Event shapes for hadron colliders

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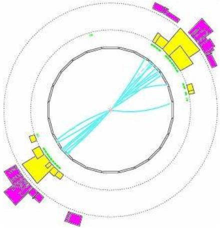
LPTHE, Universities of Paris VI and VII and CNRS

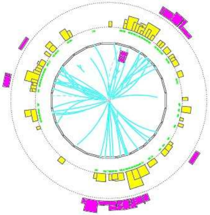
HERA–LHC workshop
CERN, Geneva, January 2005

Why event shapes?

- Perhaps the most basic class of final-state observables in e^+e^-
- Continuous measure of deviation from lowest-order 'Born' event



2-jet event: Thrust $\simeq 1$



3-jet event: Thrust $\simeq 2/3$

- Many uses: serve as a QCD 'laboratory', both in e^+e^- and DIS:
  - α_s fits
  - Tuning of Monte Carlos
  - Colour factor fits ($C_A, C_F, \ldots$)
  - Studies of analytical hadronisation models ($1/Q$, shape functions, ...)
- Largely neglected at hadronic colliders

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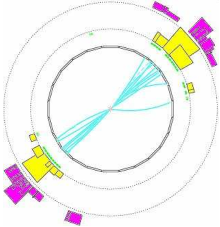

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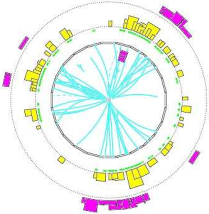

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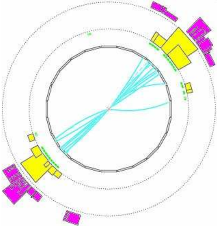

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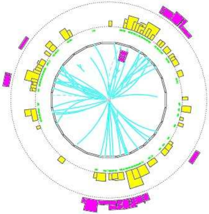

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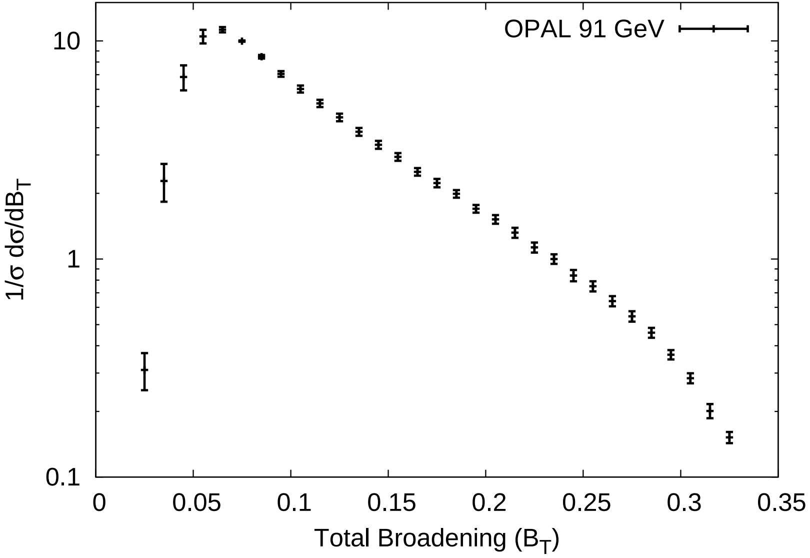

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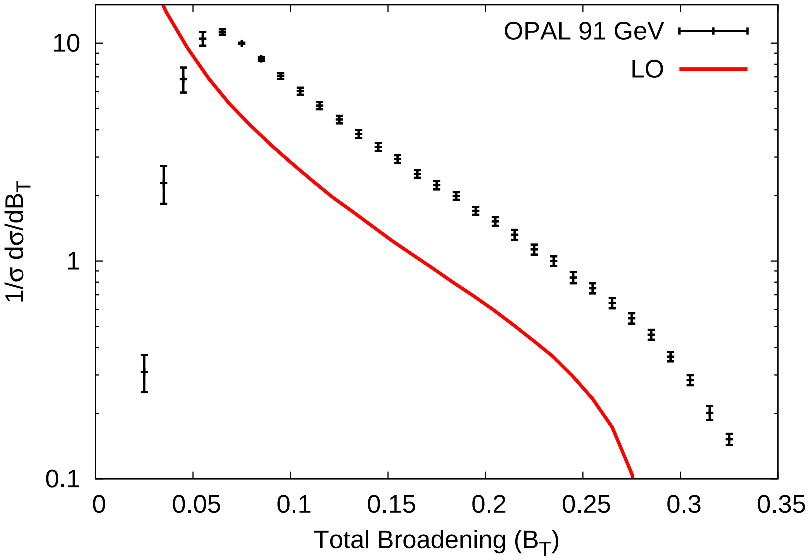

