Preasymptotics in small-x splitting functions

Gavin Salam

LPTHE — Univ. Paris VI & VII and CNRS

In collaboration with M. Ciafaloni, D. Colferai and A. Staśto

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Outline

- Introduction
  - Problem of convergence of small-x splitting functions.
  - Quick overview of renorm. group improved small-x approach .

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- Results for splitting functions
  - small-x growth
  - preasymptotics
  - explanation of 'dip'

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- Results for splitting functions
  - small-x growth
  - preasymptotics
  - explanation of 'dip'

- [Slow] Progress towards phenomenology
  - Toy convolution, $P_{gg} \otimes g$
  - Difficulties with $\overline{\text{MS}}$ scheme

- Small-x gluon splitting function has logarithmic enhancements:

$$xP_{gg}(x) = \sum_{n=1} \alpha_{\mathsf{s}}^n \ln^{n-1} \frac{1}{x}$$

$$+ \sum_{n=2} \alpha_{\mathsf{s}}^n \ln^{n-2} \frac{1}{x} + \dots$$

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Leading Logs (LLx):

$$\bar{\alpha}_{\mathsf{s}} + \frac{\zeta(3)}{3} \bar{\alpha}_{\mathsf{s}}^4 \ln^3 \frac{1}{x} + \frac{\zeta(5)}{60} \bar{\alpha}_{\mathsf{s}}^6 \ln^5 \frac{1}{x} + \cdots$$

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Next-to-Leading Logs (NLLx):

$$A_{20} \bar{\alpha}_{\sf s}^2 + A_{31} \bar{\alpha}_{\sf s}^3 \ln \frac{1}{x} + A_{42} \bar{\alpha}_{\sf s}^4 \ln^3 \frac{1}{x} + \dots$$
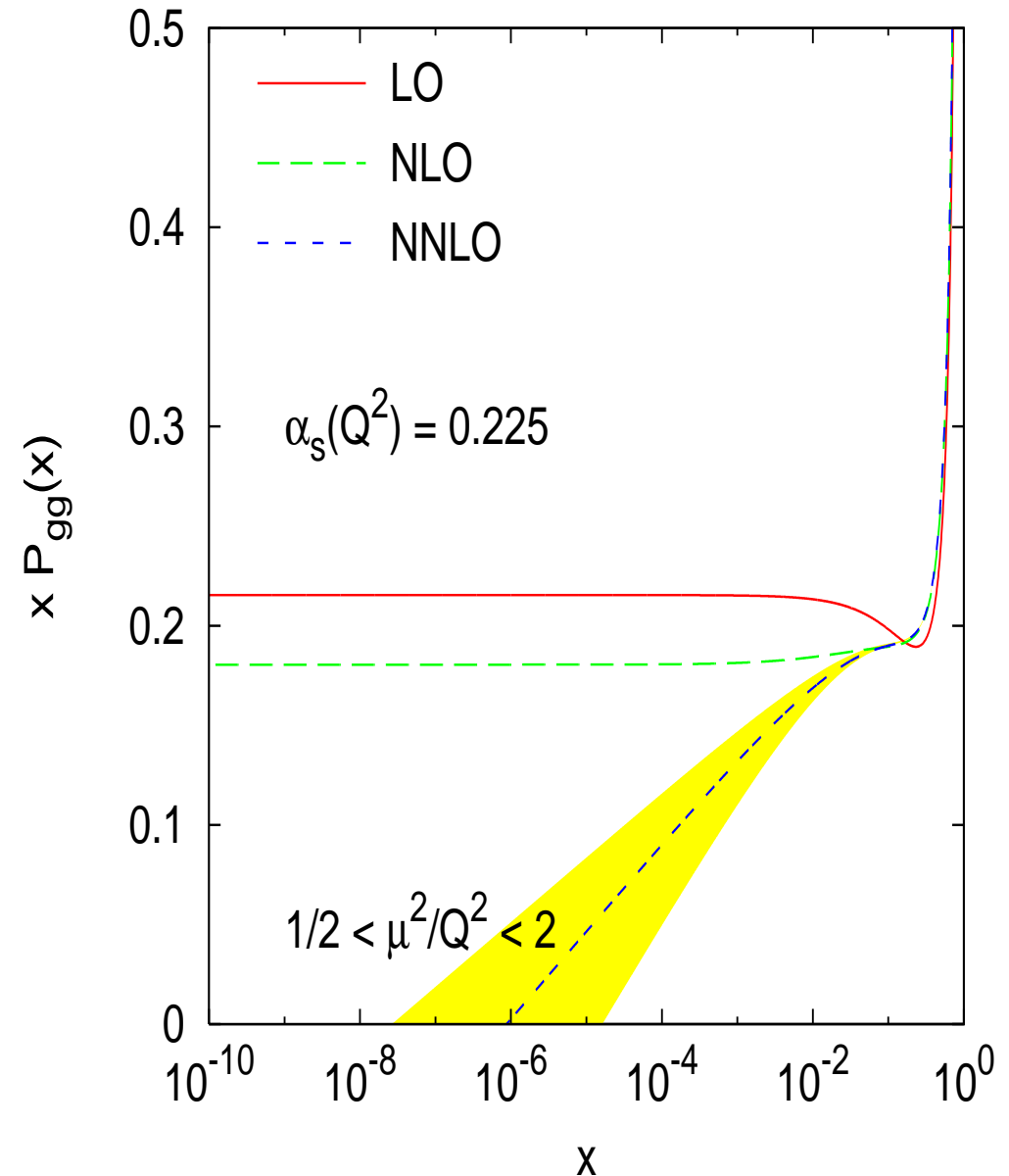
Fadin & Lipatov '98

Camici & Ciafaloni '98

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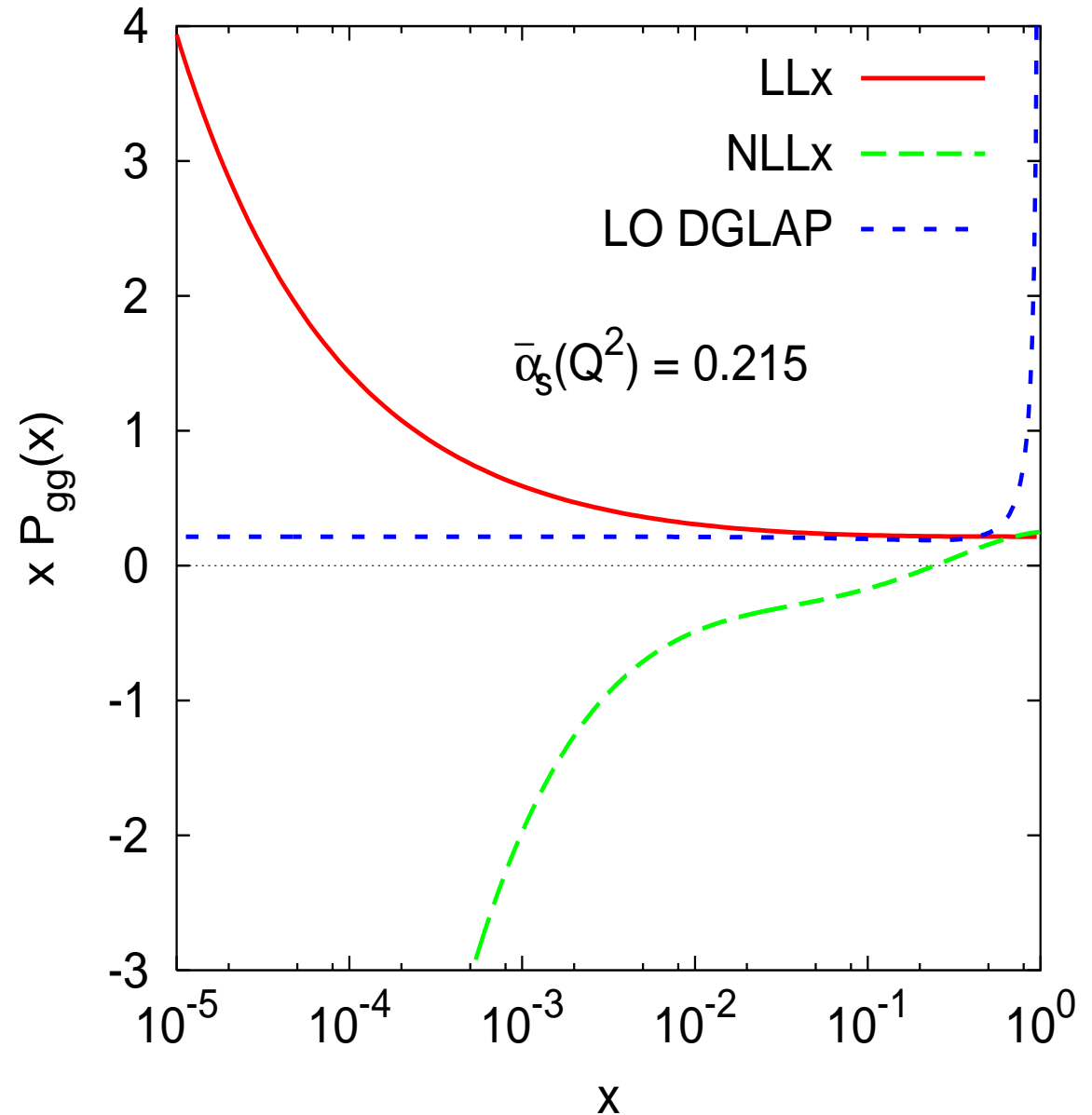
$$+ \sum_{n=2} \alpha_\mathsf{s}^n \ln^{n-2} \frac{1}{x} + \dots$$

- NNLO (α_s^3): first small-x enhancement in gluon splitting function.

Plot of $x\,P_{gg}(x)$ versus x with curves: LO (red solid), NLO (green dashed), NNLO (blue dashed). $\alpha_s(Q^2) = 0.225$. $1/2 < \mu^2/Q^2 < 2$.
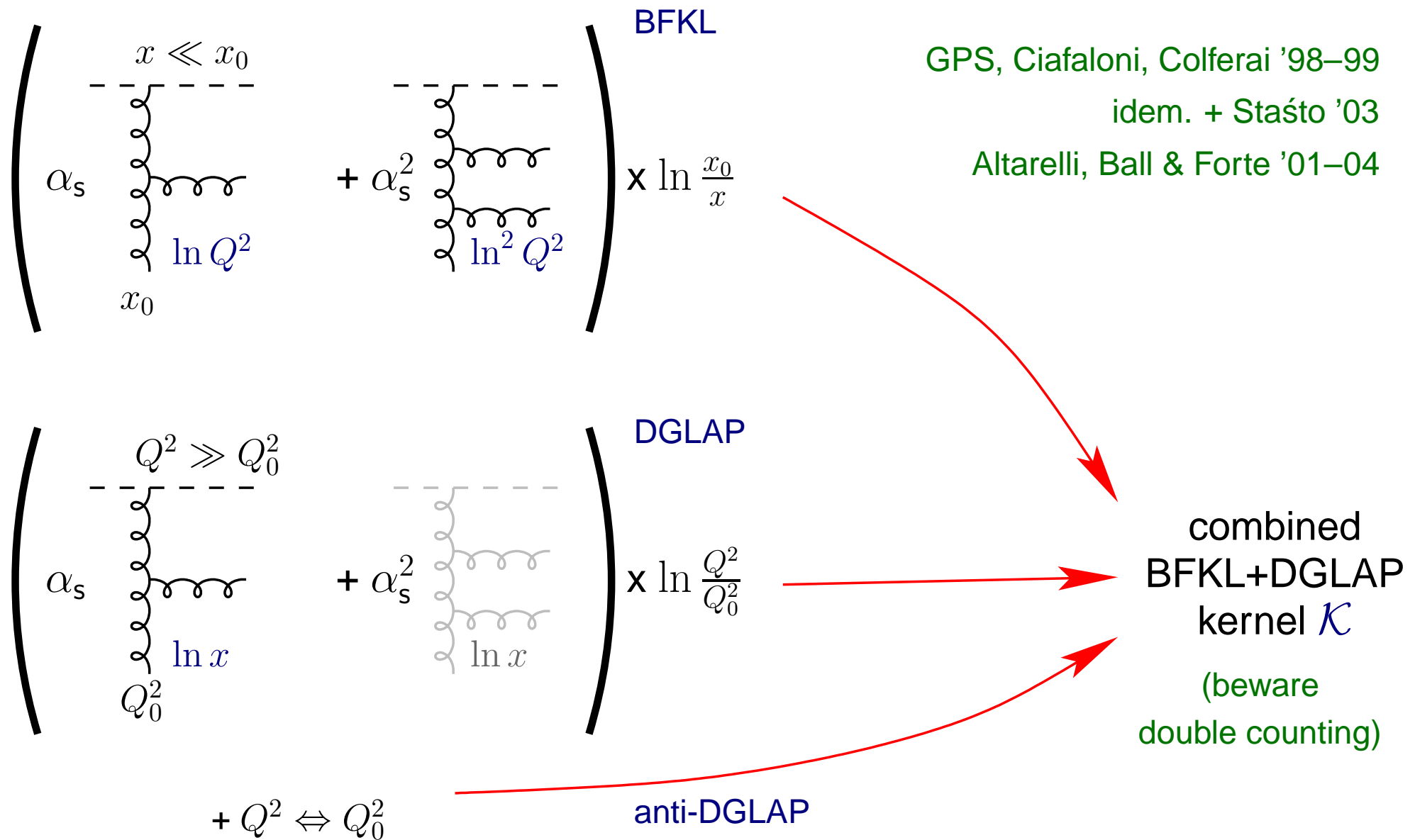
<u>Reminder</u>

- LLx terms rise very fast, $xP_{gg}(x) \sim x^{-0.5}$.

  Incompatible with data.

  Ball & Forte '95

- NLLx terms go negative very fast.

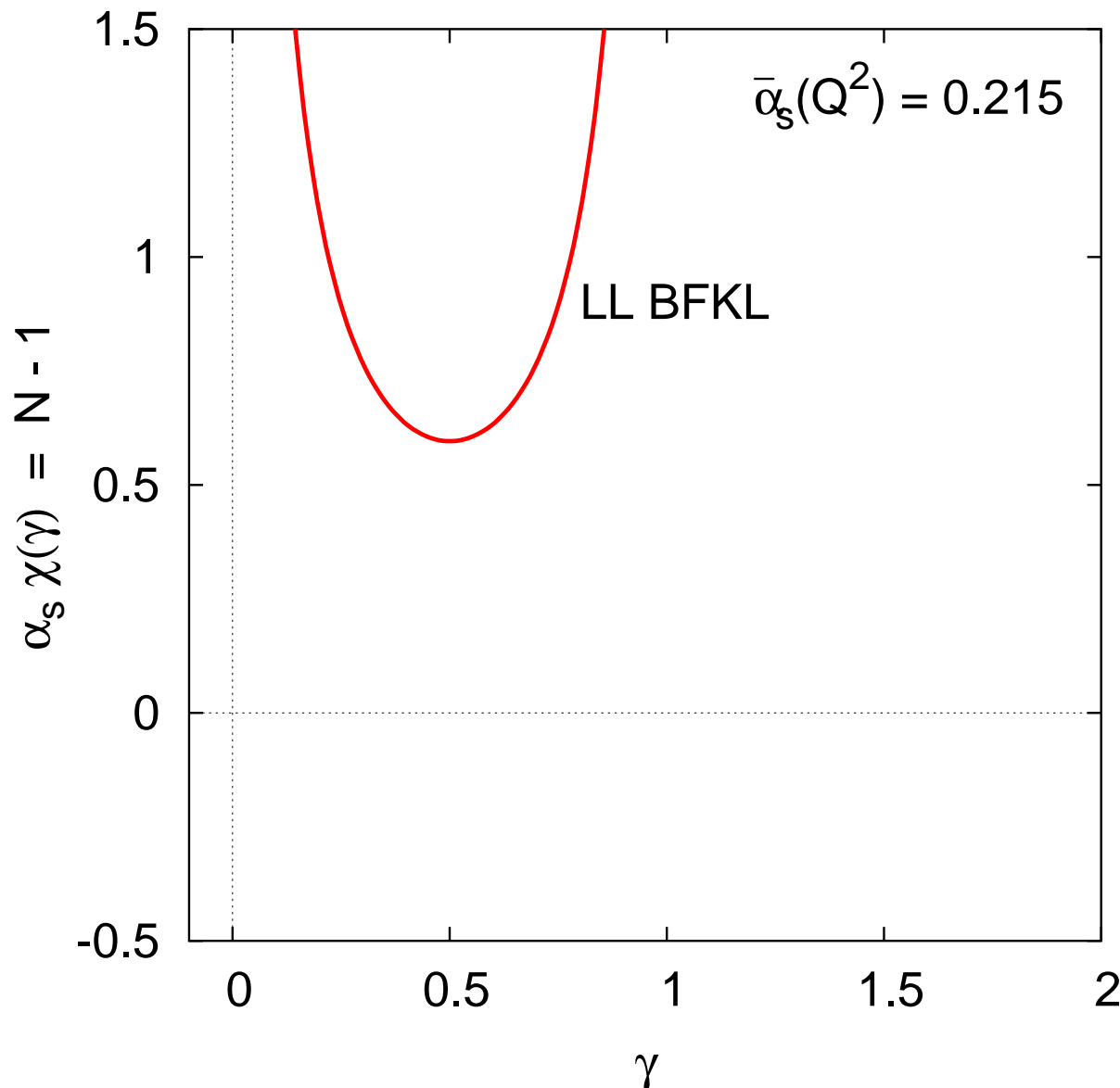 No one's even tried fitting the data!

