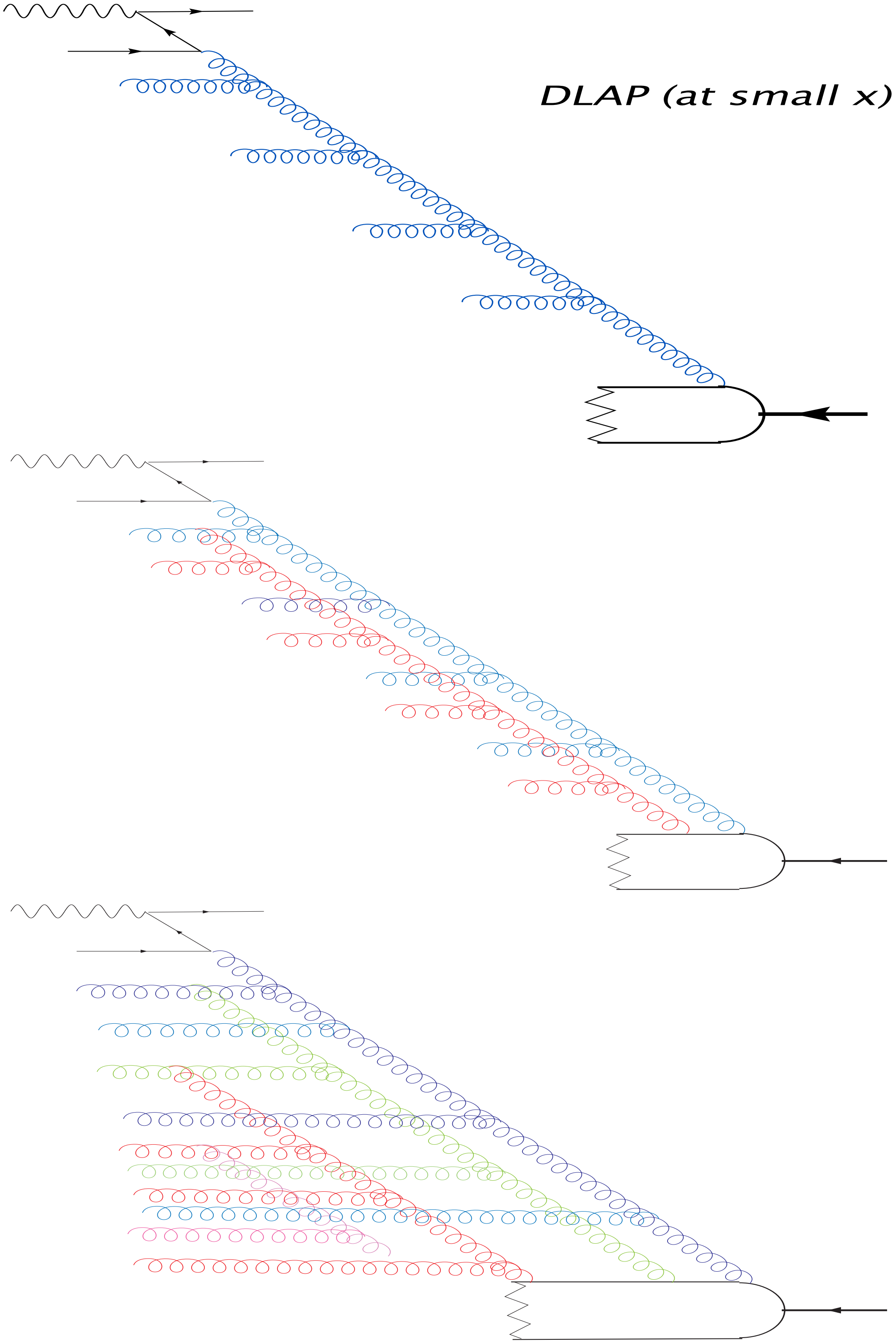
- linear evolution based upon assumption of diluteness of partons inside the hadron, one parton radiates until it enters the hard interaction.

- growth of gluon density at small x comes into conflict with diluteness assumption. Two (and more) partons inside one hadron may become relevant: multiparton contributions

- leads to a state of high gluon density
  Concept of parton density no longer valid:
  $\rightarrow$ strong classical field ('color glass condensate')

- multiparton contributions lower the growth of the parton densities (and of the total cross section).

- new scale: $Q_s^2(x) = c(1/x)^\lambda$, $\lambda \approx 0.3$


DLAP (at small x)

If true: a simplified summary:

small-x ($Q^2 < Q_s^2(x)$) gluons come from regions inside the proton where the density is high.

Has consequences:

- pp scattering at LHC: multiparton interactions (diffractive Higgs production)
- heavy ion collisions at RHIC, LHC: ‘pre-stage’ of formation of quark gluon plasma

(Experimental) Evidence at HERA

Probably not enough phase space to see saturation at medium/large Q^2 and small x.
Need eN machine!
Instead: low-Q^2 and small x, apply the same arguments.

What can be listed as evidence:

1) success in describing F_2
2) geometrical scaling
3) ratio of cross sections in DIS diffraction

1) Describe F_2 in the transition region.
Several successful saturation models for F_2:

Golec-Biernat, Wuesthoff, Phys.Rev.D 59 (1999) 014017; Phys.Rev.D 60 (1999) 114023

JB, Golec-Biernat, Kowalski, Phys.Rev.D 66 (2002) 014001

Gotsman, Levin, Lublinsky, Maor, Eur.Phys.J.C 27 (2003) 411

Iancu, Itakura, Munier, Phys.Lett.B 590 (2004) 199.

Some models provide quantitative estimates for the saturation scale $Q_s(x)$ (for F_2, averaged over $\vec{b}$):

