QED⊗QCD Exponentiation and Shower/ME Matching at the LHC

B.F.L. Ward

Department of Physics, Baylor University

Waco, TX, USA

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Papers by S. Jadach and B.F.L. Ward, S. Jadach, *et al.*, M. Phys. Lett. A **14** (1999) 491, hep-ph/0205062; *ibid.* **12** (1997) 2425; *ibid.***19** (2004) 2113

INTRODUCTION

Motivation

- FNAL/RHIC $t\bar{t}$ PRODUCTION; POLARIZED pp PROCESSES; $b\bar{b}$ PRODUCTION; J/Ψ PRODUCTION: SOFT n(G) EFFECTS ALREADY NEEDED
  $\Delta m_t = 5.1$ GeV with SOFT n(G) UNCERTAINTY $\sim$ 2-3 GeV, ..., ETC.

- FOR THE LHC/TESLA/LC, THE REQUIREMENTS WILL BE EVEN MORE DEMANDING AND OUR QCD SOFT n(G) MC EXPONENTIATION RESULTS WILL BE AN IMPORTANT PART OF THE NECESSARY THEORY – YFS EXPONENTIATED $\mathcal{O}(\alpha_s^2)L$, IN THE PRESENCE OF SHOWERS, ON AN EVENT-BY-EVENT BASIS, WITHOUT DOUBLE COUNTING AND WITH EXACT PHASE SPACE.

- HOW RELEVANT ARE QED HIGHER ORDER CORRECTIONS WHEN QCD IS CONTROLLED AT $\sim 1\%$ PRECISION?

- CROSS CHECK OF QCD LITERATURE:
  1. PHASE SPACE – CATANI, CATANI-SEYMOUR, ALL INITIAL PARTONS MASSLESS
  2. RESUMMATION – CATANI ET AL., BERGER ET AL., ...
  3. NO-GO THEOREMS

- CROSS CHECK OF QED LITERATURE:

  1. ESTIMATES BY SPIESBERGER, STIRLING, ROTH and WEINZIERL – FEW PER MILLE EFFECTS FROM QED CORRECTIONS TO STR. FN. EVOLUTION.

  2. WELL-KNOWN POSSIBLE ENHANCEMENT OF QED CORRECTIONS AT THRESHOLD, ESPECIALLY IN RESONANCE PRODUCTION

  $\Rightarrow$ HOW BIG ARE THESE EFFECTS AT THE LHC?

- TREAT QED AND QCD SIMULTANEOUSLY IN THE YFS EXPONENTIATION TO ESTIMATE THE ROLE OF THE QED AND TO ILLUSTRATE AN APPROACH TO SHOWER/ME MATCHING.

PRELIMINARIES

- WE USE THE GPS CONVENTIONS OF JWW FOR SPINORS; PHOTON-GLUON POLARIZATION VECTORS FOLLOW THEREFROM:

$$(\epsilon^{\mu}_{\sigma}(\beta))^* = \frac{\bar{u}_\sigma(k)\gamma^\mu u_\sigma(\beta)}{\sqrt{2}\;\bar{u}_{-\sigma}(k)u_\sigma(\beta)}, \quad (\epsilon^{\mu}_{\sigma}(\zeta))^* = \frac{\bar{u}_\sigma(k)\gamma^\mu u_\sigma(\zeta)}{\sqrt{2}\;\bar{u}_{-\sigma}(k)u_\sigma(\zeta)}, \qquad (1)$$

- REPRESENTATIVE PROCESSES

  $pp \rightarrow V + n(\gamma) + m(g) + X \rightarrow \bar{\ell}\ell' + n'(\gamma) + m(g) + X,$
  where $V = W^\pm, Z$,and $\ell = e, \mu,\ \ell' = \nu_e, \nu_\mu(e,\mu)$ respectively for $V = W^+(Z)$, and $\ell = \nu_e, \nu_\mu,\ \ell' = e, \mu$ respectively for $V = W^-$.