[NB: Taking NLLx terms of P_{gg} is almost the worst possible expansion]
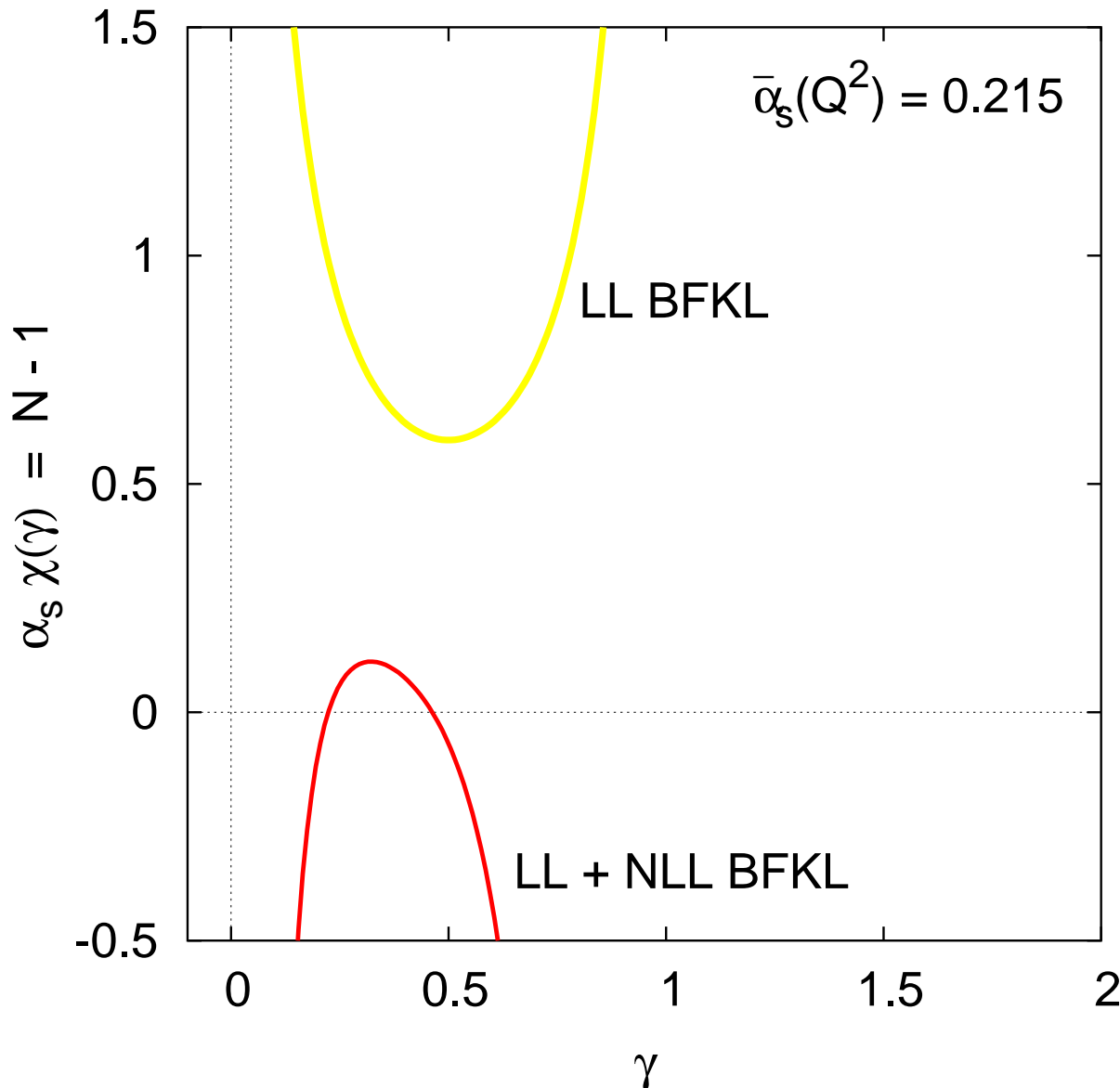
Improved NLLx? Start with kernel...

$$\left(\quad \alpha_s \quad + \alpha_s^2 \quad \right) \times \ln \frac{x_0}{x} \qquad \text{BFKL}$$

with $x \ll x_0$ and x_0

$$\left(\quad \alpha_s \quad + \alpha_s^2 \quad \right) \times \ln \frac{Q^2}{Q_0^2} \qquad \text{DGLAP}$$

with $Q^2 \gg Q_0^2$ and Q_0^2

$$+ \, Q^2 \Leftrightarrow Q_0^2 \qquad \text{anti-DGLAP}$$

Improved NLLx? Start with kernel...

BFKL

$$\left(\alpha_s \quad \underset{x_0}{\overset{x \ll x_0}{\ln Q^2}} \quad + \alpha_s^2 \quad \ln^2 Q^2 \right) \times \ln \frac{x_0}{x}$$

DGLAP

$$\left(\alpha_s \quad \underset{Q_0^2}{\overset{Q^2 \gg Q_0^2}{\ln x}} \quad + \alpha_s^2 \quad \ln x \right) \times \ln \frac{Q^2}{Q_0^2}$$

$+ \, Q^2 \Leftrightarrow Q_0^2$ anti-DGLAP

Improved NLLx? Start with kernel...

BFKL

$$\left(\alpha_{\mathsf{s}} \overset{x \ll x_0}{\underset{x_0}{\includegraphics{gluon1}}} \ln Q^2 + \alpha_{\mathsf{s}}^2 \,\ln^2 Q^2 \right) \times \ln \frac{x_0}{x}$$

GPS, Ciafaloni, Colferai '98–99

idem. + Staśto '03

Altarelli, Ball & Forte '01–04

DGLAP

$$\left(\alpha_{\mathsf{s}} \overset{Q^2 \gg Q_0^2}{\underset{Q_0^2}{\includegraphics{gluon2}}} \ln x + \alpha_{\mathsf{s}}^2 \,\ln x \right) \times \ln \frac{Q^2}{Q_0^2}$$

$$+ \, Q^2 \Leftrightarrow Q_0^2$$

anti-DGLAP

combined
BFKL+DGLAP
kernel $\mathcal{K}$

(beware
double counting)

Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)
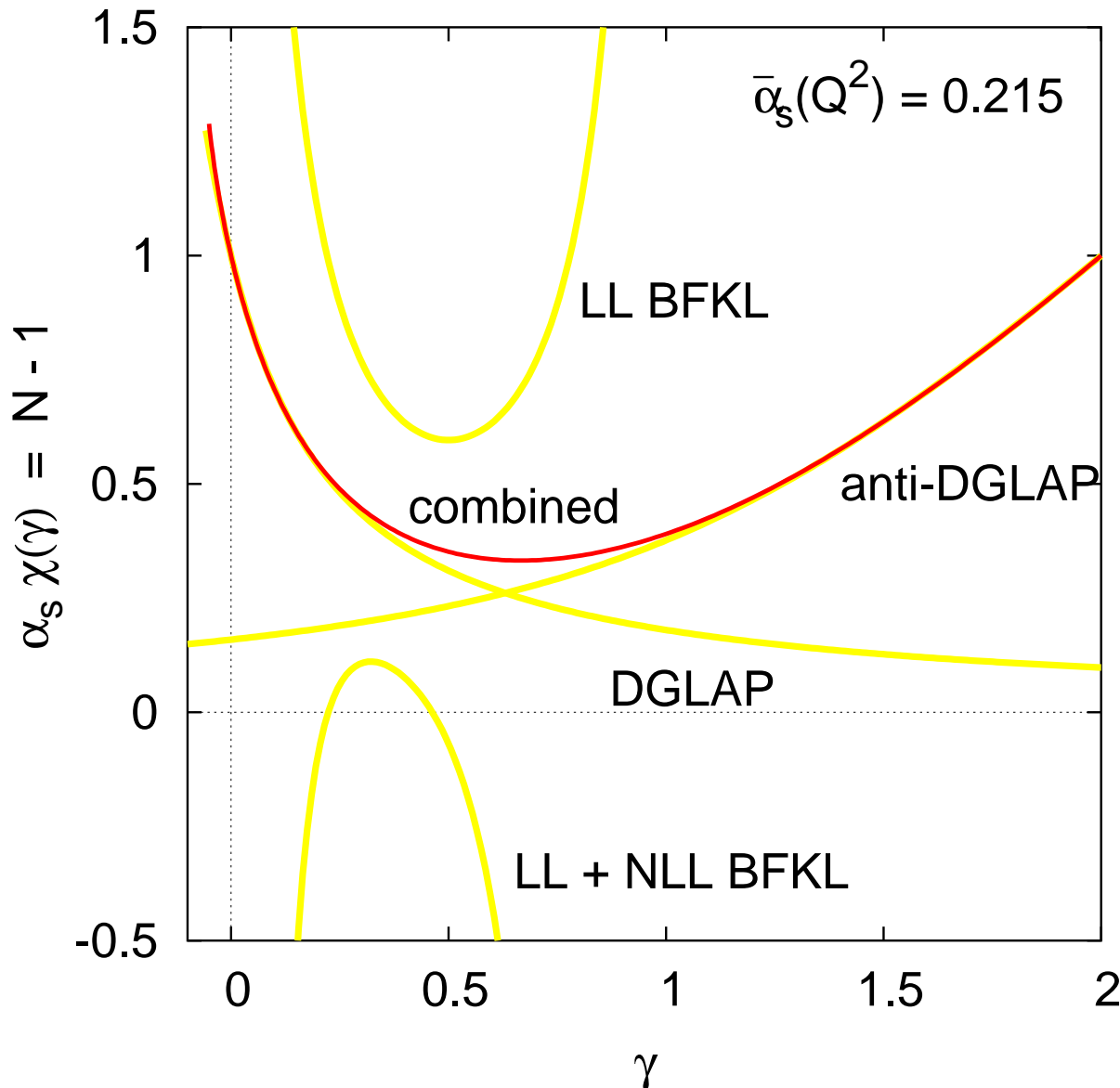
$$\bar{\alpha}_s \chi(\gamma) =$$

$$= \int \frac{dk^2}{k^2} \left(\frac{k^2}{k_0^2} \right)^{\gamma} \mathcal{K}(k, k_0)$$

Height of minimum is 'BFKL power'

In the figure:

$\bar{\alpha}_s(Q^2) = 0.215$

LL BFKL

Axes: $\alpha_s \chi(\gamma) = N - 1$ versus γ

$\bar{\alpha}_s(Q^2) = 0.215$

LL BFKL

LL + NLL BFKL

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NB: DGLAP = 'rotated' plot of

$$\gamma(N)$$

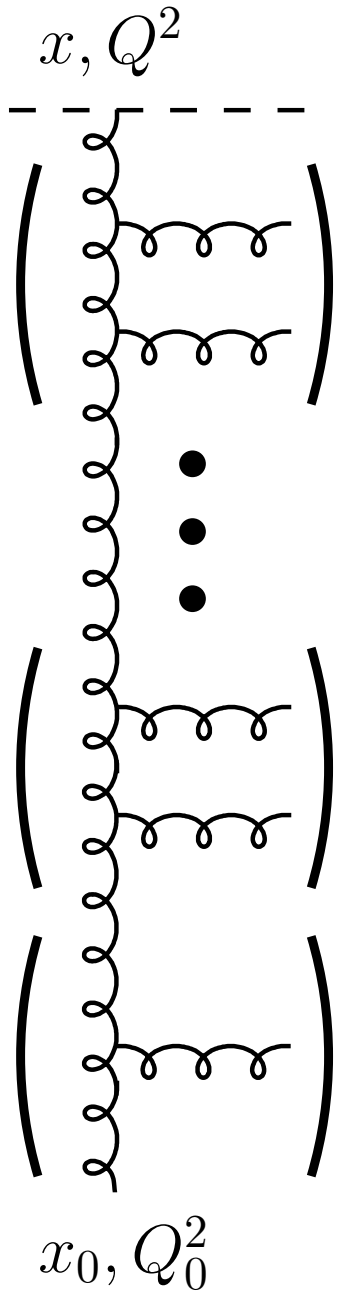Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)

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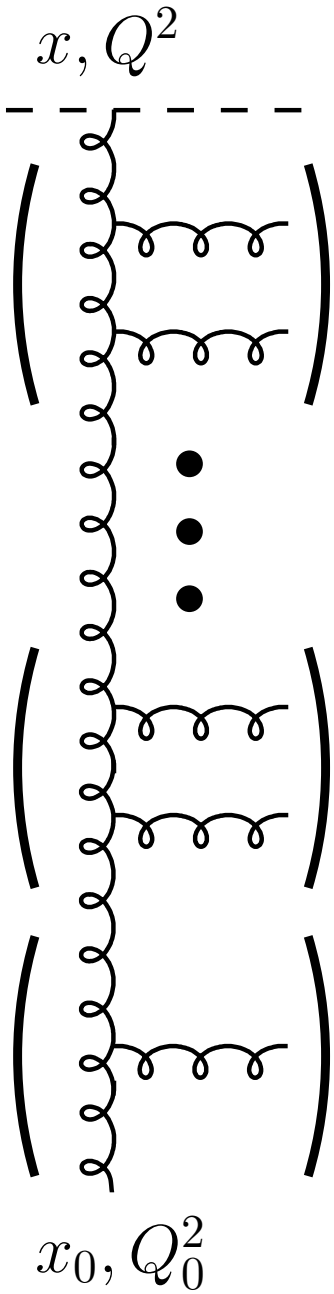
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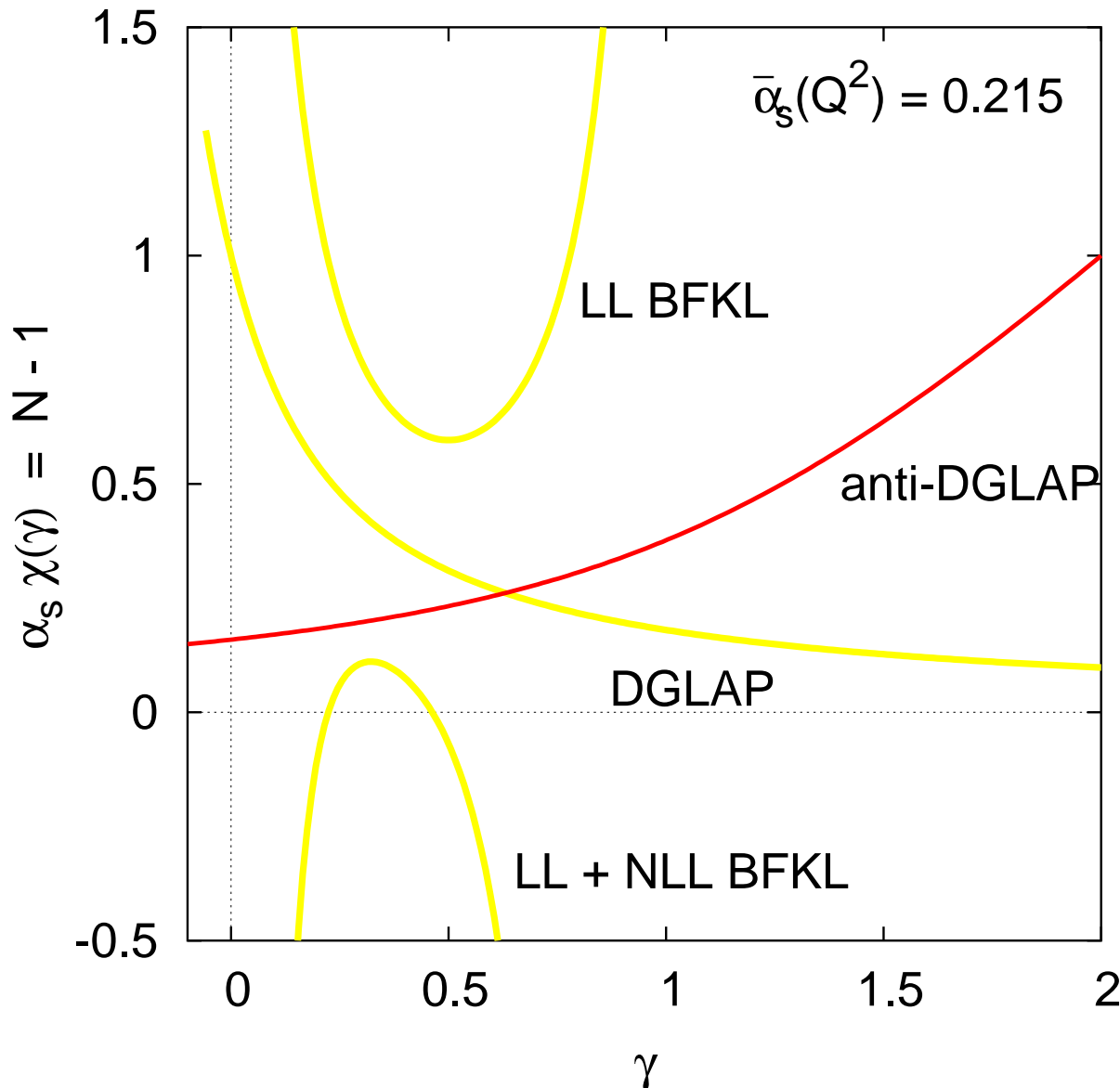
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Iteration of kernel $\Rightarrow$ Green function
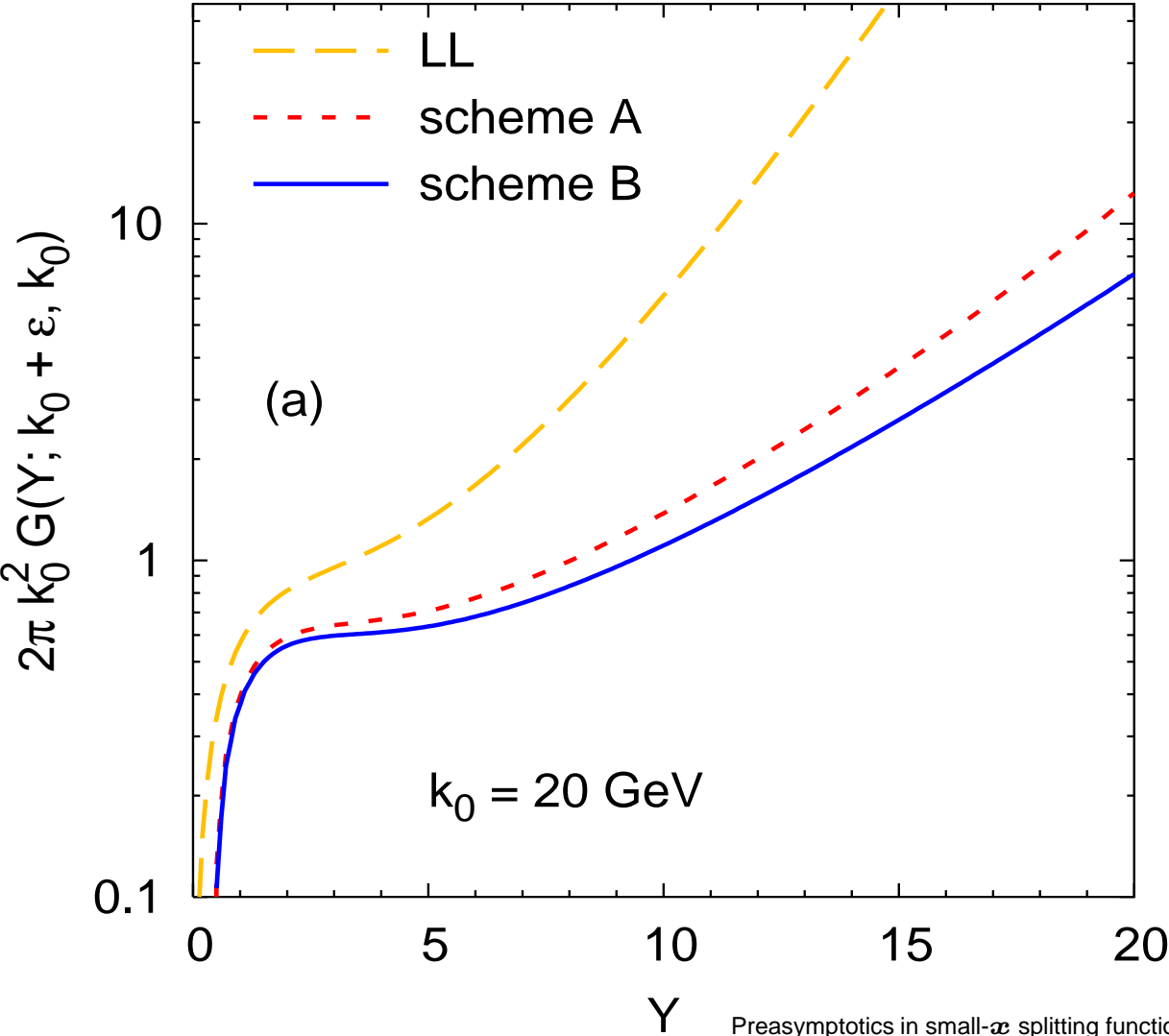
x, Q^2

