

# *$\Lambda$ -Based Perturbation Theory for Event Shapes in $e^+e^-$ Annihilation*

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# Talk Plan

- What is  $\Lambda$ -Based Perturbation Theory?

*a.k.a. RESIPE (Dhar, Gupta), Method of Effective Charges (Grunberg), RGI perturbation theory...*

- Event Shapes

*taking thrust as an example*

- Previous Applications of  $\Lambda$ -Based PT to Event Shapes

*means by the DELPHI collaboration, distributions by Burby and Maxwell*

- Our Analysis

*including resummation of logs and power correction fits*

- Results

*fits for  $\Lambda_{\overline{MS}}$  and  $C_1$*

Work done in collaboration with C. J. Maxwell

# What is $\Lambda$ -based Perturbation Theory?

## Standard Approach to Perturbation Theory

- Consider a perturbative QCD expansion of an observable normalized as an **effective charge** (with  $a = \alpha/\pi$ ):

$$R(Q) = a_S(\mu) + r_1(\mu/Q, S) * a_S^2(\mu) + r_2(\mu/Q, S) * a_S^3(\mu) + \dots$$

- $R(Q)$  is independent of  $\mu$  and  $S$ , but only because of cancellation between different orders. So, all finite order predictions depend on  $\mu$  and  $S$ .
- Normally have NLO calculation  $\rightarrow \mu$  dependence at  $O(a_S^3)$ .
- Standard fix: choose  $\mu = Q$ , vary to get **theoretical error**.
- BUT the definition of  $\mu$  depends on  $S$ ! Specifically our results depend only on the ratio  $\mu/\Lambda_S$ .

# What is $\Lambda$ -based Perturbation Theory?

## $\Lambda$ -Based Approach to Perturbation Theory

- We take the **dimensional transmutation parameter**  $\Lambda$ , defined in some scheme (e.g.  $\overline{MS}$ ) as the fundamental parameter to fit to data.
- For each effective charge  $R$  we write

$$\frac{dR(Q)}{d \ln Q} = \rho_R(R) = -bR^2(1 + cR + \rho_2^R R^2 + \dots)$$

where  $\rho$  is the **effective charge  $\beta$ -function**.

- Compare with

$$\frac{da_S(Q)}{d \ln Q} = \beta_S(a) = -ba_S^2(1 + ca_S + c^S_2 a_S^2 + \dots)$$

the  $\beta$ -function for the scheme  $S$ .

# What is $\Lambda$ -based Perturbation Theory?

## $\Lambda$ -Based Approach to Perturbation Theory

- $\Lambda$  enters as a **boundary condition**. We can integrate up to find the relation

$$\Lambda_S e^{-r_1(\mu=Q,S)/b} = Q \mathcal{F}(R(Q)) \mathcal{G}(R(Q)) \left( \frac{2c}{b} \right)^{c/b}$$

where the LHS and RHS are **scheme independent**. Explicitly:

$$\mathcal{F}(R) = e^{-1/bR} (1 + 1/cR)^{c/b}$$

$$\mathcal{G}(R) = \exp \left[ - \int_0^R dx \left( \frac{1}{\rho_R(x)} + \frac{1}{bx^2(1+cx)} \right) \right]$$

- At NLO  $\rho_R(x) = -bx^2(1+cx)$  so  $\mathcal{G} = 1$ .

