Heavy quark potentials and quarkonia binding at finite temperature

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- Introduction : color screening and quarkonium dissolution at finite temperature
- Static charges on lattice at finite temperature
- Applications to quarkonium
- Conclusions

Hard Probes, Ericeira, November 4-10, 2004

Quarkonium dissolution at finite temperature

Potential between singlet $(Q\bar{Q})_1$ pair

$$V(r) = \int \frac{d^3k}{(2\pi)^3} T^{Born}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} = -\frac{4}{3} \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} D_{00}(\mathbf{k})$$

Coulomb gauge gluon propagator

$$D_{00}(\mathbf{k}) = \frac{1}{\mathbf{k}^2} \to \frac{1}{\mathbf{k}^2 + \Pi_{00}(k_0=0, \mathbf{k}, T)}$$

$$\Pi_{00}(k_0 = 0, \mathbf{k} \to 0, T) \sim m_{D0} = gT$$

$$\Rightarrow \quad V(r) \simeq -\frac{4}{3} g^2 \frac{e^{-m_{D0} r}}{4\pi r}, \quad r > 1/T$$

Matsui and Satz, PLB 178 (1986) 416

No quarkonium binding is possible when $m_D > 1/r_{QQ} \sim mv$

Charmonium suppression right after deconfinement

Implicit assumption : instantenous modification of interaction by the medium

$$t_{med} \sim 1/T \ll 1/(mv^2)$$

Does not hold close to $T_c \sim 170 - 270 MeV$

Static quark anti-quark pair in T>0 QCD

QCD partition function in the presence of static $Q\bar{Q}$ pair

McLerran, Svetitsky, PRD 24 (1981) 450

$$\frac{Z_{Q\bar{Q}}(r,T)}{Z(T)} = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} W(\vec{r}) W^\dagger(0) e^{-\int_0^{1/T} d\tau d^3x L_{QCD}}$$

$$Z(T) = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-\int_0^{1/T} d\tau d^3x L_{QCD}}$$

temporal Wilson line: $W(\vec{x}) = \mathcal{P} e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} = \prod_{\tau=0}^{N_\tau - 1} U_0(\tau,\vec{x})$

Polyakov loop: $L(\vec{x}) = \mathrm{Tr}\, W(\vec{x})$

$$3 \otimes \bar{3} = 1 \oplus 8$$

⇨ Separate singlet and octet contributions using projection operators

P_1 and P_8 Nadkarni, PRD 34 (1986) 3904

Color singlet free energy:

$$\exp(-F_1(r,T)/T) = \frac{1}{Z(T)} \frac{\mathrm{Tr} P_1 Z_{Q\bar{Q}}}{\mathrm{Tr} P_1} = \frac{1}{3} \mathrm{Tr} \langle W(\vec{r}) W^\dagger(0) \rangle$$

Color octet free energy:

$$\exp(-F_8(r,T)/T) = \frac{1}{Z(T)} \frac{\mathrm{Tr} P_8 Z_{Q\bar{Q}}}{\mathrm{Tr} P_8} =$$

$$\frac{1}{8} \langle \mathrm{Tr} W(\vec{r}) \mathrm{Tr} W^\dagger(0) \rangle - \frac{1}{24} \mathrm{Tr} \langle W(\vec{r}) W^\dagger(0) \rangle$$

Fix the *Coulomb gauge* transfer matrix can be defined

equivalent

Dressed gauge invariant Wilson line
Philipsen, PLB 535 (2002) 138

$$W(\vec{x}) \to \tilde{W}(\vec{x}) = \Omega^\dagger W(\vec{x}) \Omega(\vec{x})$$

$$\Omega = f_\alpha^n, \; D_\mu^2 f_\alpha^{(n)} = \lambda_n f_\alpha^{(n)}, \; \tau = 0$$

At T=0 equivalent to definition through Wilson loop, Philipsen, PLB 535 (2002) 138

Free energy in the pertubative high temperature limit

$$\exp(-F_1(r,T)/T) = \frac{1}{3}\mathrm{Tr}\langle W(\vec{r})W^\dagger(0)\rangle, \quad W \simeq 1 + igA_0/T$$

$$\Rightarrow \quad F_1(r,T) = -g^2 C_F \frac{e^{-m_{D0} r}}{4\pi r} = U_1(r,T), \quad C_F = \frac{N^2-1}{2N} \qquad m_{D0} = gT$$

