Recommendations for Limits

CDF Statistics Committee

Introduction	**Louis Lyons**
Bayes	**Joel Heinrich**
Bayes'	**Luc Demortier**
Conclusions	

Introduction

Different Methods for Limits

Different Methods for Systematics

Pro's and Con's of Methods

Reading List

Scenario

n = Poisson(m) Þ $\frac{e^{-m} m^{n}}{n!}$

m = **s** e + **b**

s = signal strength

e = **efficiency *** L ³ **0**

b = background

Simplest version: e**, b precisely known**

More realistic

e**, b estimated**

Multichannel

e_i**,** b_i **correlated**

Kinematic variables

Theory Uncertainties

Philosophy

Important

Non-trivial: CERN, FNAL CLW's

Several wrong, but no unique correct method

Part of larger picture

- **2-sided limits (rate estimation)**
- **p-values for discovery**

Computable

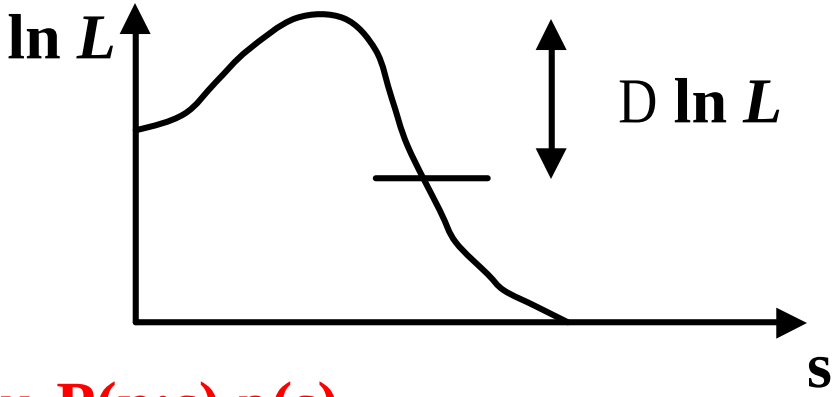
Applicable to more than 1 channel counting

Wrong Question?

- **2-sided limits**
- **Unphysical values**
- **Extent of exclusion**

Methods for Limits

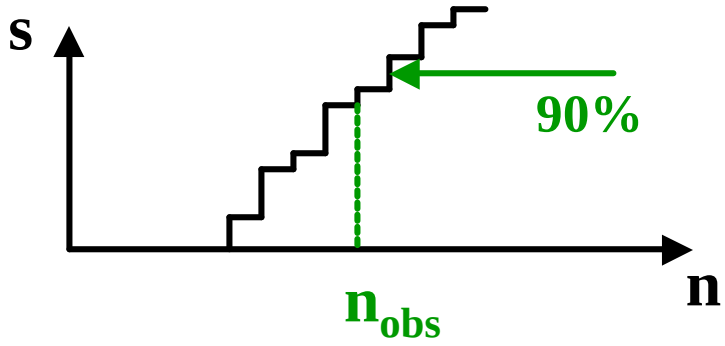
L **(or** χ^2**)**



Bayes $p(s;n) \propto P(n;s)\, p(s)$

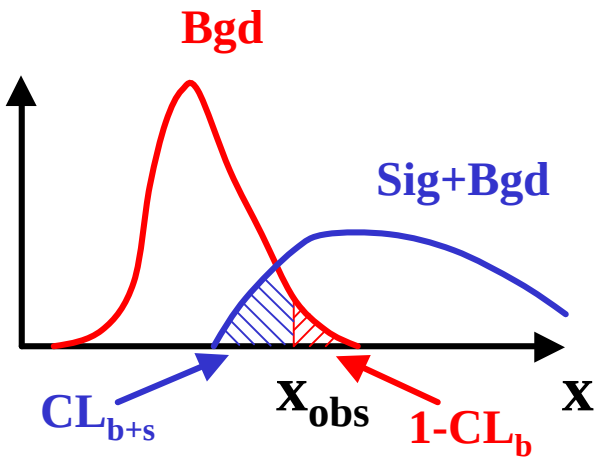
Posterior **Prior**

Frequentist



CL_s

$CL_s = CL_{b+s} / CL_b$



Methods for Systematics

1. **Shift** $\theta \pm \sigma_{syst}$
2. ***L*: Marginalise or Profile (widen)**
3. **Bayes**
4. **Frequentist**
5. **Cousins-Highland**

N.B. ε_i, b_i **can have correlated uncertainties.** σ_b **not as serious as** σ_ε

Desirable Features

Computable in practice

Coverage (no undercoverage

not too much overcoverage

no very large intervals)

No empty or very short intervals

Discriminate against incorrect values

1 sided « 2 sided « signal discovery

Adaptable to N_c channels

Incorporate (correlated) systematics

Consistent philosophy for all parameters?

Only physical values of parameters

Feature of Bayes

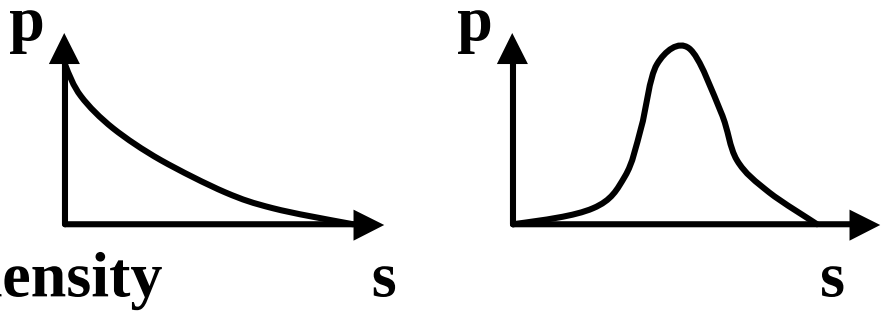
Easy to understand

"Easy" to compute

"Easy" to incorporate systematics

Limits are physical

2-sided « 1-sided



Not max posterior prob density

Box method

Priors (especially multidimensional)

Robustness, Divergent posterior*

Coverage: Average coverage theorem

Bayes Priors

Physicists favourite choice

s: constant

improper

unbelievable

e: **Gaussian (truncated at zero)**

***PROBLEM* Divergent Posterior**

(e ® **0, s** ® ¥)

Possible solutions

Truncate s (100pb « 100mb?)