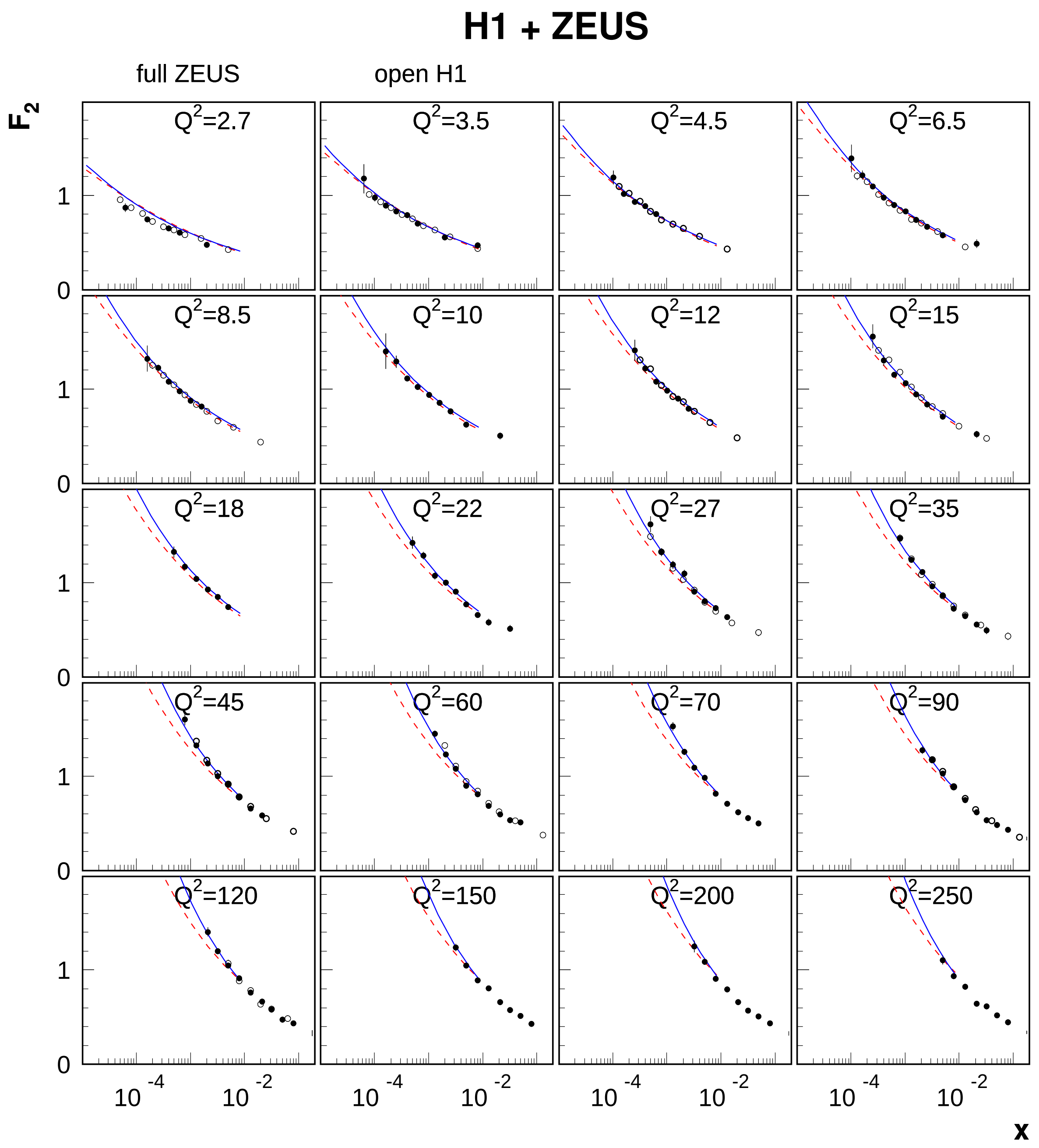
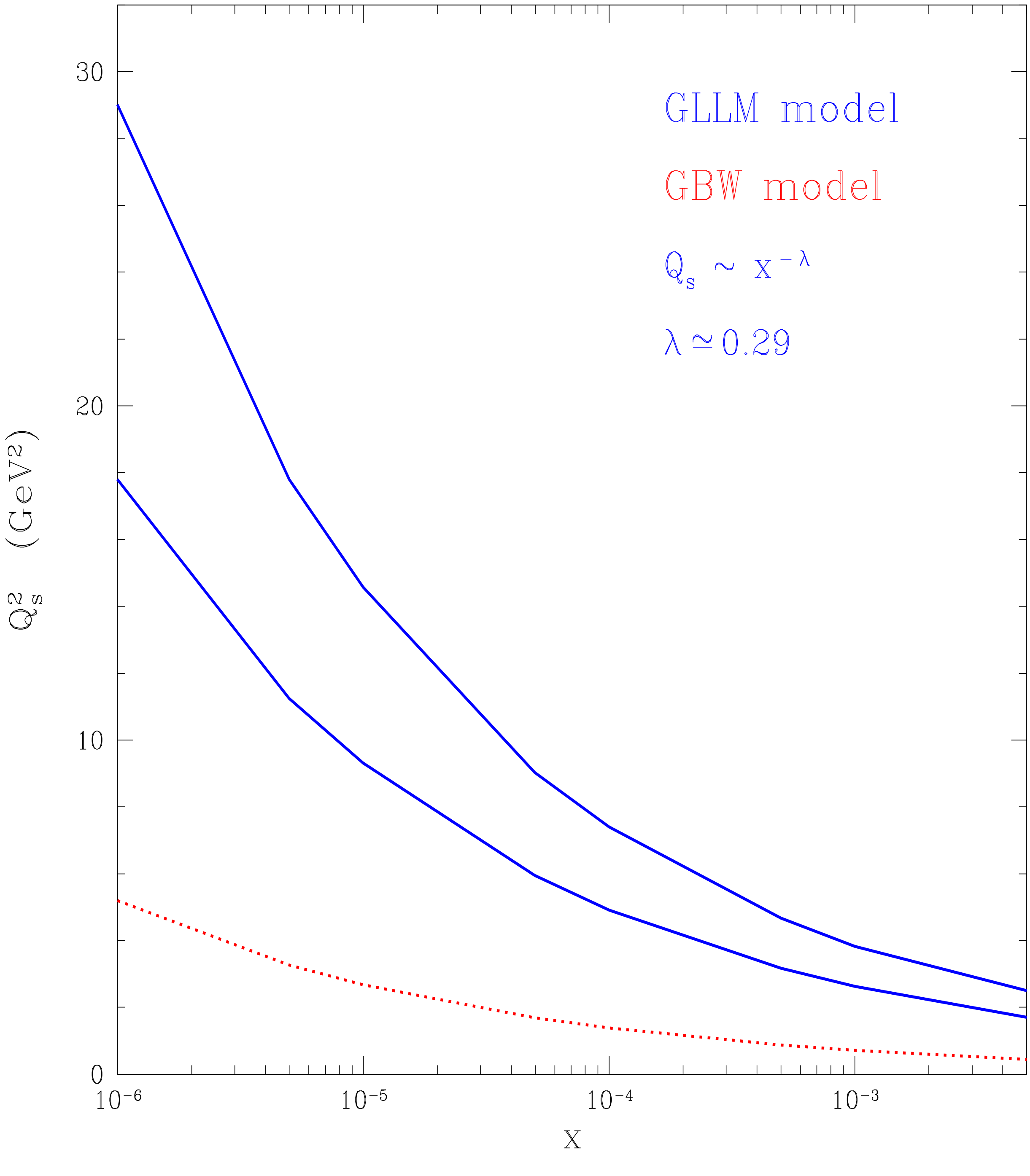


Figure 2: *H1 and ZEUS data on F_2 as a function of x for fixed values of $Q^2 > 1$ GeV^2 and the saturatiom model curves. The solid lines: the model with the DGLAP evolution (8) (FIT 1) and the dotted lines: the saturation model (2).*



No consensus on the location of saturation scale.