Green function: $G\left(\ln\frac{x}{x_0}; Q_0, Q\right)$

x_0, Q_0^2

Green function: $G\left(\ln \frac{x}{x_0} ; Q_0, Q\right)$

(a)

LL

scheme A

scheme B

$2\pi\, k_0^2\, G(Y; k_0 + \varepsilon, k_0)$

$k_0 = 20$ GeV

Y

Iteration of kernel $\Rightarrow$ Green function

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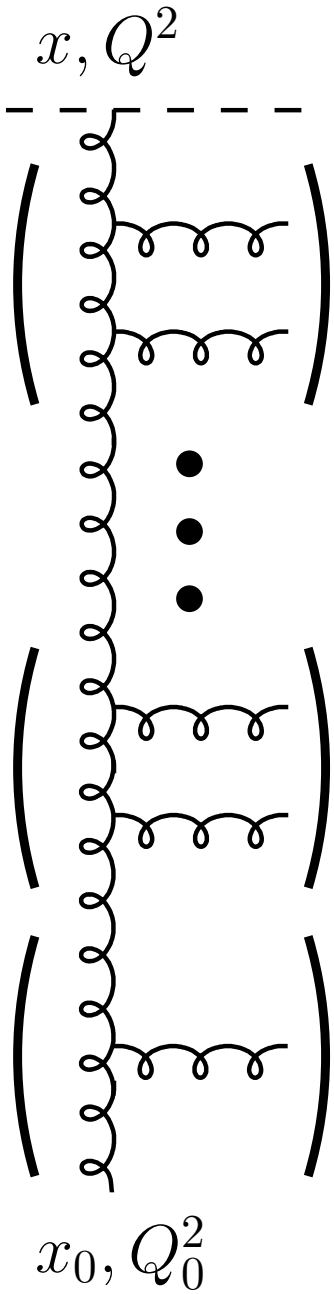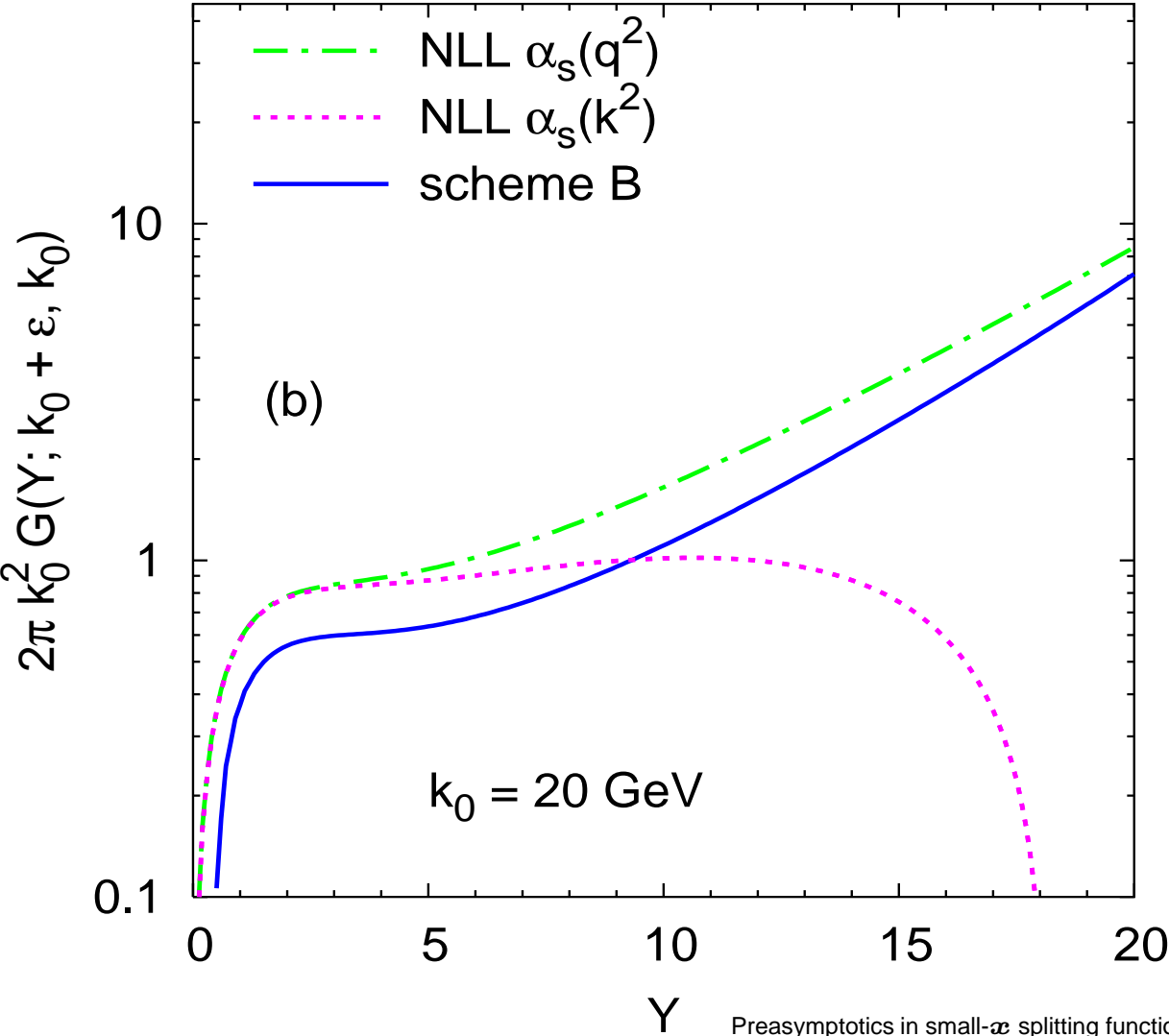
- — · — NLL $\alpha_s(q^2)$
- ········ NLL $\alpha_s(k^2)$
- ——— scheme B

(b)

$2\pi\, k_0^2\, G(Y; k_0 + \varepsilon, k_0)$

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Green function $\Rightarrow$ effective DGLAP splitting function

Construct a gluon density from Green function (take $k \gg k_0$):

$$xg(x, Q^2) \equiv \int^Q d^2k \, G^{(\nu_0=k^2)}(\ln 1/x, k, k_0)$$

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Numerically solve equation for effective splitting function, $P_{gg,\text{eff}}(z, Q^2)$:

$$\frac{dg(x, Q^2)}{d \ln Q^2} = \int \frac{dz}{z} \, P_{gg,\text{eff}}(z, Q^2) \, g\left(\frac{x}{z}, Q^2\right)$$

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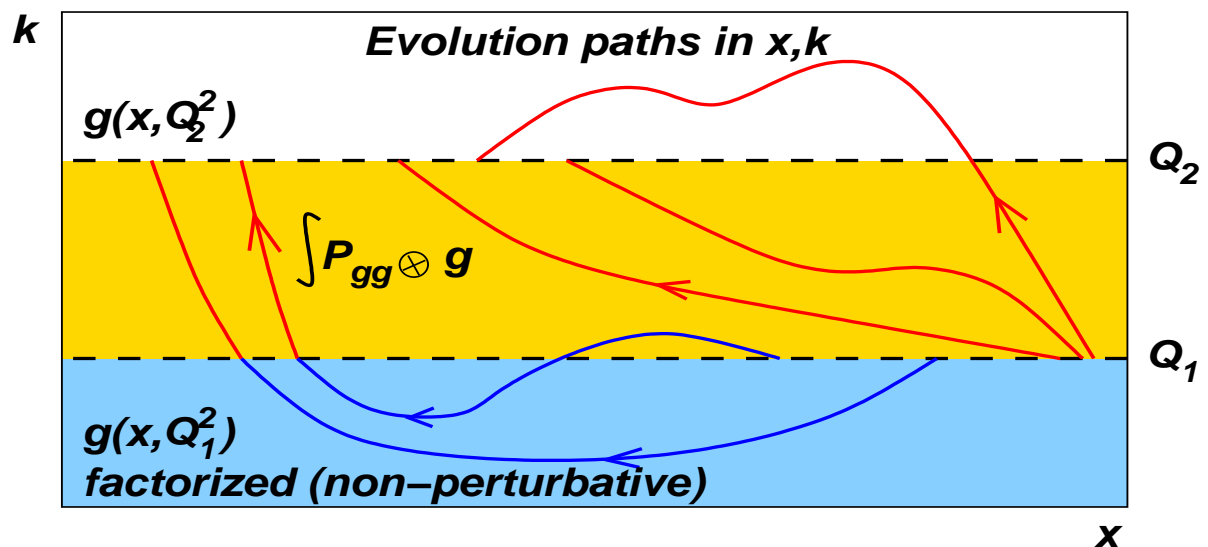Construct a gluon density from Green function (take $k \gg k_0$):

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Factorisation

- Splitting function:
  red paths
- Green function:
  all paths

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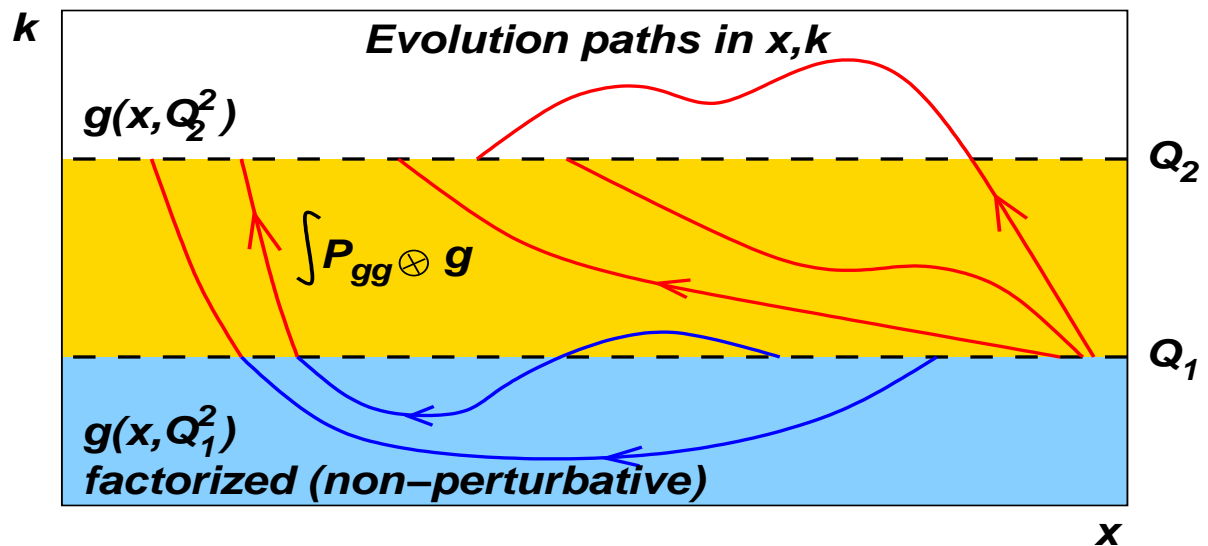
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Factorisation

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Splitting function $\equiv$ evolution with cutoff

BFKL splitting function 'power'

Two classes of correction, to power growth ω:

$$\omega = 4 \ln 2 \, \bar{\alpha}_{\mathsf{s}}(Q^2) \left(1 \; - \; \underbrace{6.5 \, \bar{\alpha}_{\mathsf{s}}}_{NLL} \; - \; \underbrace{4.0 \, \bar{\alpha}_{\mathsf{s}}^{2/3}}_{running} + \cdots \right)$$

$$\bar{\alpha}_{\mathsf{s}} = \alpha_{\mathsf{s}} N_c / \pi$$

- NLL piece is *universal*

- running piece appears only in problems with *cutoffs*

  - a consequence of *asymmetry* due to cutoff (only scales higher than cutoff contribute)

    $$\alpha_{\mathsf{s}}(Q^2) \rightarrow \alpha_{\mathsf{s}}(Q^2 e^{-X/(b\alpha_{\mathsf{s}})^{1/3}})$$

Hancock & Ross '92

BFKL splitting function 'power'

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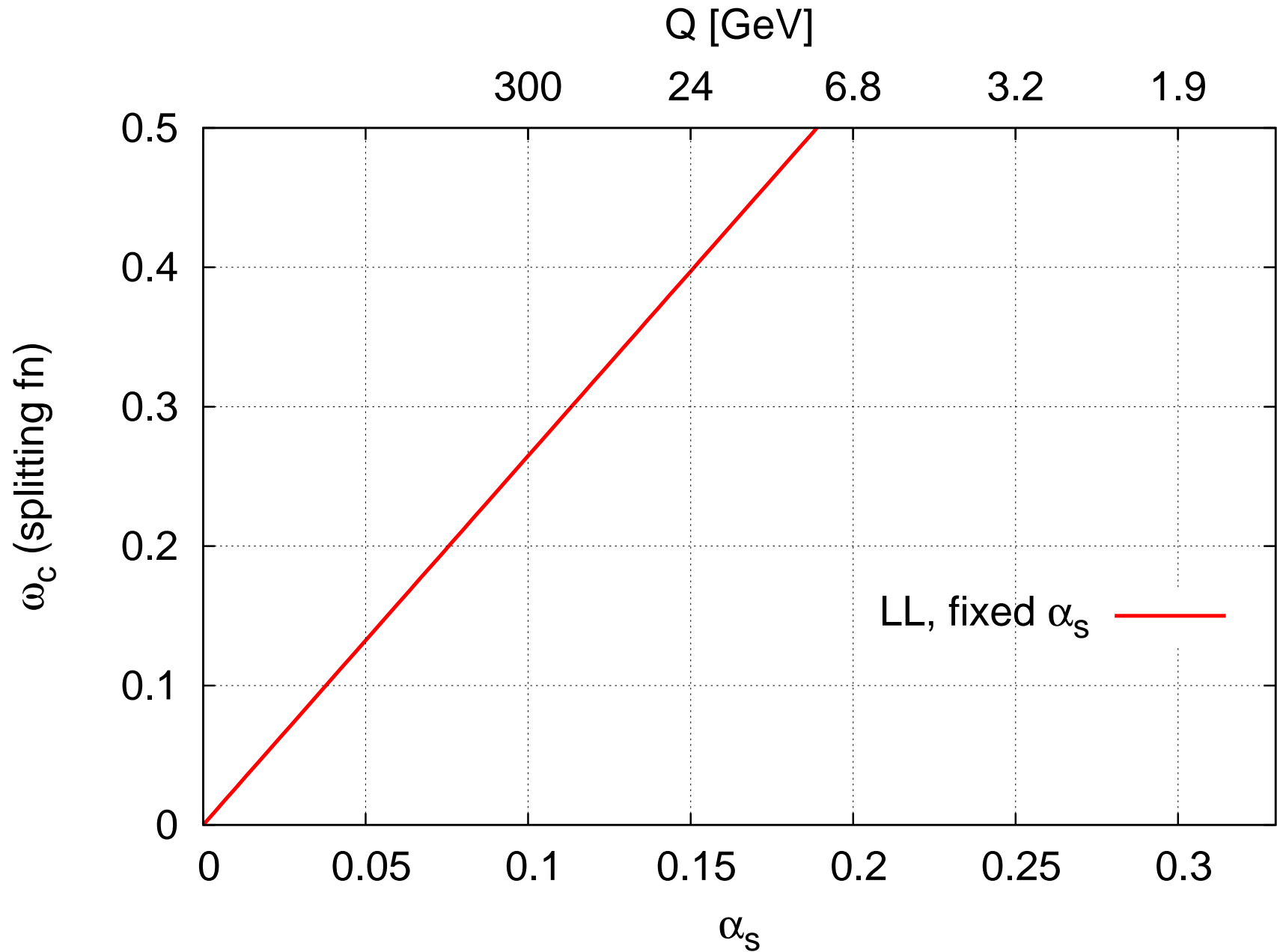$$\bar{\alpha}_{\mathsf{s}} = \alpha_{\mathsf{s}}N_c/\pi$$