# Event Shapes

- Eg. thrust

$$\tau \equiv 1 - T \equiv 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

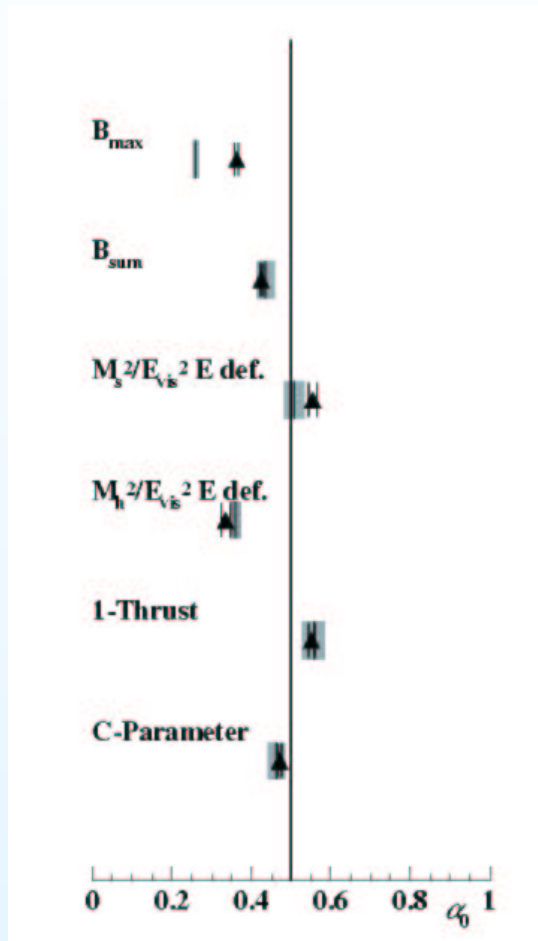
- Standard PT on its own doesn't describe  $\langle 1-T \rangle$  very well. Instead we fit for an additional “power correction” giving something like:

$$\langle \tau \rangle = \langle 1 - T \rangle = 0.335\alpha_{\overline{MS}}(Q) + 1.02\alpha_{\overline{MS}}^2(Q) + \frac{C_1}{Q}$$

where  $C_1 \sim 1\text{GeV}$ .

- More sophisticated approach: “Dokshitzer-Webber ansatz”.  
Relate the power correction to a hypothetical infrared finite coupling  $\rightarrow$  **relate power corrections to different observables!**
- Gives fairly successful description of data - “approximate universality” for new parameter  $\bar{\alpha}_0$ .

# $\Lambda$ -based PT for Event Shape Means @ DELPHI

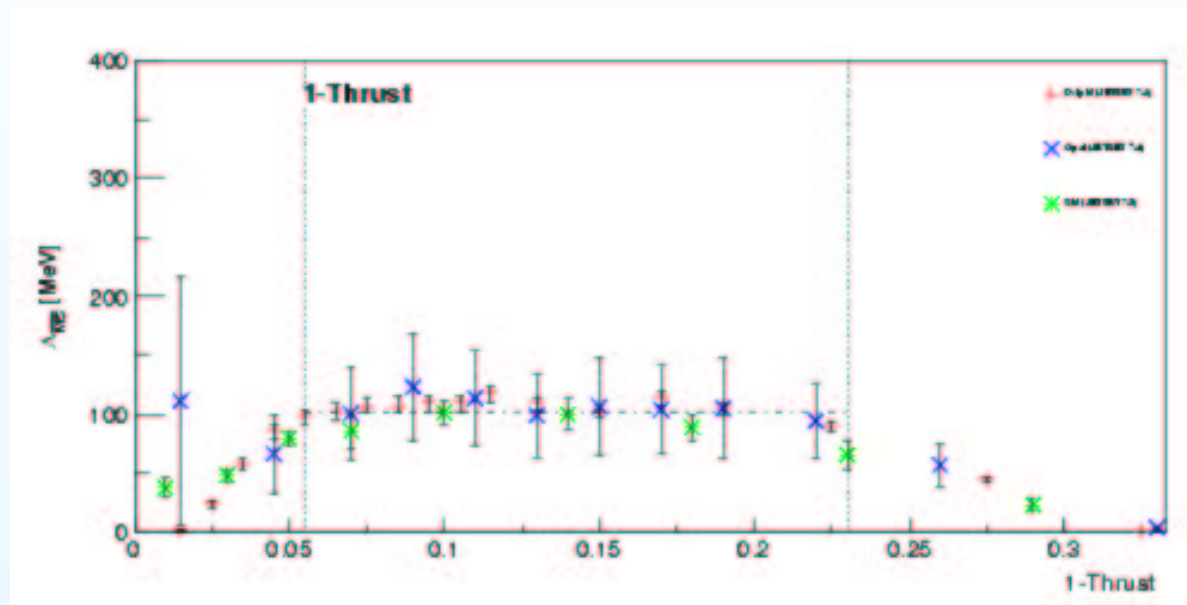


- The DELPHI collaboration have found that  $\Lambda$ -based PT describes the event shape means well without any power corrections.
- This plot shows the effective  $\overline{\alpha}_0$  induced by converting the  $\Lambda$ -based results to the  $\overline{MS}$  scheme.
- Remarkable agreement with experiment compared to universality hypothesis!