Other successful descriptions of low-Q^2 transitions.

Limit of applicability of DGLAP:
resummation lowers the limit.

Altarelli et al, Florence group

2) Scaling: $F_2(Q^2, x) \; = \; F_2(Q^2/Q_s^2(x)$.

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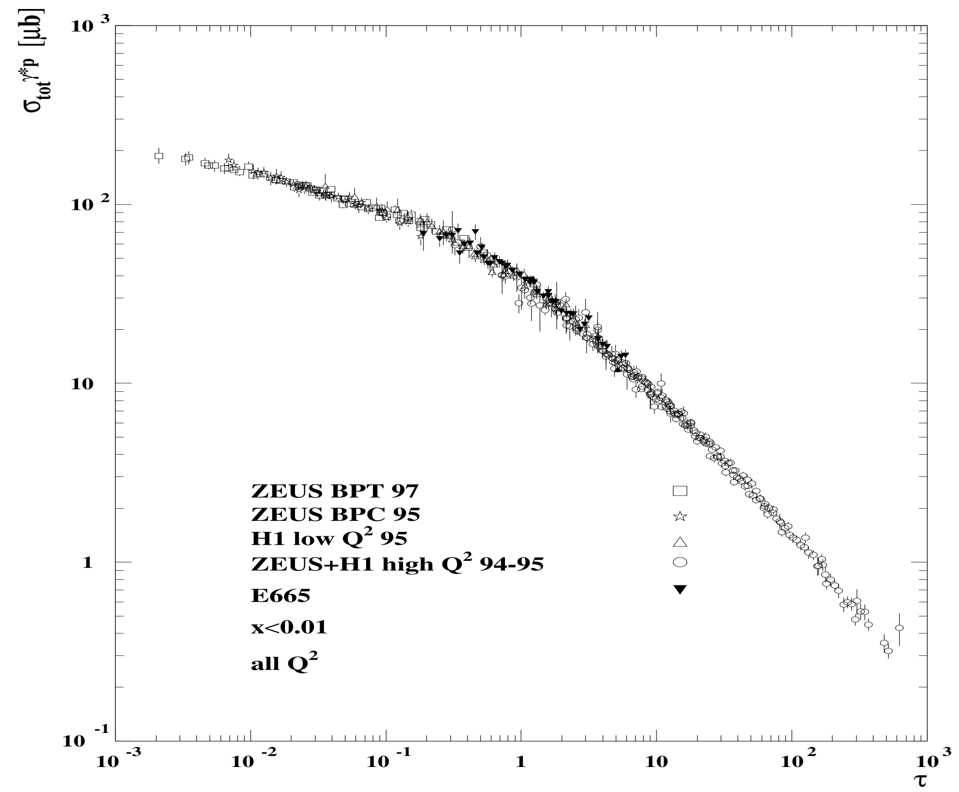

Figure 1: Experimental data on $\sigma_{\gamma^* p}$ from the region $x < 0.01$ plotted versus the scaling variable $\tau = Q^2 R_0^2(x)$.

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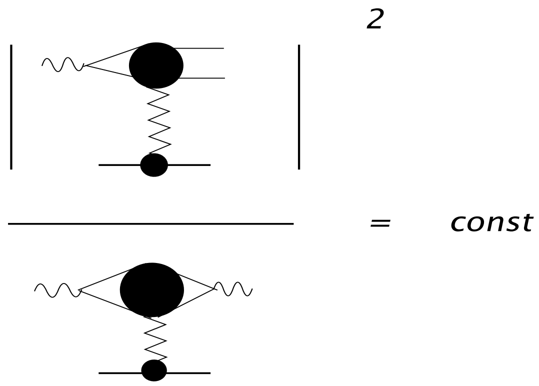
Claim: can be derived also from other models.

3) Energy dependence of $\frac{\sigma_{diff}}{\sigma_{tot}}$ at fixed Q^2, M^2:



most natural explanation: saturation models.