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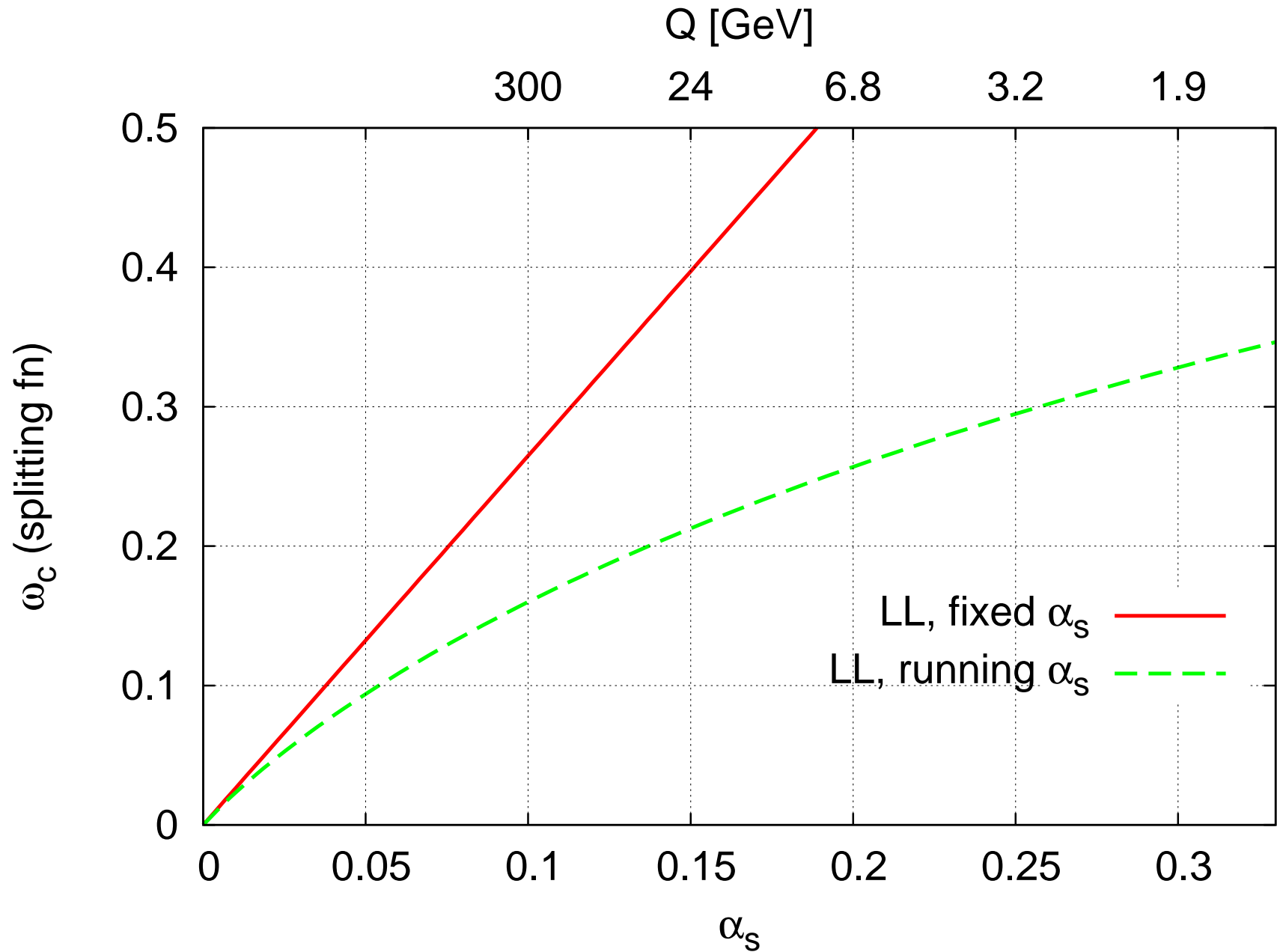$$\alpha_{\mathsf{s}}(Q^2) \rightarrow \alpha_{\mathsf{s}}(Q^2 e^{-X/(b\alpha_{\mathsf{s}})^{1/3}})$$

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- Beyond first terms, not possible to separate effects of 'pure' higher orders & running coupling
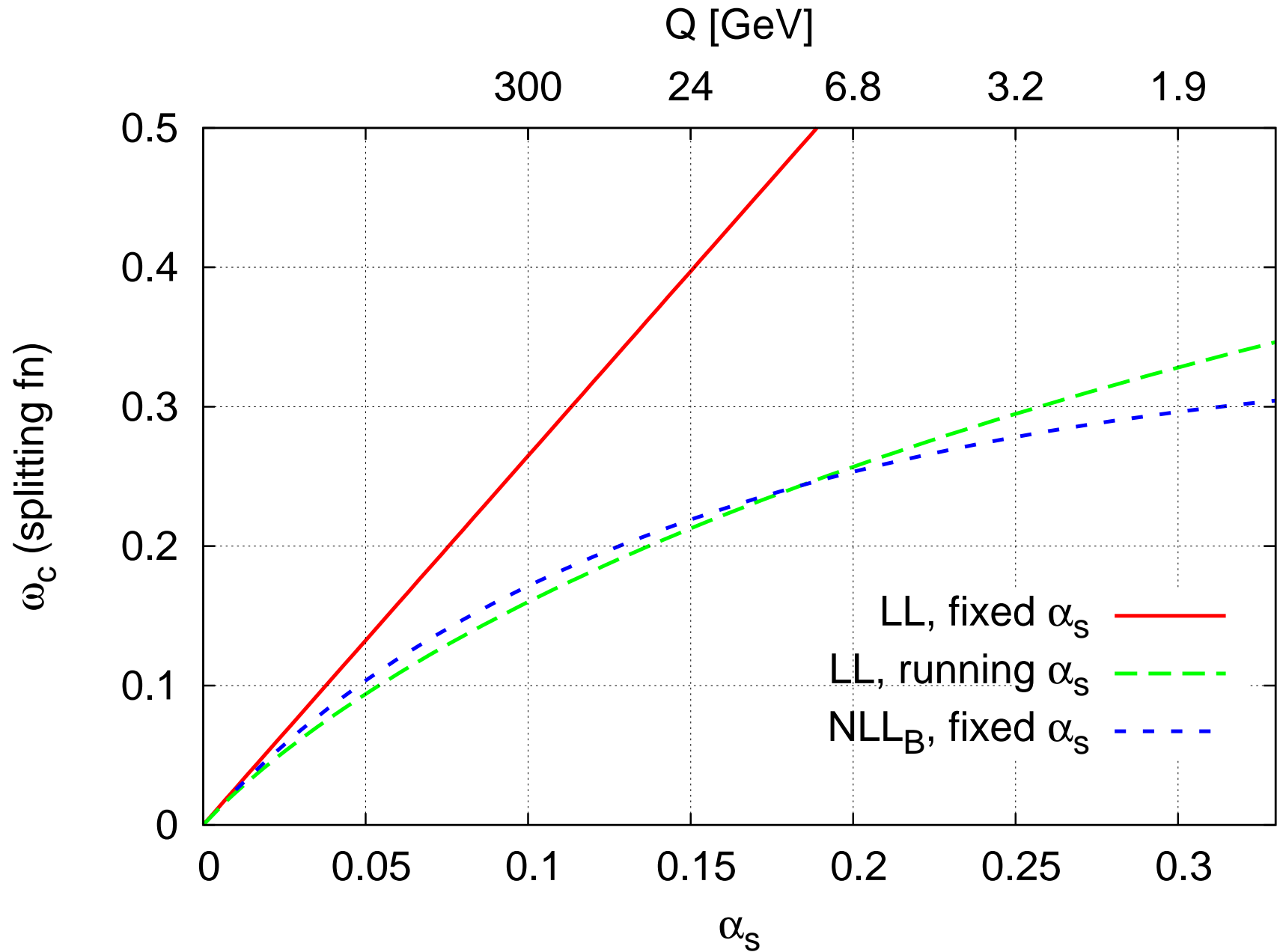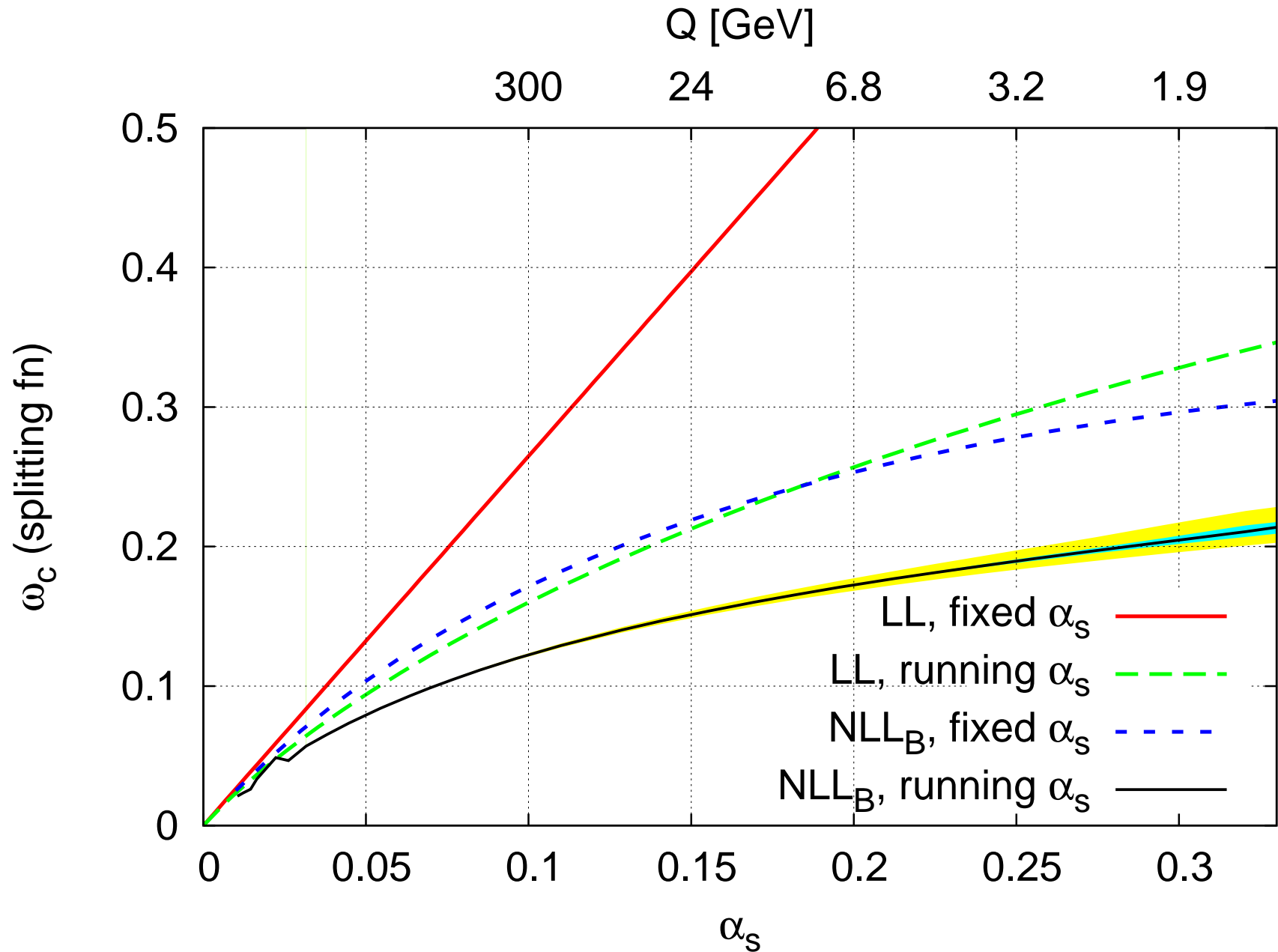
BFKL splitting function power growth – results

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Q [GeV]

ω_c (splitting fn)

α_s

LL, fixed α_s

LL, running α_s

NLL_B, fixed α_s

NLL_B, running α_s

LL (fixed $\bar{\alpha}_s$)
LO DGLAP

$Q = 4.5$ GeV
$\bar{\alpha}_s(Q^2) = 0.215$

$z\,P(z)$

z
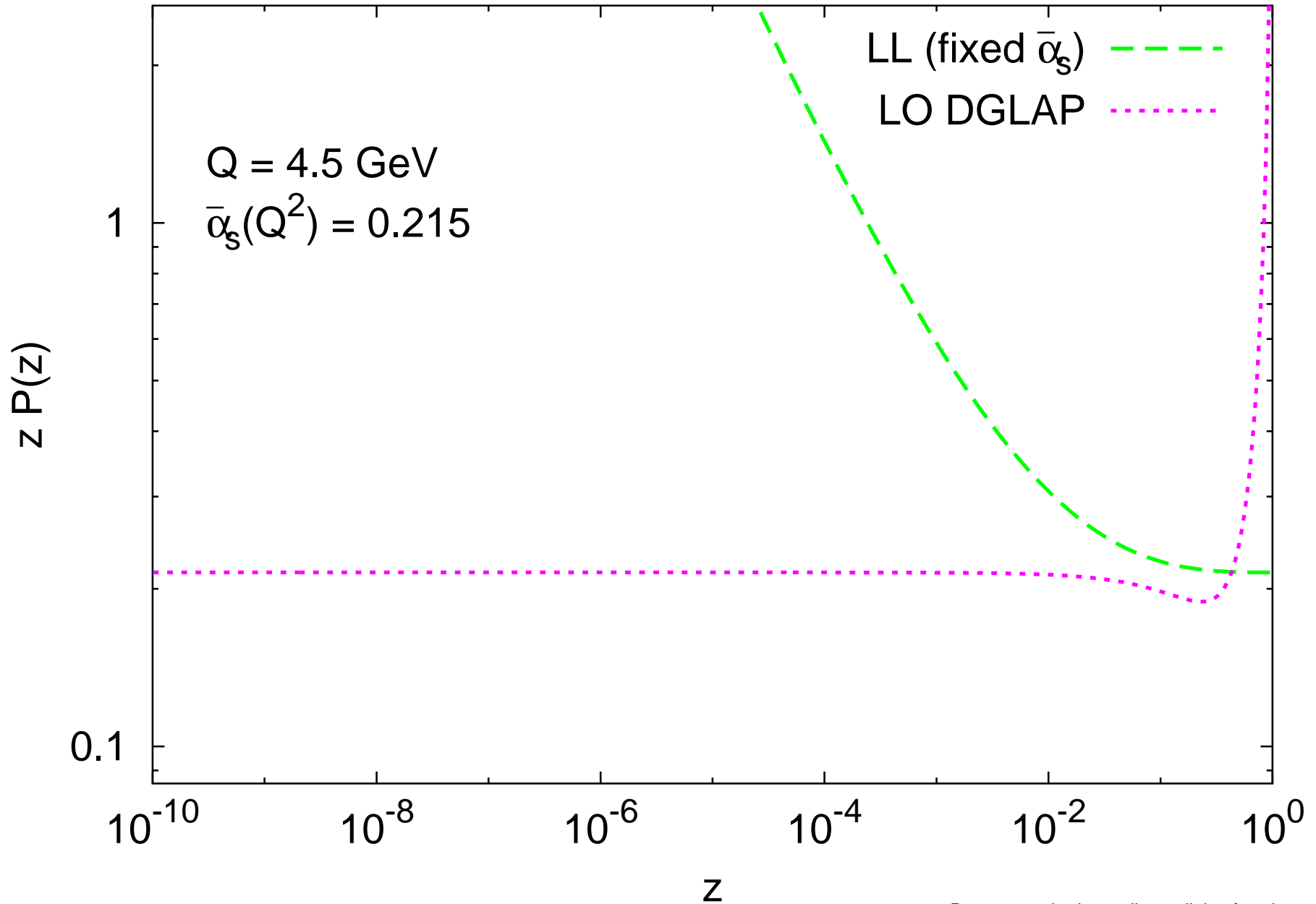
Full $P_{gg}(z)$ splitting fn

Q = 4.5 GeV
$\bar{\alpha}_s(Q^2) = 0.215$

LL (fixed $\bar{\alpha}_s$)
LL ($\bar{\alpha}_s(q^2)$)
LO DGLAP

z P(z)

z

Dominant phenomenological structure in P_{gg} is **dip**

- Rapid rise in P_{gg} is not for today's energies!

