

Mass Determination of SUSY Particles from Endpoints

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Based on:

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CONTENTS

Introduction (p. 2–3)

Endpoint Method (p. 4–9)

Applicability of Method (p. 10–15)

Simulation (p. 16–22)

Masses from Endpoint (p. 23–28)

INTRODUCTION

R-parity (to avoid proton decay)

- $P_R = (-1)^{3(B-L)+2s}$

$P_R = +1$: SM particles

$P_R = -1$: SUSY particles

- Any interaction term must have $\prod P_R = +1$

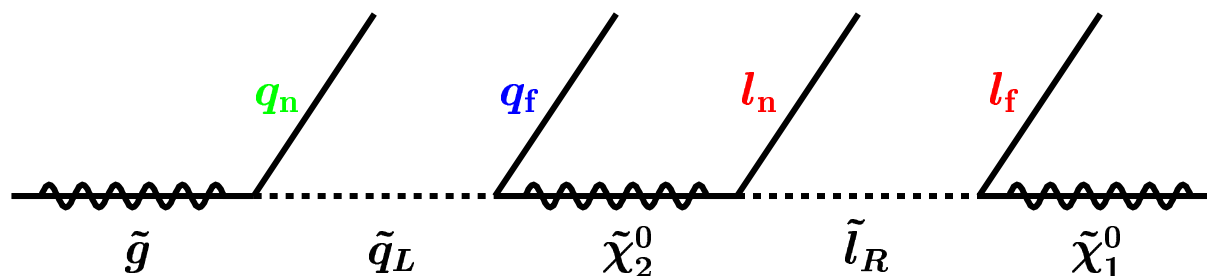
⇒ Vertex must have even number of sparticles

⇒ Lightest sparticle (LSP) is stable

⇒ Any sparticle must eventually decay into the LSP

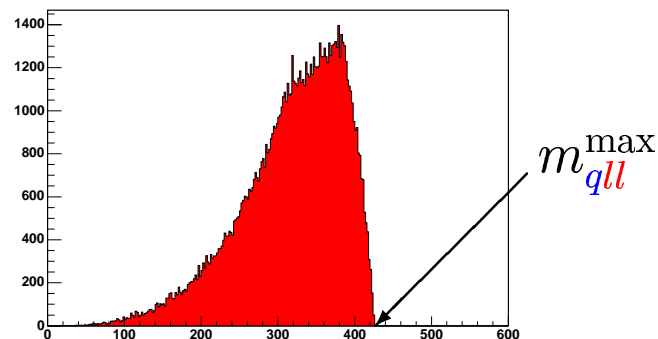
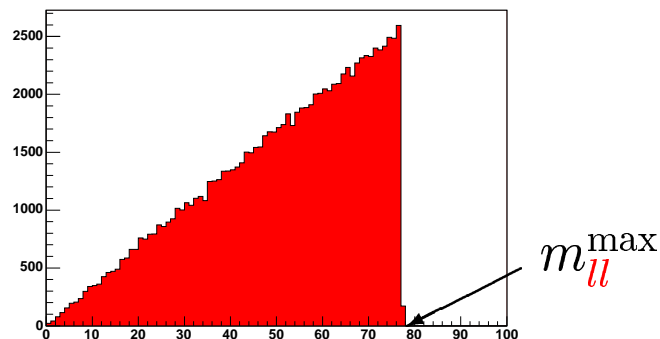
⇒ Collider: Sparticles are pair produced

ENDPOINT METHOD



Construct mass distribution of the SM particles.

Kinematical endpoints relate to masses of intermediate sparticles



2 particles (l_n, l_f) \implies 1 mass distribution (3 unknown masses)

3 particles (q_f, l_n, l_f) \implies 4 mass distributions (4 unknown masses)

[4 particles (q_n, q_f, l_n, l_f) \implies 11 mass distributions (5 unknown masses)]

Complication:

Experimentally, cannot distinguish l_n from l_f

Instead of m_{ql_n} and m_{ql_f} , use distributions

$$m_{ql(\text{high})} = \max(m_{ql_n}, m_{ql_f})$$

$$m_{ql(\text{low})} = \min(m_{ql_n}, m_{ql_f})$$

(constructed on an event by event basis)

Additional endpoint:

m_{ll} carry exact information on angle between leptons in \tilde{l}_R rest frame

Can combine m_{qll} with this info to get

$$m_{qll}(\theta > \frac{\pi}{2}) \text{ (threshold)}$$

$$(m_{ll}^{\max})^2 = (m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)/m_{\tilde{l}_R}^2$$

$$(m_{ql}^{\max})^2 = \left\{ \begin{array}{ll} \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{\chi}_2^0}^2} & \text{for } \frac{m_{\tilde{q}_L}}{m_{\tilde{\chi}_2^0}} > \frac{m_{\tilde{\chi}_1^0}}{m_{\tilde{l}_R}} \quad (1) \\ \frac{(m_{\tilde{q}_L}^2 m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_2^0}^2 m_{\tilde{\chi}_1^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)}{m_{\tilde{\chi}_2^0}^2 m_{\tilde{l}_R}^2} & \text{for } \frac{m_{\tilde{\chi}_2^0}}{m_{\tilde{l}_R}} > \frac{m_{\tilde{l}_R}}{m_{\tilde{\chi}_1^0}} \quad (2) \\ \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2} & \text{for } \frac{m_{\tilde{l}_R}}{m_{\tilde{\chi}_1^0}} > \frac{m_{\tilde{q}_L}}{m_{\tilde{\chi}_2^0}} \quad (3) \\ (m_{\tilde{q}_L} - m_{\tilde{\chi}_1^0})^2 & \text{otherwise} \quad (4) \end{array} \right\}$$

$$(m_{ql(\text{low})}^{\max}, m_{ql(\text{high})}^{\max}) = \left\{ \begin{array}{ll} (m_{ql_n}^{\max}, m_{ql_f}^{\max}) & \text{for } 2m_{\tilde{l}_R}^2 > m_{\tilde{\chi}_1^0}^2 + m_{\tilde{\chi}_2^0}^2 > 2m_{\tilde{\chi}_1^0} m_{\tilde{\chi}_2^0} \quad (1) \\ (m_{ql(\text{eq})}^{\max}, m_{ql_f}^{\max}) & \text{for } m_{\tilde{\chi}_1^0}^2 + m_{\tilde{\chi}_2^0}^2 > 2m_{\tilde{l}_R}^2 > 2m_{\tilde{\chi}_1^0} m_{\tilde{\chi}_2^0} \quad (2) \\ (m_{ql(\text{eq})}^{\max}, m_{ql_n}^{\max}) & \text{for } m_{\tilde{\chi}_1^0}^2 + m_{\tilde{\chi}_2^0}^2 > 2m_{\tilde{\chi}_1^0} m_{\tilde{\chi}_2^0} > 2m_{\tilde{l}_R}^2 \quad (3) \end{array} \right\}$$

$$(m_{ql_n}^{\max})^2 = (m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)/m_{\tilde{\chi}_2^0}^2$$

$$(m_{ql_f}^{\max})^2 = (m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)/m_{\tilde{l}_R}^2$$

$$(m_{ql(\text{eq})}^{\max})^2 = (m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)/(2m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)$$

Note 1: mass *differences*

Note 2: multiple expressions
define 9 mass regions (i, j)

Masses from endpoints: 2 methods

1. **Analytical inversion** of 4 endpoint formulae m_{ll}^{\max} , m_{ql}^{\max} , $m_{ql(\text{high})}^{\max}$, $m_{ql(\text{low})}^{\max}$
 - only possible if $\#\text{endpoints} = \#\text{masses}$
 - + transparent w.r.t. endpoint dependence
 - + transparent w.r.t. multiple mass regions
 - unique set of inversion formulae for each 6/9 mass region
 - In 3/9 regions the 4 endpoints are not independent: need 5th endpoint

2. **Numerical fit**
 - only way if $\#\text{endpoints} > \#\text{masses}$
 - not transparent
 - need to look for several minima
 - difficult near **borders** between mass regions;
 - e.g. m_{ql}^{\max} is continuous across **borders**, but not its derivatives
 - \implies challenge for some minimisers

1. Inversion formulae for mass region (1,1):

$$m_{\tilde{\chi}_1^0}^2 = \frac{[(m_{qll}^{\max})^2 - (m_{ql(\text{high})}^{\max})^2] [(m_{qll}^{\max})^2 - (m_{ql(\text{low})}^{\max})^2]}{[(m_{ql(\text{low})}^{\max})^2 + (m_{ql(\text{high})}^{\max})^2 - (m_{qll}^{\max})^2]^2} (m_{ll}^{\max})^2$$

$$m_{\tilde{l}_R}^2 = \frac{(m_{ql(\text{low})}^{\max})^2 [(m_{qll}^{\max})^2 - (m_{ql(\text{low})}^{\max})^2]}{[(m_{ql(\text{low})}^{\max})^2 + (m_{ql(\text{high})}^{\max})^2 - (m_{qll}^{\max})^2]^2} (m_{ll}^{\max})^2$$

$$m_{\tilde{\chi}_2^0}^2 = \frac{(m_{ql(\text{low})}^{\max})^2 (m_{ql(\text{high})}^{\max})^2}{[(m_{ql(\text{low})}^{\max})^2 + (m_{ql(\text{high})}^{\max})^2 - (m_{qll}^{\max})^2]^2} (m_{ll}^{\max})^2$$

$$m_{\tilde{q}_L}^2 = \frac{(m_{ql(\text{low})}^{\max})^2 (m_{ql(\text{high})}^{\max})^2 [(m_{ql(\text{low})}^{\max})^2 + (m_{ql(\text{high})}^{\max})^2 - (m_{qll}^{\max})^2 + (m_{ll}^{\max})^2]}{[(m_{ql(\text{low})}^{\max})^2 + (m_{ql(\text{high})}^{\max})^2 - (m_{qll}^{\max})^2]^2}$$

Similar expressions (sometimes more difficult) are found for the other of 6 mass regions where the four endpoints are independent.