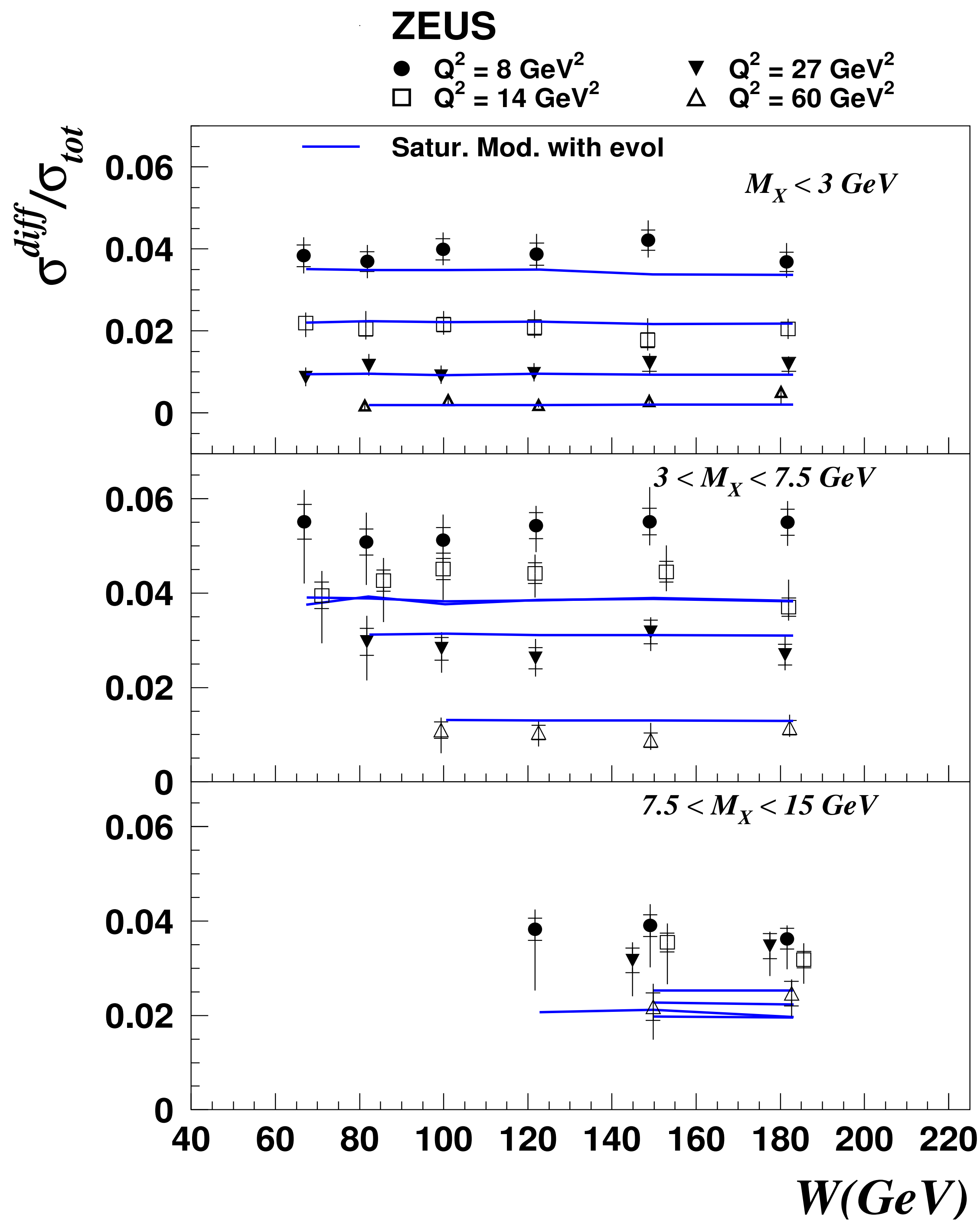


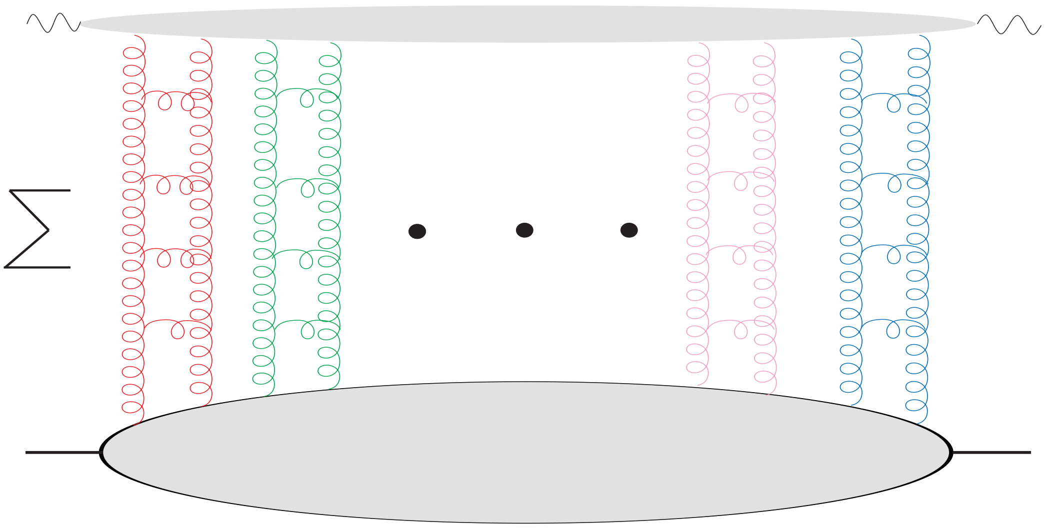
Figure 9: *The ratio of $\sigma_{diff}/\sigma_{tot}$ versus the $\gamma^* p$ energy W. The data is from ZEUS an lines correspond to the results of the DGLAP improved model with massless quarks*

More detailed tests: b-dependent dipole cross sections

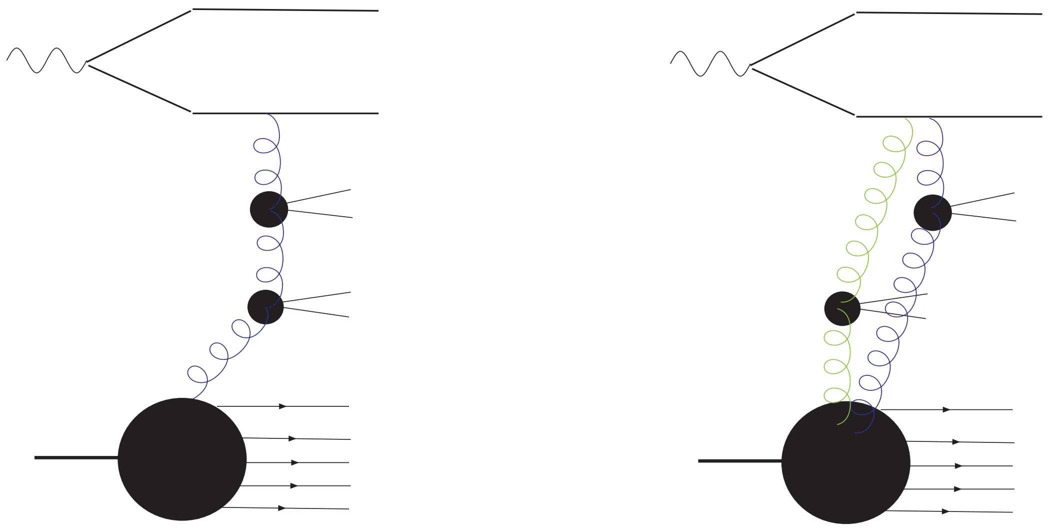
H.Kowalski, D.Teaney, Phys.Rev.D68:114005,2003

In summary: there is (indirect) evidence for saturation in HERA data,
different phenomena can be explained by the same idea,
but we need to find direct signals.