- Main feature is a *dip at $x \sim 10^{-3}$*

ω-expansion (1999) — — —
NLL_B (2003) ———
LO DGLAP ·········

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Questions:

- Various 'dips' have been seen

  Thorne '99, '01 (running α_s, NLLx)

  ABF '99–'03 (fits, running α_s)

  CCSS '01,'03 (running α_s, NLL$_B$)

  Is it always the same dip?

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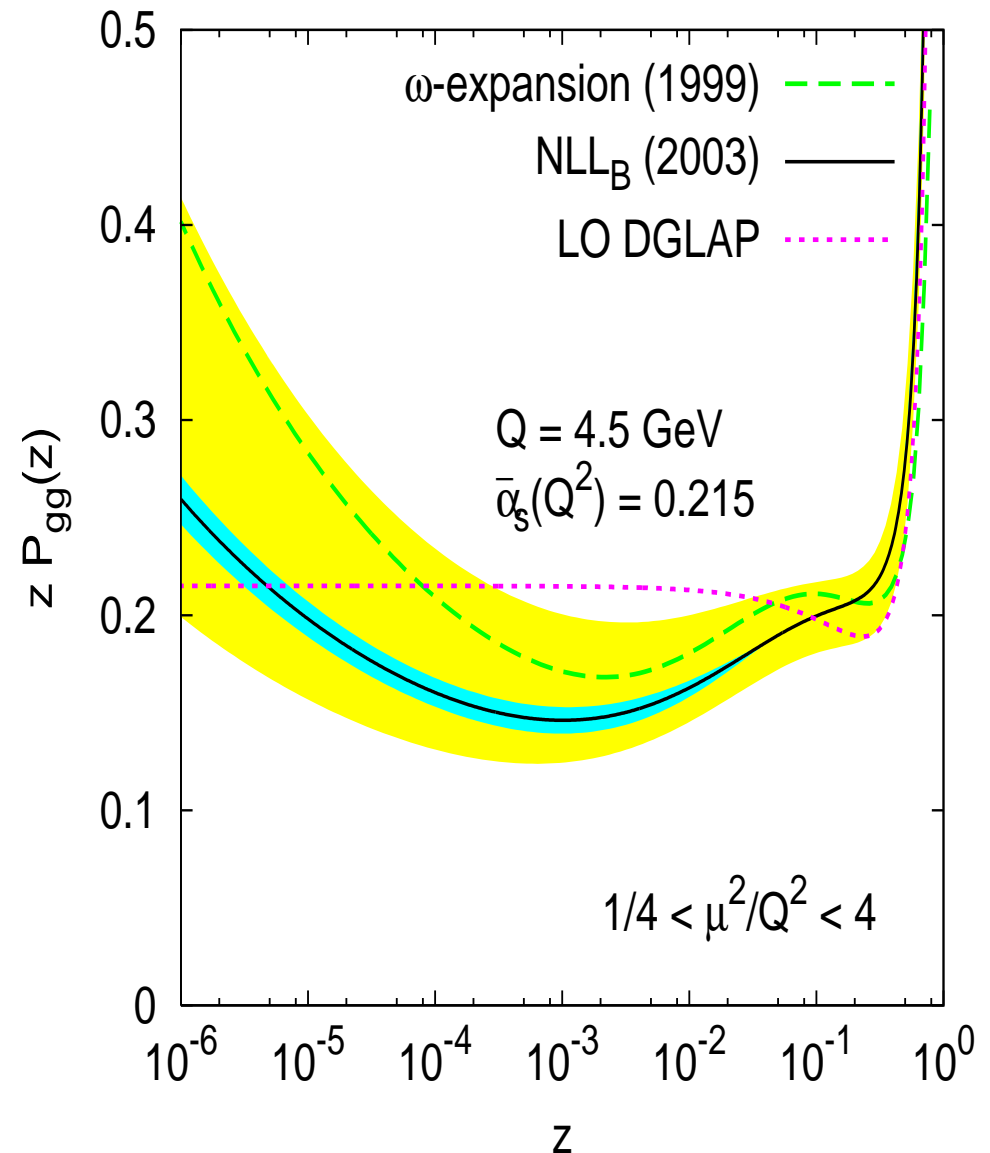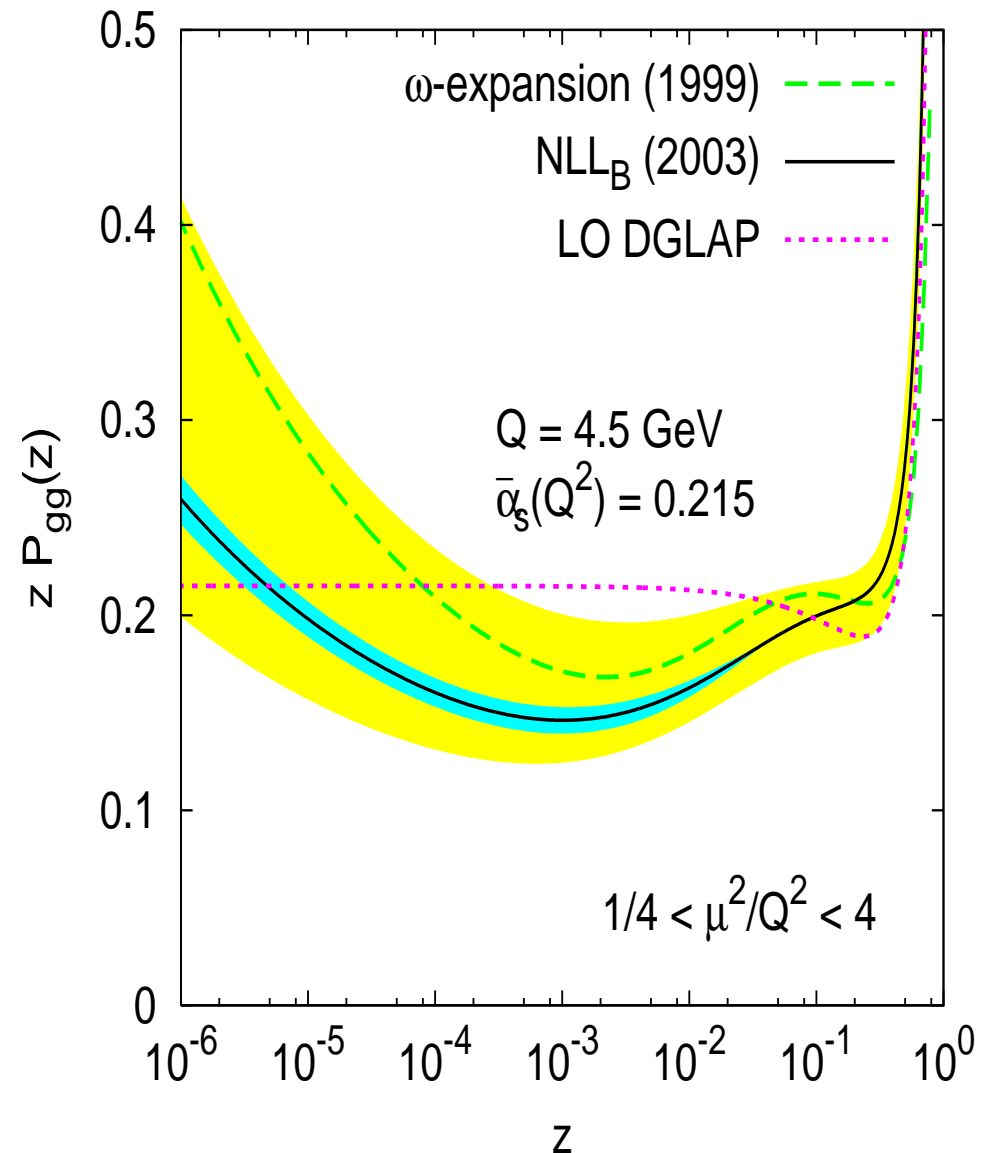
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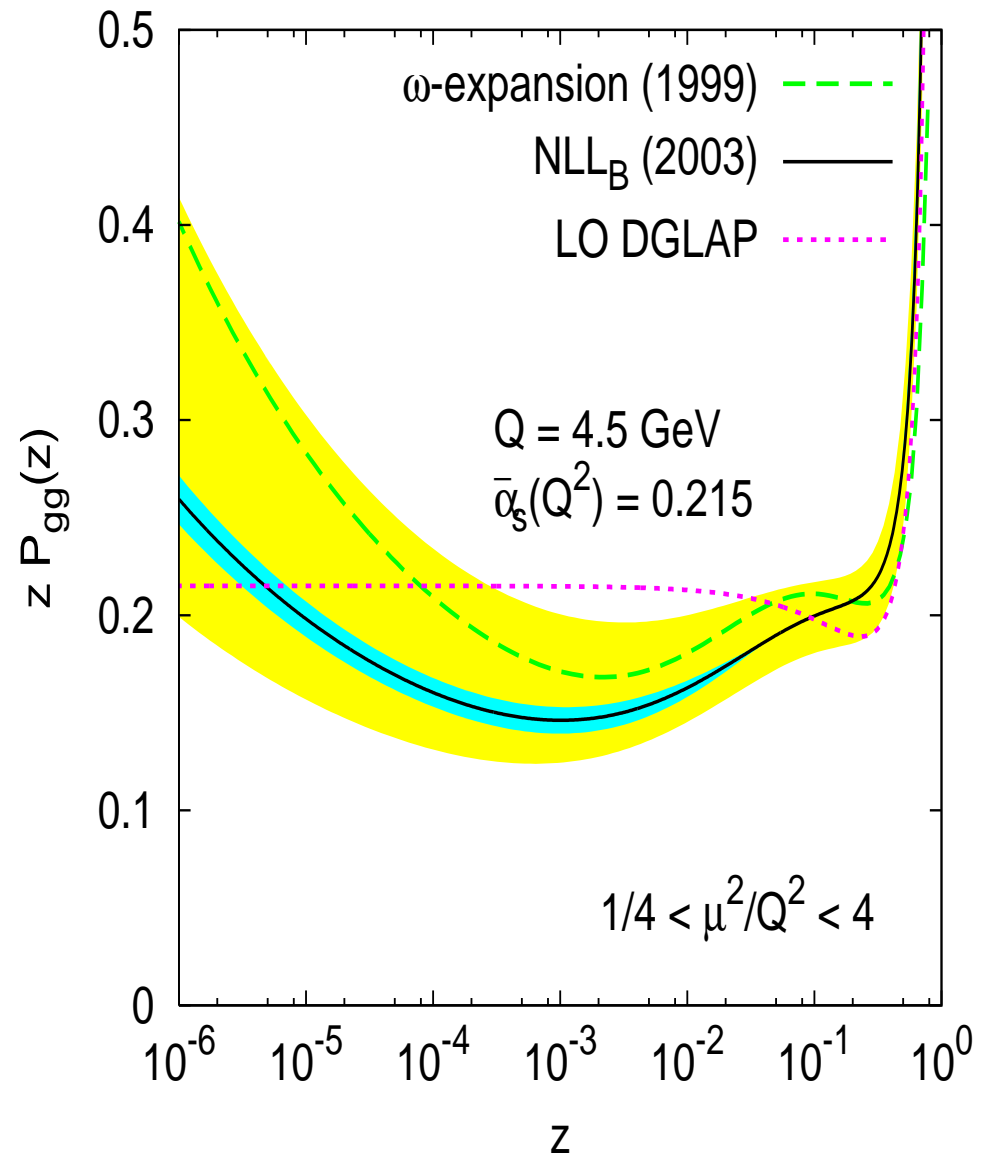$1/4 < \mu^2/Q^2 < 4$

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  Running α_s, momentum sum rule…?

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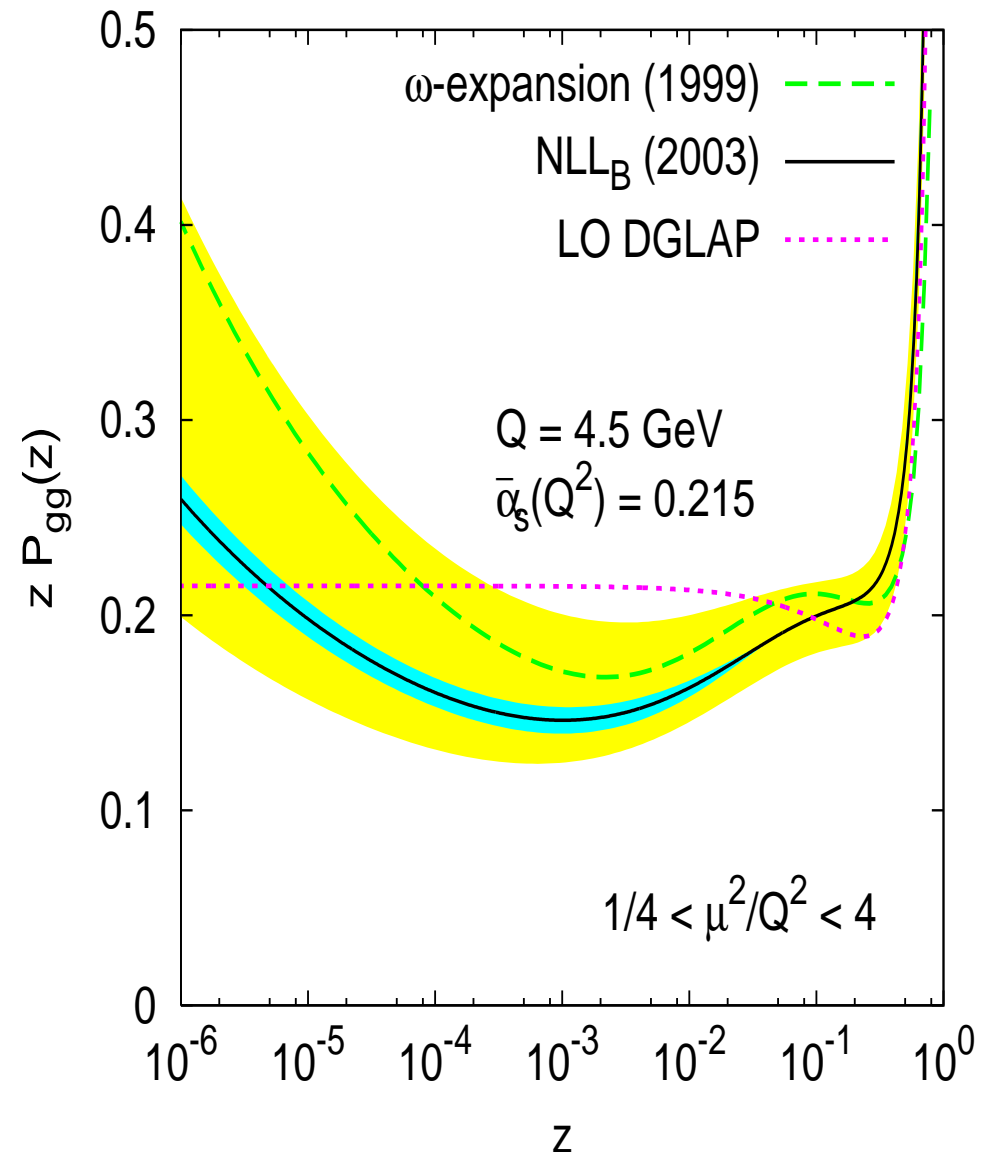
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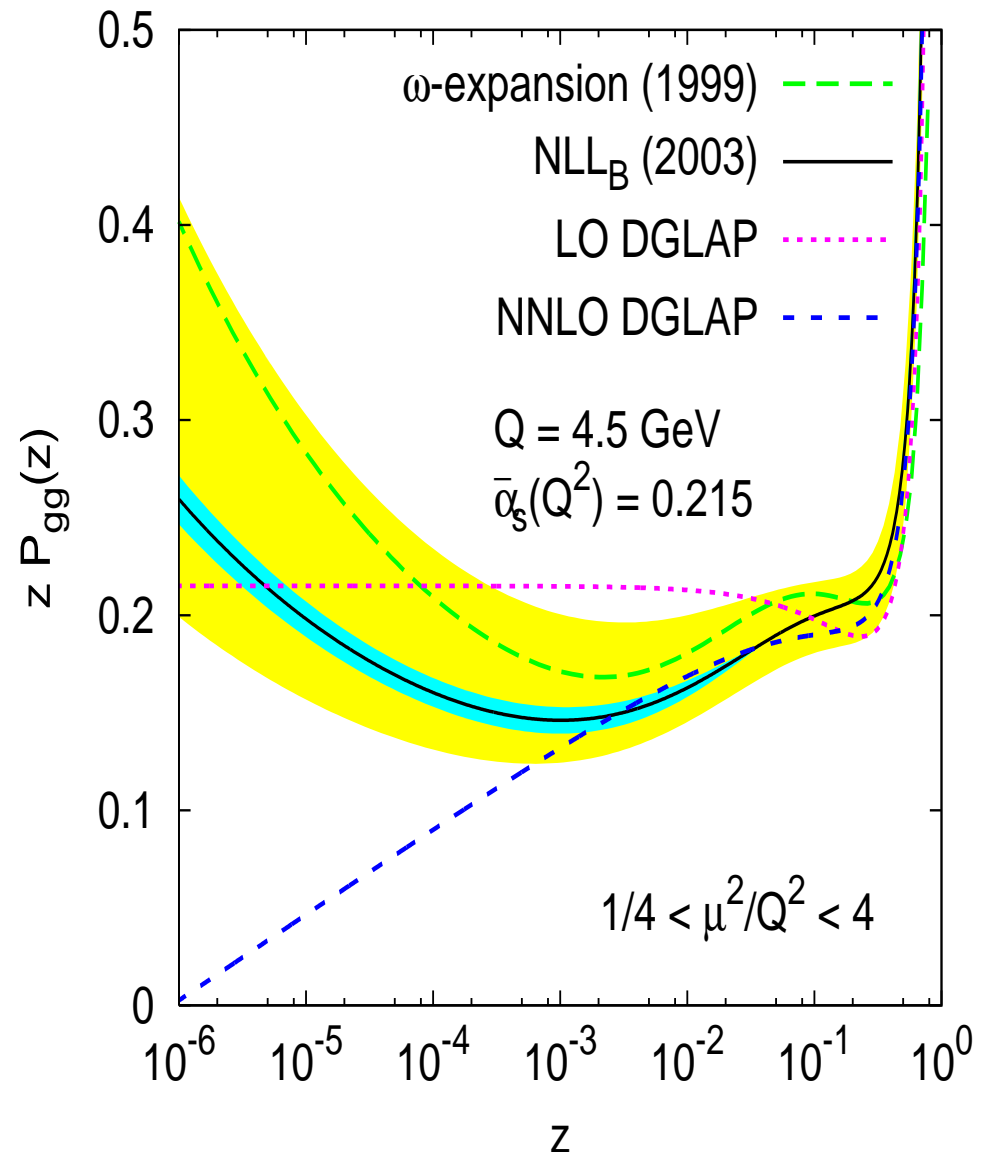# Dominant phenomenological structure in P_{gg} is **dip**

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NNLO DGLAP gives a clue...

