Radiative Higgs-Sector CP Violation

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- **Introduction: Higgs-Sector CP Violation**

- **Radiative Higgs-Sector CP Violation in the MSSM**

- **Implications:**
  - **Electric Dipole Moments**
  - **FCNC observables**

    ⋮

  - **Phenomenology of CP-violating Higgs bosons at LEP2, Tevatron, LHC, ILC and $\gamma\gamma$ colliders.**

  → **Resonant CP Violation at LHC and $\gamma\gamma$ colliders.***

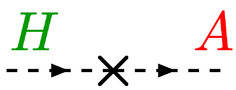
- **Conclusions**

*Recent articles:

J. Ellis, J.S. Lee and A.P., hep-ph/0404167 and hep-ph/0411379.

• Introduction: Higgs-Sector CP Violation

- Explicit or spontaneous CP violation in the Higgs potential at the tree level, e.g. 2HDM. [T.D. Lee, PRD**8** (1973) 1226; S. Weinberg, PRL**37** (1976) 657; G.C. Branco, PRL**44** (1980) 504, . . .]



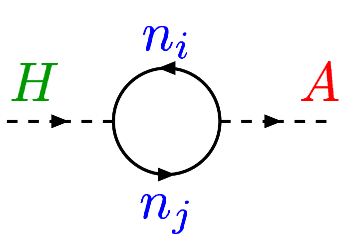
The CP-violating HA mixing occurs at the tree level, but generically $M_H \not\sim M_A$.

- Spontaneous or explicit radiative CP violation in the Higgs potential.

  – Spontaneous radiative CP violation: it generically leads to a very light 'CP-odd' scalar, with $M_A \lesssim 40$ GeV, and is phenomenologically highly disfavoured.
  [H. Georgi, G. Pais, PRD**10** (1974) 1246; J.C. Romao, PLB**173** (1986) 309]

  – Explicit radiative CP violation:

  (i) Through loop effects of heavy Majorana neutrinos in a constrained 2HDM potential [A.P., PRL**77** (1996) 4996.]



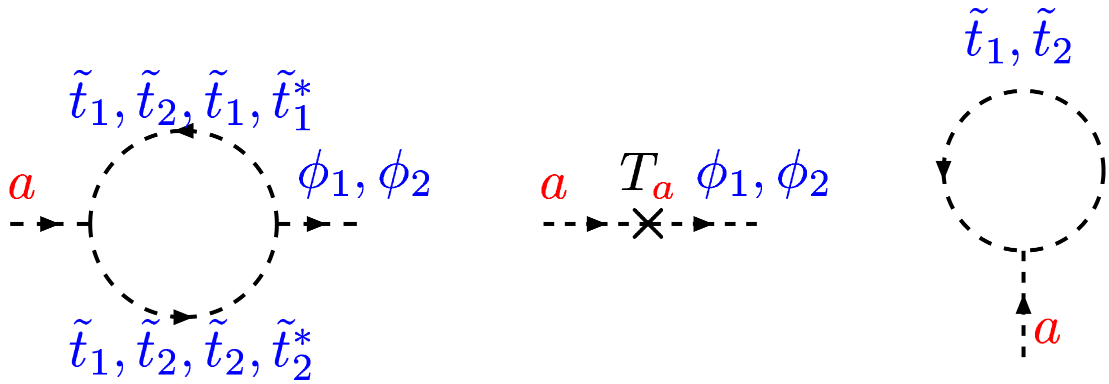
Resonant CP-violating scenarios, with $M_H - M_A \sim \Gamma_{H,A}$.

  (ii) Through radiative effects of stops/sbottoms in the MSSM. [A.P., PRD**58** (1998) 096010; PLB**435** (1998) 88]

• Radiative Higgs-sector CP violation in the MSSM

Two major effects of CP violation on the Higgs sector:
– CP-violating self-energy effects
– CP-violating vertex effects

– CP-violating self-energy effects:



$$
\begin{aligned}
\mathcal{M}^2_{SP} &\sim \frac{m_t^4}{v^2}\,\frac{\mathrm{Im}\,(\mu A_t)}{32\pi^2 Q_t^2} \\
&\quad \times \left(1,\ \frac{|A_t|^2}{Q_t^2},\ \frac{|\mu|^2}{\tan\beta\ Q_t^2},\ \frac{2\mathrm{Re}\,(\mu A_t)}{Q_t^2}\right) \\
&\lesssim (100\ \mathrm{GeV})^2
\end{aligned}
$$

[A.P., PR**D58** (1998) 096010; PL**B435** (1998) 88;
A.P., C.E.M. Wagner, NP**B553** (1999) 3;
M. Carena, J. Ellis, A.P., C.E.M. Wagner, NP**B586** (2000) 92; hep-ph/0009212;
D.A. Demir, PR**D60** (1999) 055006; S.Y. Choi, M. Drees, J.S. Lee, PL**B481** (2000) 57;
T. Ibrahim and P. Nath, hep-ph/0008237 · · ·]

The mixing of the three neutral Higgs bosons

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ a \end{pmatrix} = O \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}$$

O is a 3×3 orthogonal matrix which also describes the mixing of the Higgs bosons with different CP parities.

In analogy to the case of neutrinos and quarks, Higgs bosons with mixed CP parities are ordered according to their weights:

$$M_{H_1} \leq M_{H_2} \leq M_{H_3}$$

At the one-loop level, M_{H_i} (with $i = 1, 2, 3$) and O are analytically determined by the input parameters:

$$M_{H^+}(m_t)\,, \quad \tan\beta(m_t)\,,$$

$$\mu(Q_{tb})\,, \quad A_t(Q_{tb})\,, \quad A_b(Q_{tb})\,,$$

$$\widetilde{M}_Q^2(Q_{tb})\,, \quad \widetilde{M}_t^2(Q_{tb})\,, \quad \widetilde{M}_b^2(Q_{tb})\,.$$

CP-conserving versus CP-violating MSSM parameters:

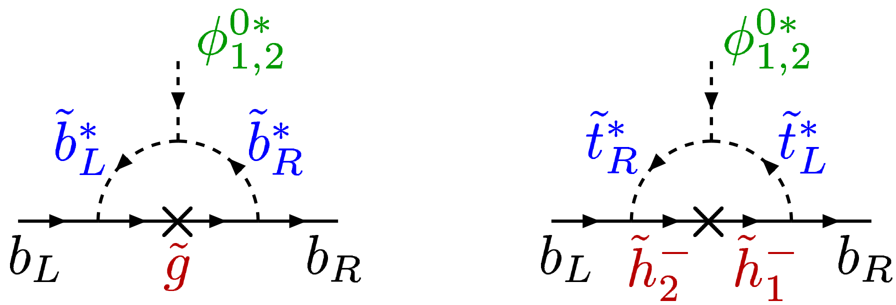
CP-conserving:	**CP-violating**:
$\tan\beta$	$\tan\beta$, with $\xi = 0$
$\widetilde{M}_Q^2$, $\widetilde{M}_t^2$, $\widetilde{M}_b^2$	$\widetilde{M}_Q^2$, $\widetilde{M}_t^2$, $\widetilde{M}_b^2$
μ	$\|\mu\|$, $\arg(\mu)$
$A_{t,b}$	$\|A_{t,b}\|$, $\arg(A_{t,b})$
M_A	M_{H^+}

At two loops:

$m_{\tilde{g}}$	$\|m_{\tilde{g}}\|$ and $\arg(m_{\tilde{g}})$

– CP-violating vertex effects:

Effective $H_1 bb$-coupling



$$-\mathcal{L}^{\mathrm{eff}}_{\phi^0\bar{b}b} = (h_b + \delta h_b)\,\phi_1^{0*}\,\bar{b}_R b_L + \Delta h_b\,\phi_2^{0*}\,\bar{b}_R b_L \;+\; \mathrm{h.c.}$$

with

$$\frac{\delta h_b}{h_b} \sim -\frac{2\alpha_s}{3\pi}\,\frac{m_{\tilde{g}}^* A_b}{\max\,(Q_b^2, |m_{\tilde{g}}|^2)} - \frac{|h_t|^2}{16\pi^2}\,\frac{|\mu|^2}{\max\,(Q_t^2, |\mu|^2)}$$

$$\frac{\Delta h_b}{h_b} \sim \frac{2\alpha_s}{3\pi}\,\frac{m_{\tilde{g}}^* \mu^*}{\max\,(Q_b^2, |m_{\tilde{g}}|^2)} + \frac{|h_t|^2}{16\pi^2}\,\frac{A_t^* \mu^*}{\max\,(Q_t^2, |\mu|^2)}$$

and

$$h_b = \frac{g_w m_b}{\sqrt{2} M_W \cos\beta\,[\,1 + \delta h_b/h_b + (\Delta h_b/h_b)\tan\beta\,]}$$

[M. Carena, J. Ellis, A.P., C. Wagner '00]

- **Implications:**

– **2-loop Higgs-mediated EDMs**

[D. Chang, W.-Y. Keung, A.P., PRL **82** (1999) 900; A.P., NP**B644** (2002) 263.]

– **FCNC observables:**
$\Delta M_{K,B}$, ϵ_K, ϵ'/ϵ, $\mathcal{B}(B_{d,s} \to \ell^+\ell^-)$,
$\mathcal{A}_{\rm CP}(B_{d,s} \to \ell^+_{L(R)}\ell^-_{L(R)})$, with $\ell = \mu\,,\tau$,
$\mathcal{B}(B \to X_s\gamma)$, . . .

[E.g. A. Dedes and A.P., PR**D67** (2003) 015012.]

– **Dark Matter**

[P. Gondolo and K. Freese, hep-ph/990839; T. Falk, A. Ferstl and K.A. Olive, Astropart. Phys. **13** (2000) 301; M. E. Gomez, T. Ibrahim, P. Nath and S. Skadhauge, PR**D70** (2004) 035014 . . .]

– **Electroweak Baryogenesis**

[E.g. M. Carena, J.M. Moreno, M. Quiros, M. Seco and C.E.M. Wagner, Nucl. Phys. **B599** (2001) 158 . . .]

– **Higgs phenomenology at LEP2, Tevatron and LHC**

[E.g. M. Carena, J. Ellis, S. Mrenna, A.P., C.E.M. Wagner, NP**B659** (2003) 145.]

⋮

→ **Resonant CP Violation at LHC and $\gamma\gamma$ colliders.**

♠ *Coupled-channel analysis of neutral Higgs bosons*

- The full 3×3 Higgs boson propagator matrix : $D(\hat{s})$

We consider a situation where two or more Higgs bosons are simultaneously involved in a process

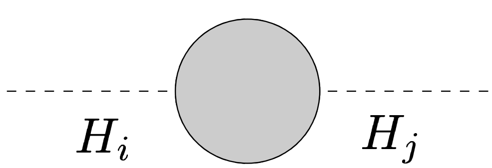
$$D(\hat{s}) = \hat{s} \times$$

$$\begin{pmatrix} \hat{s} - M_{H_1}^2 + i\Im m\widehat{\Pi}_{11}(\hat{s}) & i\Im m\widehat{\Pi}_{12}(\hat{s}) & i\Im m\widehat{\Pi}_{13}(\hat{s}) \\ i\Im m\widehat{\Pi}_{21}(\hat{s}) & \hat{s} - M_{H_2}^2 + i\Im m\widehat{\Pi}_{22}(\hat{s}) & i\Im m\widehat{\Pi}_{23}(\hat{s}) \\ i\Im m\widehat{\Pi}_{31}(\hat{s}) & i\Im m\widehat{\Pi}_{32}(\hat{s}) & \hat{s} - M_{H_3}^2 + i\Im m\widehat{\Pi}_{33}(\hat{s}) \end{pmatrix}^{-1}$$

A. Pilaftsis, Nucl. Phys. B **504** (1997) 61

- M_{H_i} : One-loop Higgs-boson pole mass
  $\rightarrow M_{H_i}\Gamma_{H_i}\Im m\widehat{\Pi}'(M_{H_i}^2)$ and $\Re e\widehat{\Pi}(\hat{s}) - \Re e\widehat{\Pi}(M_{H_i}^2)$ are neglected.
  M. Carena, J. Ellis, A. Pilaftsis and C.E.M. Wagner, Nucl. Phys. B **625** (2002) 345

- $\Im m\widehat{\Pi}_{ij}(\hat{s})$: absorptive parts of the Higgs-boson self-energies $\rightarrow \Im m\widehat{\Pi}_{ii}(\hat{s} = M_{H_i}^2) = M_{H_i}\Gamma_{H_i}$

$\hat{s} \rightarrow$  $= \; i\,\widehat{\Pi}_{ij}(\hat{s})$

♠ *Coupled-channel analysis of neutral Higgs bosons*

- The full 3×3 Higgs boson propagator matrix : $D(\hat{s})$ (cont'd)

  - Contributions from loops of fermions $(b, t, \tau, \tilde{\chi}^0, \tilde{\chi}^\pm)$, vector bosons (ZZ, WW), pairs of Higgs and vector bosons $(H_k Z, H^\pm W^\mp)$, Higgs-boson pairs $(H_k H_l)$, and sfermions $(\tilde{b}, \tilde{t}, \tilde{\tau})$ have been considered.