4) New tests to be performed: correlation functions.
Behind saturation models: multiple interactions, rescattering

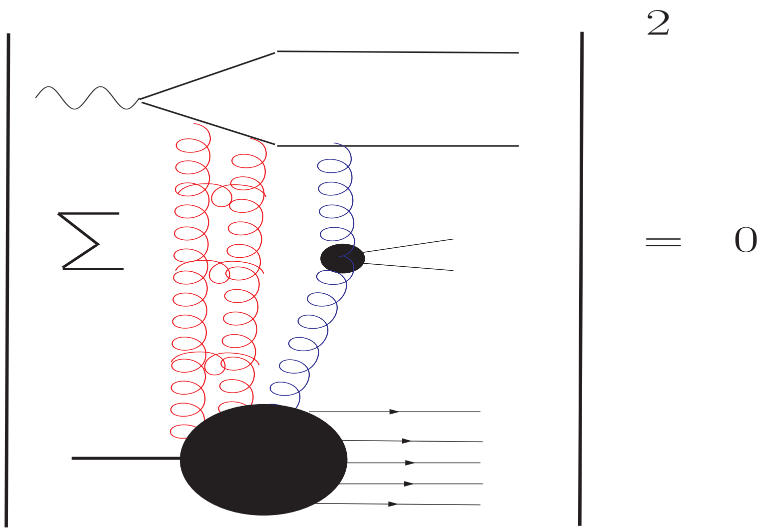


Proposal: Two-particle (two-jet) correlation:

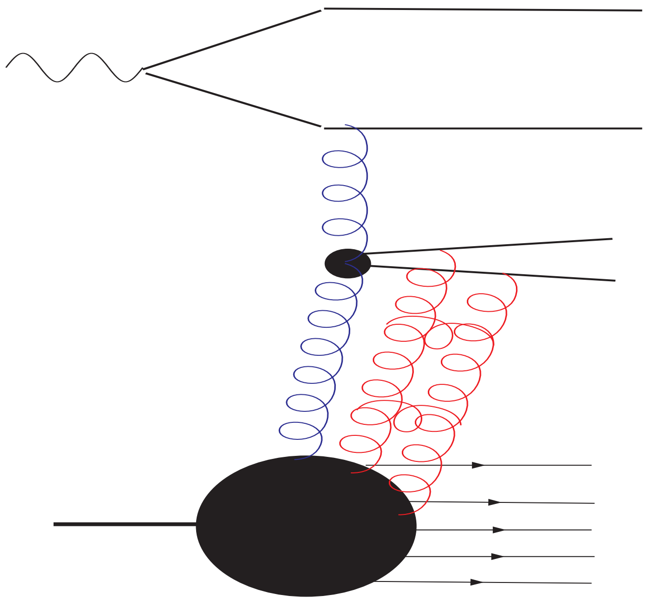


(not so simple: reggeization, AGK-rules)

AGK cancellations in inclusive cross sections:



Instead: rescattering between jet and target

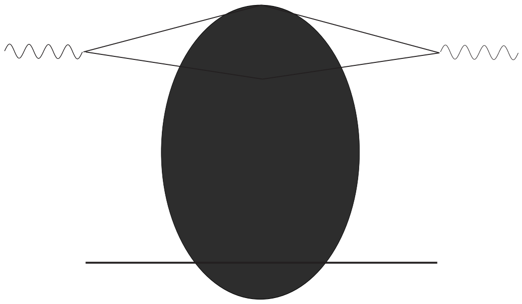


Generalization to two-jet inclusive cross section. Similar analysis applies to pp scattering at LHC.

Theory: evolution equations

Theory: very active field, connects DIS and Heavy Ion physics

- Dipole picture:



$$\sigma_{tot}^{\gamma^* p} = \int d^2r \int dz |\psi(Q, \vec{r}|^2 \sigma_{q\bar{q}}(\vec{r}, z; x)$$

- compatible with linear QCD evolution equations (DGLAP) at small x

- model for $\sigma_{q\bar{q}}$, e.g. GBW

$$\sigma_{q\bar{q}}(\vec{r}, z; x) = \sigma_0[1 - e^{-c\ \alpha_s(C/r^2)\ r^2\ xg(x,C/r^2)}]$$

- nonlinear Balitsky-Kovchegov (BK) equations

- Balitsky: set of equations

- JIMWLK: set of equations

- Reggeon field theory: field theory in 2+1 dimension: starts from momentum space

Balitsky-Kovchegov-equation:

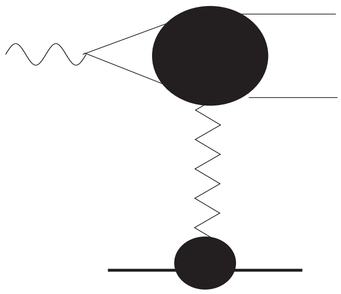
$$\frac{\partial}{\partial Y} N_{x,y} = \bar{\alpha}_s \int \frac{d^2 z}{2\pi} \frac{|x-y|^2}{|x-z|^2 \, |y-z|^2} \left(N_{x,z} + N_{y,z} - N_{x,y} - N_{x,z} N_{y,z} \right)$$

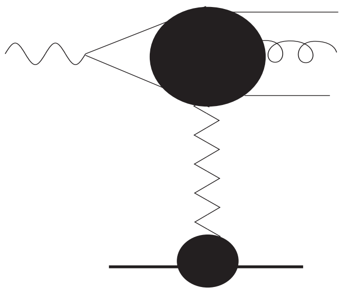
(beautiful equation, has soliton solutions, may be integrable)
Originally: for scattering on nuclei, used also for DIS.

Questions asked by a DIS person:

1) Relation between momentum space (cross sections!) and space of transverse coordinates?

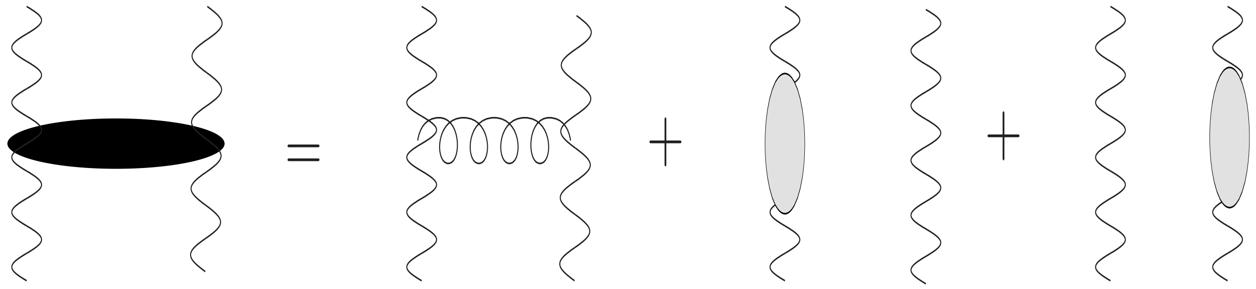
2) what is inside the BK equation,
e.g. where is diffraction: $q\bar{q}$, $q\bar{q}g$,...intermediate states?





3) Beyond the BK-equation: is it enough?