2. Numerical fit:

Minimise the least-square function,

$$\Sigma = [\mathbf{E}^{\text{exp}} - \mathbf{E}^{\text{th}}(\mathbf{m})]^T \mathbf{W} [\mathbf{E}^{\text{exp}} - \mathbf{E}^{\text{th}}(\mathbf{m})] \quad (1)$$

\mathbf{W} : ‘Weight matrix’, is the inverse of the ...

\mathbf{W}^{-1} : ‘Covariance matrix’/‘Error matrix’

$$(\mathbf{W}^{-1})_{ii} = \sigma_{ii}^{\text{stat}} + \sigma_{ii}^{\text{scale}} = (\sigma_i^{\text{stat}})^2 + (\sigma_i^{\text{scale}})^2$$

$$(\mathbf{W}^{-1})_{ij} = \sigma_{ij}^{\text{scale}} = \langle E_i^{\text{exp}} E_j^{\text{exp}} \rangle - \langle E_i^{\text{exp}} \rangle \langle E_j^{\text{exp}} \rangle = \sigma_i^{\text{scale}} \sigma_j^{\text{scale}}, \quad i \neq j$$

$$(\mathbf{W}^{-1})_{i1} = 0, \quad i \neq 1 \quad (i = 1 \text{ corresponds to } m_{ll}^{\text{max}})$$

σ_i^{stat} : statistical error of endpoint i

σ_i^{scale} : systematic error of endpoint i due to jet (and lepton) energy scale uncertainty in ATLAS

APPLICABILITY OF METHOD

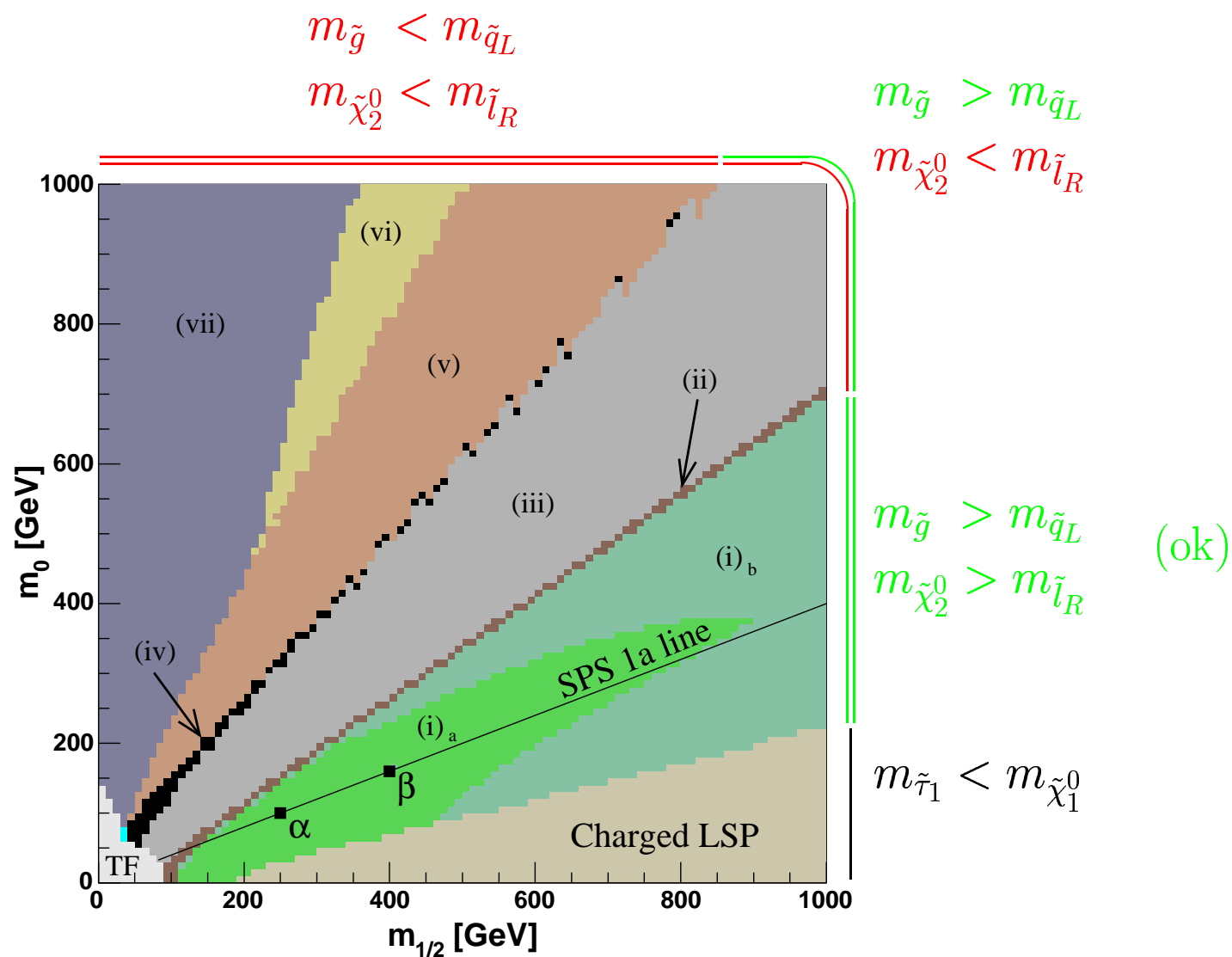
Require ($m_{\tilde{g}} >$) $m_{\tilde{q}_L} > m_{\tilde{\chi}_2^0} > m_{\tilde{l}_R} > m_{\tilde{\chi}_1^0}$