  - The PT is applied for VV and HV cases to obtain gauge-independent result and to remove the badly high-energy-behaved $\hat{s}^2$ terms consistently J. Papavassiliou and A. Pilaftsis, Phys. Rev. Lett. **80** (1998) 2785; Phys. Rev. D **58** (1998) 053002

  - Note the relation

$$\widehat{\Pi}_{ij}(\hat{s}) = \sum_{\alpha,\beta=\phi_1,\phi_2,a} O_{\alpha i} O_{\beta j} \widehat{\Pi}_{\alpha\beta}(\hat{s})$$

♠ *Coupled-channel analysis of neutral Higgs bosons*

- Three-way mixing scenario

$$\tan\beta = 50, \quad M_{H^\pm}^{\rm pole} = 155 \ \ {\rm GeV},$$
$$M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3} = 0.5 \ \ {\rm TeV},$$
$$|\mu| = 0.5 \ \ {\rm TeV}, \quad |A_{t,b,\tau}| = 1 \ \ {\rm TeV}, \quad |M_{1,2}| = 0.3 \ \ {\rm TeV},$$
$$|M_3| = 1 \ \ {\rm TeV},$$

For the choice of phases:

$$\Phi_\mu = 0^\circ, \quad \Phi_{A_t,A_b,A_\tau} = 90^\circ, \quad \Phi_{1,2} = 0^\circ, \quad \Phi_3 = -90^\circ,$$

we have (in GeV)

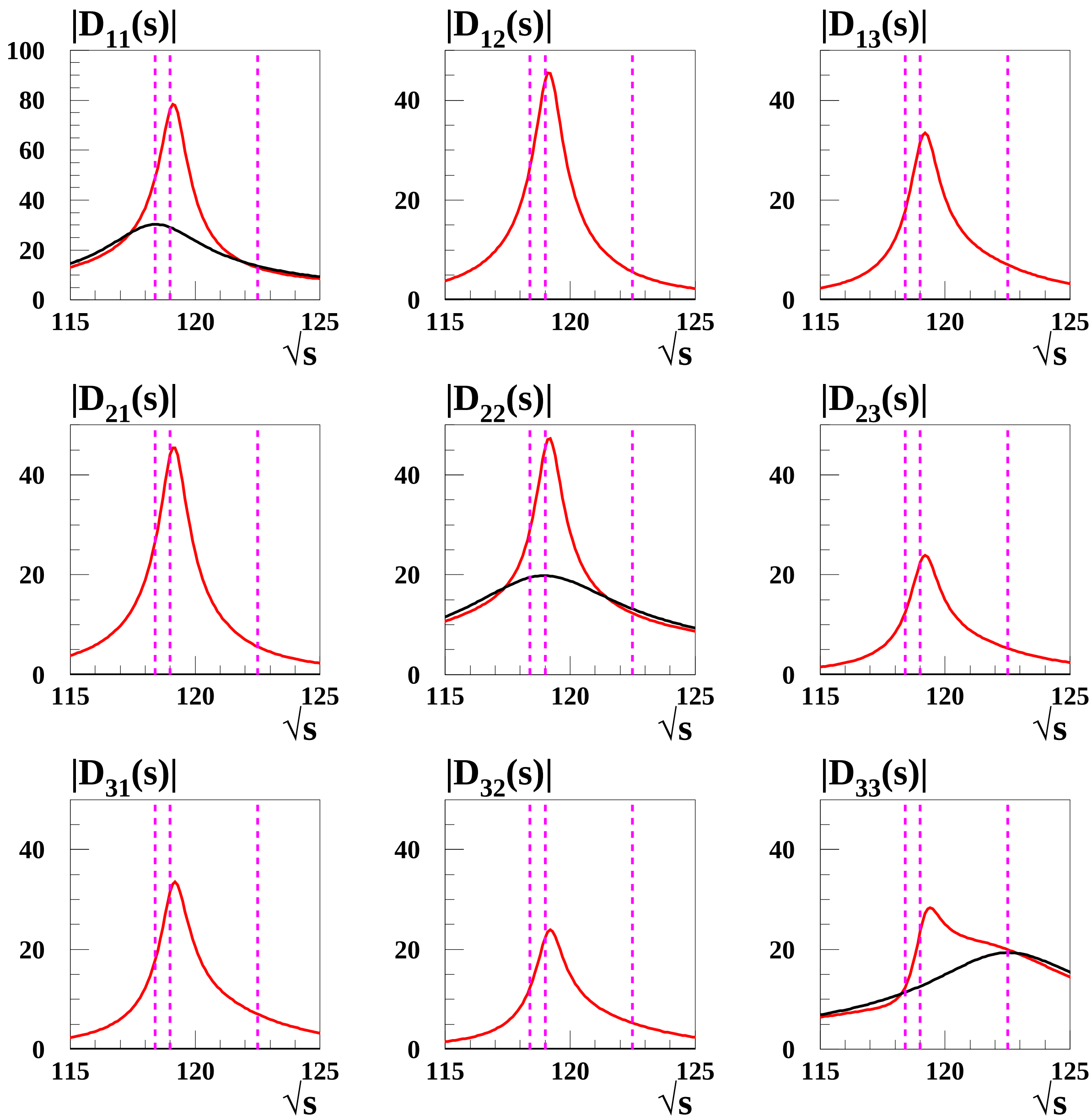
$$M_{H_1} = 118.4, \quad M_{H_2} = 119.0, \quad M_{H_3} = 122.5,$$
$$\Gamma_{H_1} = 3.91, \quad \Gamma_{H_2} = 6.02, \quad \Gamma_{H_3} = 6.34.$$

