IR Freezing of QCD Observables

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Outline

- Highlight some of the problems we have with perturbative QCD in the infrared.
- Briefly discuss how by taking large orders of perturbation theory into account we can overcome some of these problems.
- Compare standard perturbation theory with large order perturbation theory.

Perturbative QCD

- We want to evaluate perturbative corrections,

$$\mathcal{K}(\alpha_s) = \sum_{n=0}^{\infty} k_n \alpha_s^{n+1}$$

- Problems:
  - Definition of α_s in the infrared.
  - The series doesn't converge, $k_n \sim \left(\frac{b}{2}\right)^n n^\gamma n!$, where b is the first coefficient of the beta function.

$$b_{QCD} = \frac{1}{6}(11N_c - 2N_f)$$

- These issues are interlinked.

Divergent series

- After $n \sim \frac{1}{\alpha_s}$ terms the series becomes nonsense.
- We can use the Borel transform method to make more sense out of a divergent series

$$B[\mathcal{K}](z) = \sum_{n=0}^{\infty} \frac{k_n}{n!} z^n \qquad \Rightarrow \qquad \mathcal{K}(\alpha_s) = \int_0^{\infty} e^{-z/a} B[\mathcal{K}](z)$$

- For example if $k_n = \left(\frac{b}{2}\right)^n n!$

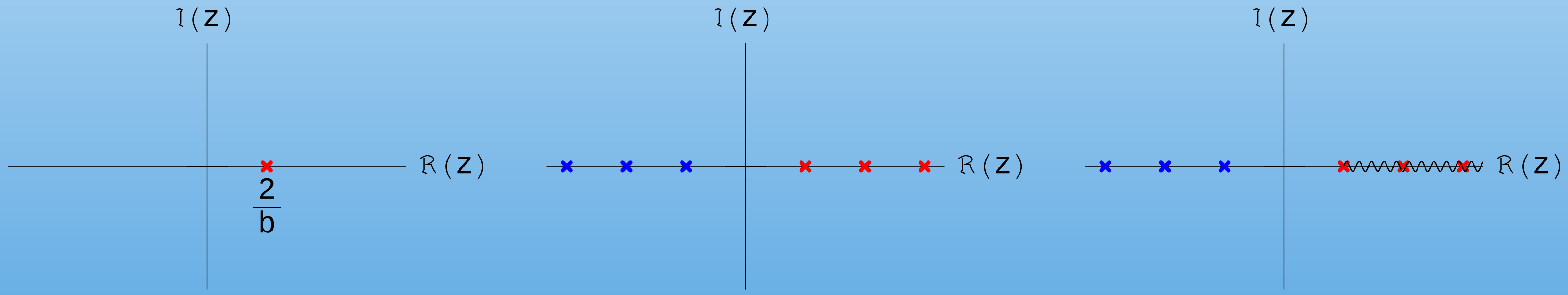
$$\Rightarrow B[\mathcal{K}](z) = \frac{1}{1 - bz/2}$$

- Singularities in the Borel transform characterize the divergent nature of the series

Borel singularity structure

$$\int_0^\infty e^{-z/a}\frac{1}{1-bz/2} \qquad \int_0^\infty e^{-z/a}\frac{\sum_l A_l}{1-bz/2l} \qquad \int_0^\infty e^{-z/a}\frac{\sum_l A_l}{(1-bz/2l)^\gamma}$$

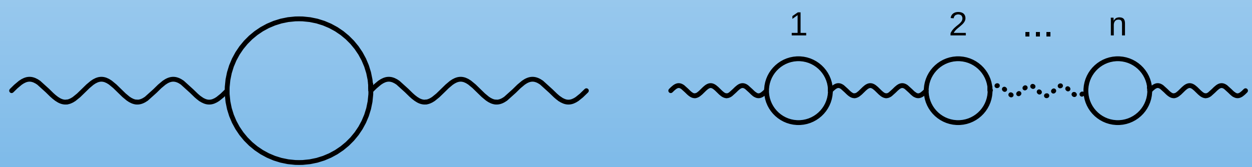
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$



- IR renormalons on the positive real semi-axis cause problems.
- UV renormalons on the negative real semi-axis are fine.

Renormalons Ia

- We can extract all orders results from the so called 't Hooft renormalon.