Require sufficient cross-section

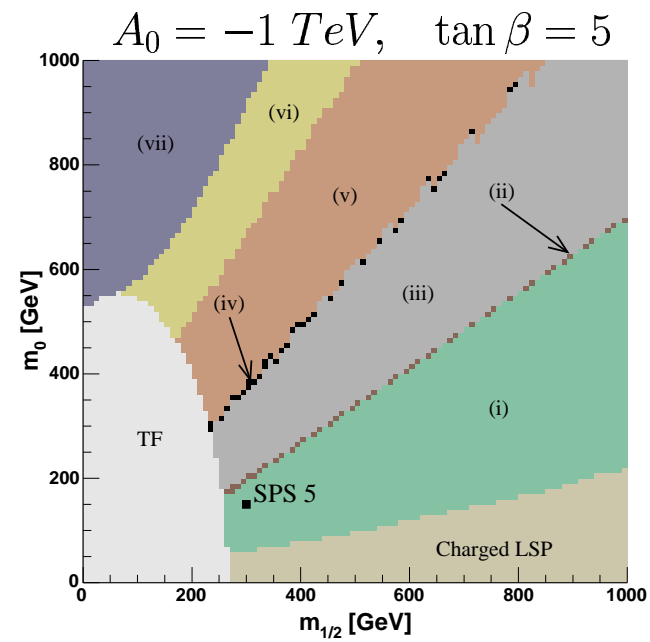
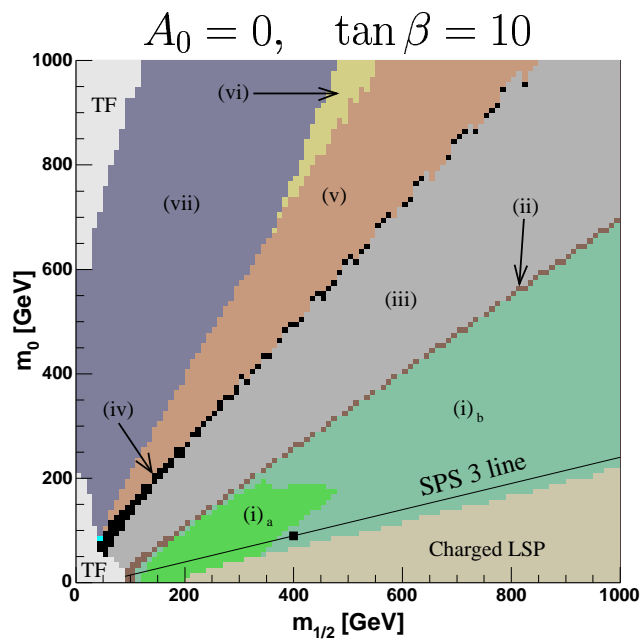
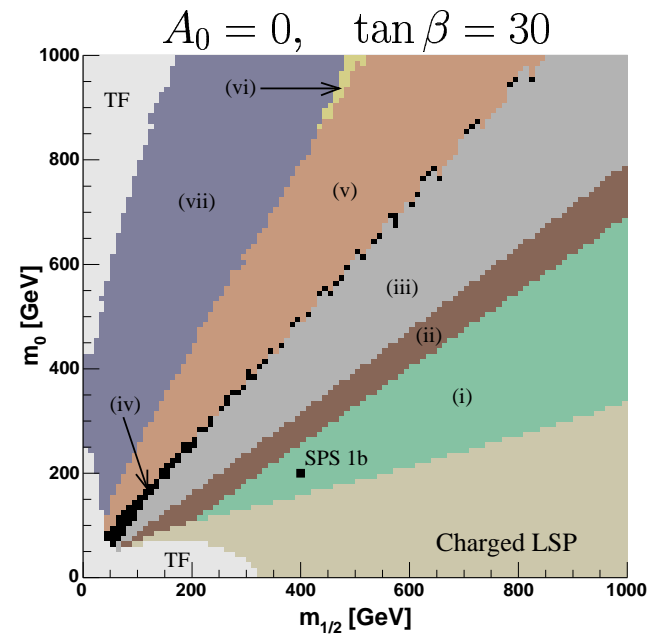
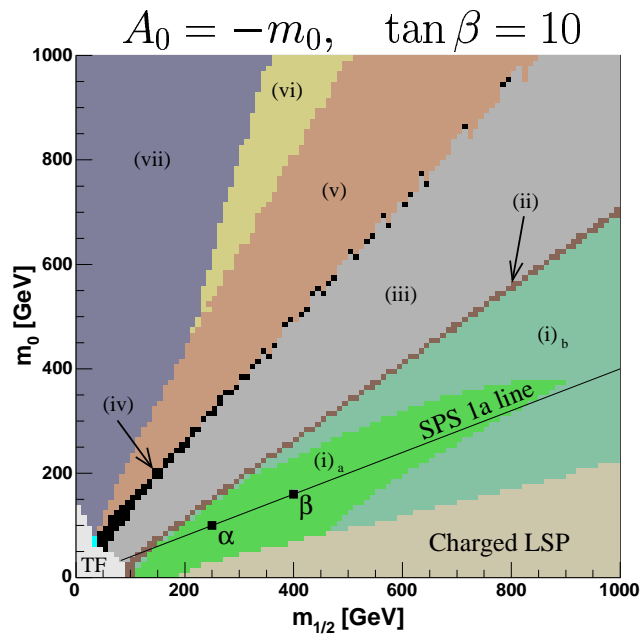
Require sufficient branching ratio

mSUGRA: 4/5 parameters: $m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu)$

Nearly always: $m_{\tilde{g}}, m_{\tilde{q}_L} > m_{\tilde{\chi}_2^0}, m_{\tilde{l}_R}$

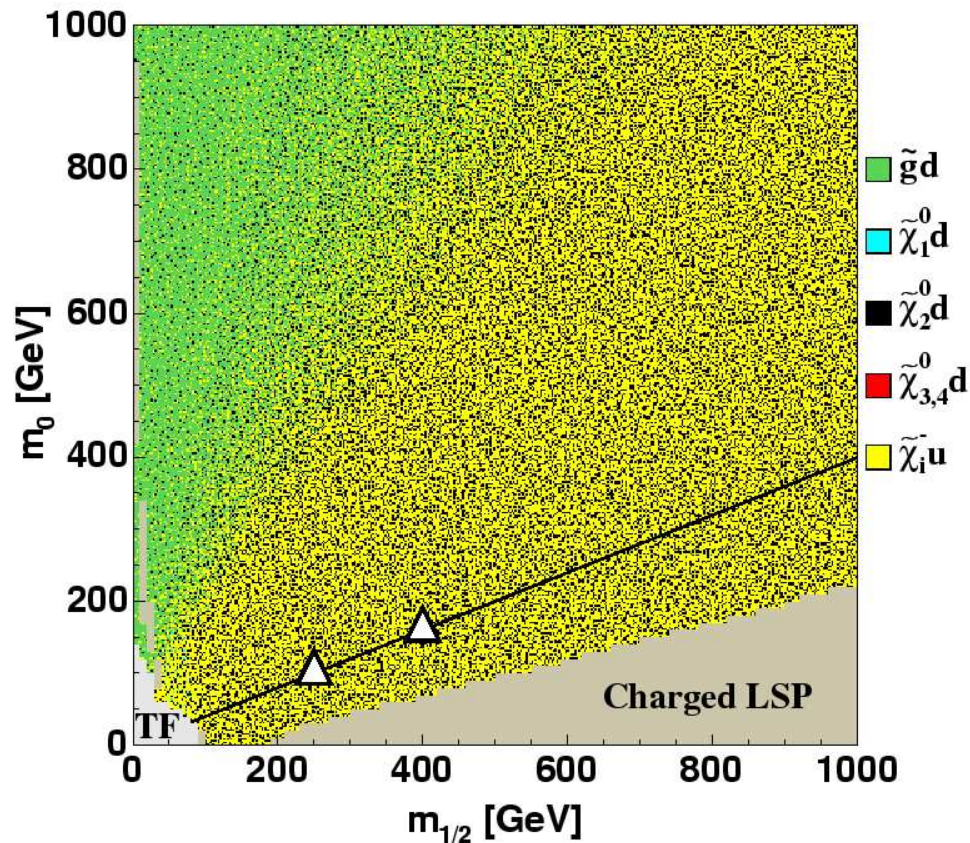
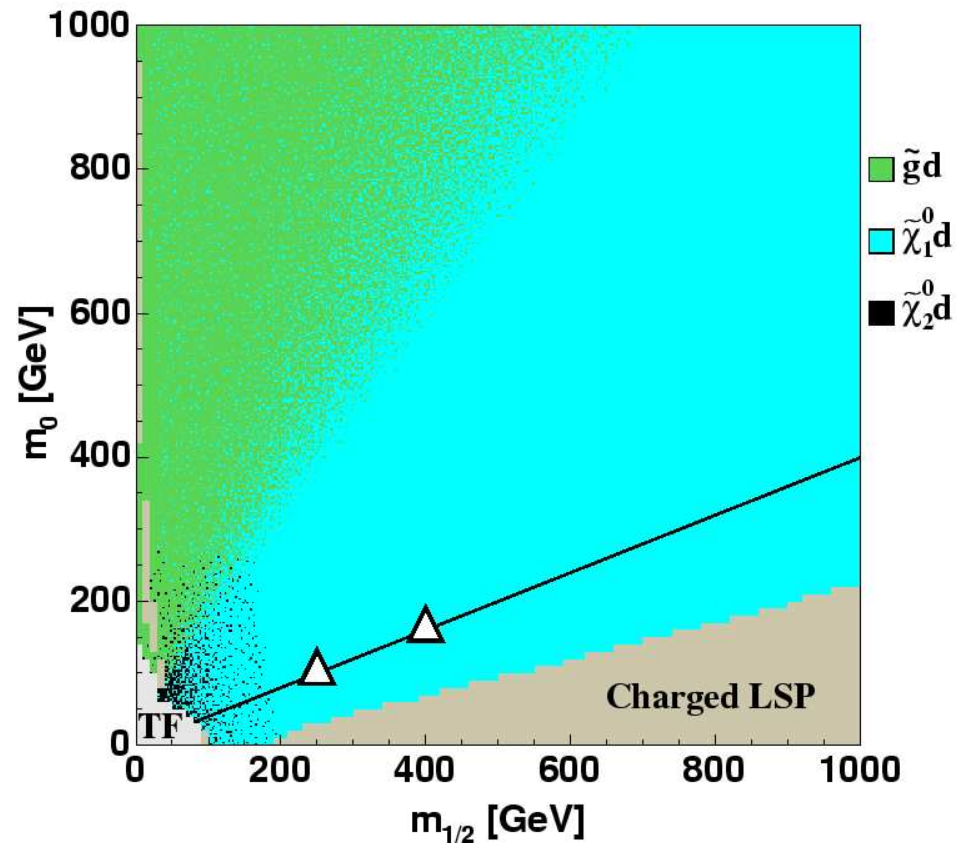


$$A_0 = -m_0, \quad \tan \beta = 10, \quad \mu > 0$$



Branching ratios ($A_0 = -m_0$, $\tan\beta = 10$, $\mu > 0$)