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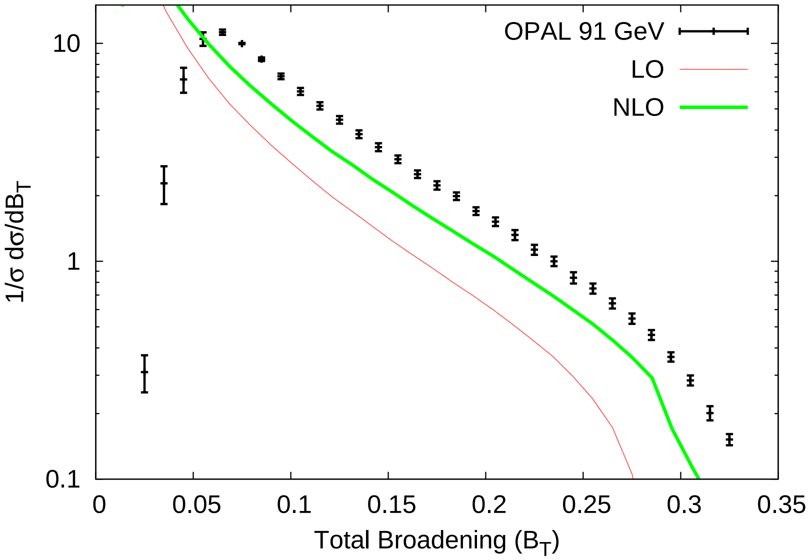

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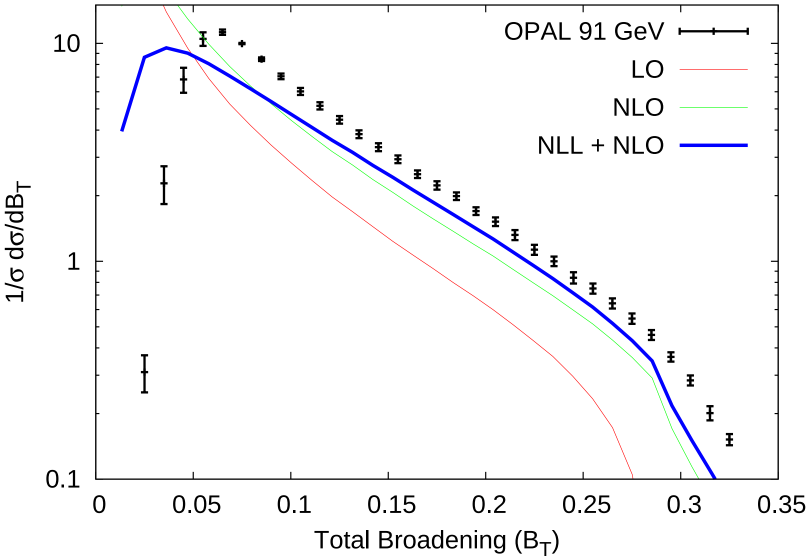

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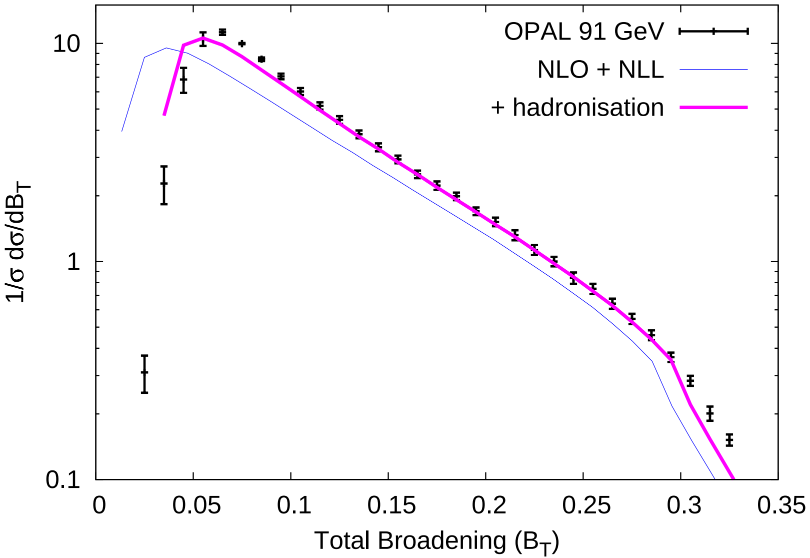

Interest of hadronic colliders?