*A study of the energy evolution of event shape distributions and their means with the DELPHI detector at LEP, DELPHI Collaboration, J. Abdallah, et al, Eur.Phys.J. C29 (2003) 285-312*

# $\Lambda$ -based PT for Event Shape Distributions

- Also interesting to look at **distributions**. At NLO:



- Problems:
  - Large **logs**  $L = \log(1/\tau)$  appear in the 2-jet region  $\tau \rightarrow 0$
  - The EC description **breaks down** as  $\tau \rightarrow 1/3$

*Direct Extraction of QCD Lambda MS-bar from e+e- Jet Observables*, S. J. Burby, C. J. Maxwell,  
Nucl.Phys. B609 (2001) 193-224



## Our Analysis

We wanted to build on this analysis in 2 ways:

- Large logs in the perturbative series for the distribution give large logs in the perturbative series for  $\rho \rightarrow$  need for **resummation**:

$$\rho(R, L) = -bR^2(\rho_{LL}(RL) + L^{-1}\rho_{NLL}(RL) + \dots).$$

- To see if the  $\Lambda$ -based approach needs smaller power corrections we wanted to **fit for  $1/Q$  shifts**:

$$\left. \frac{1}{\sigma} \frac{d\sigma}{d\tau} \right|_{\tau} = \left. \frac{1}{\sigma} \frac{d\sigma}{d\tau} \right|_{PT, \tau - C_1/Q}$$