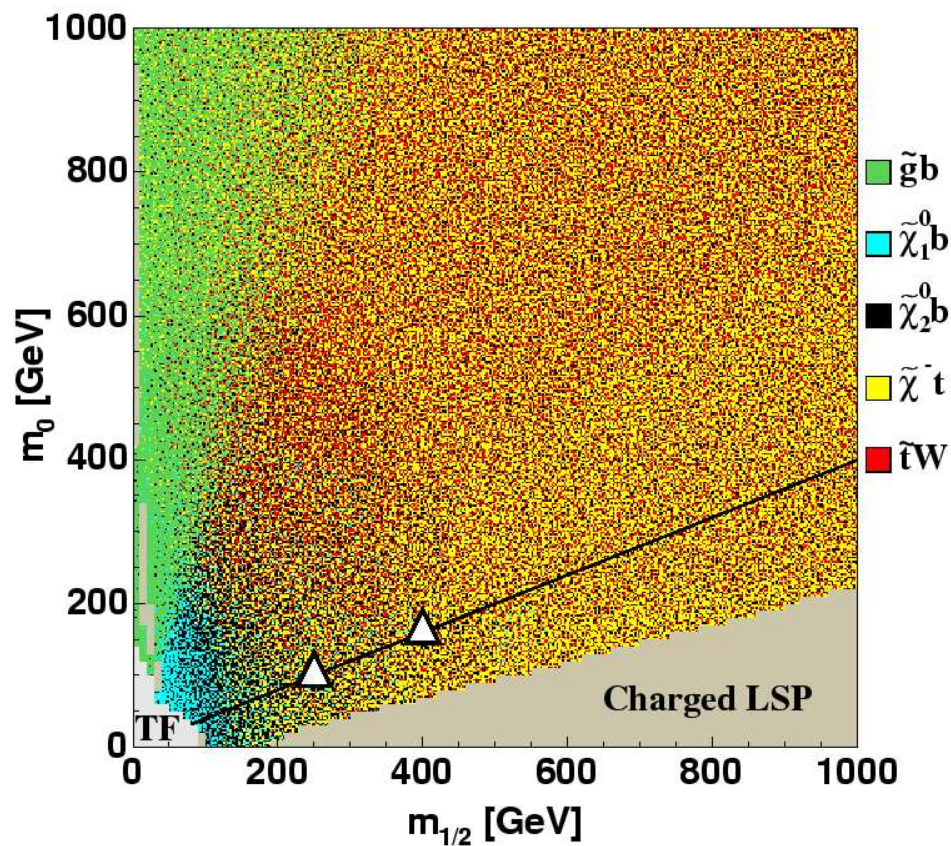
How to read the plots: The **branching ratio** of a given **decay channel** in a specific region of the $m_{1/2}$ - m_0 plane is equal to the **fraction** which the **corresponding colour** occupies in that region.

BR(\tilde{u}_L)BR(\tilde{u}_R)

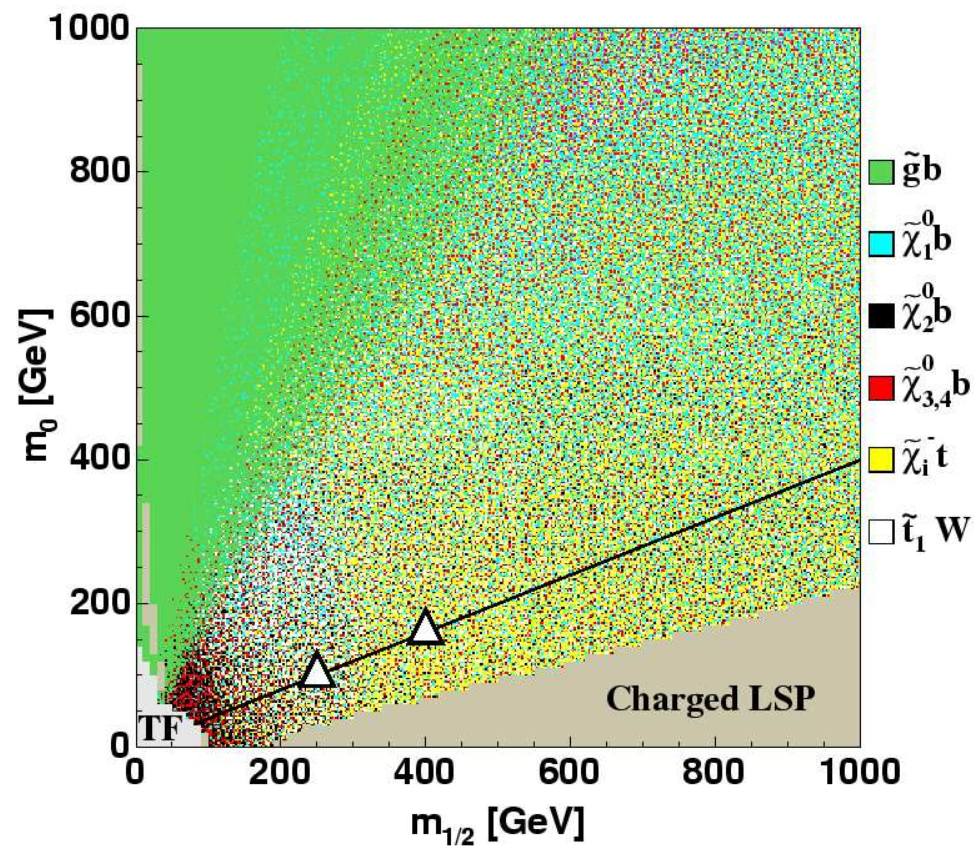
Two triangles are **SPS1a** (α) and (β).