Review of YFS Theory and Its Extension to QCD

QED CASE – S. Jadach et al., YFS2, YFS3, BHLUMI, BHWIDE, KORALZ, KKMC, YFSWW3, YFSZZ, KoralW

For $e^+(p_1)e^-(q_1) \to \bar{f}(p_2)f(q_2) + n(\gamma)(k_1, \cdot, k_n)$**, renormalization group improved YFS theory (PRD36(1987)939) gives**

$$d\sigma_{exp} = e^{2\alpha\, Re\, B + 2\alpha\, \tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D}$$
$$\bar{\beta}_n(k_1, \ldots, k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}$$

where the YFS real infrared function $\tilde{B}$ and the virtual infrared function B are known and where we note the usual connections

$$2\alpha\, \tilde{B} = \int^{k \le K_{max}} \frac{d^3 k}{k_0} \tilde{S}(k)$$

$$D = \int d^3 k \frac{\tilde{S}(k)}{k^0} \left(e^{-iy \cdot k} - \theta(K_{max} - k) \right) \qquad (2)$$

for the standard YFS infrared emission factor

$$\tilde{S}(k) = \frac{\alpha}{4\pi^2}\left[Q_f Q_{(\bar{f})'}\left(\frac{p_1}{p_1\cdot k} - \frac{q_1}{q_1\cdot k}\right)^2 + (\ldots)\right] \quad (3)$$

if Q_f is the electric charge of f in units of the positron charge. For example, the YFS hard photon residuals $\bar{\beta}_i$ in (1), $i = 0, 1, 2$, are given in S. Jadach *et al.*,CPC102(1997)229 for BHLUMI 4.04 $\Rightarrow$ YFS exponentiated exact $\mathcal{O}(\alpha)$ and LL $\mathcal{O}(\alpha^2)$ cross section for Bhabha scattering via a corresponding Monte Carlo realization of (1).

In hep-ph/0210357(ICHEP02), Acta Phys.Polon.B33,1543-1558,2002, we have extended the YFS theory to QCD:

$$d\hat{\sigma}_{\rm exp} = \sum_n d\hat{\sigma}^n$$
$$= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}}$$
$$* \tilde{\bar{\beta}}_n(k_1,\ldots,k_n)\frac{d^3P_2}{P_2^{\,0}}\frac{d^3Q_2}{Q_2^{\,0}} \tag{4}$$

where now the hard gluon residuals $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ defined by

$$\tilde{\bar{\beta}}_n(k_1,\ldots,k_n) = \sum_{\ell=0}^{\infty} \tilde{\bar{\beta}}_n^{(\ell)}(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$.

- We stress that the arguments in the earlier papers (**DeLaney** *et al.* **PRD52(1995)108, PLB342(1995)239**) are not really sufficient to derive the respective analog of eq.(4); for, **they did not really expose the compensation between the left over genuine non-Abelian IR virtual and real singularities between** $\int dPh\bar{\beta}_n$ **and** $\int dPh\bar{\beta}_{n+1}$ **respectively that really allows us to isolate** $\tilde{\bar{\beta}}_j$ **and distinguishes QCD from QED, where no such compensation occurs.**

Extension to QED⊗QCD and QCED

Simultaneous exponentiation of QED and QCD higher order effects,

hep-ph/0404087,

gives

$$\begin{aligned} B^{nls}_{QCD} &\rightarrow B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED}, \\ \tilde{B}^{nls}_{QCD} &\rightarrow \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED}, \\ \tilde{S}^{nls}_{QCD} &\rightarrow \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED} \equiv \tilde{S}^{nls}_{QCED} \end{aligned} \tag{5}$$

which leads to

$$d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^0}\frac{d^3q_2}{q_2^0}, \tag{6}$$

where the new YFS residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$**, with n hard gluons and m hard photons,**

represent the successive application of the YFS expansion first for QCD and subsequently for QED.

The infrared functions are now

$$\mathrm{SUM_{IR}(QCED)} = 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED}$$
$$D_{\mathrm{QCED}} = \int \frac{dk}{k^0}\left(e^{-iky} - \theta(K_{max} - k^0)\right)\tilde{S}^{nls}_{QCED} \quad (7)$$

where K_{max} is a dummy parameter – here the same for QCD and QED.

Infrared Algebra(QCED):

$x_{avg}(QED) \cong \gamma(QED)/(1 + \gamma(QED))$
$x_{avg}(QCD) \cong \gamma(QCD)/(1 + \gamma(QCD))$
$\gamma(A) = \frac{2\alpha_A \mathcal{C}_A}{\pi}(L_s - 1)$, $A = QED, QCD$
$\mathcal{C}_A = Q_f^2, C_F$, respectively, for $A = QED, QCD$

$\Rightarrow$ QCD dominant corrections happen an order of magnitude earlier than those for QED.