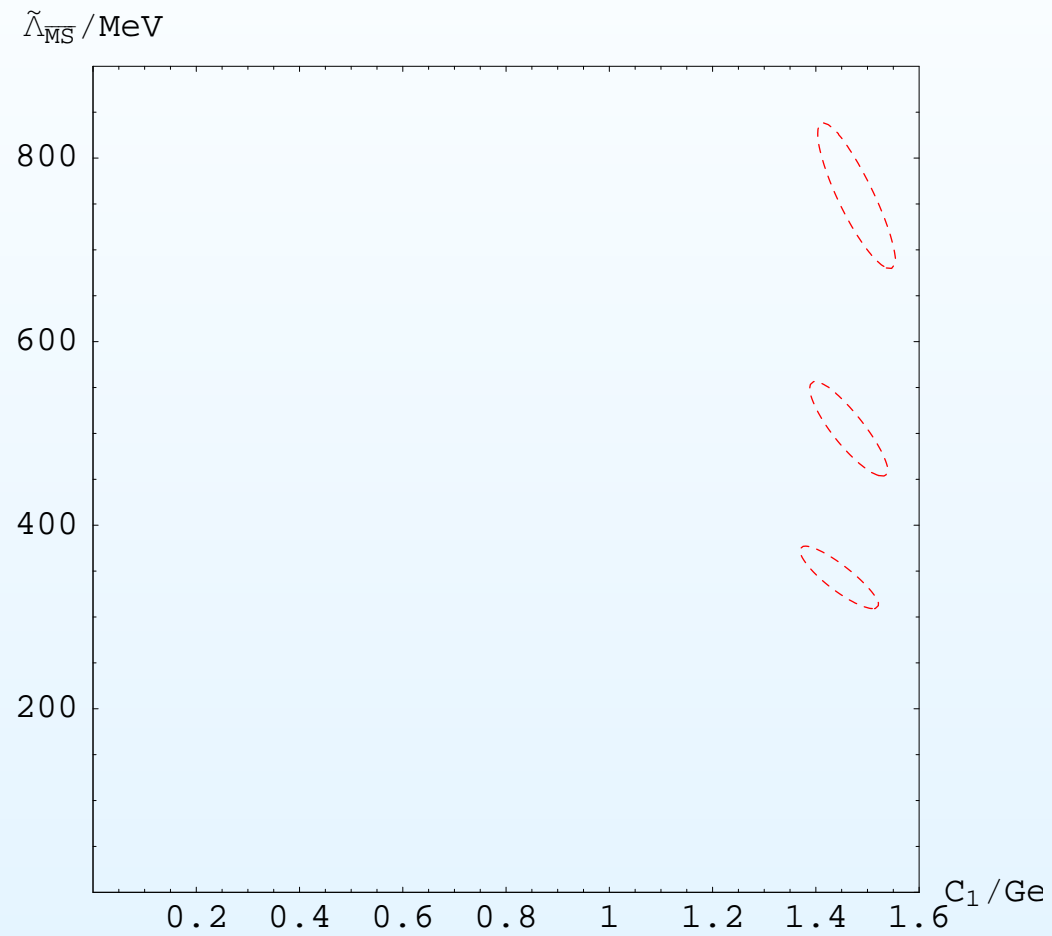
- Define an effective charge via

$$R(Q, \tau) = \ln \left( \int_0^{\tau} \frac{1}{\sigma} \frac{d\sigma}{d\tau} \right) / LO$$

## Our Analysis

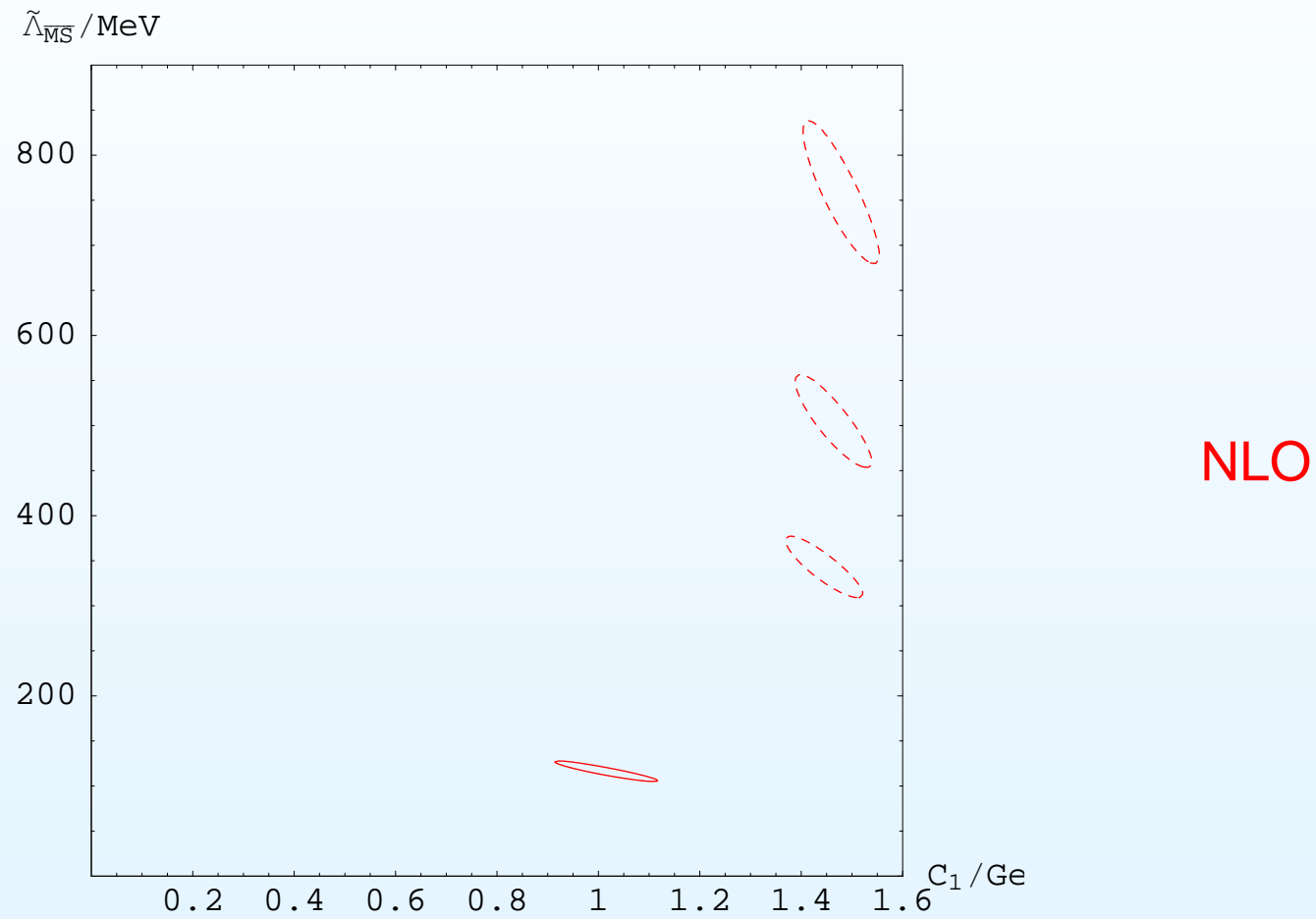
- We need to select a limited fit range to avoid problems in the extreme 2-jet region and as  $\tau \rightarrow 1/3$  (made worse by exponentiating).
- Found good fit quality with  $\tau = 0.04 - 0.16$  at  $Q = 91.2 GeV$  with the lower bound scaling  $\propto 1/Q$ .
- Take data from TASSO, JADE, DELPHI, L3, SLD at energies  $Q = 35 - 189 GeV$ .
- Note that the “standard approach” in the  $\overline{MS}$  scheme we can fit over a large range of  $\tau$ !