At leading order: $F_8(r,T)/F_1(r,T) = -1/8$ $\quad S_1(r,T) = 0$

At next to leading order: $F_1(r,T) = -g^2 C_F \frac{e^{-m_{D0} r}}{4\pi r} - \frac{C_F m_{D0} g^2}{4\pi}$

$$F_1(r=\infty,T) = F_\infty(T) \neq 0$$

and the entropy appears: $S_1(r,T) = \frac{C_F g^2 m_{D0}}{4\pi T} - \frac{C_F g^2 m_{D0}}{4\pi T} e^{-m_{D0} r} \sim \mathcal{O}(g^3)$

The internal energy is different from the free energy

$$U_1(r,T) = F_1(r,T) + TS_1(r,T) = -g^2 C_F \frac{e^{-m_{D0} r}}{4\pi r} - \frac{C_F g^2 m_{D0}}{4\pi} e^{-m_{D0} r}$$

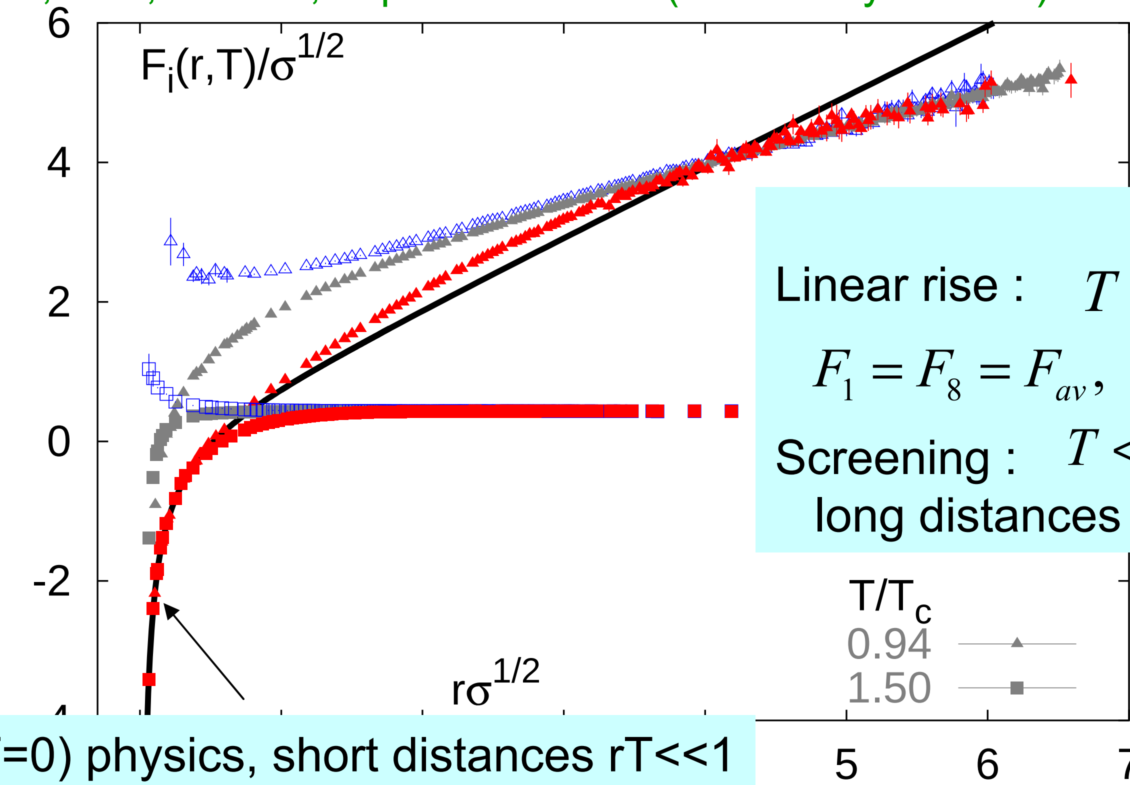
$$U_1(r=\infty,T) = U_\infty(T) = 0$$

Color averaged free energy:

$$\exp(-F_{av}(r,T)/T) = \frac{1}{Z(T)} \frac{\mathrm{Tr}(P_1 + P_8) Z_{Q\bar{Q}}}{\mathrm{Tr}(P_1 + P_8)} = \frac{1}{9} \langle \mathrm{Tr} W(\vec{r}) \mathrm{Tr} W^\dagger(0) \rangle$$

$$= \frac{1}{9} \exp(-F_1(r,T)/T) + \frac{8}{9} \exp(-F_8(r,T)/T)$$

Kaczmarek, Karsch, P.P., Zantow, hep-lat/0309121 (see talk by Zantow)