$\Rightarrow$ Leading $\tilde{\bar{\beta}}^{(0,0)}_{0,0}$ -level gives a good estimate of the size of the effects we study.

QED⊗QCD Threshold Corrections and Shower/ME Matching

We shall apply the new simultaneous QED⊗QCD exponentiation calculus to the sinlge Z production with leptonic decay at the LHC (and at FNAL) to focus on the ISR alone, for definiteness. See also the work of Baur *et al.***, Dittmaier and Kramer, Zykunov for exact $\mathcal{O}(\alpha)$ results and Hamberg** *et al.***, van Neerven and Matsuura and Anastasiou** *et al.* **for exact $\mathcal{O}(\alpha_s^2)$ results.**

For the basic formula

$$d\sigma_{exp}(pp \to V + X \to \bar{\ell}\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s), \tag{8}$$

we use the result in (6) here with semi-analytical methods and structure functions from Martin *et al.***.**

A MC realization will appear elsewhere.

SHOWER/ME MATCHING

- Note the following: In (8) WE DO NOT ATTEMPT TO REPLACE HERWIG and/or PYTHIA –
  WE INTEND TO COMBINE OUR EXACT YFS CALCULUS, $d\hat{\sigma}_{exp}(x_i x_j s)$,
  WITH HERWIG and/or PYTHIA BY USING THEM/IT "IN LIEU" OF $\{F_i\}$.
- THIS COMBINATION OF THEORETICAL CONSTRUCTS CAN BE SYSTEMATICALLY IMPROVED WITH EXACT RESULTS
  ORDER-BY-ORDER IN α_s, α, WITH EXACT PHASE SPACE.
- THE RECENT ALTERNATIVE PARTON SHOWER ALGORITHM BY JADACH and SKRZYPEK,Acta. Phys. Pol.B35, 745 (2004), CAN ALSO BE USED.
- LACK OF COLOR COHERENCE $\Rightarrow$ ISAJET NOT CONSIDERED HERE.

With this said, we compute , with and without QED, the ratio

$r_{exp} = \sigma_{exp}/\sigma_{Born}$

to get the results (We stress that we *do not* use the narrow resonance approximation here.)

$$r_{exp} = \begin{cases} 1.1901 & , \text{QCED} \equiv \text{QCD+QED, LHC} \\ 1.1872 & , \text{QCD, LHC} \\ 1.1911 & , \text{QCED} \equiv \text{QCD+QED, Tevatron} \\ 1.1879 & , \text{QCD, Tevatron} \end{cases} \quad (9)$$

$\Rightarrow$

$*$***QED IS AT .3% AT BOTH LHC and FNAL.***

$*$***THIS IS STABLE UNDER SCALE VARIATIONS.***

$*$***WE AGREE WITH BAUR ET AL., HAMBERG ET AL., van NEERVEN and ZIJLSTRA.***

$*$***QED EFFECT SIMILAR IN SIZE TO STR. FN. RESULTS.***

$*$***DGLAP SYNTHESIZATION HAS NOT COMPROMISED THE NORMALIZATION.***

Conclusions

YFS THEORY (EEX AND CEEX) EXTENDS TO NON-ABELIAN GAUGE THEORY AND ALLOWS SIMULTANEOUS EXPN OF QED AND QCD WITH PROPER SHOWER/ME MATCHING BUILT-IN.

FOR QED$\otimes$QCD

- FULL MC EVENT GENERATOR REALIZATION IS POSSIBLE.
- SEMI-ANALYTICAL RESULTS FOR QED (AND QCD) THRESHOLD EFFECTS AGREE WITH LITERATURE ON Z PRODUCTION
- AS QED IS AT THE .3% LEVEL, IT IS NEEDED FOR 1% LHC THEORY PREDICTIONS.
- A FIRM BASIS FOR THE COMPLETE $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ MC RESULTS NEEDED FOR THE FNAL/LHC/RHIC/TESLA/LC PHYSICS HAS BEEN DEMONSTRATED AND ALL THE LATTER IS IN PROGRESS.