# Fits: Thrust

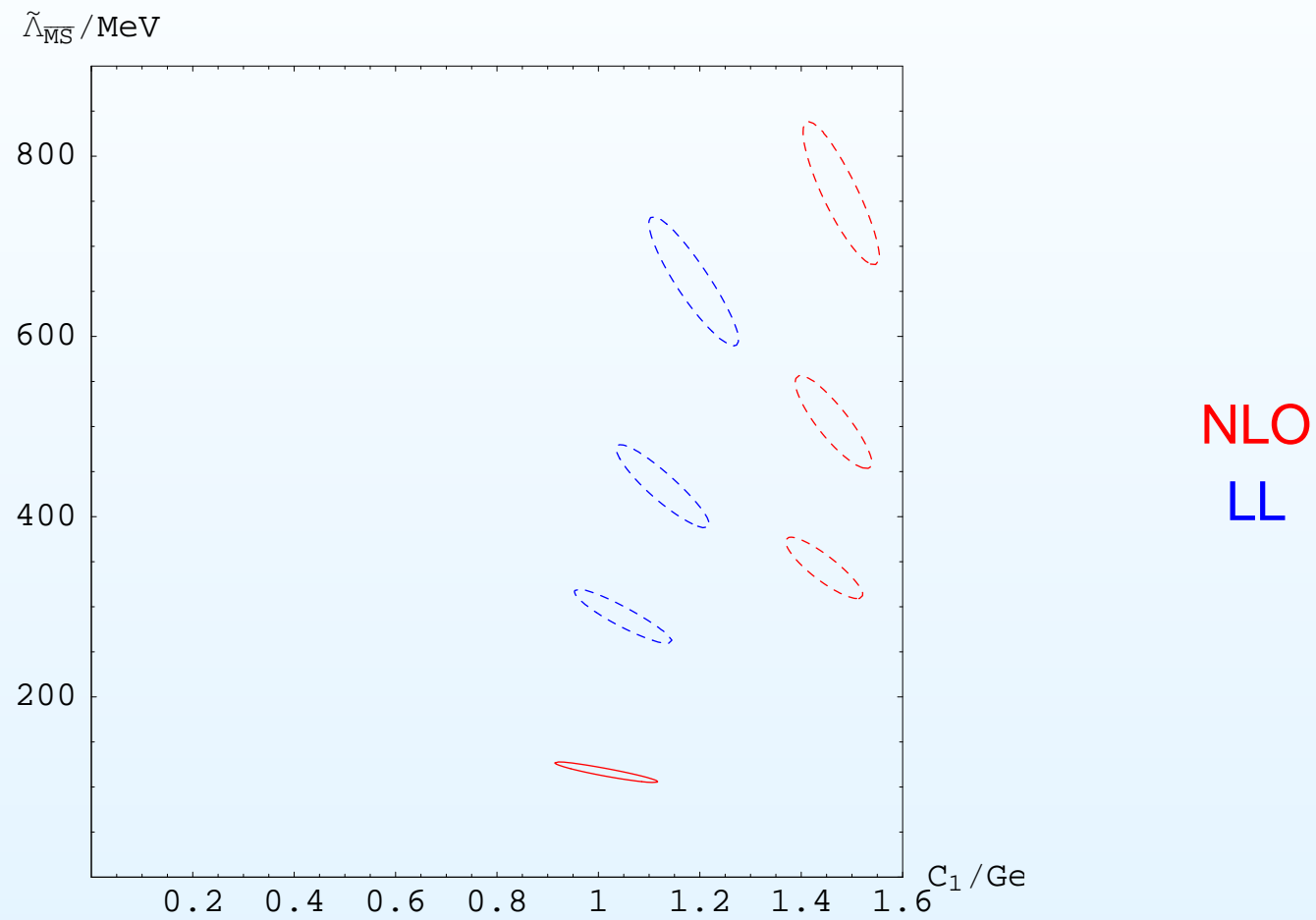


NLO

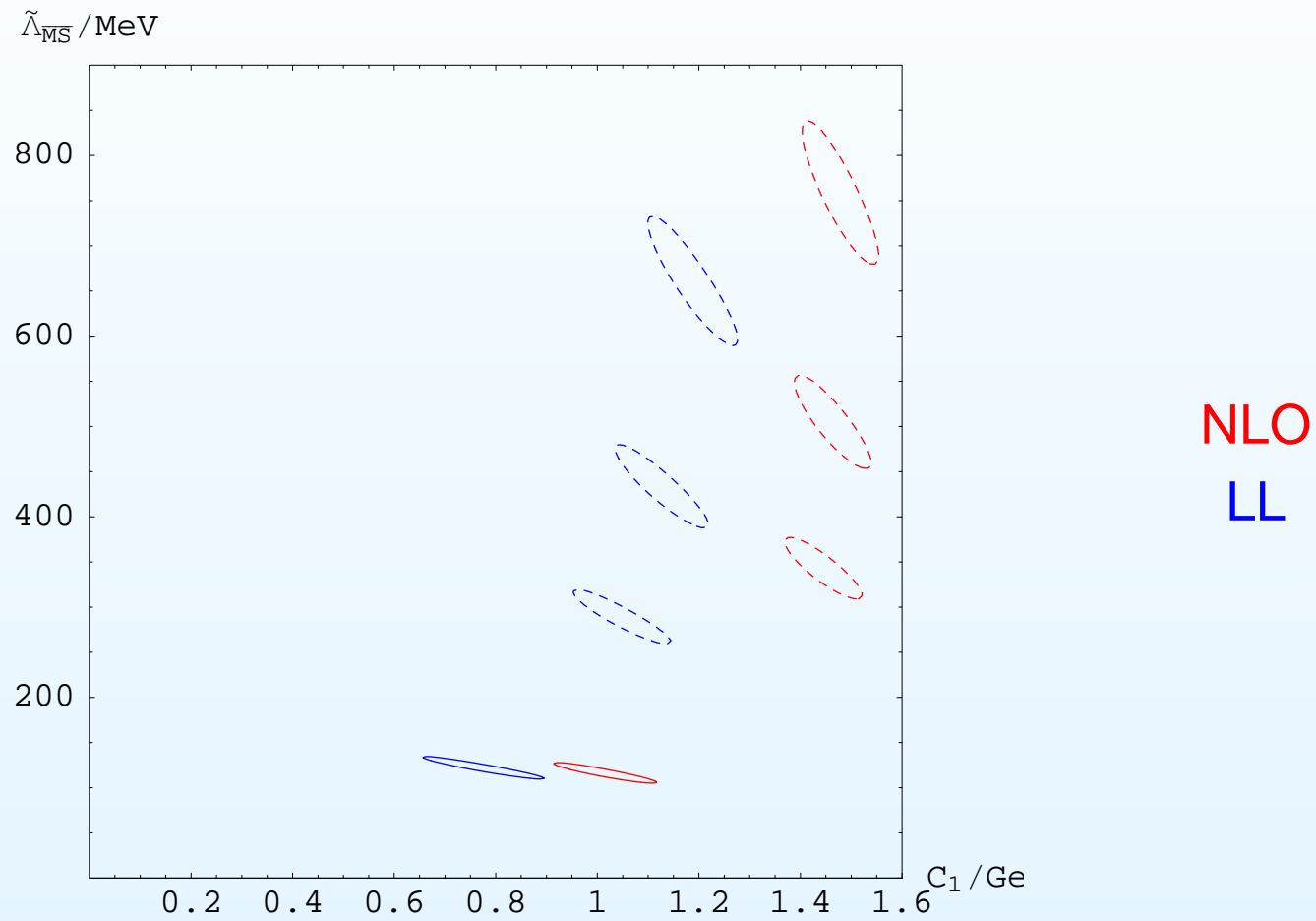
# Fits: Thrust



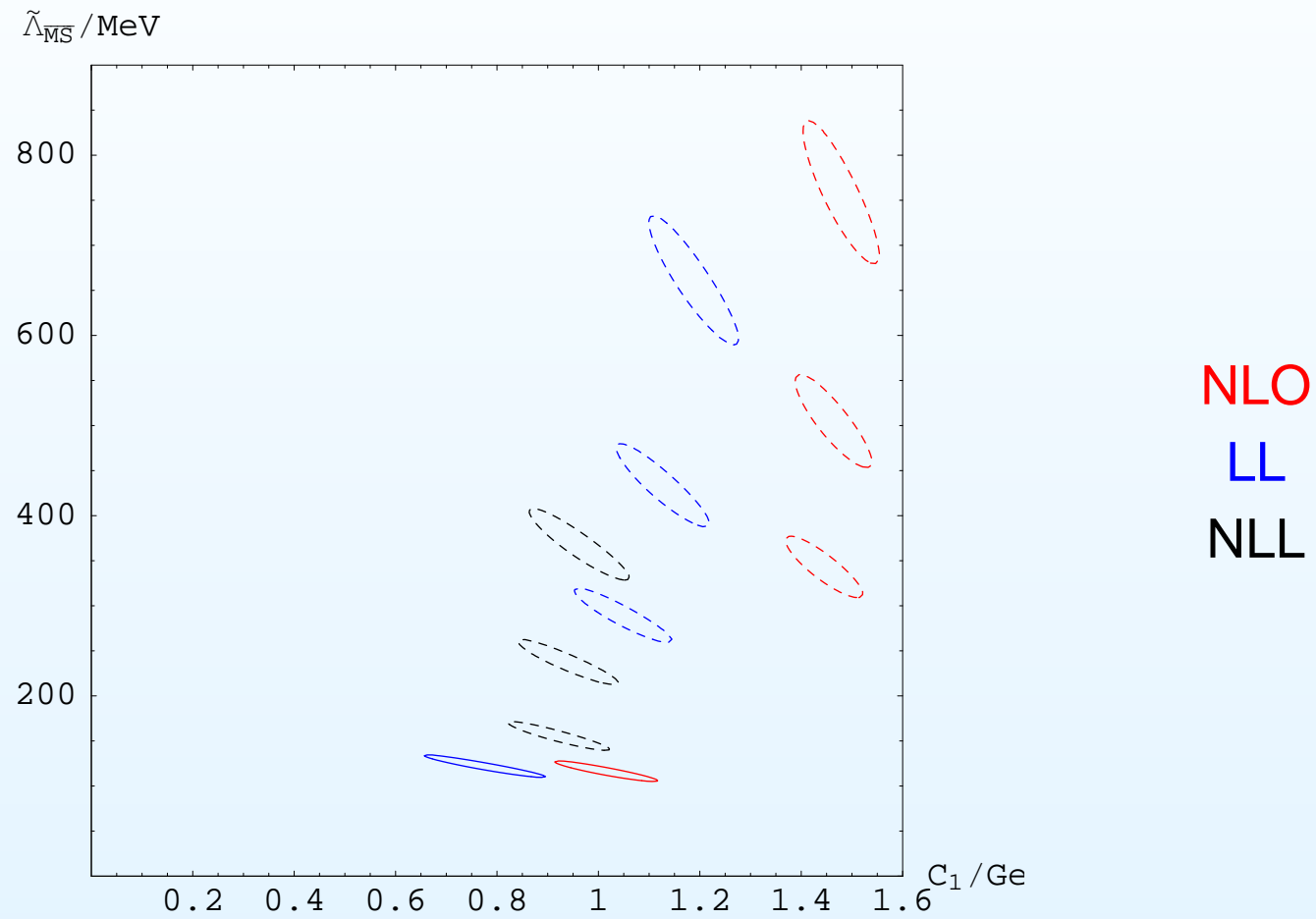
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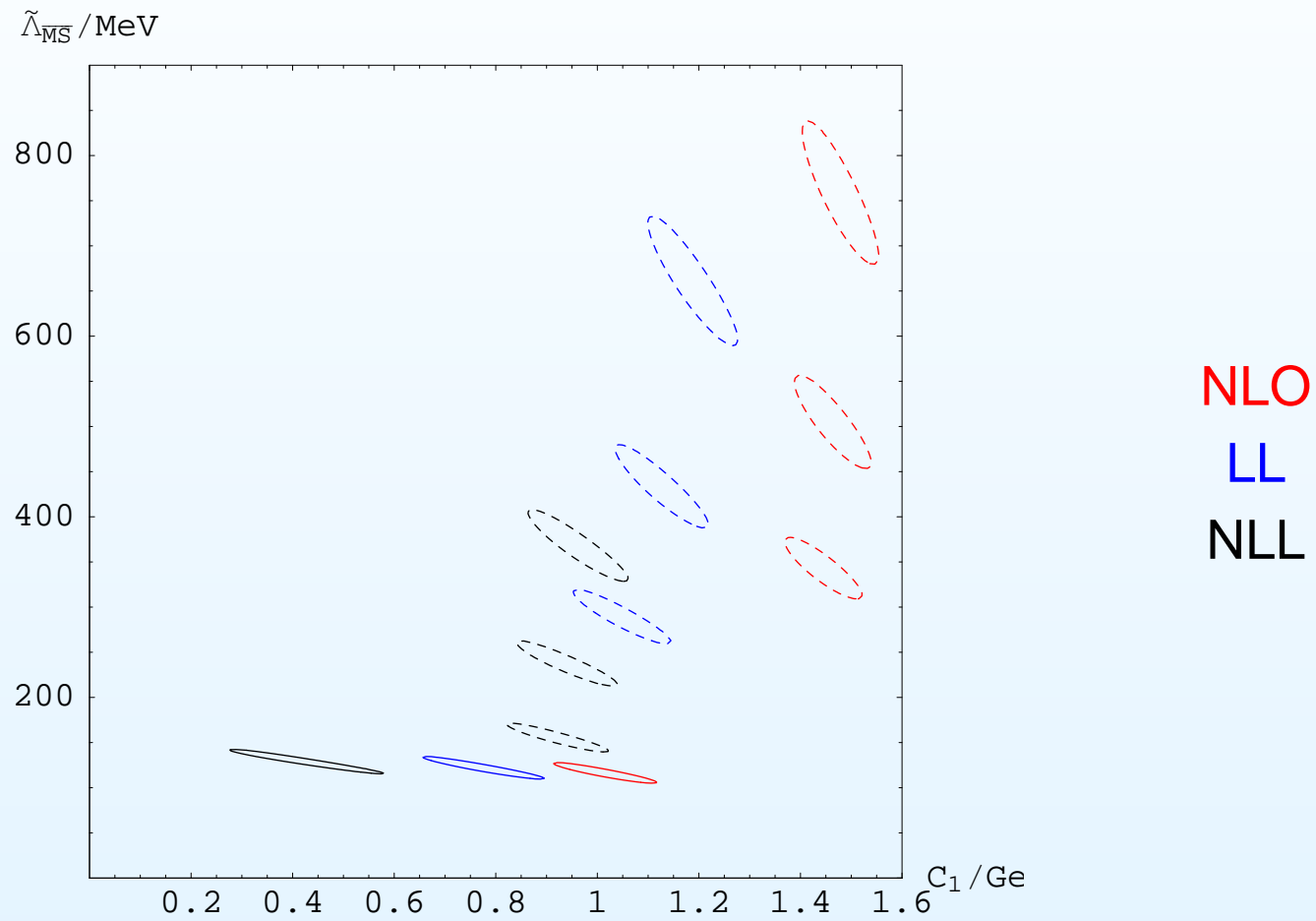
# Fits: Thrust



# Fits: Thrust



# Fits: Thrust





## Conclusions

- $\Lambda$ -based PT provides us with a way to make perturbative predictions without needing to fix  $\mu$  by some kinematical argument.
- Analysis of event shape means has suggested this may give more accurate predictions than the standard approach, with significantly reduced power corrections needed to fit the data.
- We have applied this approach to some event shape distributions, and after significantly restricting the fit range we find good agreement with reduced (though still significant) power corrections.
- Adding resummed logs further reduces the fitted power correction.