Linear rise : $T < T_c$

$F_1 = F_8 = F_{av}, \quad r \to \infty$

Screening : $T < T_c$

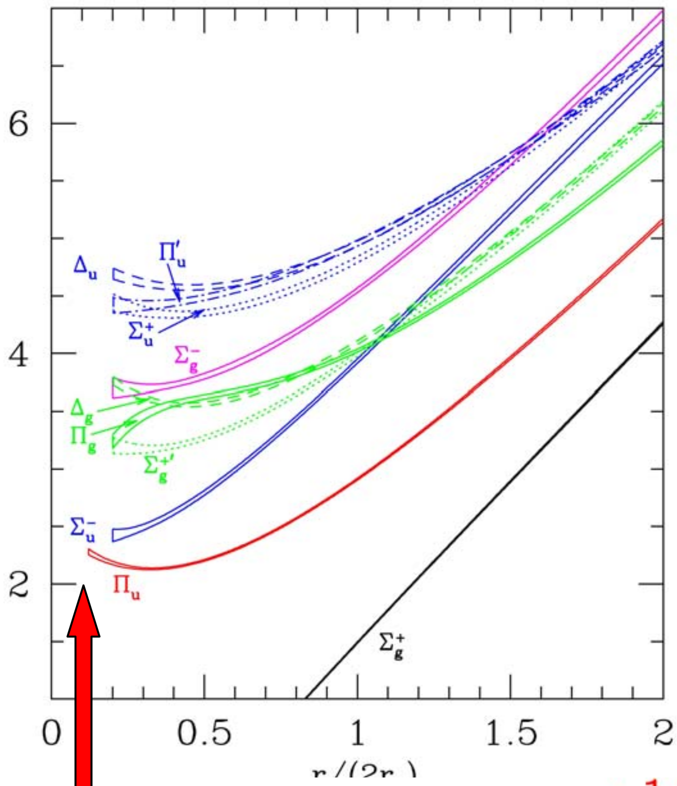
long distances rT>>1

Vacuum (T=0) physics, short distances rT<<1

Color octet free energy ?

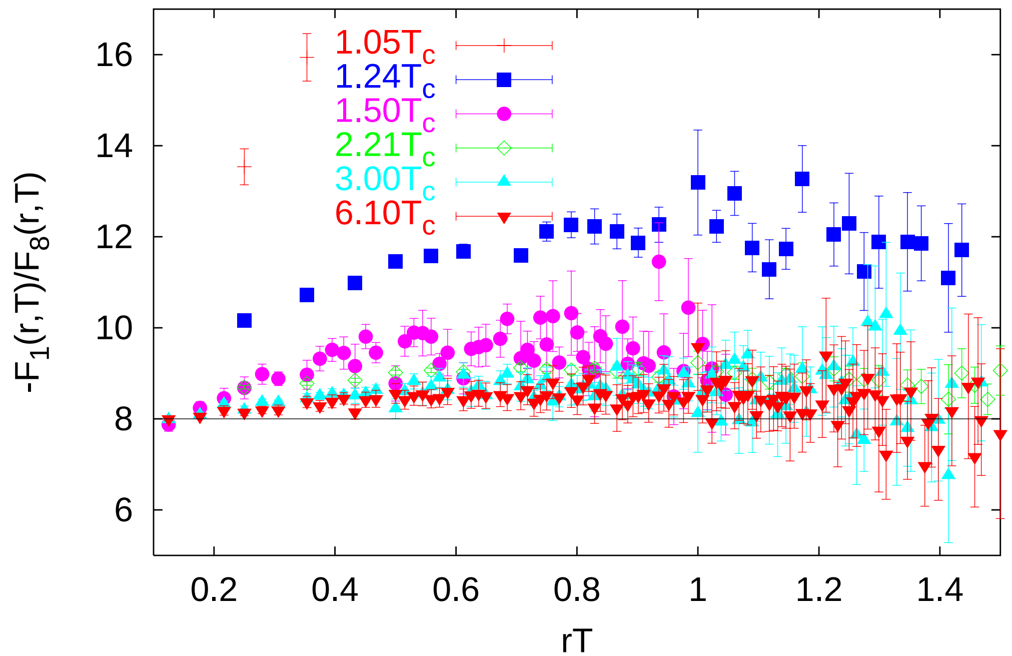
The color octet correlator only fixes the relative color orientation of static quark anti-quark pair.