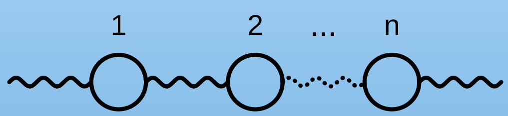


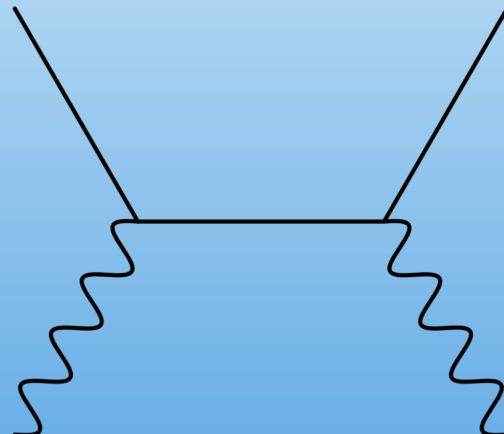
$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

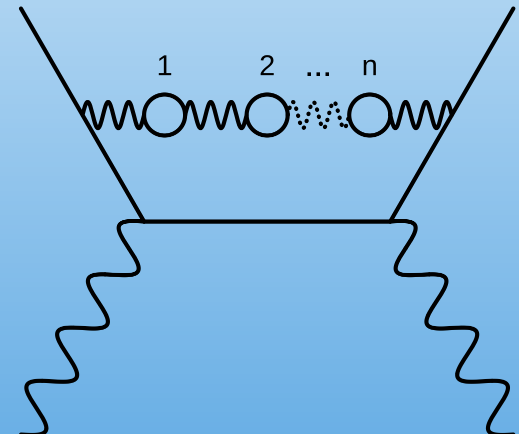
$$\sim a N_f \ln k^2 \qquad\qquad \sim a^n N_f^n (\ln k^2)^n$$

Renormalons Ib

- $\Xi(K^2)$ represents the skeleton diagram and can be obtained from a 1-loop calculation in some cases.







$\downarrow$ $a^n N_f^n (\ln k^2)^n$

$\downarrow$ $a\Xi(k^2)$

$\downarrow$ $a^{n+1} N_f^n \int dk^2 \Xi(k^2)(\ln k^2)^n$

Renormalons II

$$\sum_{n=0}^{\infty} a^{n+1} \int_0^{\infty} \Xi(k^2) \Big(\ln k^2 \Big)^n$$

- If we isolate a power of k^2 in the kernel

$$\sum_n a^{n+1} N_f^n \int dk^2 k^{2l} (\ln k^2)^n \quad \longrightarrow \quad \sum_n a^{n+1} N_f^n \int dz e^{-z} z^n = a^n n!$$

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$$\downarrow$$

$$\sum_n \int dz e^{-z/a} z^n \quad \longrightarrow \quad \int dz e^{-z/a} \frac{1}{1-z}$$

- So we can use $\Xi(k^2)$ to partially determine the Borel transform.

QCD Observables

$$K_{pBj} \equiv \int_0^1 g_1^{ep-en}(x, Q^2)dx \qquad K_{GLS} \equiv \frac{1}{6}\int_0^1 F_3^{\bar{\nu}p+\nu p}(x, Q^2)dx$$

$$K = A + a + k_1 a^2 + k_2 a^3$$

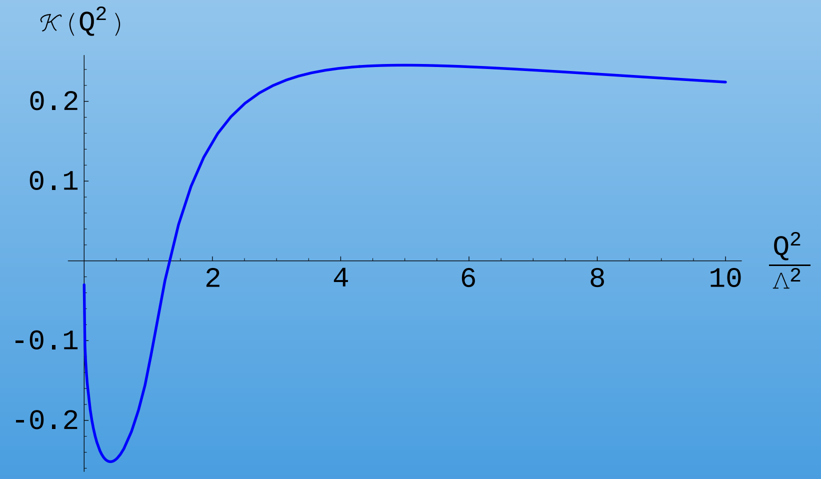
$$\begin{aligned} k_1 &= k_1^{[1]} N_f + k_1^{[0]} \\ k_2 &= k_2^{[2]} N_f^2 + k_2^{[1]} N_f + k_2^{[0]} \end{aligned}$$

$$B[\mathcal{K}](z) = \frac{4/9}{1+bz/2} - \frac{1/18}{1+bz/4} + \frac{8/9}{1-bz/2} - \frac{5/18}{1-bz/4}$$