Various processes:

- $pp \to$ W/Z/H boson + jet — Banfi Marchesini Smye Zanderighi '01
- $pp \to$ 2 jets — Main subject of this talk

Standard applications (*e.g.*)

- Measure α_s
- As for 3-jet/2-jet ratio in $p\bar{p}$, reduce dependence on PDFs
- But for event-shapes → *distribution*
- Far more information than 3-jet/2-jet ratio

New territory

- 4-jet (2 + 2) topology → novel perturbative structures
  soft colour evln matrices
- 3 & 4-jet topologies (& g-jets) → rich environment for analytical non-pert. studies
- Underlying event — test models (analytical & MC).

Variety of event-shape observables → complementary information → disentangle the different physics issues.

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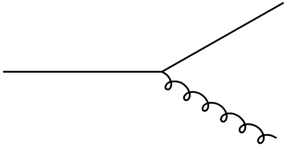
Soft colour evolution

Multi-jet final states: relative colour of pairs of hard partons determines soft large-angle radiation.

2 jets: always in a *colour singlet*



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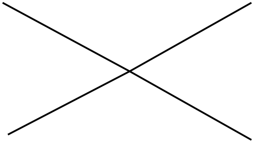

4 jets: a given pair can be in various colour states. Soft virtual corrections mix colour states.

Resummation leads to *matrix evolution equation for colour state of amplitudes* ('soft anomalous dimenions')

Developed at Stony Brook: Botts, Kidonakis, Oderda & Sterman '89–99

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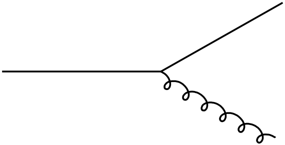

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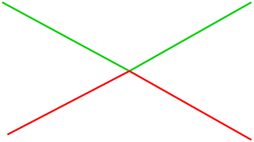

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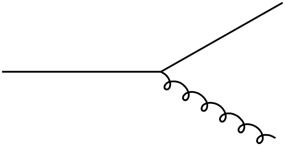

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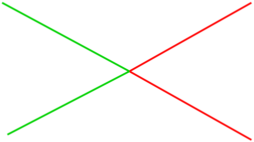

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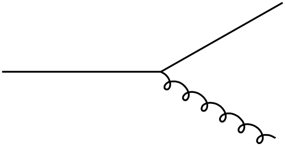

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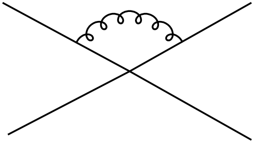

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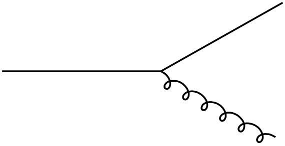

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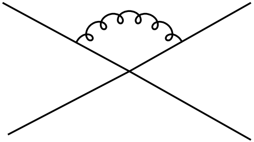

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Theoretical tools

Fixed order

- Event shapes trivial for Born events (*e.g.* $p\bar{p} \to 2$ jets, thrust=1)
- First non-trivial order (LO) is Born + 1 parton, *i.e.* $p\bar{p} \to 3$ jets
- For NLO, need a program like NLOJET++ ($p\bar{p} \to 3$ jets @ NLO)
  Nagy, '01 & '03
- Also:
  - Kilgore-Giele code ($p\bar{p} \to 3$ jets @ NLO),
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Resummation

- In e^+e^- it was always done by hand, one observable at a time.
- *Next-to-leading logs* (NLL) are tedious, complicated, error-prone.
- Recently automated: Computer-Automated Expert Semi-Analytical Resummer (CAESAR). Banfi, GPS & Zanderighi '01–'04
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Automated resummation

Analytical work (done once and for all)

A1. derive a master formula for a generic observable in terms of simple properties of the observable

A2. formulate the exact applicability conditions for the master formula

Numerical work (to be repeated for each observable)

N1. let an "expert system" investigate the applicability conditions

N2. it also determines the inputs for the master formula

N3. straightforward evaluation of the master formula, including phase space integration etc.

Note: N1 and N2 are core of automation

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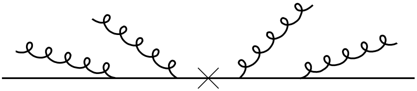
Globalness

Global observable:

e.g. total e^+e^- Broadening, B



making $B \ll 1$ restricts emissions everywhere.