The branching ratios are

$$\begin{aligned} B(H_1 \to b\bar{b}/\tau^+\tau^-) &= 0.908/0.09 \\ B(H_2 \to b\bar{b}/\tau^+\tau^-) &= 0.909/0.09 \\ B(H_3 \to b\bar{b}/\tau^+\tau^-) &= 0.908/0.09 \end{aligned}$$

♠ *Coupled-channel analysis of neutral Higgs bosons*

- $|D_{ij}(s)|$ for three-way mixing scenario with $\Phi_A \equiv \Phi_{A_t,A_b,A_\tau} = 90^\circ$ and $\Phi_3 = -90^\circ$
  - Black Line : Set $i\Im\mathrm{m}\widehat{\Pi}_{ij} = 0$ when $i \neq j$
  - Red Line : Absorptive parts fully considered



♠ *Production* $\rightarrow$ *Mixing* $\rightarrow$ *Decay*

- The (differential) CPV cross section : $\tau = \hat{s}/s$

$$\Delta\sigma_{\rm CP} \;=\; \sigma_{RR} - \sigma_{LL}$$

  - b-quark fusion

$$\tau\,\frac{d\Delta\sigma_{\rm CP}}{d\tau} = 4\,\hat{\sigma}_2(b\bar{b} \to H \to \tau^+\tau^-)\;\tau\,\frac{d\mathcal{L}^{bb}}{d\tau}$$

  - gluon fusion

$$\tau\,\frac{d\Delta\sigma_{\rm CP}}{d\tau} = 4\;K\,\hat{\sigma}_2(gg \to H \to \tau^+\tau^-)\;\tau\,\frac{d\mathcal{L}^{gg}}{d\tau}$$

  - W-boson fusion

$$\tau\,\frac{d\Delta\sigma_{\rm CP}}{d\tau} = 4\,\hat{\sigma}_2(W^+W^- \to H \to \tau^+\tau^-)\;\tau\,\frac{d\mathcal{L}^{WW}}{d\tau}$$

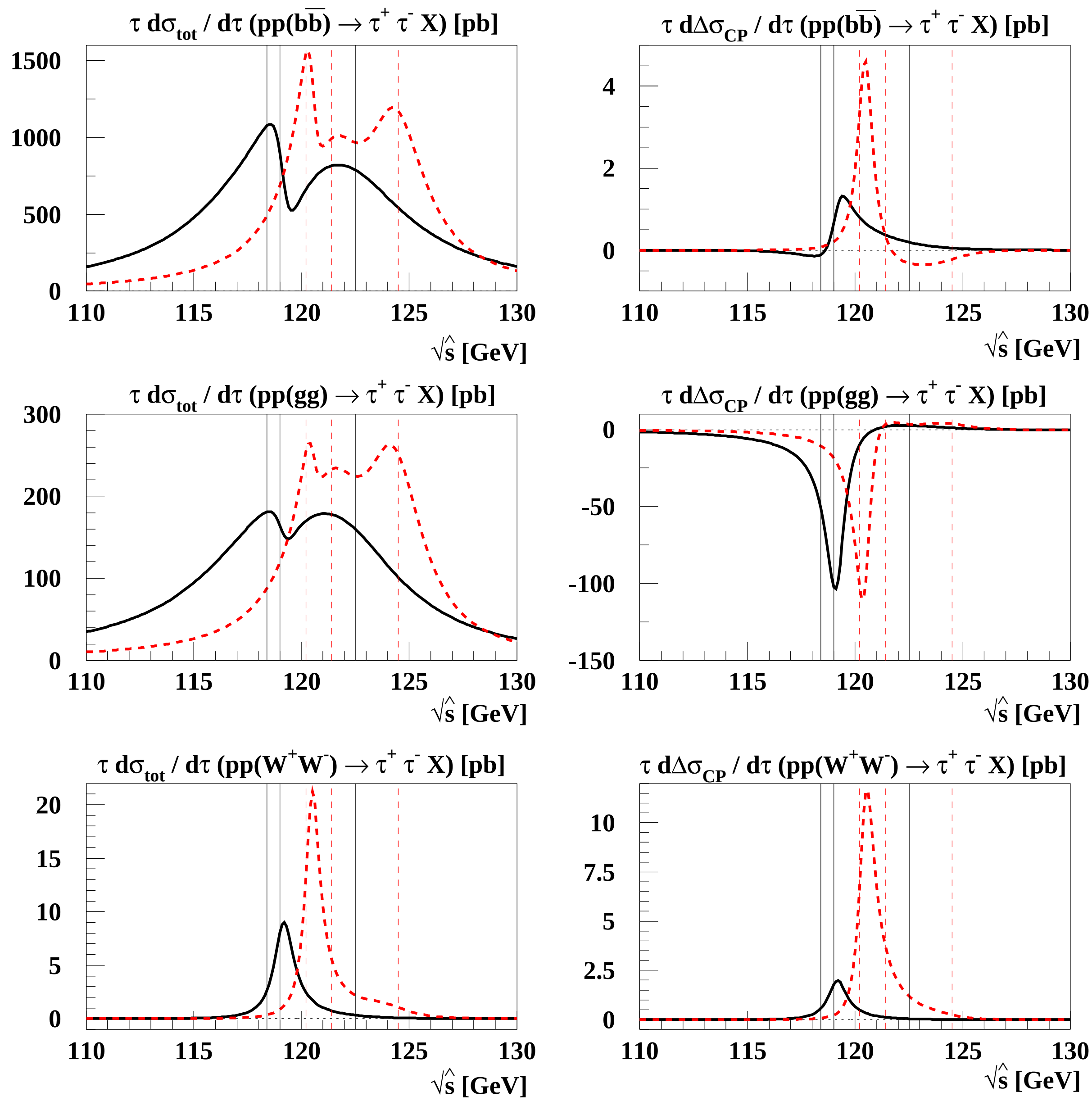
* The other CP-odd cross section $\hat{\sigma}_4$? $\longrightarrow$ It's quite sizable but difficult to measure at the LHC due to the large boosts of $\tau^{\pm}$ and an alternating sign ($b\bar{b}$, gg)

♠ Production → Mixing → Decay

- $\tau \frac{d\sigma_{\rm tot}}{d\tau}$ and $\tau \frac{d\Delta\sigma_{\rm CP}}{d\tau}$ for three-way mixing scenario