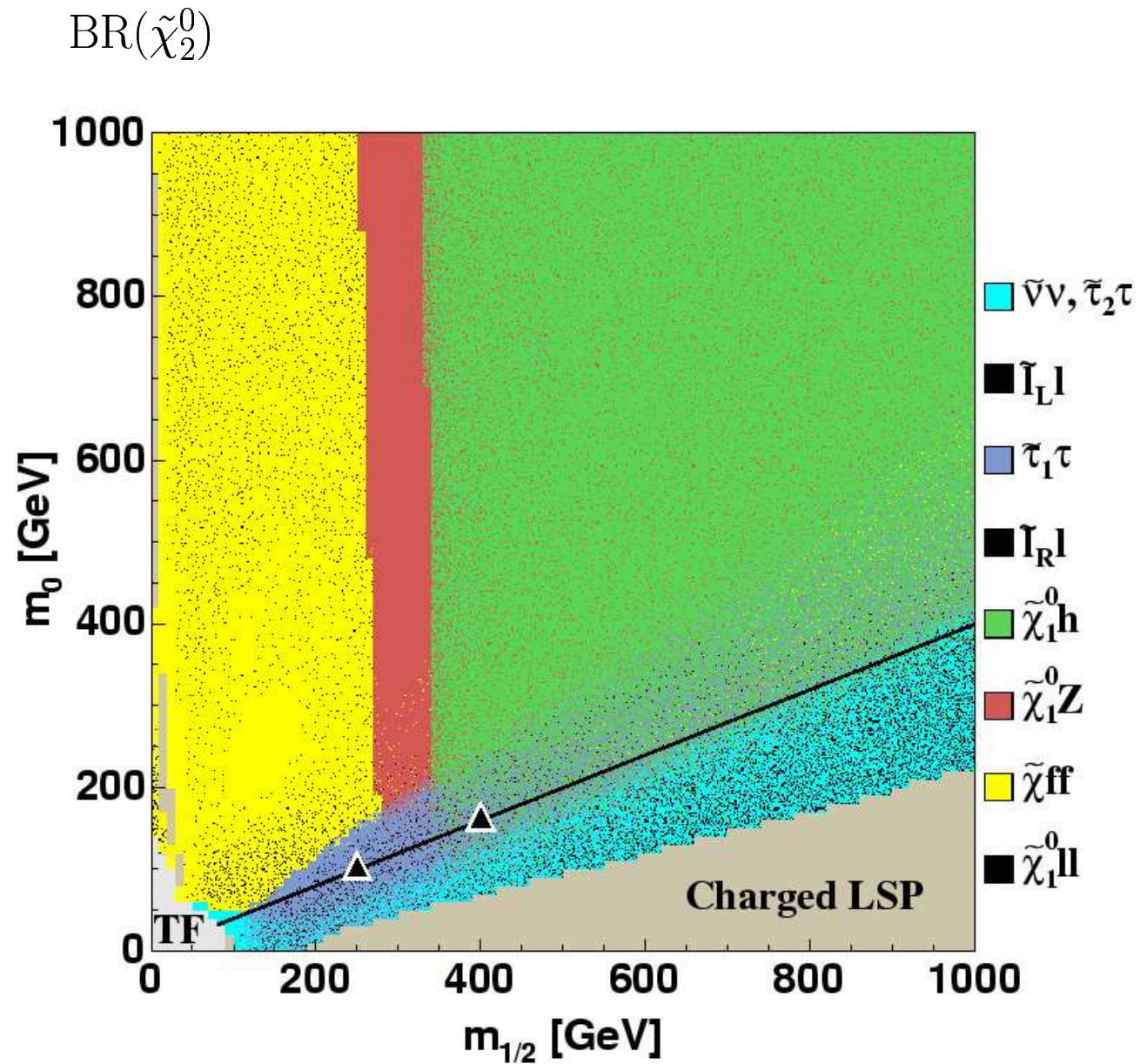
Branching ratios

$BR(\tilde{b}_1)$



$BR(\tilde{b}_2)$

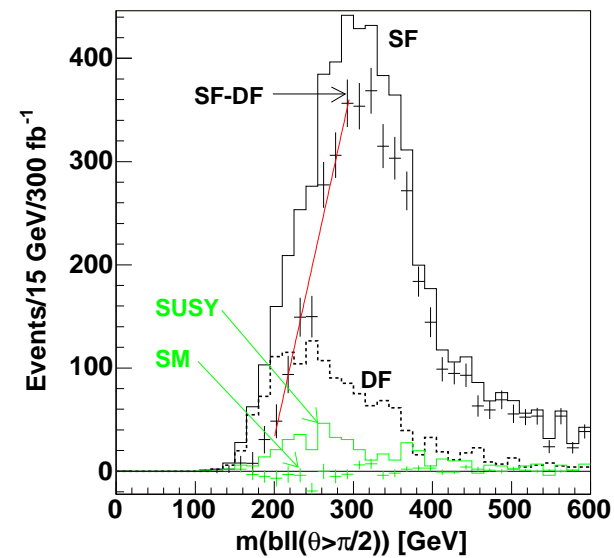
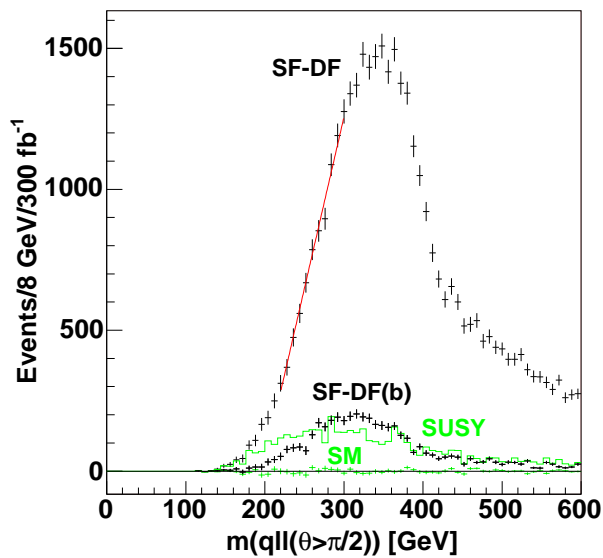
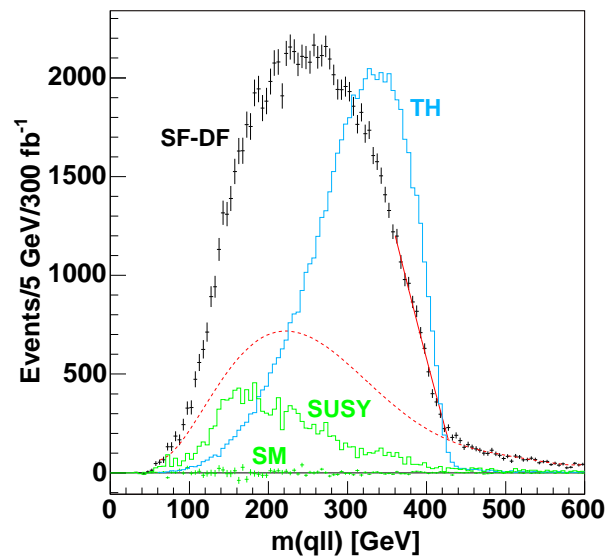
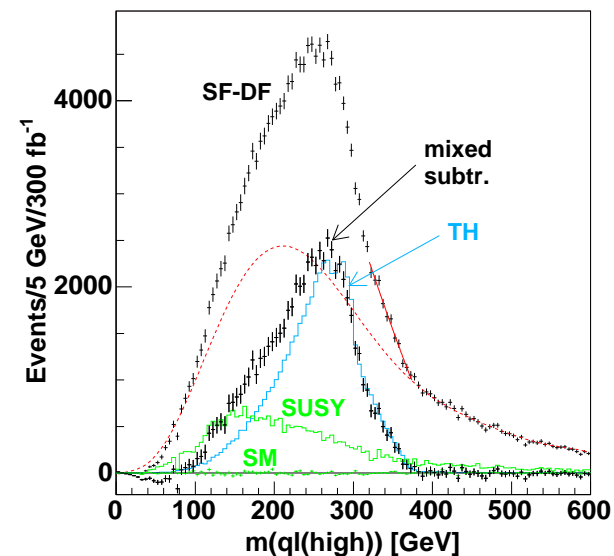
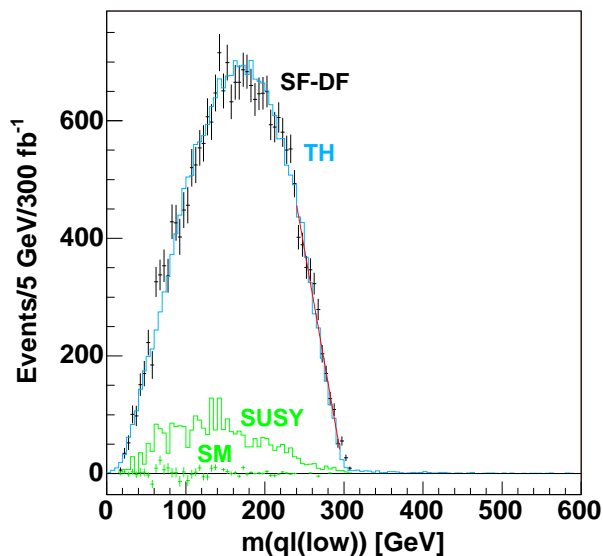
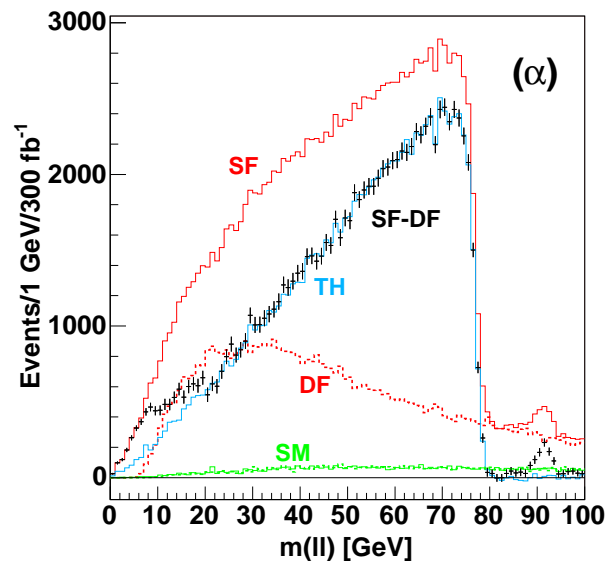