Ad 1), 2): Difficulty:
Fourier transform can be applied only to sums of QCD Feynman diagrams, e.g. BFKL:



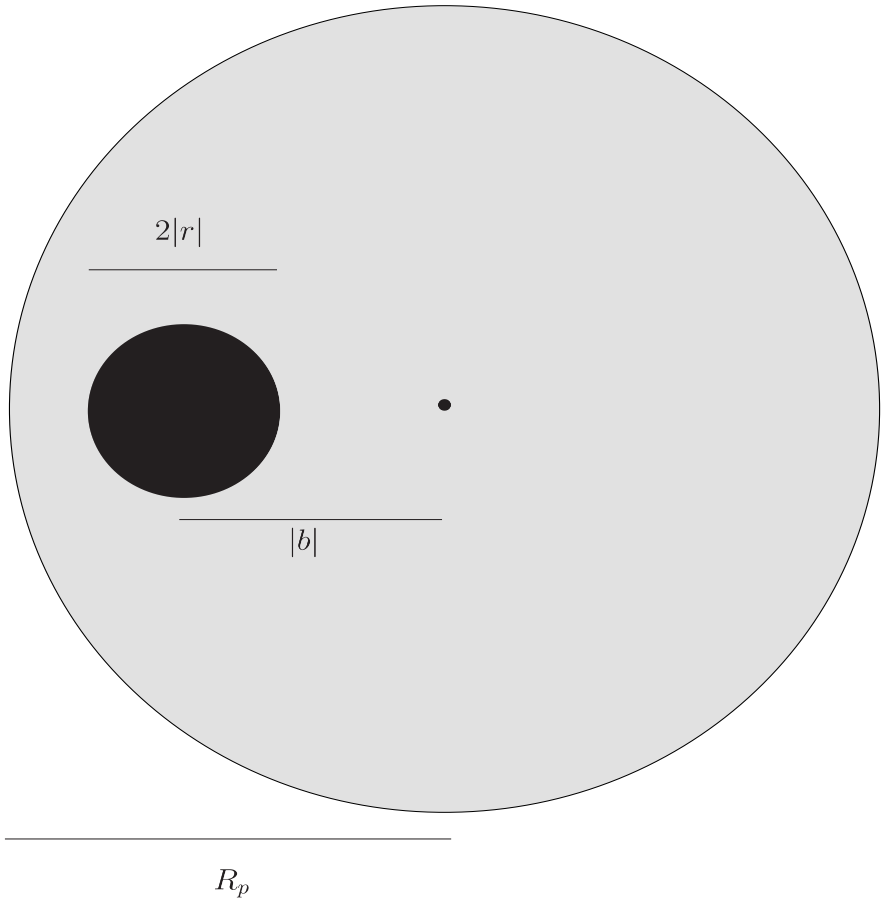
After Fourier transform connection with diagrams is obscure.

Partial answer to these questions:
1) For large-N_c, BK-kernel is Fourier transform of $2 \to 4$ gluon vertex (triple Pomeron vertex)

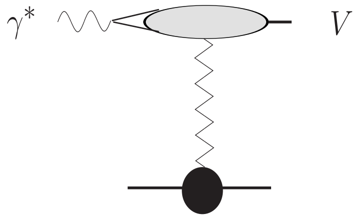
Consequences: opens the door for computing corrections:
$1/N_c^2$ corrections, NLO corrections $2 \to 4$ gluon vertex, Pomeron loops,...

2) For suitable initial conditions (and large N_c): BK equation contains $q\bar{q}$, $q\bar{q}g$,... states.
Consequence: do not add.

3) Beyond the BK-equation? No answer:
most urgent: b-dependence of dipole cross section, large b physics.
Schematic view in the transverse plane:



HERA data on diffractive DIS allow to adress dependence upon dipole size and impact parameter:



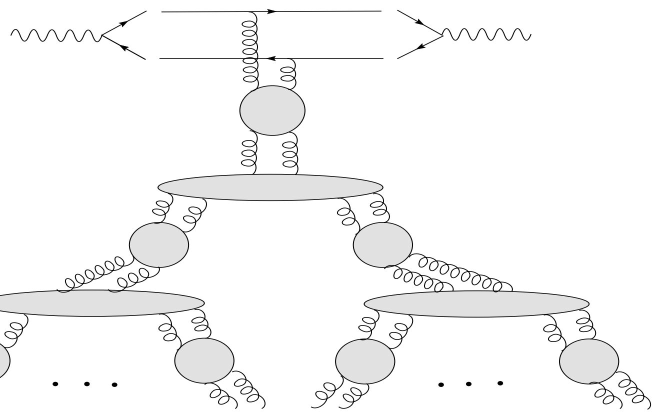
BK-equation (perturbation theory!) predicts
power-law fall-off at large b,
nonperturbative strong interactions needs
exponential fall-off:
need to modify the evolution equation!

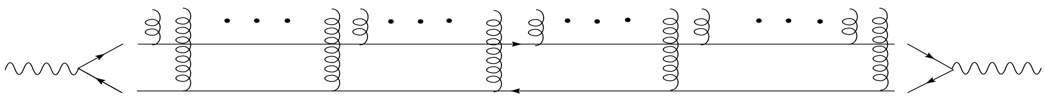
Calculations can be tested by comparing with HERA data.

Fan Diagram Equation vs.BK Equation

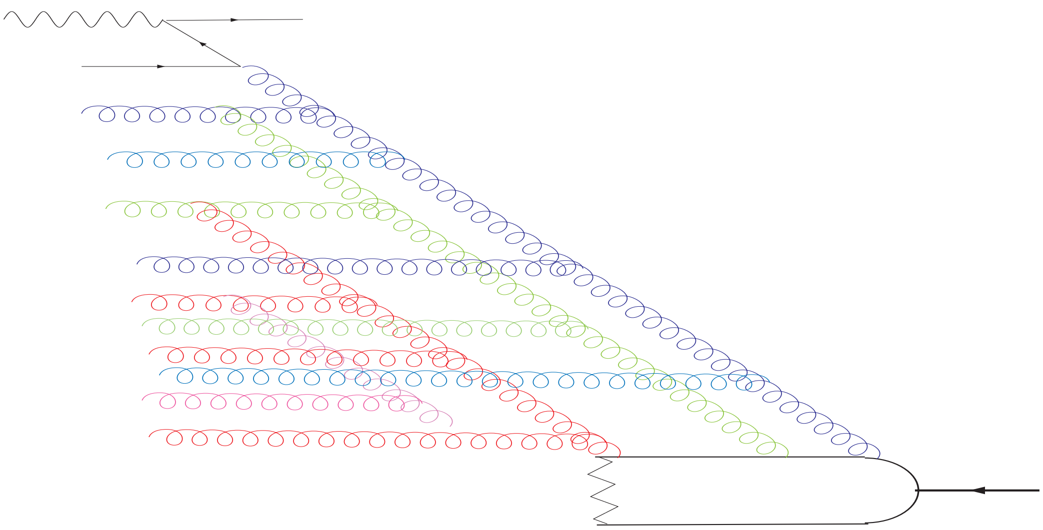
JB, L.N.Lipatov, G.P.Vacca, hep-ph/0404110