GLS Sum Rule

- Performing the Borel integral for the GLS sum rule gives

$$\mathcal{K}(a) = \frac{1}{9b}[-8e^{2/ba}\mathrm{Ei}(-2/ba) + 2e^{4/ba}\mathrm{Ei}(-4/ba) + 16e^{-2/ba}\mathrm{Ei}(2/ba) - 10e^{-4/ba}\mathrm{Ei}(4/ba)]$$



- Where we have used $a(Q^2) = \frac{1}{b\ln(Q^2/\Lambda^2)}$

Perturbative 'scheme'

- The 1-loop form of the coupling is rather simplistic. We need to make our model more realistic.

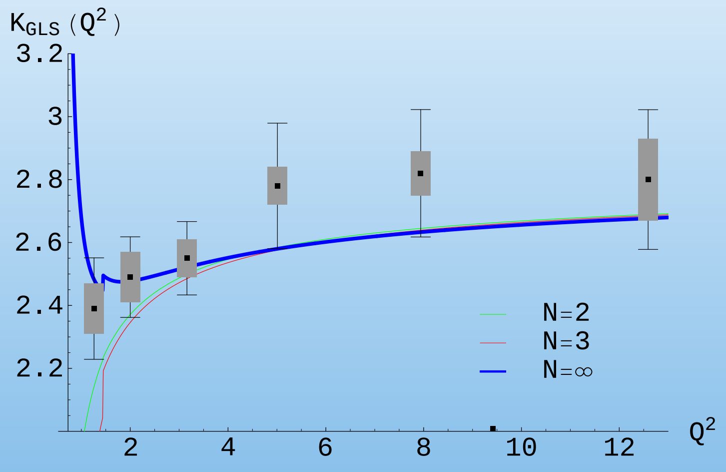
$$a_0(q) = \frac{-1}{c\left(1 + W_{-1}\left[-\frac{1}{e}\left(\frac{q}{\Lambda_{\overline{MS}}}\right)^{-b/c}\right]\right)}$$

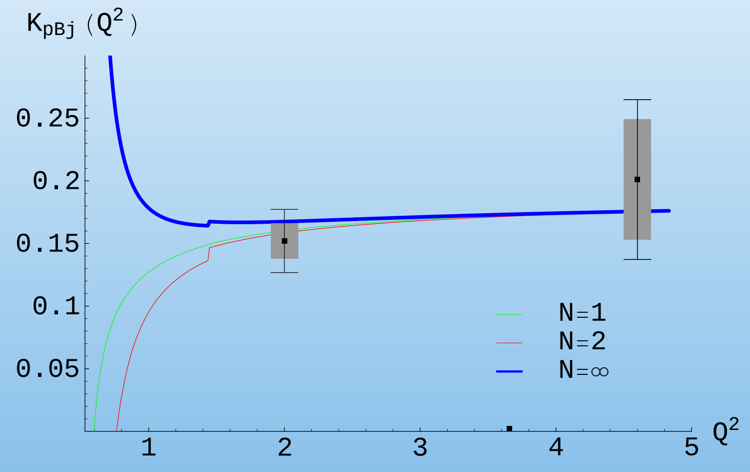
- We also need to make this a QCD calculation. We make the replacement.

$$N_f \to -3b_{QCD} = N_f - \frac{11}{2}N_c$$

- This is known as naïve non-abelianization and approximates those terms subleading in N_f rather well.

Results





- Fixed order and large order perturbation theory begin to disagree at about 2 GeV.
- We can do the same analysis for the unpolarized Bjorken sum rule but we have no experimental data.

Conclusions

- The Borel transform can tell us about perturbation theory at large orders and can also tell us about non-perturbative physics.
- Resumming part ot the perturbative coefficients can give us a result which ‘makes sense’ in the infrared.
- Fixed order perturbation theory and large order perturbation theory seem to disagree at low Q^2.