Coherence + globalness:

➥ emissions can be resummed as if independent *(proved)*

Answers guaranteed to NLL accuracy

Non-Global observable:

Right-hemisphere Broadening, B_R

making $B_R \ll 1$ restricts emissions in right-hand hemisphere ($\mathcal{H}_\mathcal{R}$).

Tempting to *assume* one can:

- ignore left hemisphere ($\mathcal{H}_\mathcal{L}$)
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WRONG AT NLL ACCURACY

Dasgupta & GPS '01

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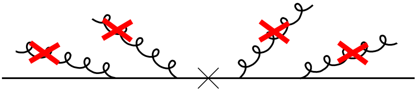

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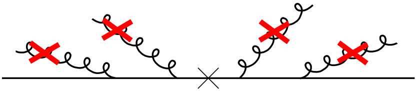

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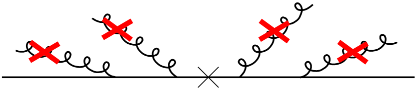

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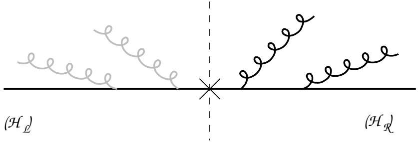

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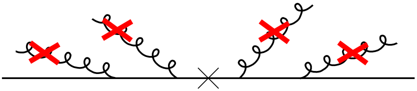

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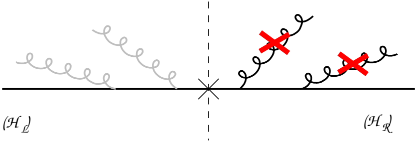

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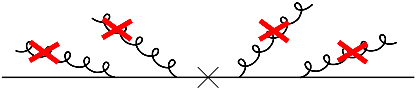

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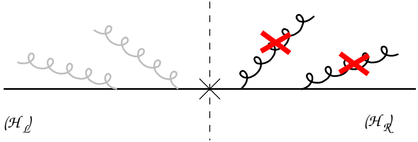

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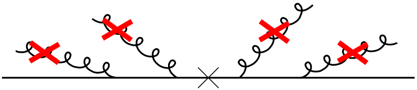

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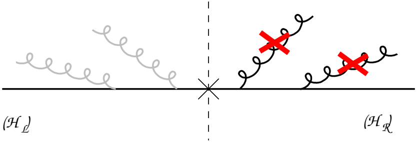

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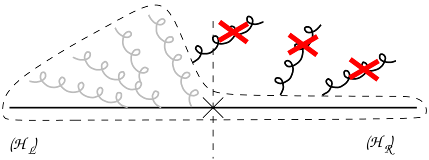
Resummation of NG observables

All-orders:

Forbid coherent radiation from energy-ordered ensembles of large-angle gluons



➥ $\alpha_s^n \ln^n B_R$

Difficulties:

- Logarithms resummed so far only in large-N_c limit
- In general, boundary between the two regions may have arbitrary shape.
- It may depend on the pattern of emissions (*e.g.* with jet algorithm).

Appleby & Seymour '02, '03

Resummation of a general non-global observable is tricky.
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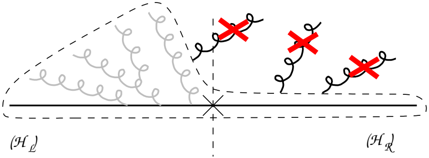

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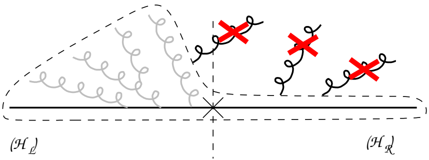

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Experimental considerations

Contradiction?

Theoretical calculations are for global observables.
But experiments only have detectors in limited rapidity range.
(Strictly: series of sub-detectors, of worsening quality as rapidity increases)

Model by cut around beam $|\eta| < \eta_{\text{max}}$
➥ Problems with globalness

Take cut as being edge of most forward detector with momentum or energy resolution:

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η_{max}	3.5	5.0

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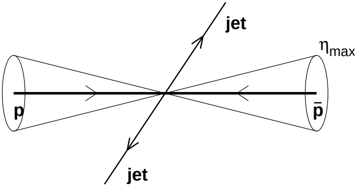

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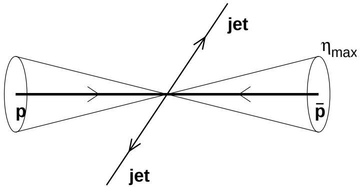

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Sidestepping non-globalness

Select events with central, hard jets (x_1, x_2 not too small), with transverse momentum $P_\perp$.