– Black Line : $\Phi_A = 90^\circ$ and $\Phi_3 = -90^\circ$

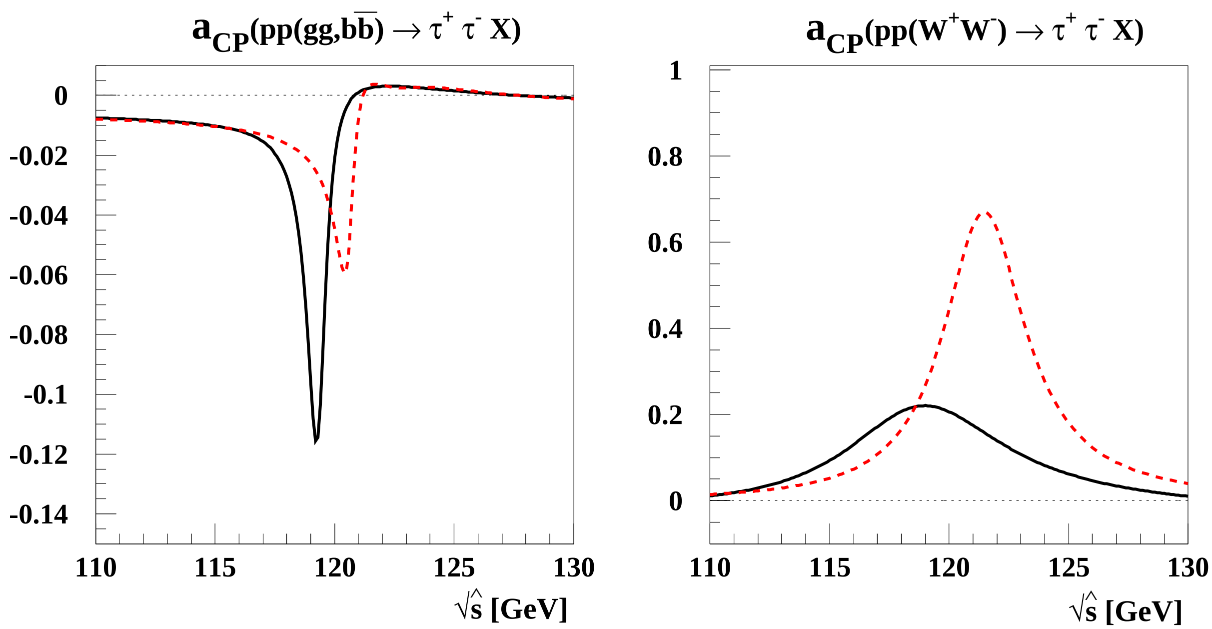
– Red Dashed Line : $\Phi_A = 90^\circ$ and $\Phi_3 = -10^\circ$



♠ *Production* → *Mixing* → *Decay*

- CP asymmetry : $a_{\rm CP}$ and $\mathcal{A}_{\rm CP}$

$$a_{\rm CP}(\tau) \equiv \frac{\tau \frac{d\Delta\sigma_{\rm CP}}{d\tau}}{\tau \frac{d\sigma_{\rm tot}}{d\tau}}, \qquad \mathcal{A}_{\rm CP} \equiv \frac{\Delta\sigma_{\rm CP}}{\sigma_{\rm tot}}$$



– The b-quark fusion can not be distinguished from gluon fusion : $\mathcal{A}_{\rm CP}^{b\bar{b}+gg} = -1.4\,(-1.0)\,\%$ for $\Phi_3 = -90^\circ\,(-10^\circ)$

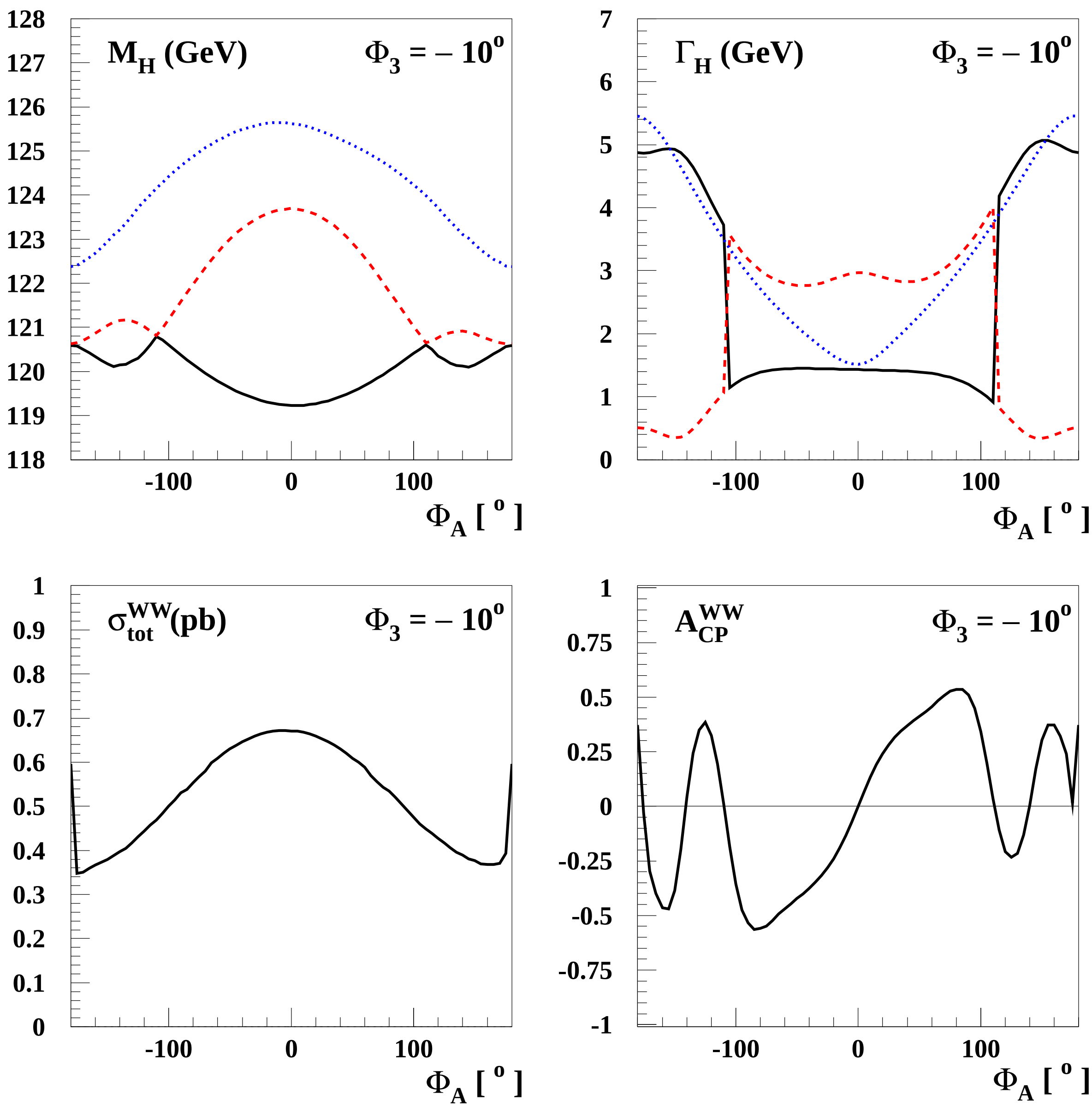
$[\mathcal{A}_{\rm CP}^{gg} = -8.4\,(-6.2)\,\%]$