Simple model: consider the sum of fan diagrams





Corresponds to the Feynman diagrams shown in the introduction, with special initial conditions:



Fouriertransform:

$$\alpha_s^2 \mathcal{V}(q_1, q_2; k_1, k_2, k_3, k_4) =$$

$$\frac{1}{(N_c^2-1)^2}\, \Big(\delta_{a_1a_2}\delta_{a_3a_4} V(1234) + \delta_{a_1a_3}\delta_{a_2a_4} V(1324) + \delta_{a_1a_4}\delta_{a_2a_3} V(1423) \Big) \ ,$$

$$V(1234) D_2 = \frac{1}{2} g^2 \left(G(1,2+3,4) + G(2,1+3,4) + G(1,2+4,3) + G(2,1+4,3) \right.$$
$$\left. - G(1+2,3,4) - G(1+2,4,3) - G(1,2,3+4) - G(2,1,3+4) + G(1+2,0,3+4) \right)$$

$$A_1 = \frac{g^2 N_c}{8\pi^3} \Big(2\pi \delta^2(r_{23}) \partial_3^2 (c - \ln r_{13}) \partial_3^{-2} + 2\pi \delta^2(r_{12}) \partial_1^2 (c - \ln r_{13}) \partial_1^{-2}$$
$$-2 \frac{\mathbf{r}_{12}\mathbf{r}_{23}}{r_{12}^2 r_{23}^2} - 2\pi (c - \ln r_{13}) (\delta^2(r_{12}) + \delta^2(r_{23})) -$$
$$- 4\pi^2 \delta^2(r_{12}) \delta^2(r_{23}) (\partial_1 + \partial_3)^2 \partial_1^{-2} \partial_3^{-2} \Big) \ .$$

Still far from the BK-kernel!

But: use properties of BFKL kernel and of triple Pomeron vertex (Möbius invariance, gauge invariance). Redefine:

$$\Psi(\rho_1, \rho_2) \to \tilde{\Psi}(\rho_1, \rho_2) = \Psi(\rho_1, \rho_2) - \frac{1}{2}\Psi(\rho_1, \rho_1) - \frac{1}{2}\Psi(\rho_2, \rho_2)$$

$$\tilde{\Psi}(\rho_1, \rho_1) = 0$$

Call this the 'Möbius representation. Go to the large N_c limit and use the Möbius representation:

$$V(1234) \to -4 \frac{g^2}{2} \frac{g^2 N_c}{8\pi^3} \frac{\rho_{12}^2}{\rho_{13}^2 \rho_{23}^2} .$$

Agrees with BK-kernel.

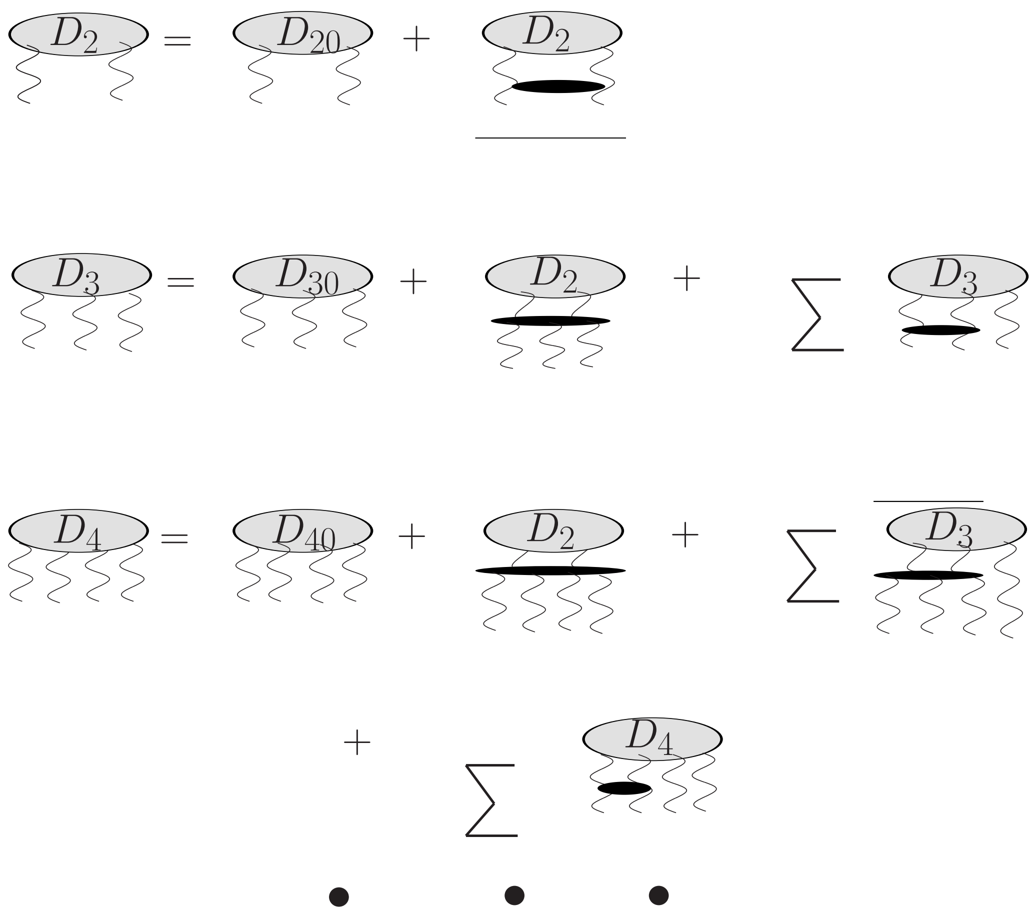
BK Equation appears as special case of fan-equation!

Similar analysis for DIS on nucleus.

Where is diffractive $q\bar{q}$ production?

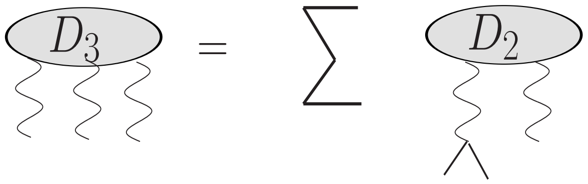
Should it be added to the BK equation?
Answer: no - is part of the triple vertex
and of initial conditions.