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$

Figure: plot of $z\,P_{gg}(z)$ versus z with curves labelled ω-expansion (1999), NLL$_B$ (2003), LO DGLAP, NNLO DGLAP. $Q = 4.5$ GeV, $\bar{\alpha}_s(Q^2) = 0.215$, $1/4 < \mu^2/Q^2 < 4$.

	LLx	NLLx	NNLLx	...
α_s	X	−	−	
α_s^2	0	n_f	−	
α_s^3	0	X	X	
α_s^4	X	X	X	
α_s^5	0	X	X	
⋮				

$const.$

$\ln 1/x$

$\ln^2 1/x$

$\ln^3 1/x$

	LLx	NLLx	NNLLx	...
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$$const.$$
$$\ln 1/x$$
$$\ln^2 1/x$$
$$\ln^3 1/x$$

$\bar{\alpha}_s = 0.05$

$x\,P_{gg}(x)$

10^{-4} 10^{-3} 10^{-2} 0.1   1

x

Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	X	–	–	
α_s^2	0	n_f	–	
α_s^3	0	**X**	X	
α_s^4	X	X	X	
α_s^5	0	X	X	
⋮				

const.

$\ln 1/x$

$\ln^2 1/x$

$\ln^3 1/x$

At moderately small x, first terms with x-dependence are
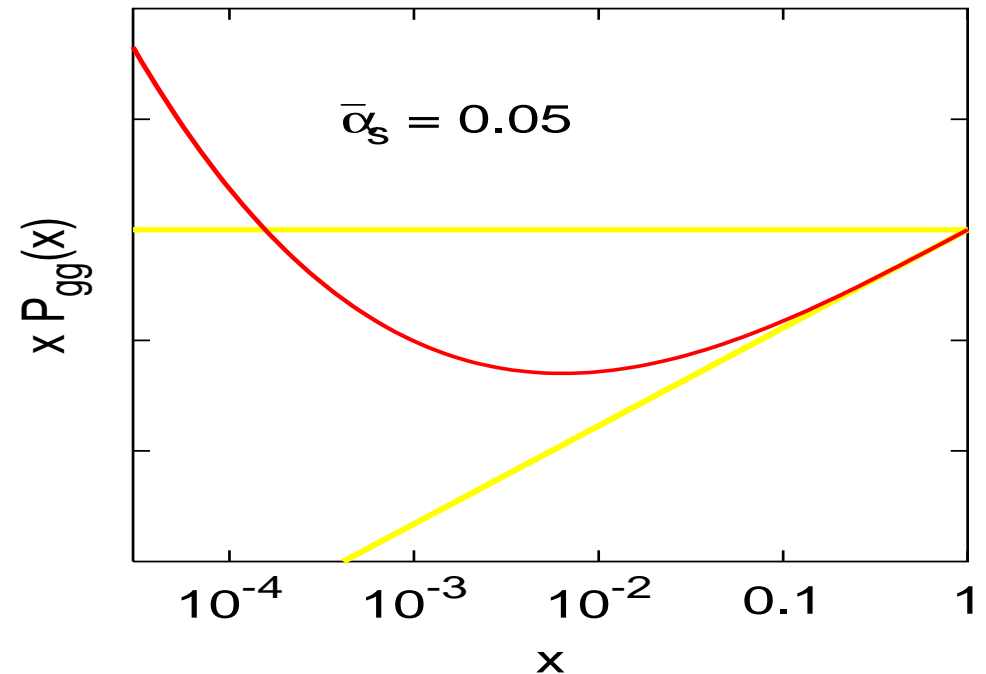
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$\bar{\alpha}_s = 0.05$

Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	**X**	X	X	*const.*
α_s^5	0	X	X	$\ln 1/x$
⋮				$\ln^2 1/x$
				$\ln^3 1/x$

At moderately small x, first terms with x-dependence are

$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x} + \mathbf{0.401}\,\bar{\alpha}_s^4 \ln^3\frac{1}{x}$$

$\bar{\alpha}_s = 0.05$

$x\,P_{gg}(x)$

x

Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	X	–	–	
α_s^2	0	n_f	–	
α_s^3	0	X	X	
α_s^4	X	X	X	
α_s^5	0	X	X	

const.

$\ln 1/x$

$\ln^2 1/x$

$\ln^3 1/x$

At moderately small x, first terms with x-dependence are

$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x} + 0.401\,\bar{\alpha}_s^4 \ln^3\frac{1}{x}$$

Minimum when

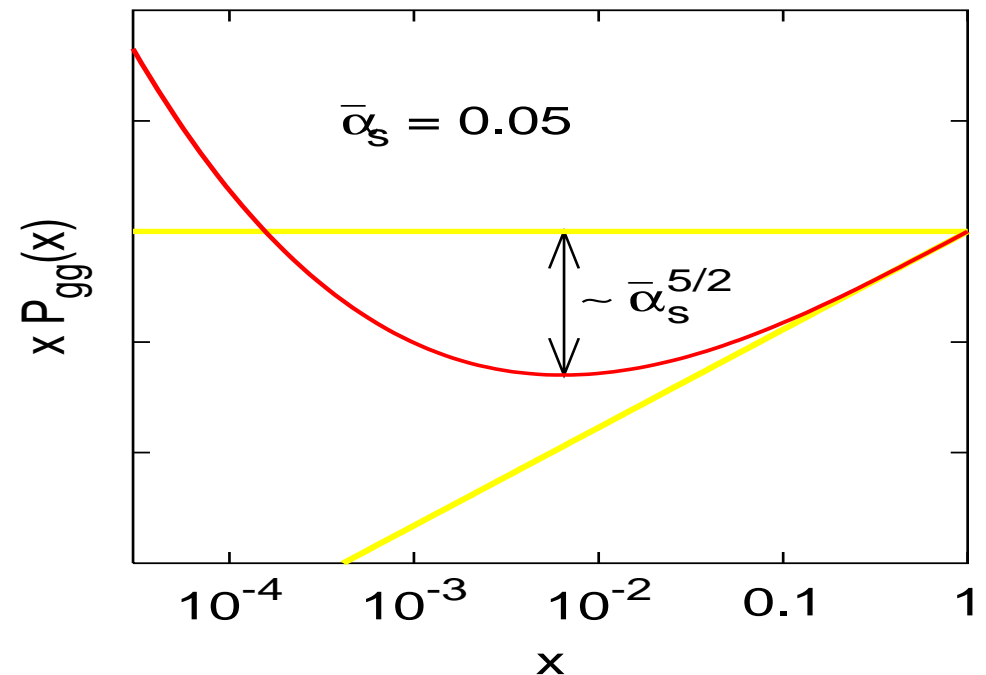$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln\frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$

$\bar{\alpha}_s = 0.05$

$\sim \bar{\alpha}_s^{5/2}$

$x\,P_{gg}(x)$

	LLx	NLLx	NNLLx	...
α_s	X	−	−	
α_s^2	0	n_f	−	
α_s^3	0	X	X	
α_s^4	X	X	X	
α_s^5	0	X	X	
⋮				

const.

ln 1/x

ln² 1/x

ln³ 1/x

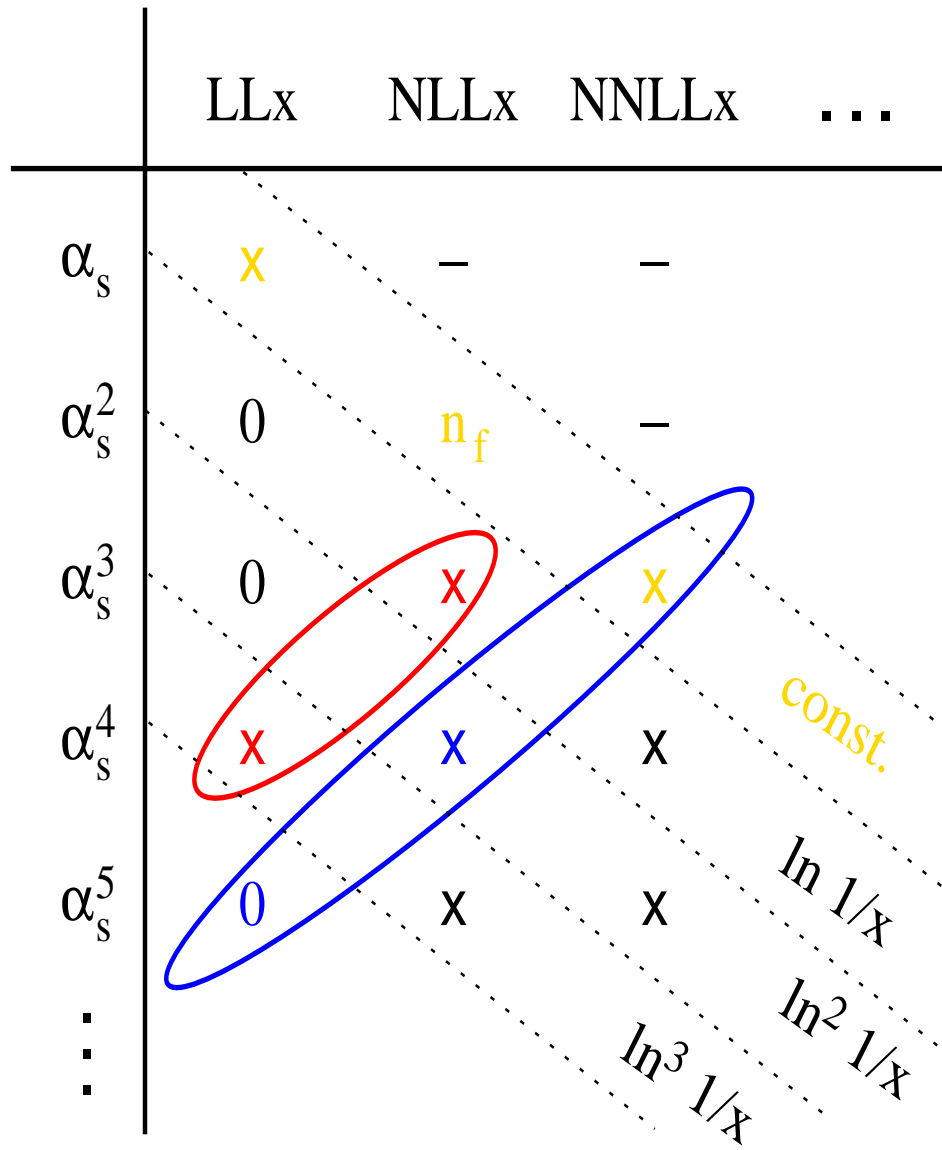Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}}$$

Depth of dip

$$-d \simeq -1.237 \bar{\alpha}_s^{5/2}$$

	LLx	NLLx	NNLLx	...
α_s	X	–	–	
α_s^2	0	n_f	–	
α_s^3	0	X	X	
α_s^4	X	X	X	
α_s^5	0	X	X	
⋮				

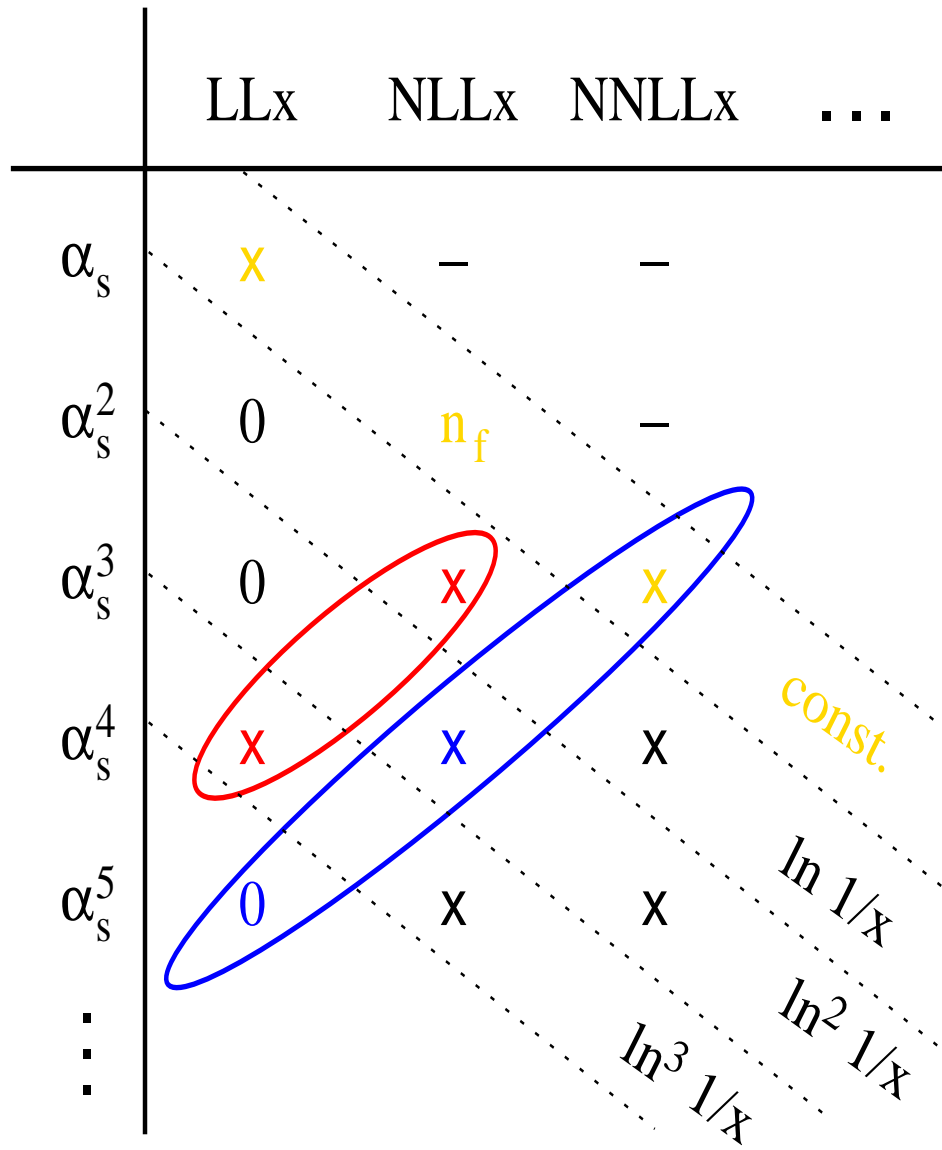*const.* *ln 1/x* *ln² 1/x* *ln³ 1/x*

Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}} + 6.947$$

Depth of dip

$$-d \simeq -1.237\,\bar{\alpha}_s^{5/2} - 11.15\,\bar{\alpha}_s^3$$

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
⋮				

const.

ln 1/x

ln² 1/x

ln³ 1/x

Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}} + 6.947 + \cdots$$
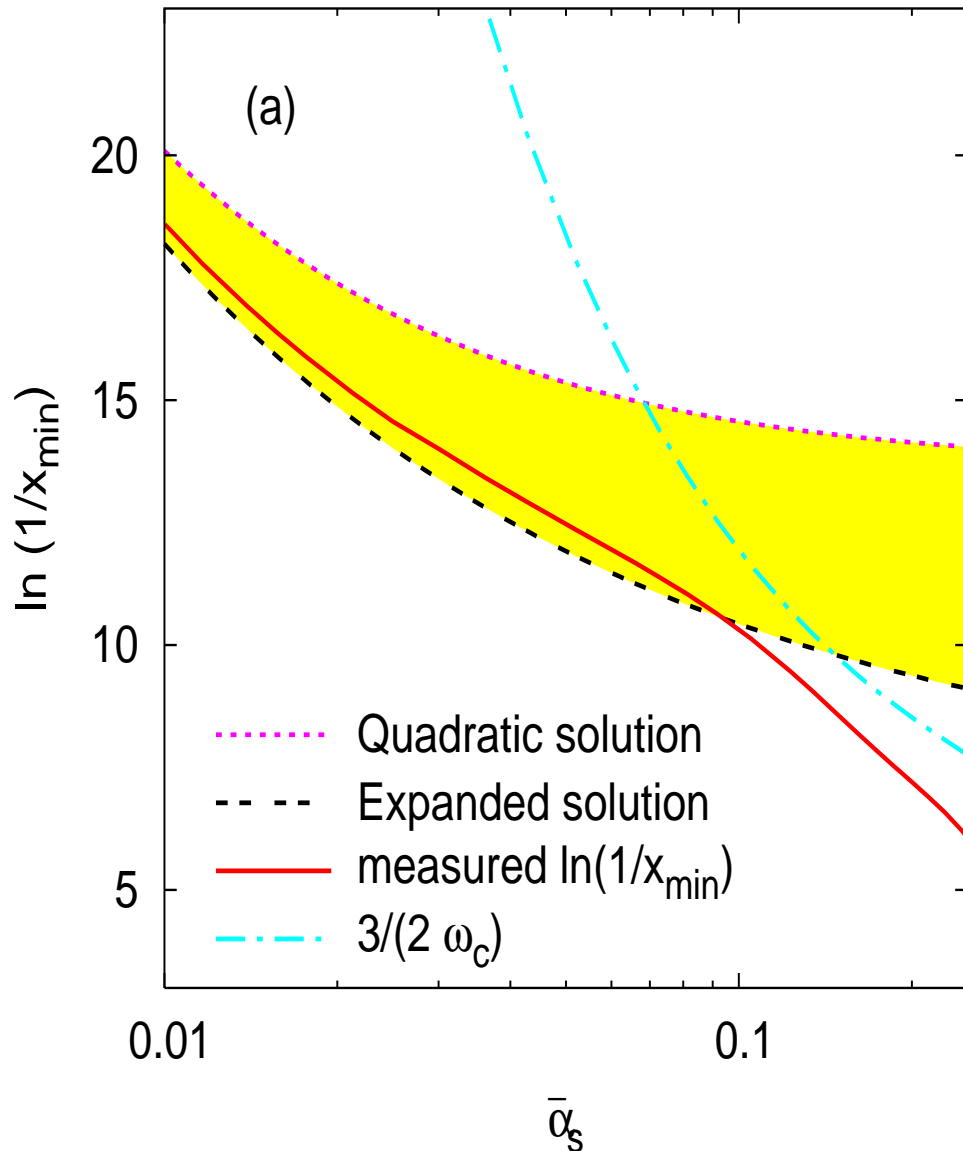
Depth of dip

$$-d \simeq -1.237\bar{\alpha}_s^{5/2} - 11.15\bar{\alpha}_s^3 + \cdots$$

<u>NB:</u>

- convergence is very poor
  As ever at small x!