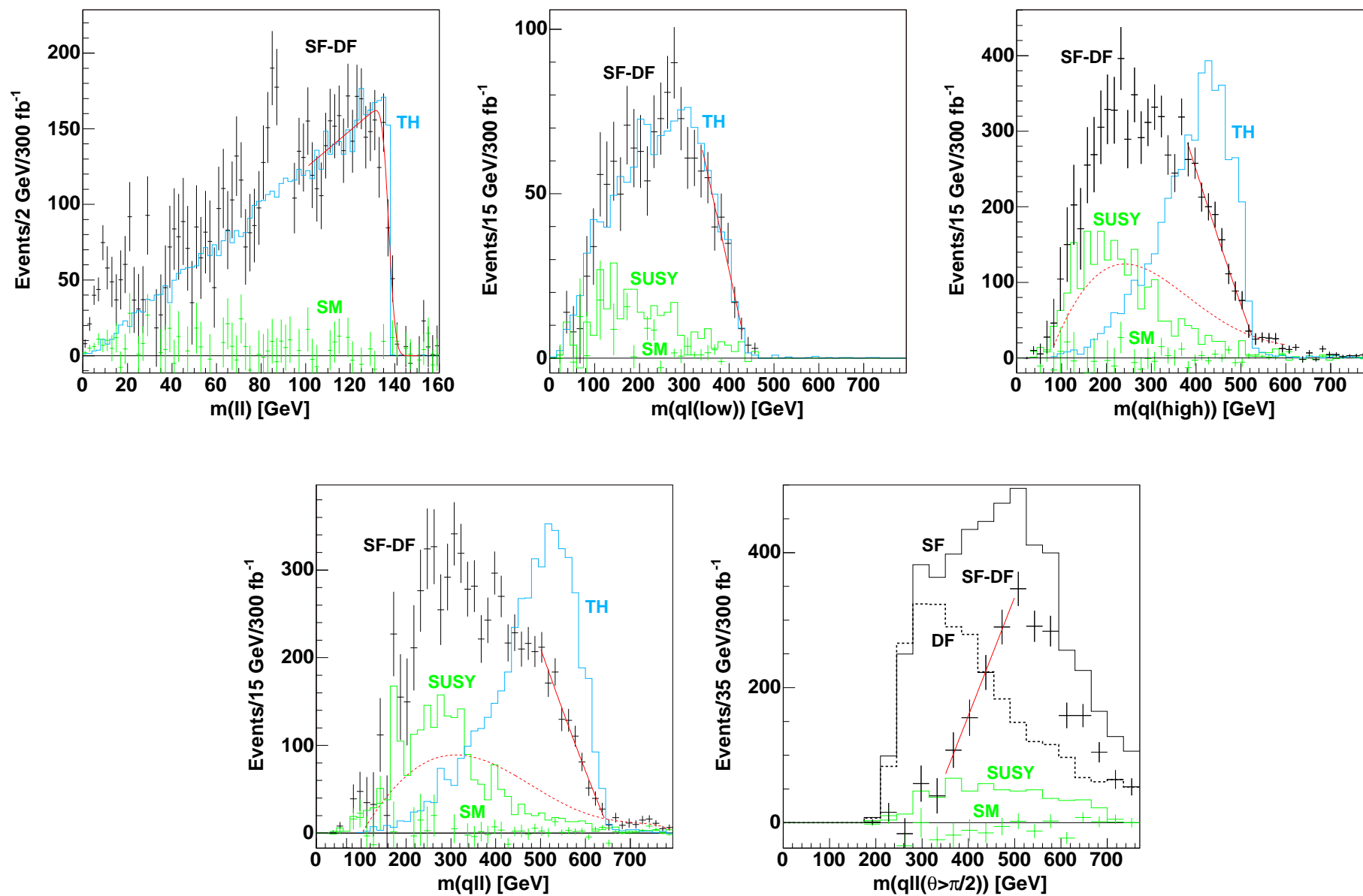


SIMULATION: SPS 1a (α) and (β)

- ‘Standard precuts’:
 - At least three jets, satisfying: $p_T^{\text{jet}} > 150, 100, 50$ GeV
 - $E_{T,\text{miss}} > \max(100 \text{ GeV}, 0.2M_{\text{eff}})$ with $M_{\text{eff}} \equiv E_{T,\text{miss}} + \sum_{i=1}^3 p_{T,i}^{\text{jet}}$
 - Two isolated **opposite-sign same-flavour (SF)** leptons, $p_T^{\text{lep}} > 20, 10$ GeV
- Lepton selection: ok
- Jet selection: selecting incorrect jet gives ‘Combinatorial background’
- Multiple squark masses, b -tagging, ... [mSUGRA: degenerate squarks]
- Different-Flavour (DF) subtraction:
 - ‘Lepton-uncorrelated background’ produces same amount of **SF** = $e^+e^- + \mu^+\mu^-$ and **DF** = $e^+\mu^- + \mu^+e^-$.
 - **SF** and **DF** have same characteristics (event shape etc.)
 - **SF**–**DF** = **SF**(signal)+**SF**(bck)–**DF**(bck) = **SF**(signal) [statistically]
- Standard Model background is tiny for (α), sizeable for (β)
- Main (‘lepton-correlated’) background comes from other SUSY processes

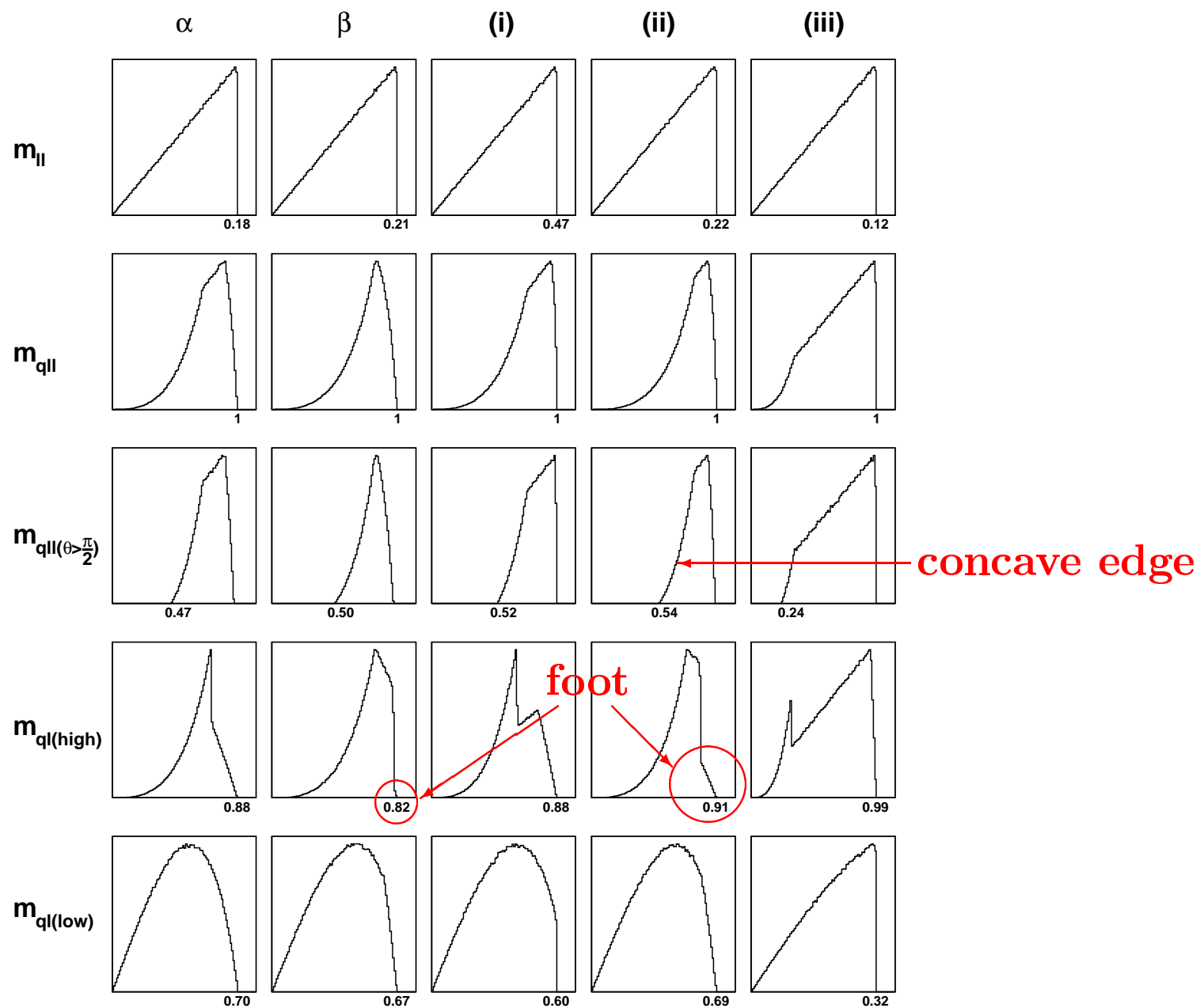
SPS 1a (α)



SPS 1a (β)

- Fit to obtain endpoints:
 - **signal** hypothesis: **straight line**
 - **background** hypothesis: (unknown) use **polynomial/exponential** or **mixed events**
 - At **present**: **systematics** of fitting procedure are **sizeable**
 - **Future**: **systematics below statistical error** for most endpoints
- ⇒ (Need more study)
- Danger:
 - **non-linearity of theory distribution**
 - **‘foot’ in theory distribution**

Theory Distributions



Energy Scale Error (differs from LHC/ILC report)

Lepton: 0.1%

Jet: 1%

$\Rightarrow m_{ll}$: 0.1%

$\Rightarrow m_{qq}$: 1%

$\Rightarrow m_{ql}$: 0.5% (!)

$$\left(\frac{\sigma(m_{ql})}{m_{ql}} = \frac{\sigma(m_{ql}^2)}{2m_{ql}^2} = \frac{1}{2} \sqrt{\left(\frac{\sigma(E_j)}{E_j}\right)^2 + \left(\frac{\sigma(E_l)}{E_l}\right)^2} = 0.50\% \right)$$

$\Rightarrow m_{qll} \in (0.1\%, 0.5\%)$ (!)