Juge et al, hep-lat/0103008

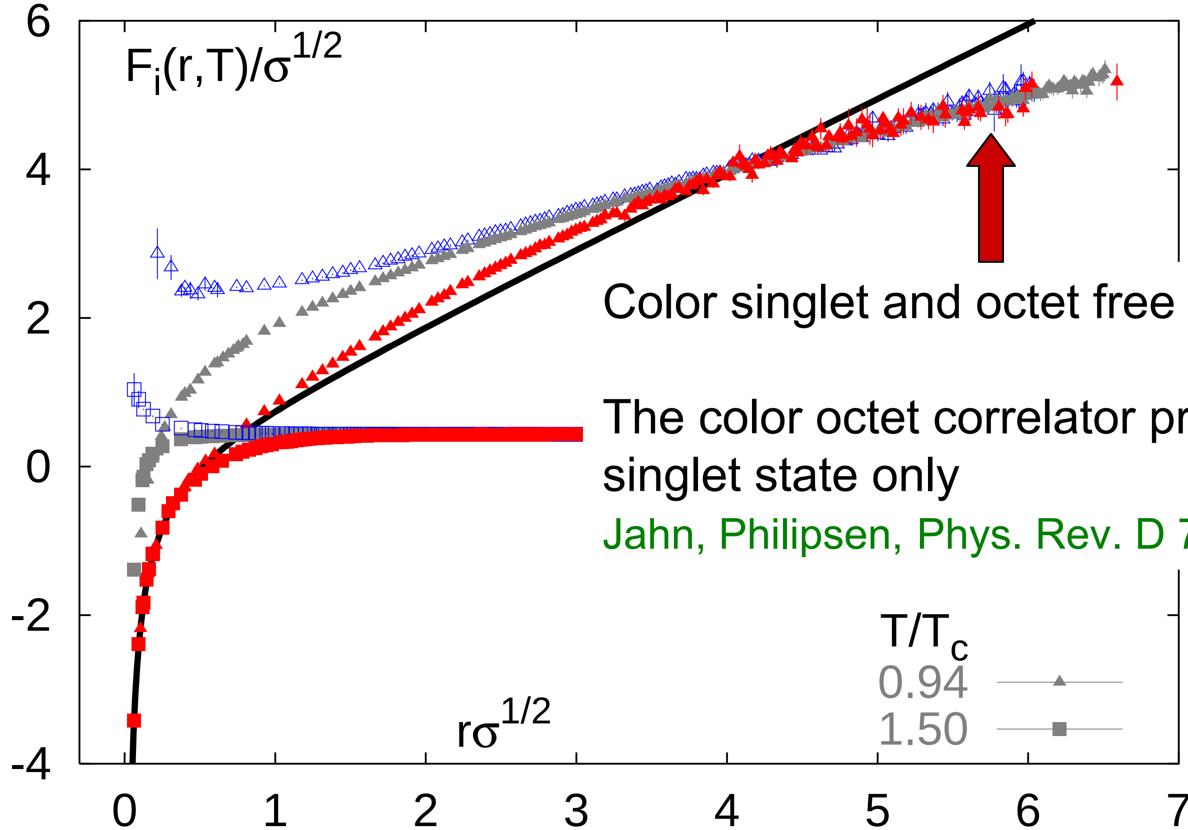


Excited potentials: $+\frac{1}{6}\frac{\alpha_s}{r}$,

$r \ll 1/\Lambda_{QCD}$



Perturbative relation between to the color singlet free energy at high temperatures



Color singlet and octet free energy are the same !