– The W^+W^--fusion can be distinguished from b-quark and gluon fusions

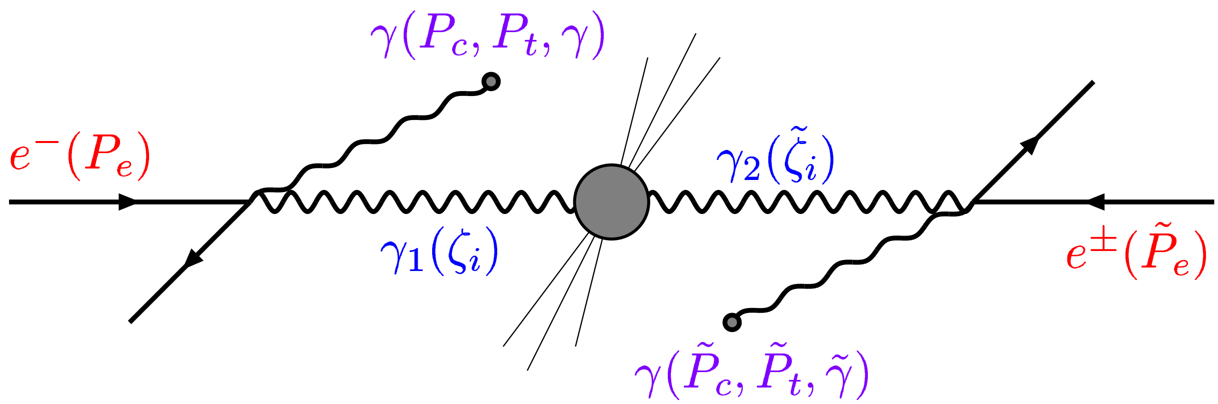
⟶ CONCENTRATE ON W^+W^- FUSION !!!

♠ Production → Mixing → Decay

- $\sigma^{WW}_{\rm tot}$ and CP asymmetry $\mathcal{A}^{WW}_{\rm CP}$ in W^+W^- fusion for three-way mixing scenario with $\Phi_3 = -10^\circ$ as functions of Φ_A
  - Black : H_1 Red Dashed : H_2 Blue Dotted : H_3



♠ A Photon Collider



- **Luminosity and polarizations** I.F. Ginzburg, G.L. Kotkin, S.L. Panfil, V.G. Serbo, and V.I. Telnov, Nucl. Instrum. Meth. **A219** (1984) 5
  - Luminosity

$$\frac{d^2\mathcal{L}_{\gamma\gamma}}{dy_1 dy_2} = f(y_1)f(y_2)\mathcal{L}_{ee}$$

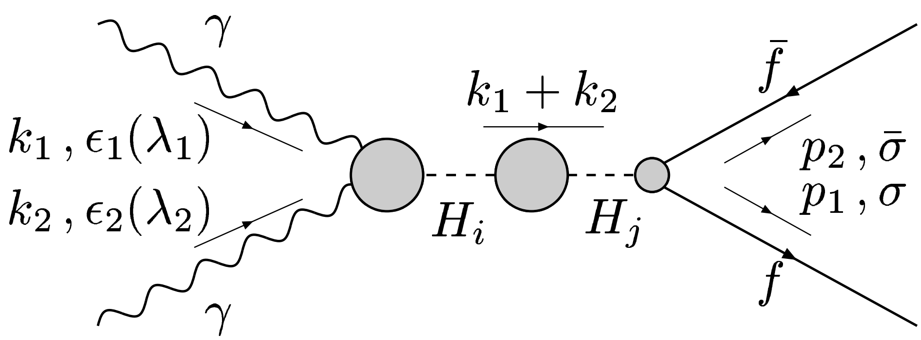
$$f(y) = \frac{1}{\sigma_c}\frac{d\sigma_c}{dy} = \mathcal{F}\left(y = \frac{E_\gamma}{E_b}, x = \frac{4E_b\omega_0}{m_e^2}, P_e, P_c\right)$$

$$\frac{1}{\mathcal{L}_{ee}}\frac{d\mathcal{L}_{\gamma\gamma}}{dz} = 2z\int_{\tau/y_m}^{y_m}\frac{dx}{x}f(x)f(\tau/x)$$

where $y_m = x/(1+x)$ and $\tau = z^2 = \hat{s}/s$

♠ A Photon Collider

- Observables in $\gamma(\lambda_1)\gamma(\lambda_2) \to f(\sigma)\bar{f}(\bar{\sigma})$ process E. Asakawa, S.Y. Choi, K. Hagiwara, JSL, PRD**62** (2000) 115005;R.M. Godbole, S.D. Rindani, R.K. Singh, PRD**67** (2003) 095009



The helicity amplitude is ($g_f = \frac{g m_f}{2M_W}$ when $f = \tau^-$ or t)