From kinematics, emissions (k) near forward detector edges typically have small transverse momentum:

$$k_\perp \sim P_\perp e^{-\eta_0} \ll P_\perp$$

If event-shape value is always sufficiently large that such an emission contributes negligibly, then:
we can ignore rapidity cut & pretend measurement is global

Proceed as follows:

- Calculate distribution without any rapidity cutoff
- Determine smallest 'typical' value of observable
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Event selection cuts

Results that follow based on this (illustrative) event selection:

- Run longitudinally invariant inclusive k_t jet algorithm (could also use midpoint cone)
- Require hardest jet to have $P_{\perp,1} > P_{\perp,\min} = 50$ GeV
- Require two hardest jets to be central $|\eta_1|, |\eta_2| < \eta_c = 0.7$

Pure resummed results
no matching to NLO (or even LO)
Shown for Tevatron run II

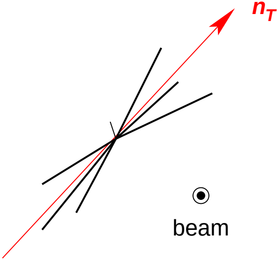
Transverse thrust & minor

Some observables are naturally defined in terms of all particles in the event, *e.g.* *Global Transverse Thrust*

$$T_{\perp,g} \equiv \max_{\vec{n}_T} \frac{\sum_i |\vec{q}_{\perp i} \cdot \vec{n}_T|}{\sum_i q_{\perp i}}\,, \qquad \tau_{\perp,g} = 1 - T_{\perp,g}\,,$$

and *Global Thrust Minor*

$$T_{m,g} \equiv \frac{\sum_i |\vec{q}_i . \vec{n}_m|}{\sum_i q_{\perp i}}\,, \qquad \vec{n}_m \cdot \vec{n}_T = 0$$



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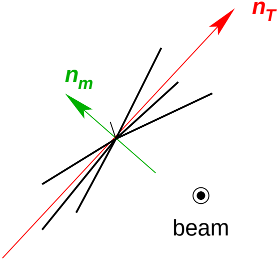


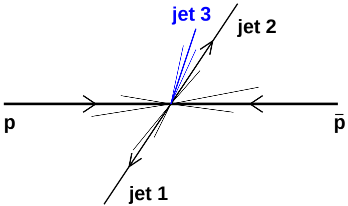
3-jet resolution threshold

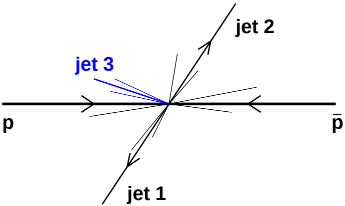
Use *exclusive* long. inv. k_t algorithm: successive recombination of pair with smallest closeness measure d_{kl}, d_{kB}:

$$d_{kB} = q_{\perp k}^2 \,, \qquad d_{kl} = \min\{q_{\perp k}^2, q_{\perp l}^2\} \left((\eta_k - \eta_l)^2 + (\phi_k - \phi_l)^2\right) \,.$$

Define $d^{(n)}$ as smallest d_{kl}, d_{kB} when only n pseudo-jets left. Examine (normalised) 3-jet resolution threshold

$$y_{23} = \frac{1}{(E_{\perp,1} + E_{\perp,2})^2} d^{(3)}$$





Generalisation of 3-jet cross section

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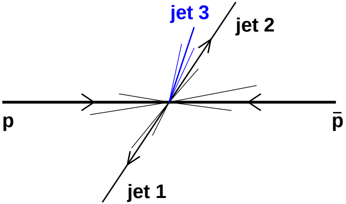

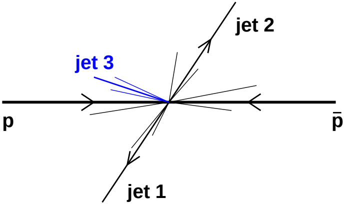

Generalisation of 3-jet cross section

Results

Probability $P(v)$ that event shape is smaller than some value v:

$$P(v) = \exp\left[-G_{12}\frac{\alpha_s L^2}{2\pi} + \cdots\right], \quad L = \ln\frac{1}{v}$$