The color octet correlator projects onto singlet state only

Jahn, Philipsen, Phys. Rev. D 70 (2004) 074504

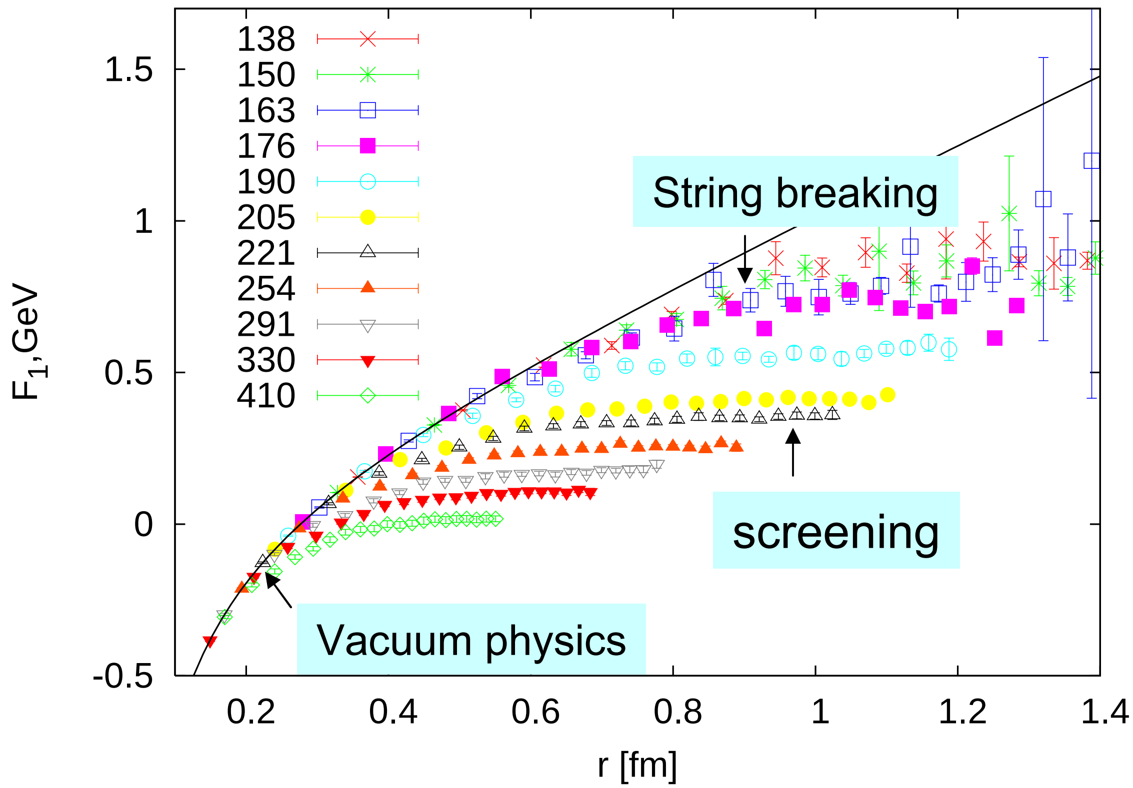
Static free energy in full QCD

3F: Improved staggered Asqtad action, $8^3 \times 4,\ 12^3 \times 4,\ 12^3 \times 6$ lattices

$m_q = 0.2 m_s,\ 0.4 m_s,\ 0.6 m_s,\quad m_s \simeq 70 MeV$ K. Petrov, P.P, PRD 70 (04) 054503