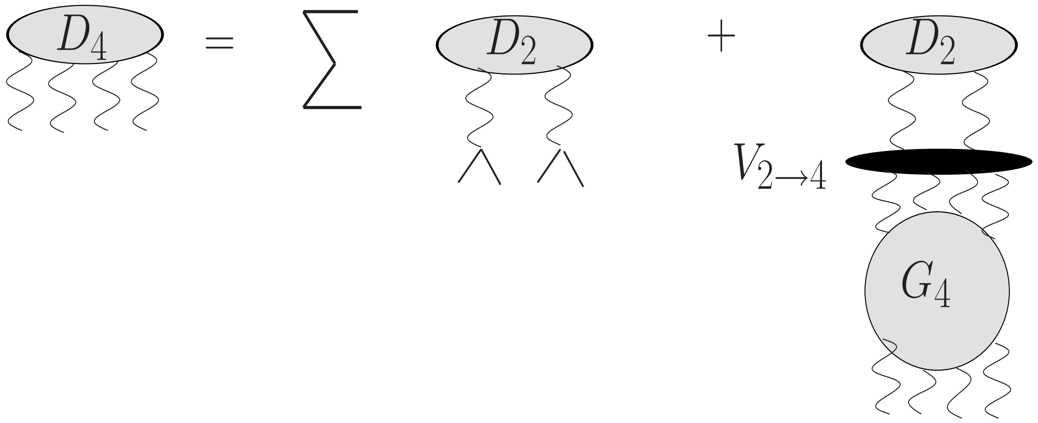
Analysis of amplitudes with n reggeized gluons:



Contains $q\bar{q}$ diffractive intermediate state! Need to symmetrize, use bootstrap properties of QCD:

Bootstrap identities:





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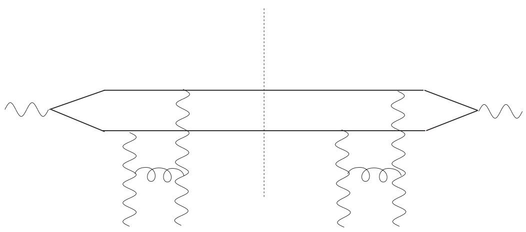
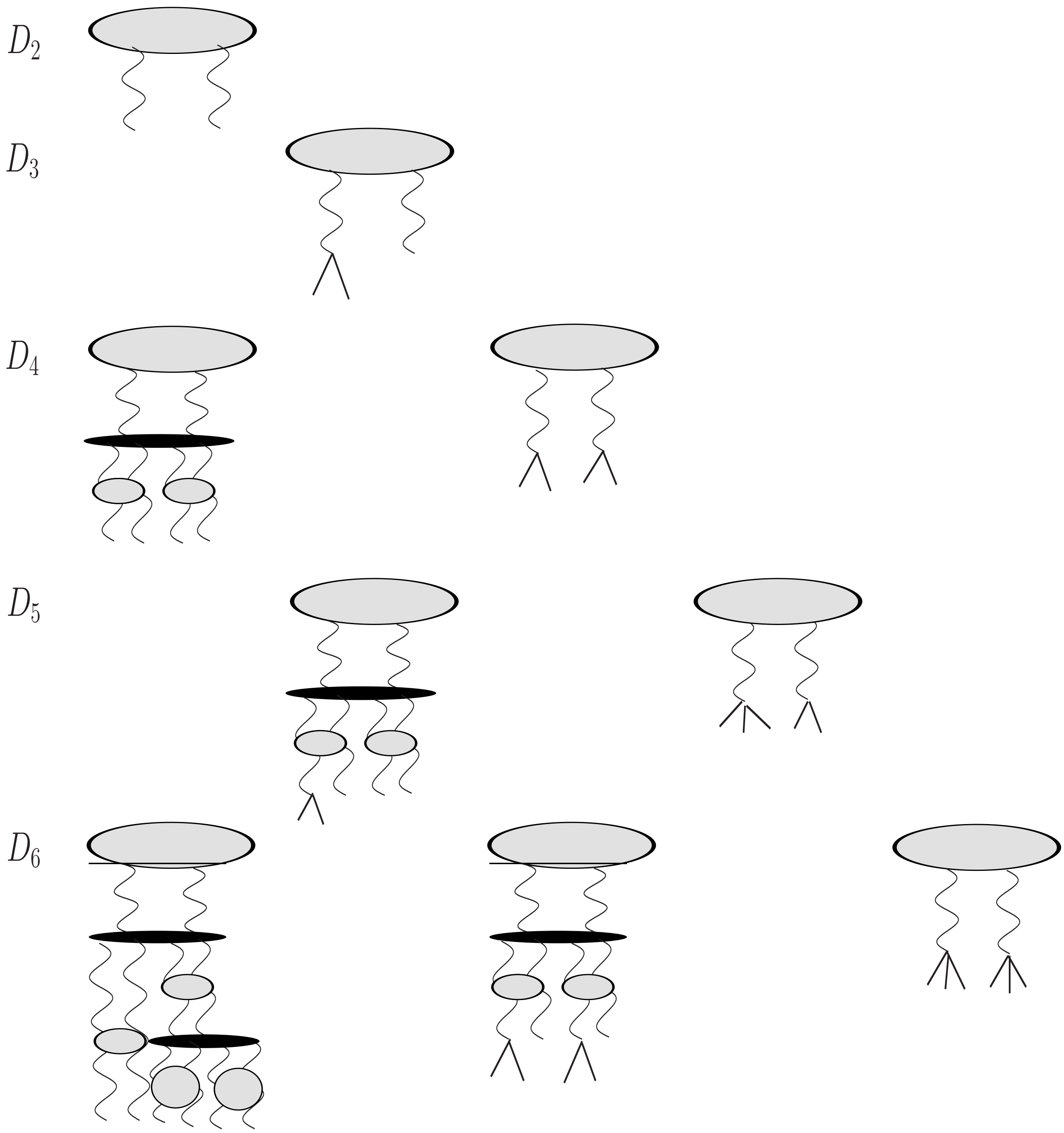
Corresponds to re-ordering of sum of diagrams.
The diffractive $q\bar{q}$ state



hidden in two different places.

General pattern:



All reggeization is part of initial conditions

Conclusions

DIS allows the most precise measurement of the parton content of hadrons (nuclei).

Saturation is a very attractive piece of QCD: transition from pQCD to strong interactions.

What has been achieved:

- There is evidence for saturation at HERA
- Theoretical tools exist:
  start from pQCD, add new pieces,
  not yet in confinement region

What is needed:

- More direct signals in DIS: final states
- Theory: further clarification
- Work on consequences for hadron collider (heavy ion collider)