See Luc's plot

Truncate Gaussian harder

Replace Gaussian by G**,** b**, ln-normal**

See Joel's talk *

p**(s**,e) ¹ p**(s)** p(e)

See Luc's talk *

Features of Frequentism

No Prior

Coverage **Clifford, Luc**

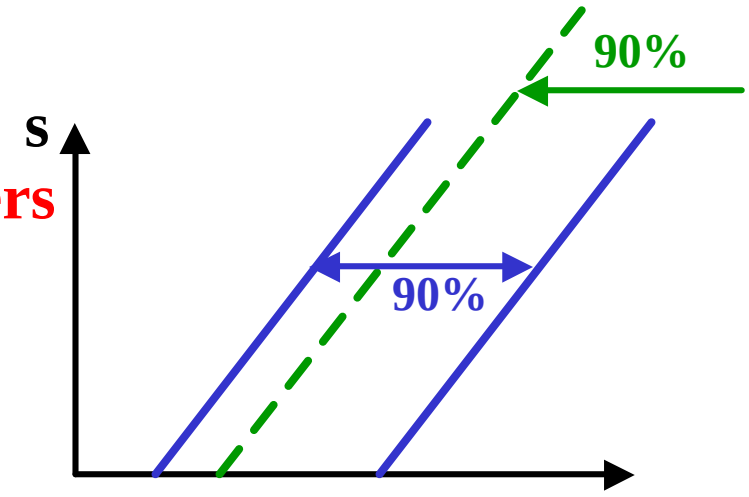
"Necessary" but not sufficient

Harder to understand/interpret

Hard to compute in several parameters

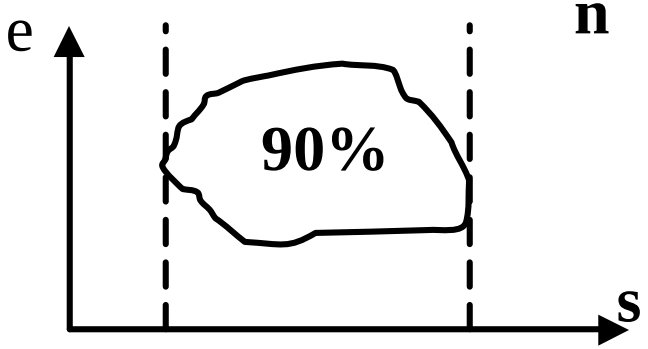
Systematics

Ordering rule



Over-coverage in several dimensions from projection

Empty intervals



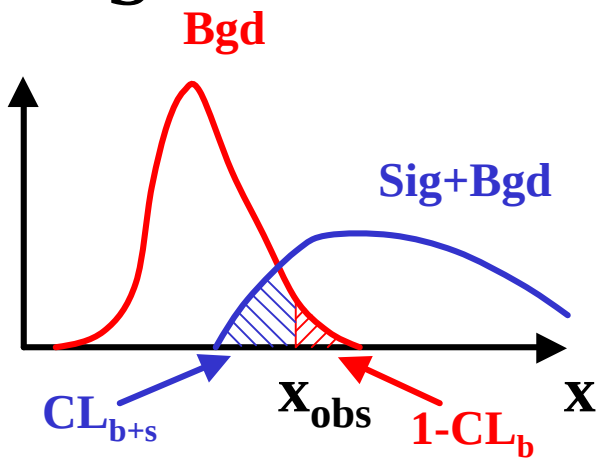
Features of Feldman-Cousins

Frequentist Þ Coverage
Unified Þ No Flip-flop
Fewer zero-length intervals
Resolves arbitrariness of ordering rule

Computation in several paramters
Systematics
Pathologies
Decreasing limit for "unphysical result"
Standard frequentist is worse
Fast exclusion of s = 0?

Features of CL_S

Hypothesis exclusion method



Coverage

- **Overcoverage**
- **"Conservative freq."**

No false sensitivity claims

LEP H; D0; Tevatron H sensitivity

Understanding

Systematics

- **Needs Bayes**

Only upper limits from CL_S

Features of Cousins-Highland

Nim A320 (1992) 331

Computable
Widely used

Mixed philosophy
Coverage not guaranteed
Approximate formulae not always accurate

Reading Material

CERN & FNAL CLW's §

CL_S: Alex Read, J Phys G 28 2693 (Durham Statistics Conf 2002) §

Setting limits CDF Stat. Comm. Web page §

Bob Cousins "Why Isn't . . ." Am J Phys 63 (1995) 398

D0 Limit Recommendations D0 Note 4629

BaBar Stat. Working Group

http://www.slac.stanford.edu/BFROOT/www/statistics §

CDF notes

Joel Heinrich "Coverage . . . Poisson" CDF 6438 §

Manhattan Project

Bayes	**CDF 7117**	**J.H.** §
Bayes'	**CDF 5928**	**L.D.** §
Cousins-Highland (soon)		**C.B.**
Frequentist (soonish)		**G.P.**

Profile *L*: W. Rolke *et al.*, physics/0403059 §

Cousins-Highland: Tegenfeldt & Conrad physics/0408039

§ **Available via CDF Statistics Committee Web page**