2F: Improved staggered p4 action $16^3 \times 4$ lattices, $m_q \simeq m_s$

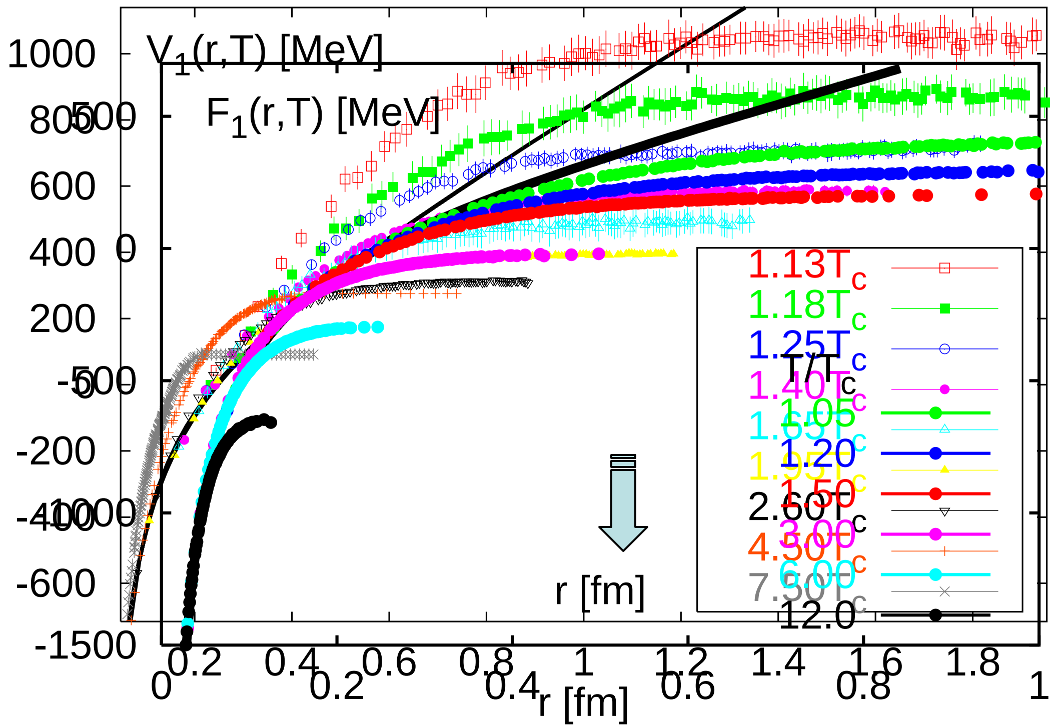
O. Kaczmarek et al, Progr. Theor. Phys. Suppl 153 (04) 287



The entropy contribution and the internal energy

$$S_1(r,T) = \frac{\partial}{\partial T} \ln \left(T \frac{Z^1_{Q\bar{Q}}(r,T)}{Z(T)} \right)$$

$$V_1(r,T) = T^2 \frac{\partial}{\partial T} \ln \left(\frac{Z^1_{Q\bar{Q}}(r,T)}{Z(T)} \right) = F_1(r,T) - TS_1(r,T)$$



$$V_1(r,T) > F_1(r,T)$$

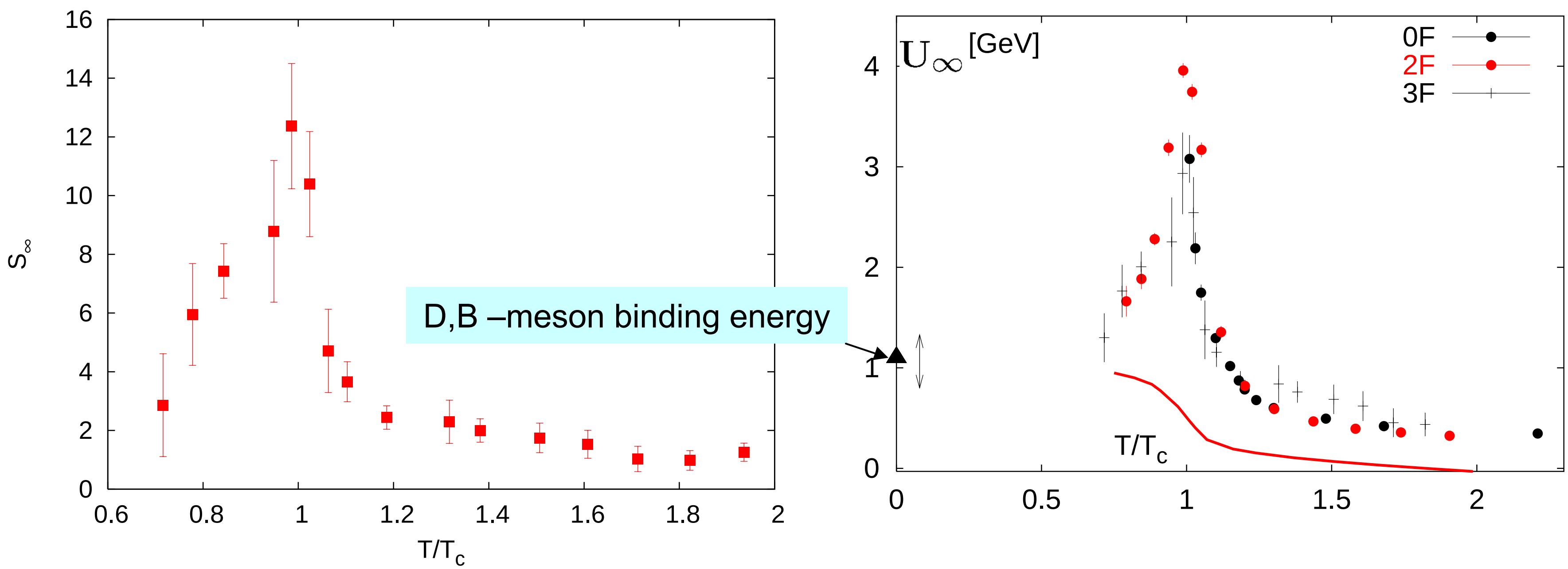
$$F_1(r,T) = \infty, \;\; T \to \infty!!!$$