- higher-order terms in expansion need NNLLx info

Test dip properties v. BFKL+DGLAP resummation

(a)

Quadratic solution
Expanded solution
measured $\ln(1/x_{min})$
$3/(2\,\omega_c)$

x-axis: $\bar{\alpha}_s$

y-axis: $\ln(1/x_{min})$

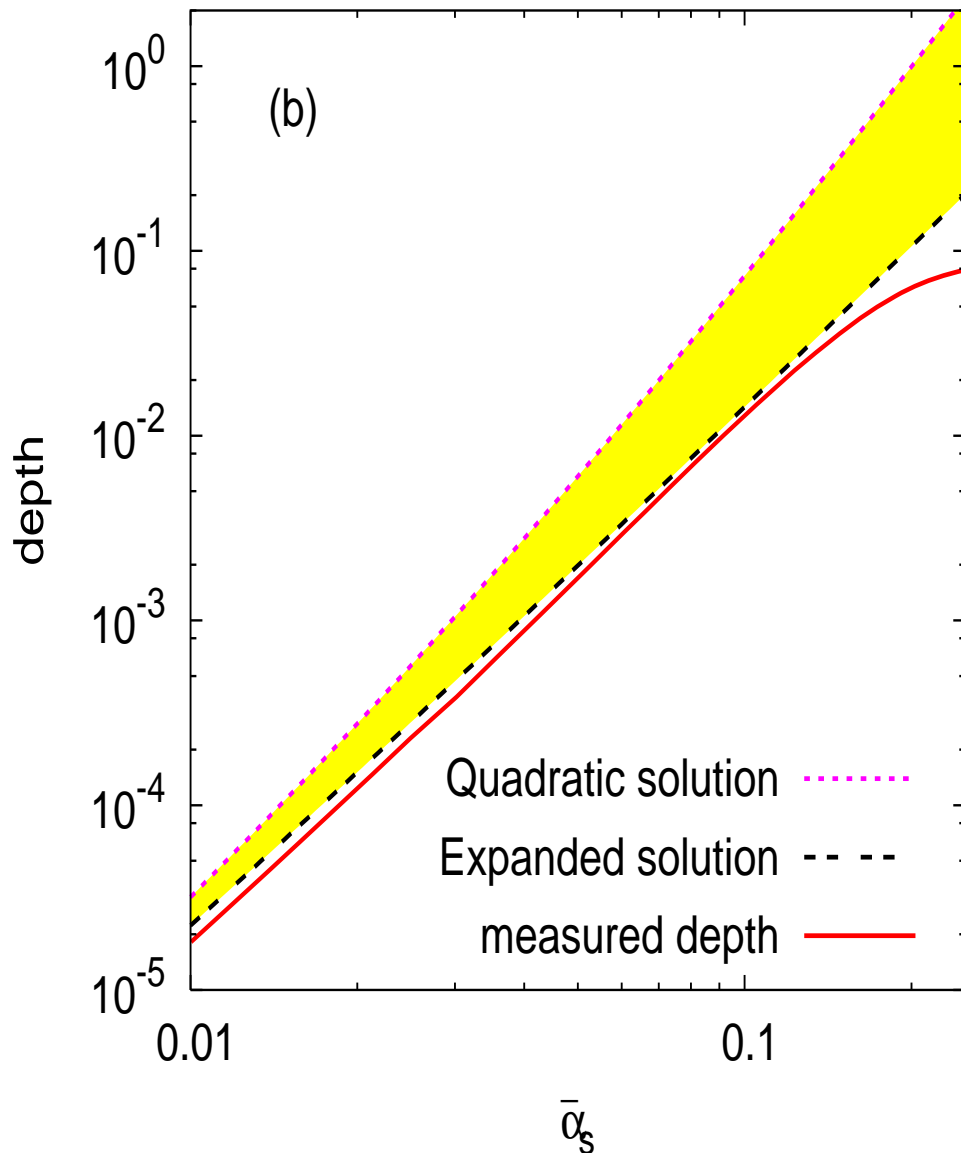## Test position of dip v. α_s

- Band is uncertainty due to higher orders in $\sqrt{\alpha_s}$

- At small α_s, good agreement $\rightarrow$ confirmation of 'dip mechanism'

- At moderate α_s, normal small-x resummation effects 'collide' with dip

$$\ln\frac{1}{x_{\min}} \lesssim \frac{3}{2\omega_c}$$

Dip then comes from interplay between $\alpha_s^3 \ln x$ (NNLO) term and full resummation.

[Actually, story more complex]

Test dip properties v. BFKL+DGLAP resummation

(b)

depth (y-axis): 10^0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}

$\bar{\alpha}_s$ (x-axis): 0.01, 0.1

Quadratic solution ······
Expanded solution - - - -
measured depth ——

Test depth of dip v. α_s

- similar conclusions!

Phenomenological relevance comes through impact on growth of small-x gluon with Q^2.

$$\frac{\partial g(x, Q^2)}{d \ln Q^2} = P_{gg} \otimes g + P_{gq} \otimes q$$

Phenomenological relevance comes through impact on growth of small-x gluon with Q^2.
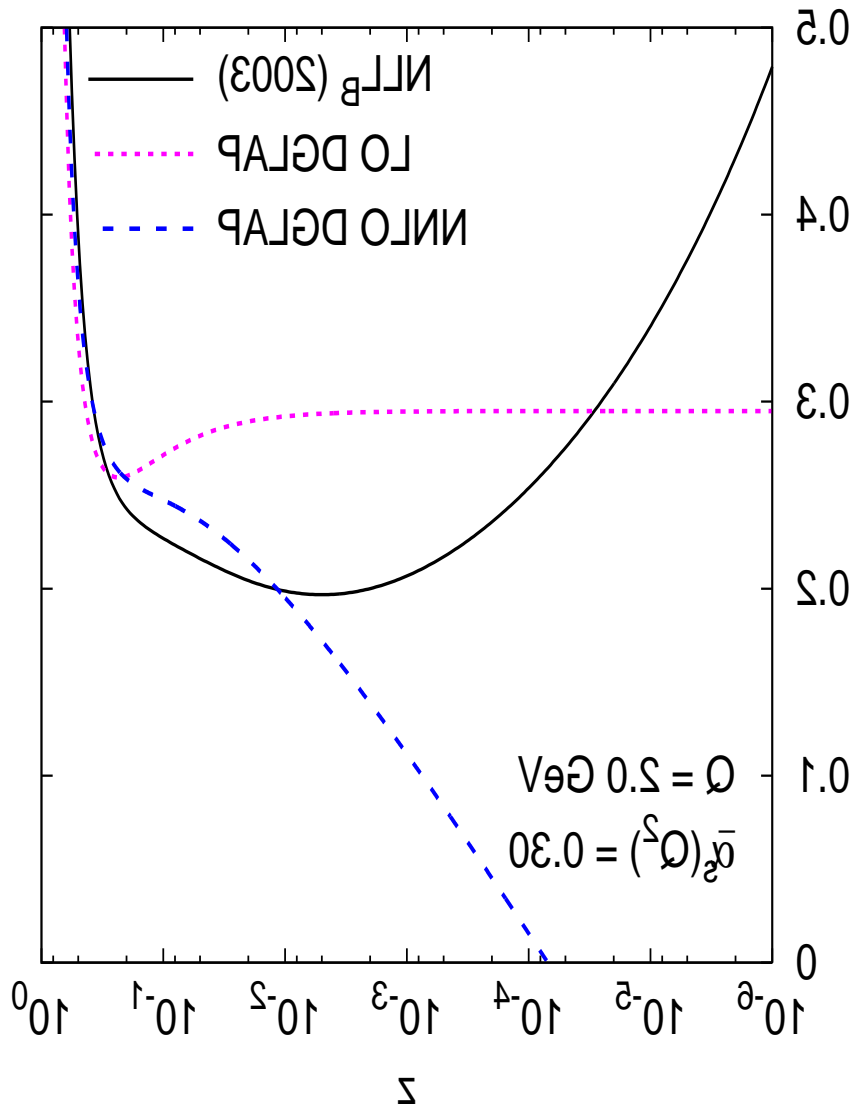
$$\frac{\partial g(x, Q^2)}{d \ln Q^2} = P_{gg} \otimes g + P_{gq} \otimes q$$

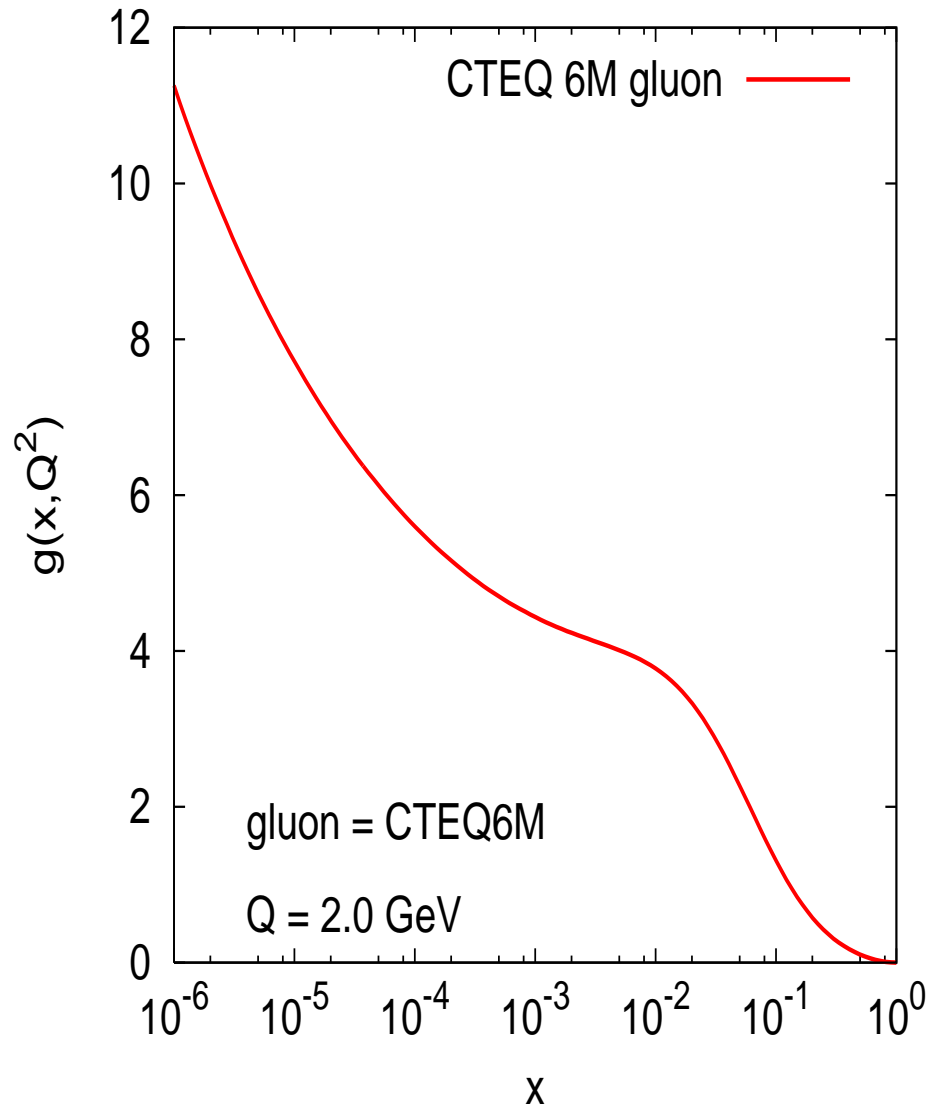At small x, $P_{gg} \otimes g$ dominates.