Ev.Shp.	G_{12}
$\tau_{\perp,g}$	$2C_B + C_J$
$T_{m,g}$	$2C_B + 2C_J$
y_{23}	$\frac{1}{2}C_B + \frac{1}{2}C_J$

C_B = total colour of Beam partons
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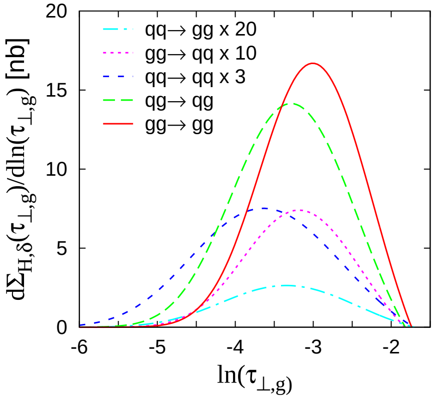

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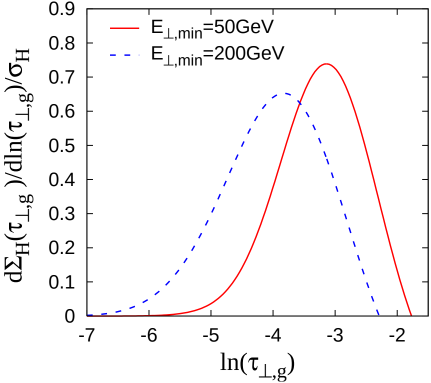

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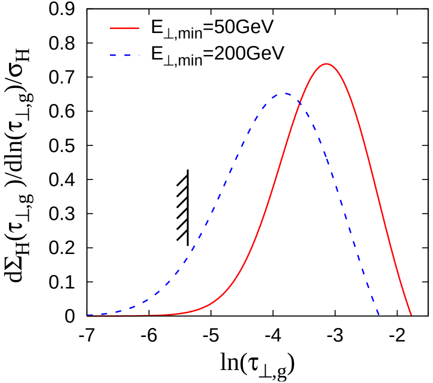

Beam cut: $\tau_{\perp,g} \gtrsim 0.15 e^{-\eta_{\max}}$

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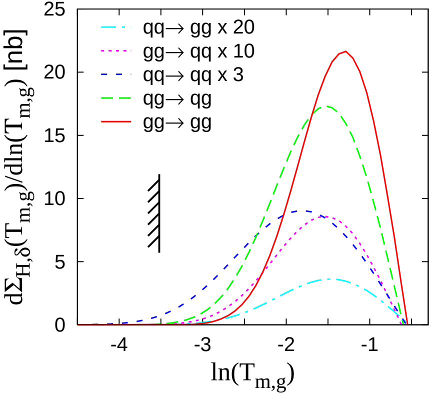

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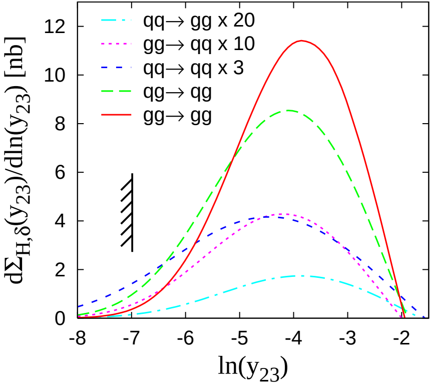


Beam cut: $y_{23} \gtrsim e^{-2\eta_{\max}}$ [because $y_{23} \sim k_t^2$]

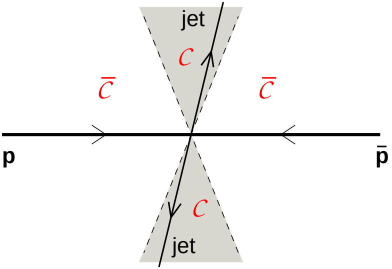
Forward-suppressed observables

Divide event into central region ($\mathcal{C}$, say $|\eta| < 1.1$) and rest of event ($\bar{\mathcal{C}}$).

[NB: $\exists$ considerable freedom in definition of $\mathcal{C}$: *e.g.* can also be two hardest jets]

Define central $\perp$ mom., and rapidity:

$$Q_{\perp,\mathcal{C}} = \sum_{i\in\mathcal{C}} q_{\perp i}\,, \quad \eta_{\mathcal{C}} = \frac{1}{Q_{\perp,\mathcal{C}}} \sum_{i\in\mathcal{C}} \eta_i\, q_{\perp i}$$



and an *exponentially suppressed forward term,*

$$\mathcal{E}_{\bar{\mathcal{C}}} = \frac{1}{Q_{\perp,\mathcal{C}}} \sum_{i\notin\mathcal{C}} q_{\perp i}\, e^{-|\eta_i - \eta_{\mathcal{C}}|}\,.$$

Define a non-global event-shape in $\mathcal{C}$. Then add on $\mathcal{E}_{\bar{\mathcal{C}}}$.