$$U_1(r \to \infty, T) \simeq 0, \;\; T \gg T_c$$

$$-\frac{\partial F_1}{\partial T} > 0$$

Kaczmarek, Karsch, P.P., Zantow, hep-lat/0309121
see talk by Kaczmarek

There is a large increase in the entropy and internal energy at the transition temperature !



K. Petrov, P.P, PRD 70 (04) 054503

P.P. , hep-lat/0409139

Adding an extra static meson increases the entropy and the internal energy like in the production of extra hadrons in resonance gas model. This increase is not related to the increase of the strength of interaction between the quark-anti-quark pair.

$$U \neq \text{potential for } T \sim T_c$$

Quarkonium binding in potential models (I)

Matsui and Satz 1986: perturbative screened Coulomb potential

J/ψ dissolves at $T < 1.2T_c$

Karsch, Mehr, Satz 1988: screened Cornell potential with lattice screening masses

$J/\psi,\ \psi',\ \chi_c$ dissolve at $T \simeq T_c$

Digal, P.P. Satz, 2001: lattice free energy as the potential

$\psi',\ \chi_c$ dissolves at $T \simeq T_c$, J/ψ melts at $T = 1.1T_c$

Shuryak and Zahed; Wong, 2004 : lattice internal energy as the potential

J/ψ dissolves only at $T \simeq 1.7T_c$

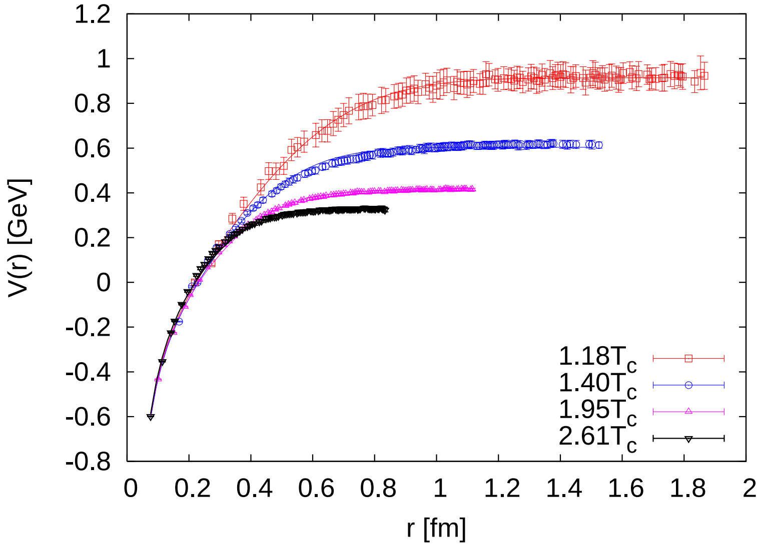
Lattice calculations of the spectral functions 2002-2004:
(see talks by Karsch, Hatsuda, Petrov) J/ψ dissolves only at $(1.7 - 2.5)T_c$

Is there an agreement bewteen potential models and lattice spectral functions ?