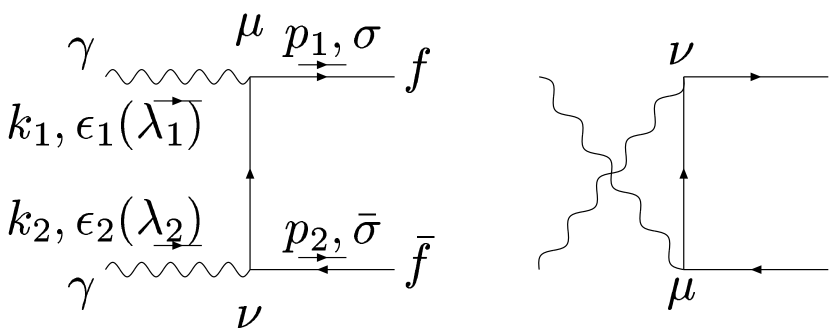
$$\mathcal{M}_H = \frac{\alpha\, m_f \sqrt{\hat{s}}}{4\pi v^2} \langle\sigma;\lambda_1\rangle_H \delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2}$$

$$\langle\sigma;\lambda\rangle_H = \sum_{i,j=1}^{3} \left[S_i^\gamma(\sqrt{\hat{s}}) + i\lambda P_i^\gamma(\sqrt{\hat{s}})\right] D_{ij}(\hat{s}) \times(\sigma\beta_f g^S_{H_j\bar{f}f} - i g^P_{H_j\bar{f}f})$$

where $\beta_f = \sqrt{1-4m_f^2/\hat{s}}$

- Observables in $\gamma(\lambda_1)\gamma(\lambda_2) \to f(\sigma)\bar{f}(\bar{\sigma})$ process (Cont'd)

E. Asakawa, J. Kamoshita, A. Sugamoto, I. Watanabe, EPJ**C14** (2000) 335; E. Asakwa, K. Hagiwara, EPJ**C31** (2003) 31



The helicity amplitude is

$$\mathcal{M}_C = 4\pi\alpha Q_f^2 \, \langle \sigma\, \bar{\sigma}; \lambda_1\, \lambda_2 \rangle_C$$

where

$$\langle \sigma\, \sigma; \lambda\, \lambda \rangle_C = \frac{4m_f}{\sqrt{\hat{s}}} \frac{1}{1-\beta_f^2 c_\theta^2} (\lambda + \sigma\beta_f)$$

$$\langle \sigma\, \sigma; \lambda\, -\lambda \rangle_C = -\frac{4m_f}{\sqrt{\hat{s}}} \frac{s_\theta^2}{1-\beta_f^2 c_\theta^2} \sigma\beta_f$$

$$\langle \sigma\, -\sigma; \lambda\, \lambda \rangle_C = 0$$

$$\langle \sigma\, -\sigma; \lambda\, -\lambda \rangle_C = -2\beta_f \frac{s_\theta}{1-\beta_f^2 c_\theta^2} (\sigma\lambda + c_\theta)$$

and $\hat{s} = (k_1+k_2)^2 = (p_1+p_2)^2$ and θ is for an angle between $\mathbf{p}_1$ and $\mathbf{k}_1$.

- Observables in $\gamma(\lambda_1)\gamma(\lambda_2) \to f(\sigma)\bar{f}(\bar{\sigma})$ process (Cont'd)
  - Three cases considered

    I. $\lambda_1 = \lambda_2$ and $\sigma = \bar{\sigma}$

    II. $\sigma = \bar{\sigma}$: spin–spin correlations of initial photons

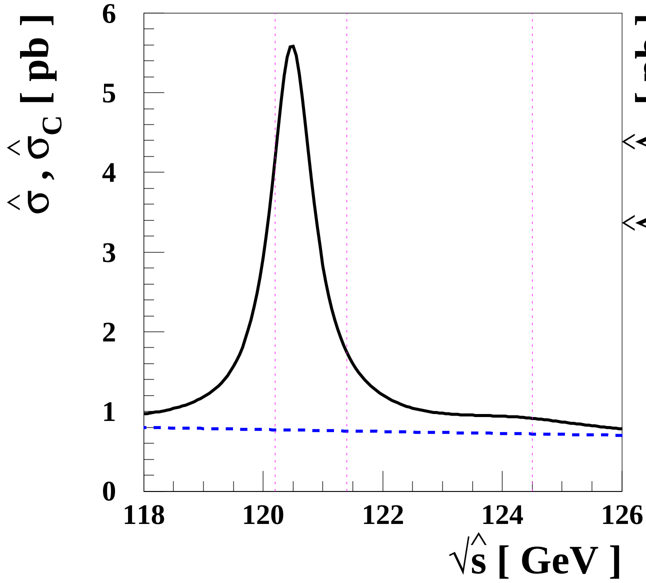
    III. $\lambda_1 = \lambda_2$: spin–spin correlations of final fermions

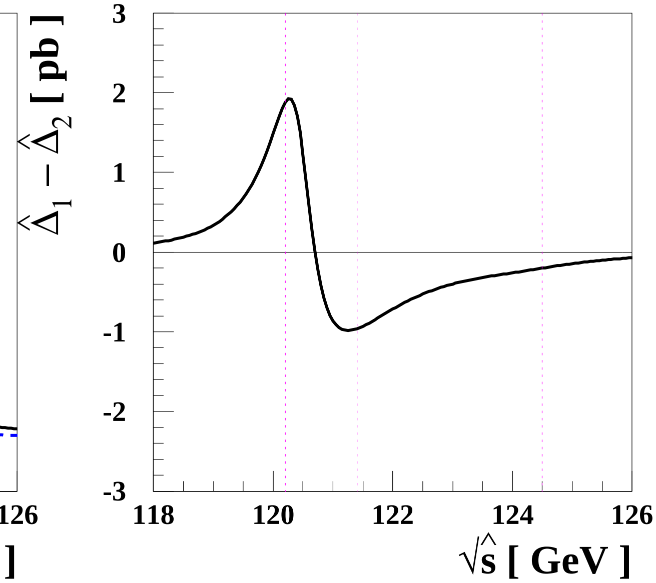
There are $4 \oplus 18 \oplus 8 = 30$ observables at a $\gamma\gamma$ collider for a typical $2 \to 2$ process!