Result is a global event shape, with suppressed sensitivity to forward region.

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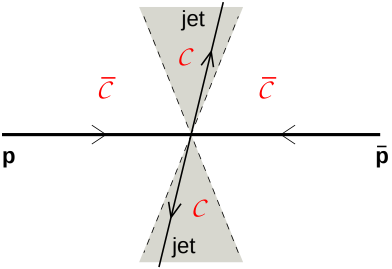

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Examples

- Split $\mathcal{C}$ into two pieces: *Up, Down*
- Define *jet masses* for each

$$\rho_{X,\mathcal{C}} \equiv \frac{1}{Q_{\perp,\mathcal{C}}^2}\Big(\sum_{i\in\mathcal{C}_X} q_i\Big)^2 , \qquad X = U, D ,$$

Define sum and heavy-jet masses

$$\rho_{S,\mathcal{C}} \equiv \rho_{U,\mathcal{C}} + \rho_{D,\mathcal{C}} , \qquad \rho_{H,\mathcal{C}} \equiv \max\{\rho_{U,\mathcal{C}}, \rho_{D,\mathcal{C}}\} ,$$

Define global extension, with extra forward-suppressed term

$$\rho_{S,\mathcal{E}} \equiv \rho_{S,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}} , \qquad \rho_{H,\mathcal{E}} \equiv \rho_{H,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}} .$$

- Similarly: *total and wide jet-broadenings*

$$B_{T,\mathcal{E}} \equiv B_{T,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}} , \qquad B_{W,\mathcal{E}} \equiv B_{W,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}} .$$

Results

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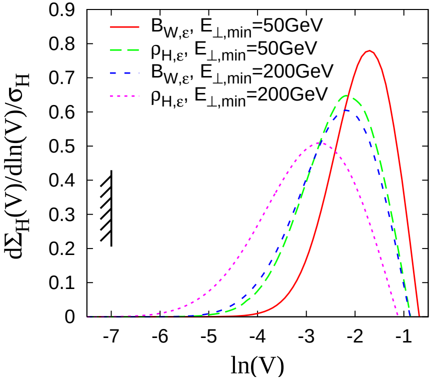

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Beam cuts: $B_{X,\mathcal{E}}, \rho_{X,\mathcal{E}} \gtrsim e^{-2\eta_{\max}}$ [because $\mathcal{E}_{\bar{\mathcal{C}}} \sim k_t e^{-|\eta|}$]

Recoil observables

By momentum conservation

$$\sum_{i\in\mathcal{C}} \vec{q}_{\perp i} = -\sum_{i\notin\mathcal{C}} \vec{q}_{\perp i}$$

Use central particles to define *recoil term*, which is *indirectly sensitive* to non-central emissions

$$\mathcal{R}_{\perp,\mathcal{C}} \equiv \frac{1}{Q_{\perp,\mathcal{C}}} \left| \sum_{i\in\mathcal{C}} \vec{q}_{\perp i} \right| ,$$

Define event shapes exclusively in terms of *central particles:*

$$\rho_{X,\mathcal{R}} \equiv \rho_{X,\mathcal{C}} + \mathcal{R}_{\perp,\mathcal{C}} \,, \qquad B_{X,\mathcal{R}} \equiv B_{X,\mathcal{C}} + \mathcal{R}_{\perp,\mathcal{C}} \,, \ldots$$

These observables are *indirectly global*

First studied at HERA (B_{zE} broadening)

Results

CAESAR resummation works for observables having *direct exponentiation*:

$$P(v) = e^{Lg_1(\alpha_s L)+g_2(\alpha_s L)+\dots}$$

For recoil observables, exponentiation holds fully only after Fourier & other integral transforms (generalised *b*-space resummation).

Manifestation: NLLs ($g_2(\alpha_s L)$) diverge at some $\alpha_s L \sim 1$.