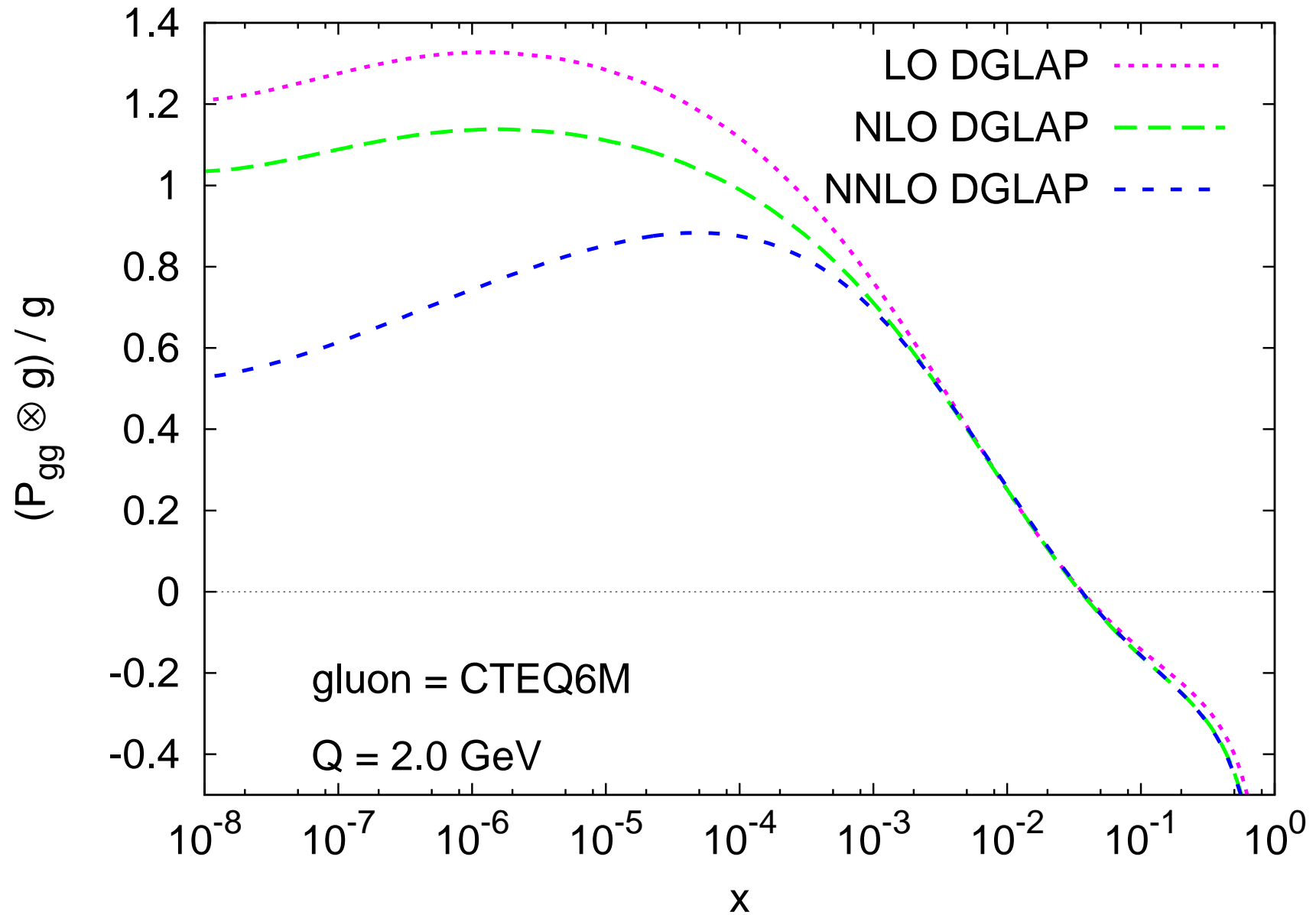Take CTEQ6M gluon as 'test' case for convolution.
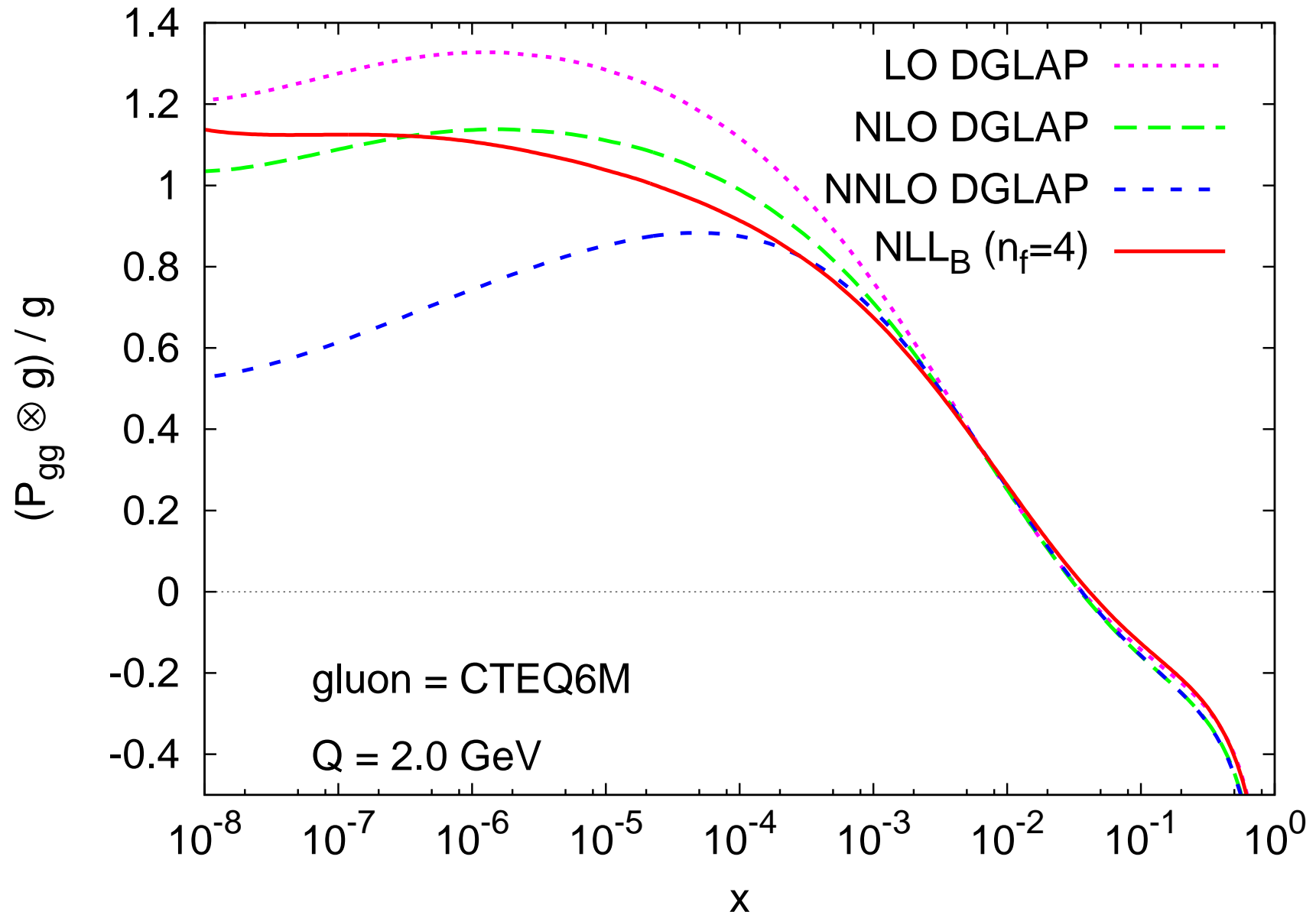
Because it's nicely behaved at small-x

Phenomenological impact? $P_{gg} \otimes g(x)$

Left panel: $z\, P_{gg}(z)$ versus z, with curves labeled NLL$_B$ (2003), LO DGLAP, NNLO DGLAP. Q = 2.0 GeV, $\bar{\alpha}_s(Q^2) = 0.30$.

$\otimes$

Right panel: $g(x,Q^2)$ versus x, CTEQ 6M gluon. gluon = CTEQ6M, Q = 2.0 GeV.

Phenomenological impact? $P_{gg} \otimes g(x)$

$P_{gg} \otimes g(x)$

gluon = CTEQ6M

Q = 2.0 GeV

LO DGLAP
NLO DGLAP
NNLO DGLAP
NLL_B (n_f=4)

$(P_{gg} \otimes g) / g$

x

Towards phenomenology?

Steps missing for 'full' phenomenology:

- Resummation of all entries of singlet matrix & coefficient functions.

- Put results in $\overline{\text{MS}}$ factorisation scheme

  ➥illustrate nature of surprises that arise...

Results shown so far in Q_0 scheme.   [Catani, Ciafaloni & Hautmann '93]

$$xg(x, Q^2) \equiv \int d^2k \, G(\ln 1/x, k, k_0)\Theta(Q - k) \qquad G^{(0)} = f(x)\delta^2(k - k_0)$$

To translate to $\overline{\text{MS}}$ scheme

$$xg(x, Q^2) \equiv \int d^2k \, G(\ln 1/x, k, k_0) r\left(\frac{k^2}{Q^2}\right), \qquad r\left(\frac{k^2}{Q^2}\right) = \int \frac{d\gamma \, e^{\gamma \ln \frac{Q^2}{k^2}}}{2\pi i \, \gamma R(\gamma)}$$

Should be easy?!

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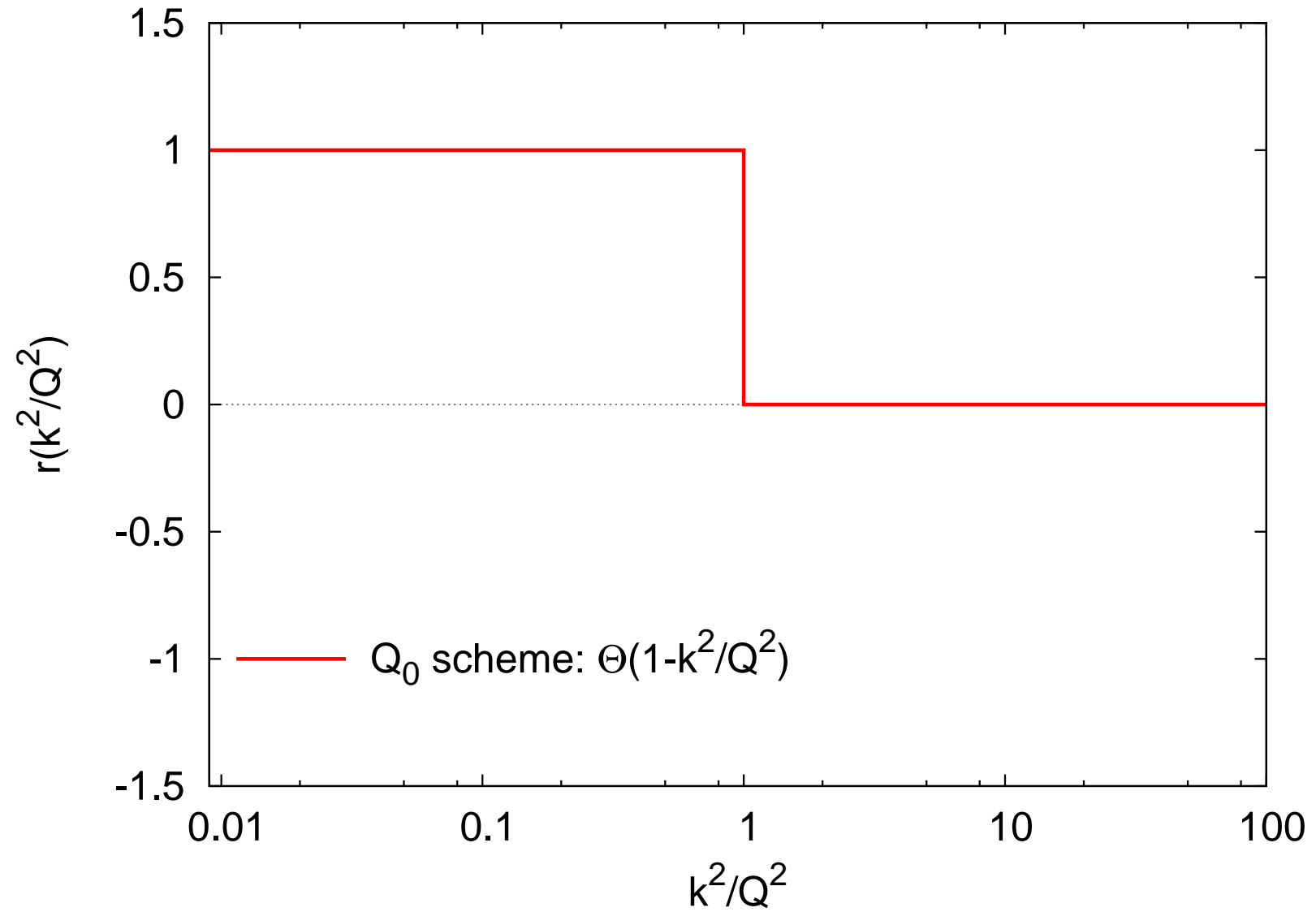
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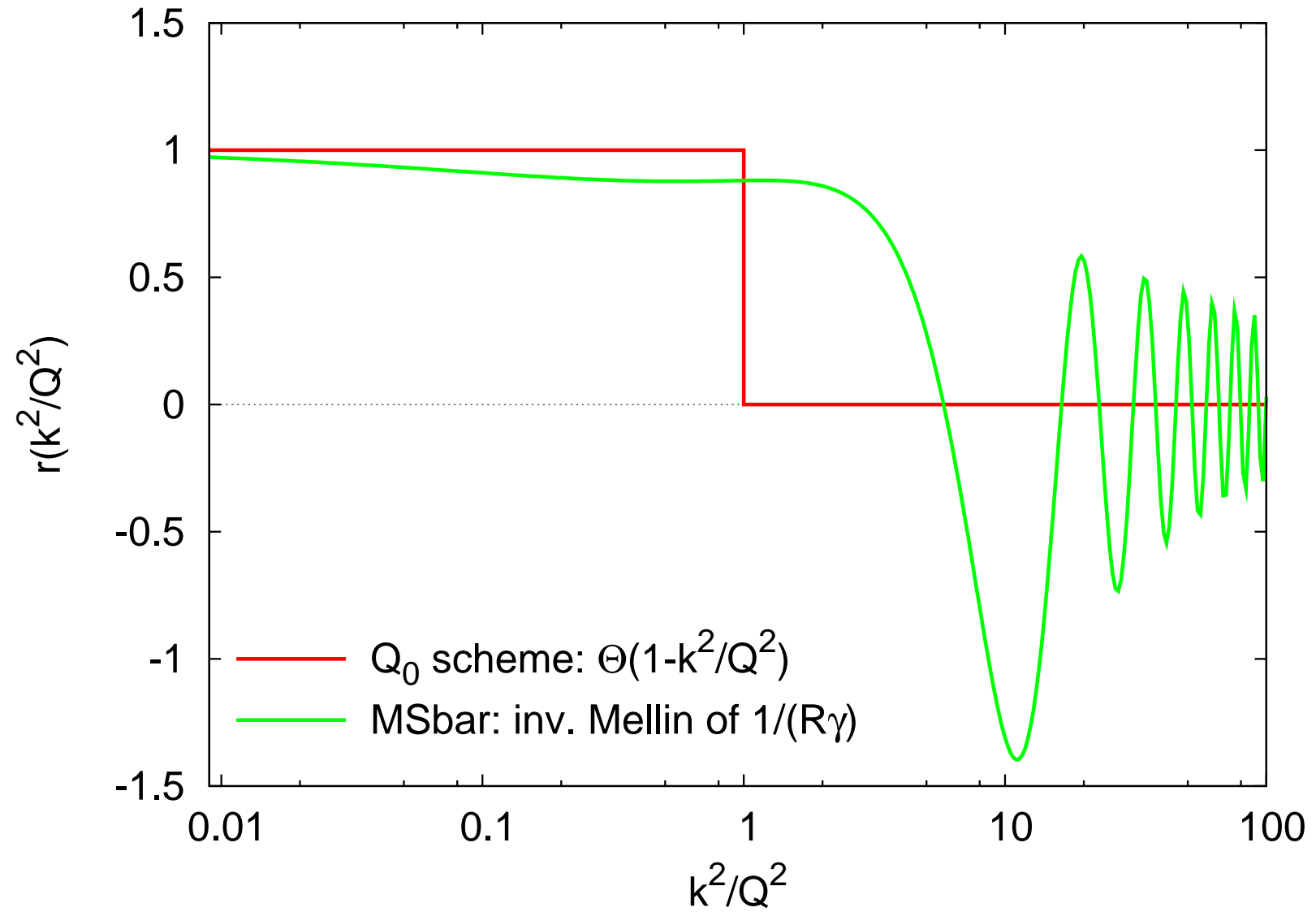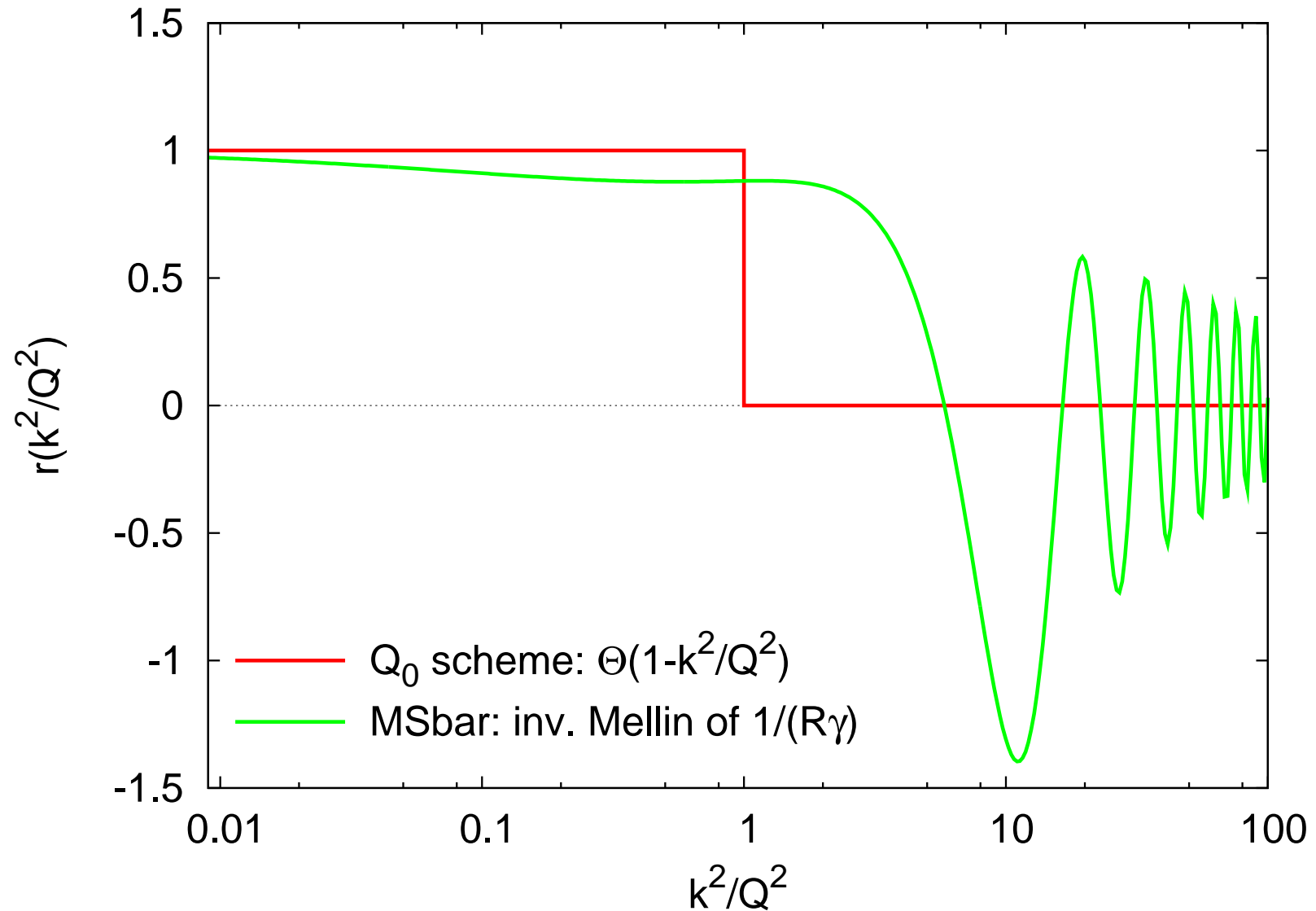
Should be easy?!

$$R(\gamma) = \left\{\frac{\Gamma(1 - \gamma)\chi(\gamma)}{\Gamma(1 + \gamma)[-\gamma\chi'(\gamma)]}\right\}^{\frac{1}{2}} \exp\left\{\int_0^\gamma d\gamma' \frac{\psi'(1) - \psi'(1 - \gamma')}{\chi(\gamma')}\right\}$$

Catani & Hautmann '94

[NB: involves $\chi(\gamma)$ — does this need to be collinearly improved? Ignore problem for now. . .]

Numerically, $\overline{\text{MS}}$ is much more difficult.

Conceptually, the oscillations are disturbing.

Conclusions

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Detailed phenomenology still needs considerably more work