not even constant

different for different mass scenarios, but always $\in (0.1\%, 0.5\%)$

we have used a constant value 0.5%

correct for SPS 1a is a little lower $\sim (0.4-0.5)\%$

Edge	Nominal Value [GeV]	Fit Value [GeV]	Energy Scale Error (σ^{scale}) [GeV]	Statistical Error (σ^{stat}) [GeV]	Syst. Fit Error [GeV]
(α)					
m_{ll}^{max}	77.07	76.72	0.08	0.04	0.1
m_{qll}^{max}	425.9	427.7	2.1	0.9	0.5
$m_{ql(\text{low})}^{\text{max}}$	298.5	300.7	1.5	0.9	0.5
$m_{ql(\text{high})}^{\text{max}}$	375.8	374.0	1.9	1.0	0.5
$m_{qll(\theta > \frac{\pi}{2})}^{\text{min}}$	200.7	-	1.0	2.2	2.0
$m_{bll(\theta > \frac{\pi}{2})}^{\text{min}}$	183.1	-	0.9	4.5	4.0
(β)					
m_{ll}^{max}	137.9	137.4	0.14	0.5	0.1
m_{qll}^{max}	649.1	647.0	3.2	5.0	3.0
$m_{ql(\text{low})}^{\text{max}}$	436.6	443.0	2.2	6.3	4.0
$m_{ql(\text{high})}^{\text{max}}$	529.9	520.5	2.6	5.5	3.0
$m_{qll(\theta > \frac{\pi}{2})}^{\text{min}}$	325.7	-	1.6	13.0	10.0

Masses from Endpoints

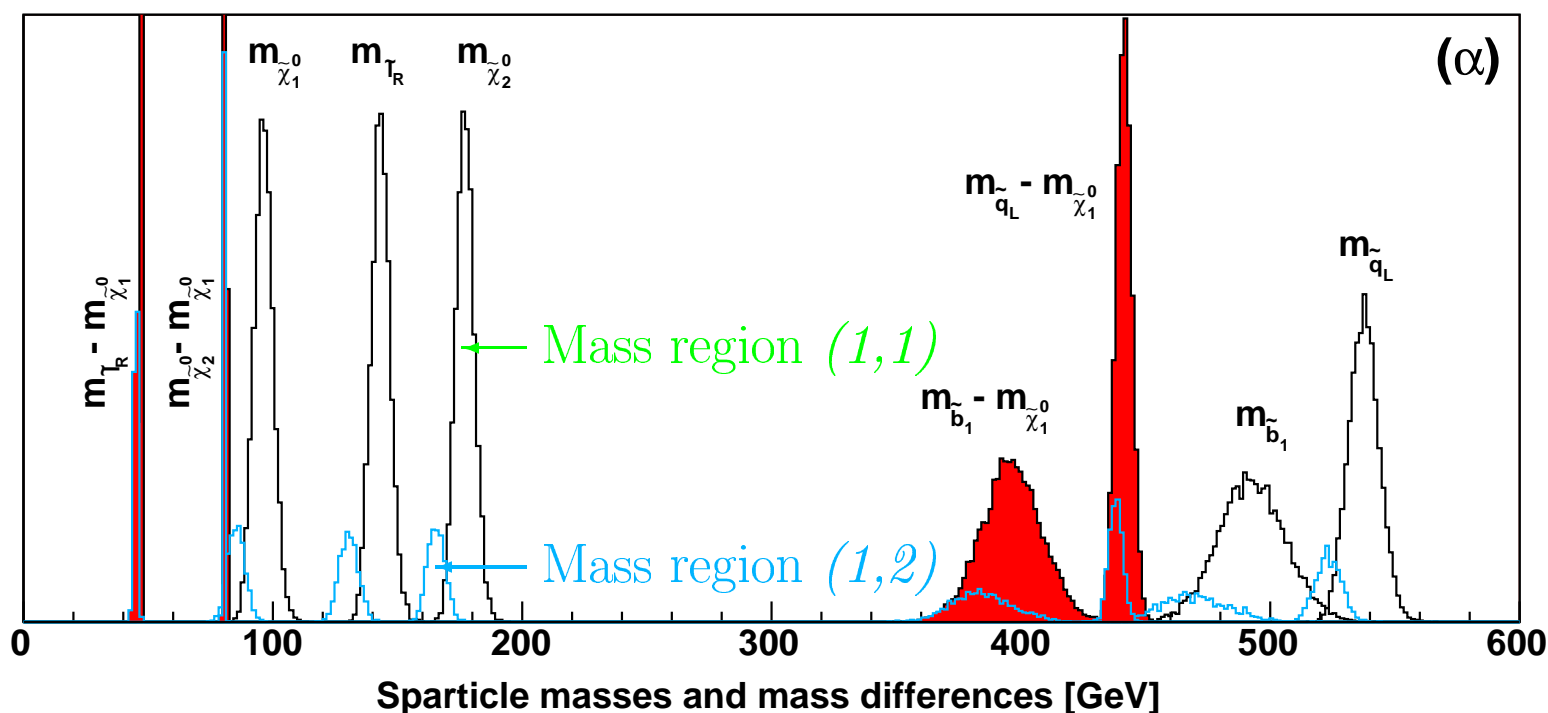
Method of 10,000 LHC experiments

- To estimate LHC potential, do not use **fit values** obtained
- Look at an **ensemble** of LHC experiments:
- Assume errors are normal distributed around nominal values for the ensemble
- For each experiments:
 - use (known) errors to construct measurements:

$$E_i^{\text{exp}} = E_i^{\text{nom}} + A_i \sigma_i^{\text{stat}} + B \sigma_i^{\text{scale}}$$
 where A_i and B are picked from Gaussian distribution, σ_i^{stat} and σ_i^{scale} are from table
 - find corresponding set(s) of masses
- Investigate **ensemble distributions**

SPS 1a (α)

	# Minima	(1,1)	(1,2)
$\Delta\Sigma \leq 0$	1.00	90%	10%
$\Delta\Sigma \leq 1$	1.12	94%	17%
$\Delta\Sigma \leq 3$	1.30	97%	33%
$\Delta\Sigma \leq \infty$	1.88	99%	88%



Note 1: Multiple minima for a given set of endpoint measurements.

Note 2: Mass differences more accurately determined than masses.

SPS 1a (α)

	Nom	(1,1) $\langle m \rangle$	σ	(1,2) $\langle m \rangle$	σ
LHC					
$m_{\tilde{\chi}_1^0}$	96.1	96.3	3.8	85.3	3.4
$m_{\tilde{l}_R}$	143.0	143.2	3.8	130.4	3.7
$m_{\tilde{\chi}_2^0}$	176.8	177.0	3.7	165.5	3.4
$m_{\tilde{q}_L}$	537.2	537.5	6.1	523.2	5.1
$m_{\tilde{b}_1}$	491.9	492.4	13.4	469.6	13.3
$m_{\tilde{l}_R} - m_{\tilde{\chi}_1^0}$	46.9	46.9	0.3	45.1	0.7
$m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$	80.8	80.8	0.2	80.2	0.3
$m_{\tilde{q}_L} - m_{\tilde{\chi}_1^0}$	441.2	441.3	3.1	438.0	2.7
$m_{\tilde{b}_1} - m_{\tilde{\chi}_1^0}$	395.9	396.2	12.0	384.4	12.0
LHC+LC	\Rightarrow (1,2) solution reduced to $\sim 1\%$				
$\tilde{\chi}_1^0$	96.05	96.05	0.05	96.04	0.05
\tilde{l}_R	142.97	142.97	0.29	141.90	0.79
$\tilde{\chi}_2^0$	176.82	176.82	0.17	176.34	0.36
\tilde{q}_L	537.25	537.22	2.49	537.55	2.67
\tilde{b}_1	491.92	492.06	11.68	488.87	11.59

SPS 1a (β)