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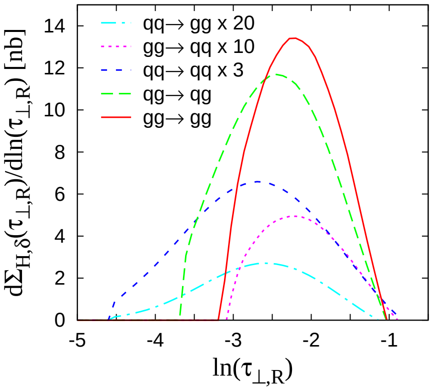
recoil transverse thrust



Quite large effect: $\sim 15\%$ of X-sct is beyond cutoff

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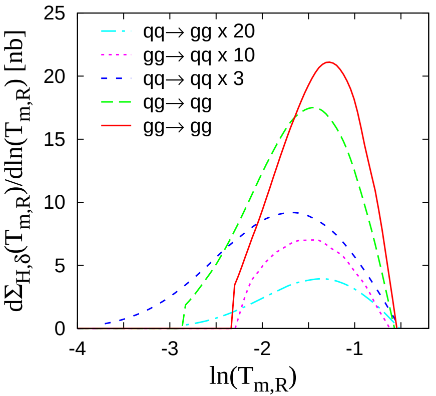

Moderate effect: few % of X-sct is beyond cutoff

Summary of observables

Event-shape	Impact of $\eta_{\max}$	Resummation breakdown	Underlying Event	Jet hadronisation
$\tau_{\perp,g}$	tolerable	none	$\sim \eta_{\max}/Q$	$\sim 1/Q$
$T_{m,g}$	tolerable	none	$\sim \eta_{\max}/Q$	$\sim 1/(\sqrt{\alpha_s}Q)$
y_{23}	tolerable	none	$\sim \sqrt{y_{23}}/Q$	$\sim \sqrt{y_{23}}/Q$
$\tau_{\perp,\mathcal{E}}$, $\rho_{X,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/Q$
$T_{m,\mathcal{E}}$, $B_{X,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/(\sqrt{\alpha_s}Q)$
$y_{23,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim \sqrt{y_{23}}/Q$
$\tau_{\perp,\mathcal{R}}$, $\rho_{X,\mathcal{R}}$	none	serious	$\sim 1/Q$	$\sim 1/Q$
$T_{m,\mathcal{R}}$, $B_{X,\mathcal{R}}$	none	tolerable	$\sim 1/Q$	$\sim 1/(\sqrt{\alpha_s}Q)$
$y_{23,\mathcal{R}}$	none	intermediate	$\sim \sqrt{y_{23}}/Q$	$\sim \sqrt{y_{23}}/Q$

NB: there may be surprises after more detailed study, *e.g.* matching to NLO...

Grey entries are definitely subject to uncertainty

Note complementarity between observables

Summary of observables

Event-shape	Impact of $\eta_{\max}$	Resummation breakdown	Underlying Event	Jet hadronisation
$\tau_{\perp,g}$	tolerable	none	$\sim \eta_{\max}/Q$	$\sim 1/Q$
$T_{m,g}$	tolerable	none	$\sim \eta_{\max}/Q$	$\sim 1/(\sqrt{\alpha_s}Q)$
y_{23}	tolerable	none	$\sim \sqrt{y_{23}}/Q$	$\sim \sqrt{y_{23}}/Q$
$\tau_{\perp,\mathcal{E}},\ \rho_{X,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/Q$
$T_{m,\mathcal{E}},\ B_{X,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/(\sqrt{\alpha_s}Q)$
$y_{23,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim \sqrt{y_{23}}/Q$
$\tau_{\perp,\mathcal{R}},\ \rho_{X,\mathcal{R}}$	none	serious	$\sim 1/Q$	$\sim 1/Q$
$T_{m,\mathcal{R}},\ B_{X,\mathcal{R}}$	none	tolerable	$\sim 1/Q$	$\sim 1/(\sqrt{\alpha_s}Q)$
$y_{23,\mathcal{R}}$	none	intermediate	$\sim \sqrt{y_{23}}/Q$	$\sim \sqrt{y_{23}}/Q$

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- Essential that multijet event shapes also be studied in DIS and e^+e^-.
- Measurements recently published by LEP and in progress at HERA.
- Theoretical comparisons in pipeline.

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- technology exists (NLOJET++) for *poor-man's* matching, all channels ($gg \to gg$, $qq \to qq$, ...) mixed together.
- To be 'sensible', matching must be done *channel-by-channel.*
- Requires *flavour information* in fixed-order codes — but seldom there...

Please, PLEASE, **PLEASE**, could authors of fixed-order codes include information on *flavours* of partons, not just momenta

Further info: hep-ph/0407287 and `http://qcd-caesar.org`

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