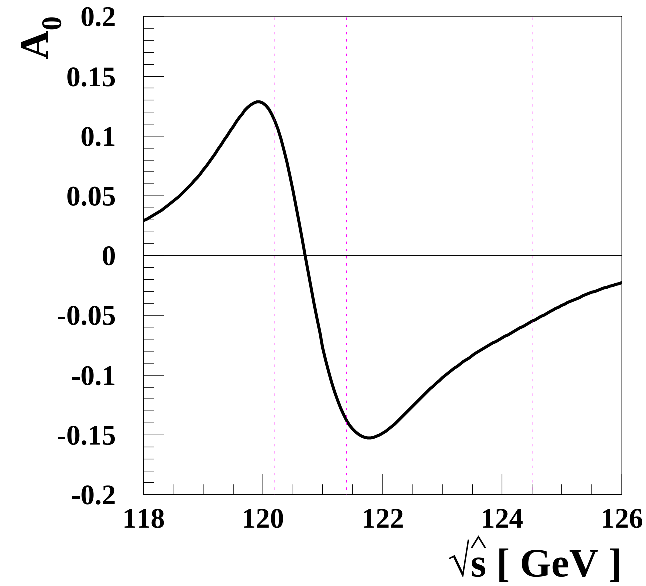
♠ *Numerical Example*

- $\gamma(\lambda_1)\gamma(\lambda_2) \to b\bar{b}$ process

  Three-way mixing scenario with $\Phi_A = 90°$ and $\Phi_3 = -10°$







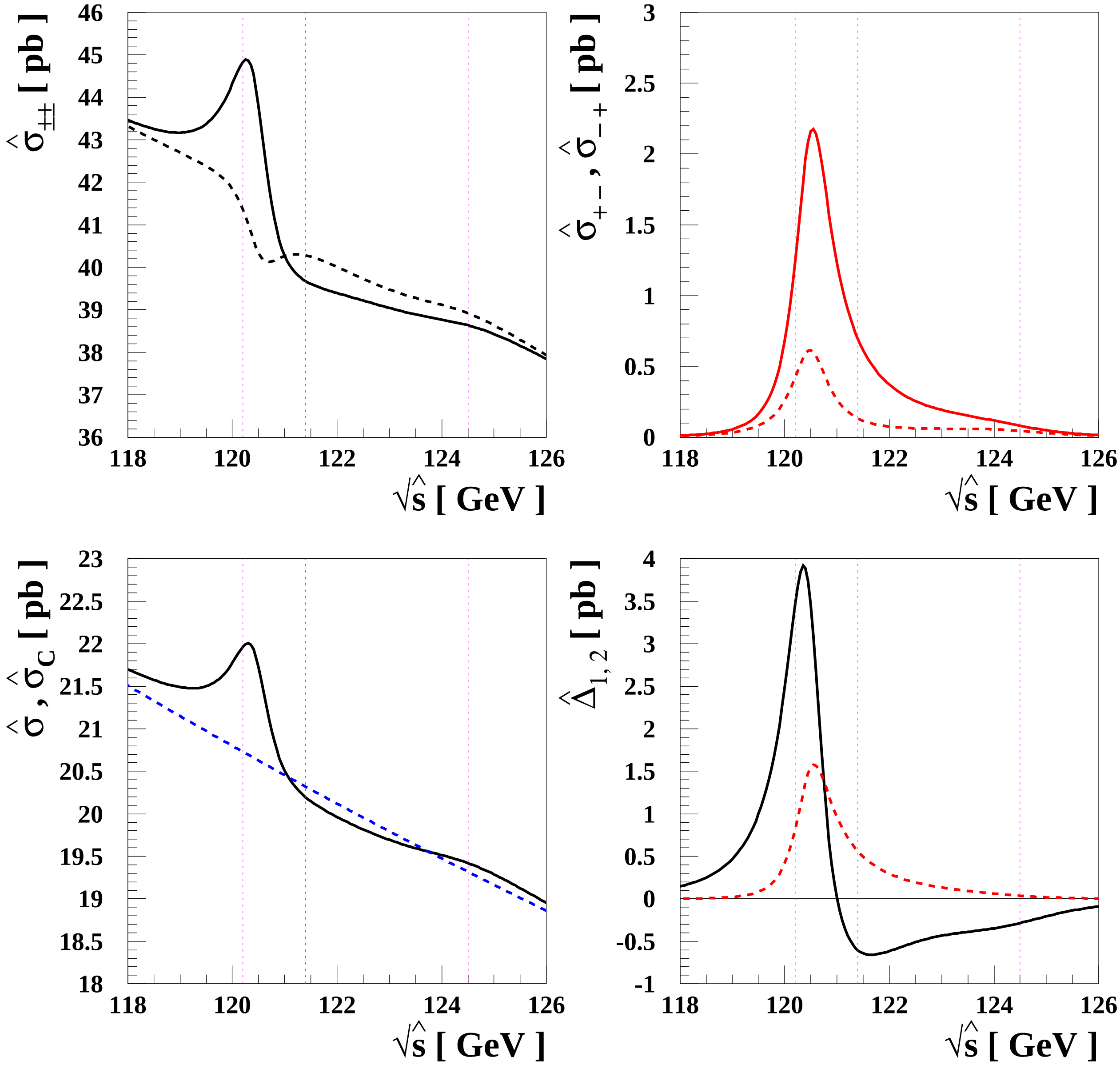
(Sld,Dsh'd) $=$ $(\hat{\sigma}\,,\hat{\sigma}_C)$

Asymmetry $\mathcal{A}_0 \equiv \frac{\hat{\Delta}_1 - \hat{\Delta}_2}{4\hat{\sigma}}$

♠ *Numerical Example*

- $\gamma(\lambda_1)\gamma(\lambda_2) \to \tau^-(\sigma)\tau^+(\bar{\sigma})$ process

Three-way mixing scenario with $\Phi_A = 90^\circ$ and $\Phi_3 = -10^\circ$



(Sld,Dsh'd)=$(\hat{\sigma}_{++}, \hat{\sigma}_{--})$, $(\hat{\sigma}_{+-}, \hat{\sigma}_{-+})$, $(\hat{\sigma}, \hat{\sigma}_C)$, (Δ_1, Δ_2)

$\mathcal{M} = \mathcal{M}_H + \mathcal{M}_C$, $\mathcal{M}_C \propto \lambda + \sigma\beta_\tau$, $|\mathcal{M}_H| \sim |\mathcal{M}_C|/3$

♠ *Numerical Example*

- $\gamma(\lambda_1)\gamma(\lambda_2) \to \tau^-(\sigma)\tau^+(\bar{\sigma})$ process (Cont'd)
  Asymmetries :

$$\mathcal{A}_1 \equiv \frac{\hat{\Delta}_1}{\hat{\sigma}_{++} + \hat{\sigma}_{--}}, \qquad \mathcal{A}_2 \equiv \frac{\hat{\Delta}_2}{\hat{\sigma}_{+-} + \hat{\sigma}_{-+}}$$