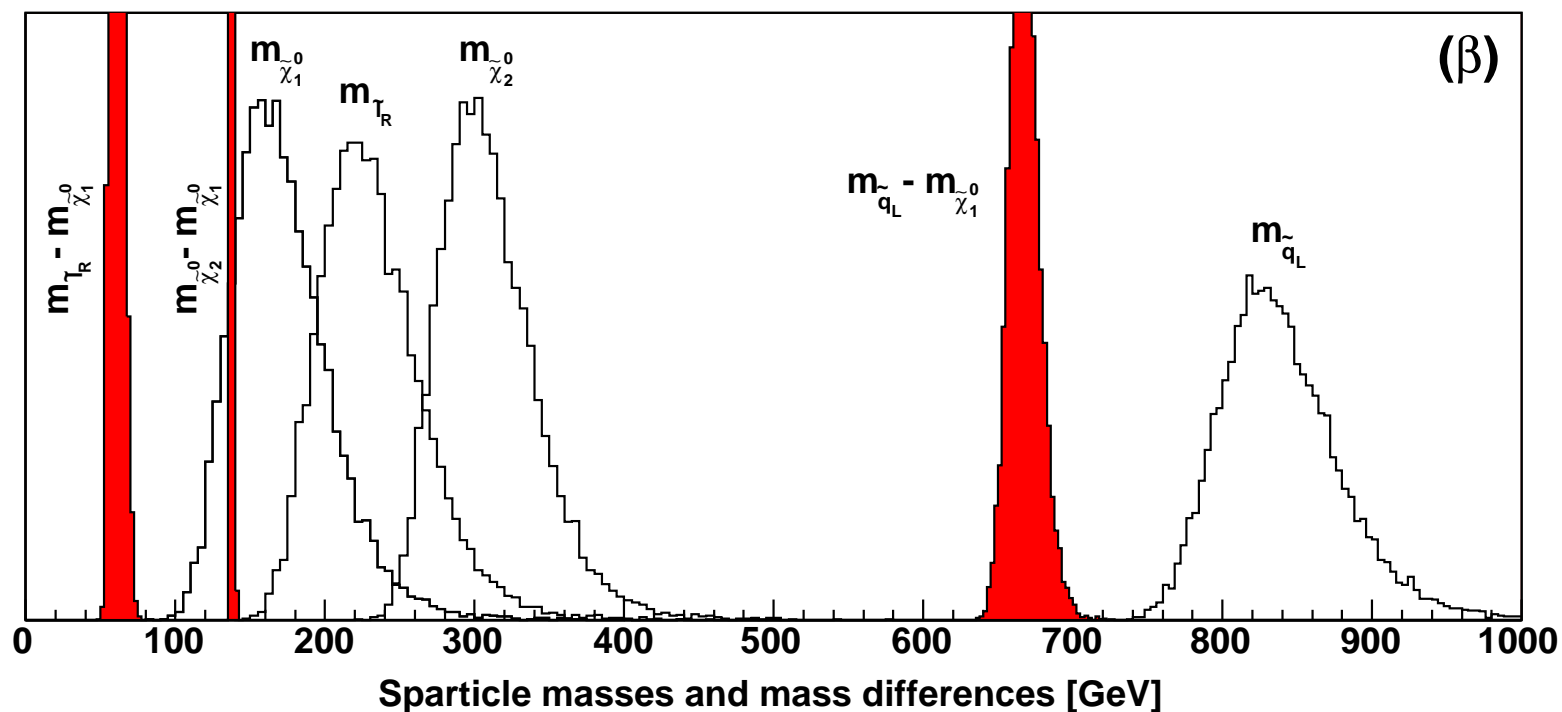
LHC						
	# Min	(1,1)	(1,2)	1 sol (1,3)	B	2 sol (1,2)&(1,3)
$\Delta\Sigma \leq 0$	1.0	3%	60%	25%	12%	0%
$\Delta\Sigma \leq 1$	1.2	5%	52%	18%	12%	16%
$\Delta\Sigma \leq 3$	1.4	13%	46%	14%	12%	28%
$\Delta\Sigma \leq 99$	2.3	99%	41%	13%	12%	34%
LHC+LC						
$\Delta\Sigma \leq 0$	1.0	-	65%	26%	9%	0%
$\Delta\Sigma \leq 1$	1.2	-	52%	18%	9%	21%
$\Delta\Sigma \leq 3$	1.4	-	36%	10%	9%	45%
$\Delta\Sigma \leq 99$	1.6	-	24%	7%	9%	59%

SPS 1a (β)

					1 solution			2 solutions					
		$(1,1)$			$(1,2)/(1,3)/B$			$(1,2)$			$(1,3)$		
	Nom	$\langle m \rangle$	σ	γ_1	$\langle m \rangle$	σ	γ_1	$\langle m \rangle$	σ	γ_1	$\langle m \rangle$	σ	γ_1
LHC													
$\tilde{\chi}_1^0$	161	438	88	0.9	175	35	1.0	161	22	0.3	166	27	0.6
\tilde{l}_R	222	518	85	0.7	236	37	0.8	221	24	0.3	223	28	0.5
$\tilde{\chi}_2^0$	299	579	85	0.7	313	35	1.0	299	22	0.3	304	27	0.6
\tilde{q}_L	826	1146	104	0.8	843	44	0.9	826	30	0.3	835	36	0.5
$\tilde{l}_R - \tilde{\chi}_1^0$	61	81	1.8	-0.3	61	4.4	0.4	61	1.9	-0.2	57	1.3	-0.2
$\tilde{\chi}_2^0 - \tilde{\chi}_1^0$	138	141	0.9	0.1	138	0.6	0.2	138	0.5	0.0	138	0.5	0.0
$\tilde{q}_L - \tilde{\chi}_1^0$	665	708	17	0.1	668	10	0.5	665	9	0.1	669	10	0.2
LHC+LC													
	Nom				1 sol		$(1,2)$			$(1,3)$			
		$\langle m \rangle$	σ		$\langle m \rangle$	σ	$\langle m \rangle$	σ		$\langle m \rangle$	σ		
$\tilde{\chi}_1^0$	161.02	-	-	-	161.02	0.05	-	161.02	0.05	-	161.02	0.05	-
\tilde{l}_R	221.86	-	-	-	221.15	3.26	-	222.22	1.32	-	217.48	1.01	-
$\tilde{\chi}_2^0$	299.05	-	-	-	299.15	0.57	-	299.11	0.53	-	299.05	0.52	-
\tilde{q}_L	826.29	-	-	-	826.12	6.33	-	825.89	5.78	-	828.60	5.52	-

SPS 1a (β)

Both (1,2) and (1,3) solutions with $\Delta\Sigma \leq 1$, only LHC:



Skewed distributions, due to

- Large measurement errors
- ‘Border effect’

SUMMARY ('New' things in red)

Introduction (p. 2–3)

Endpoint Method (p. 4–9):

Standard things (endpoint expressions, ...)

Analytic inversion formulae

Least-square function (correlations)

Emphasis on multiple regions

Applicability of Method (p. 10–15):

Hierarchy scans in $m_{1/2}$ - m_0 plane

Branching-ratio scans in $m_{1/2}$ - m_0 plane

Alternative way of presenting branching ratios in 2D

Simulation (p. 16–22):

Attempt: Mixed event sample as background hypothesis

Standard point: Sps 1a (α)

New point: SPS 1a (β)

Study of theory curves (foot, non-linearity)

Energy Scale Error: new for m_{gl} and m_{gll}

Masses from Endpoints (p. 23–28):

10,000 LHC experiments

Multiple minima

Skewed ensemble distributions for SPS 1a (β)

LC fixes scale

LC reduces number of minima for (α), less so for (β)

OUTLOOK

Add gluino to the chain:

- Calculation of endpoints (done)
- Analysis at SPS 1a (α) (done)

Paper nearly ready

The End

GLUINO

$$\left(m_{qql}^{\max} \right)^2 = \left\{ \begin{array}{ll} \frac{(m_{\tilde{g}}^2 - m_{\tilde{q}_L}^2)(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{q}_L}^2} & \text{for } \frac{m_{\tilde{g}}}{m_{\tilde{q}_L}} > \frac{m_{\tilde{q}_L} m_{\tilde{\chi}_2^0} m_{\tilde{l}_R}}{m_{\tilde{\chi}_2^0} m_{\tilde{l}_R} m_{\tilde{\chi}_1^0}} \quad (1) \\ \frac{(m_{\tilde{g}}^2 m_{\tilde{\chi}_2^0}^2 - m_{\tilde{q}_L}^2 m_{\tilde{\chi}_1^0}^2)(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)}{m_{\tilde{q}_L}^2 m_{\tilde{\chi}_2^0}^2} & \text{for } \frac{m_{\tilde{q}_L}}{m_{\tilde{\chi}_2^0}} > \frac{m_{\tilde{\chi}_2^0} m_{\tilde{l}_R} m_{\tilde{g}}}{m_{\tilde{l}_R} m_{\tilde{\chi}_1^0} m_{\tilde{q}_L}} \quad (2) \\ \frac{(m_{\tilde{g}}^2 m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_2^0}^2 m_{\tilde{\chi}_1^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)}{m_{\tilde{\chi}_2^0}^2 m_{\tilde{l}_R}^2} & \text{for } \frac{m_{\tilde{\chi}_2^0}}{m_{\tilde{l}_R}} > \frac{m_{\tilde{l}_R} m_{\tilde{g}} m_{\tilde{q}_L}}{m_{\tilde{\chi}_1^0} m_{\tilde{q}_L} m_{\tilde{\chi}_2^0}} \quad (3) \\ \frac{(m_{\tilde{g}}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2} & \text{for } \frac{m_{\tilde{l}_R}}{m_{\tilde{\chi}_1^0}} > \frac{m_{\tilde{g}} m_{\tilde{q}_L} m_{\tilde{\chi}_2^0}}{m_{\tilde{q}_L} m_{\tilde{\chi}_2^0} m_{\tilde{l}_R}} \quad (4) \\ (m_{\tilde{g}} - m_{\tilde{\chi}_1^0})^2 & \text{otherwise} \quad (5) \end{array} \right.$$