Potential models can provide more detailed information (masses, wave functions etc.) than existence of bound states. ⇨ confront these with lattices correlators

Quarkonium binding in potential models (II)

Imaginary time quarkonium correlators $G_{lat}(\tau, T)$ can be reliably calculated on lattice



$V_\infty(T)$

$\downarrow$

$$2m_{pole}(T) = 2m_{c,b} + V_\infty(T) = s_0(T)$$

Schroedinger equation

$\downarrow$

$$M_i, \quad |R_i(0)|^2, \quad |R_i'(0)|^2, \; i = J/\psi, \chi_b, \Upsilon ...$$

$$\sigma(\omega, T) = 2M_i F_i(T)\delta(\omega^2 - M_i^2(T)) + \theta(\omega - s_0(T))\omega^2, \; F_i \sim |R(0)|^2$$

$\Downarrow$

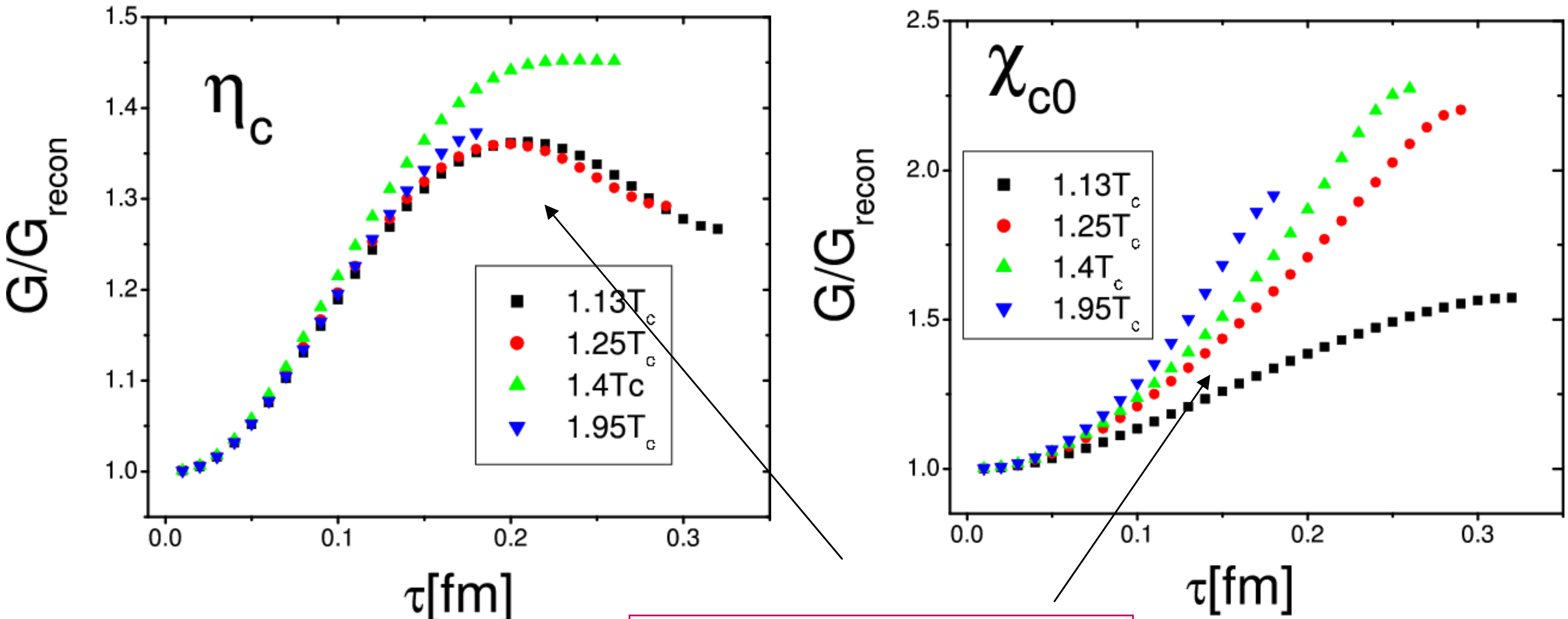
$$G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) \frac{\cosh(\omega \cdot (\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

compare with $G_{lat}(\tau, T)$

See talk by A. Mócsy

Moderate enhancement for
the pseudo-scalar correlator

Large enhancement for
the scalar correlator



Effects due to the shift of
the continuum threshold

$$G_{recon}(\tau,T) = \int_0^\infty d\omega\sigma(\omega,T=0)\frac{\cosh(\omega\cdot(\tau-1/(2T))}{\sinh(\omega/(2T))}$$

Summary

- In medium modification of inter quark forces can be studied in terms of internal energy of static quark free energies but not too close to transition region.

- There is no quantitative agreement between predictions of potential models and direct lattice calculations of the quarkonium correlator. Extra thermal effects are present which are not detected in lattice spectral functions (because of lattice artifacts ?)