Minimal Flavour Violation in the Lepton Sector

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Plan:

- The flavour sector of the SM
- The flavour problem
- The EFT approach to Minimal Flavour Violation
- MFV in the quark sector
- MFV in the lepton sector
- Conclusions

The flavour sector of the SM

$$\mathscr{L}_{SM} = \mathscr{L}_{gauge}(A_i, \psi_i) + \mathscr{L}_{Higgs}(\phi_i, A_i, \psi_i ; v)$$

3 identical replica of the basic fermion family

[$\psi_i = Q_L, u_R, d_R, L_L, e_R$] $\Rightarrow$ huge flavour-degeneracy [$U(3)^5$ group]

Within the SM the flavour-degeneracy is broken
only by the Yukawa interaction

Quark sector:

$$\bar{Q}_L{}^i Y_D{}^{ik} d_R{}^k \phi \to \bar{Q}_L{}^i M_D{}^{ik} d_R{}^k$$

$$\bar{Q}_L{}^i Y_U{}^{ik} u_R{}^k \phi_c \to \bar{Q}_L{}^i M_U{}^{ik} u_R{}^k$$

Thanks to the residual flavour symm.:

$$M_D = \text{diag}(m_d, m_s, m_b)$$

$$M_U = V_{CKM} \times \text{diag}(m_u, m_c, m_t)$$

Nowadays we have a good knowledge
of all the 10 observables entries
[6 masses + 4 CKM angles]
of the quark mass matrices

The flavour sector of the SM

$$\mathscr{L}_{SM} = \mathscr{L}_{gauge}(A_i, \psi_i) + \mathscr{L}_{Higgs}(\phi_i, A_i, \psi_i ; v)$$

Lepton sector: $\bar{L}_L{}^i \, Y_L{}^{ik} \, e_R{}^k \, \phi \rightarrow \bar{L}_L{}^i \, M_L{}^{ik} \, e_R{}^k$

$$M_L = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

No neutrino masses (& mixing angles) unless we extend the model

The flavour sector of the SM

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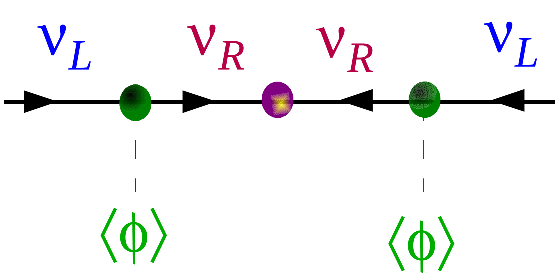
No neutrino masses (& mixing angles) unless we extend the model

right-handed neutrinos

dim.-5 Majorana mass term

completely equivalent at low energies if we assume $M_{\nu_R} \gg \langle\phi\rangle$ [see-saw]



The flavour sector *beyond* *the SM*

$$\mathscr{L}_{SM} = \mathscr{L}_{gauge}(A_i, \psi_i) + \mathscr{L}_{Higgs}(\phi_i, A_i, \psi_i; v) + \sum_{d\geq 5} \frac{c_n}{\Lambda^{d-4}} O_n^{\,d}(\phi_i, A_i, \psi_i)$$

Lepton sector: $\bar{L}_L^{\,i}\, Y_L^{\,ik}\, e_R^{\,k}\, \phi \rightarrow \bar{L}_L^{\,i}\, M_L^{\,ik}\, e_R^{\,k}$

$$M_L = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

No neutrino masses (& mixing angles) unless we extend the model

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dim.-5 Majorana mass term

$$\frac{1}{\Lambda_{LN}} (L_L^T)^i\, g_{LL}^{\,ik}\, L_L^{\,k}\, \phi^T \phi \rightarrow (L_L^T)^i\, M_\nu^{\,ik}\, L_L^{\,k}$$

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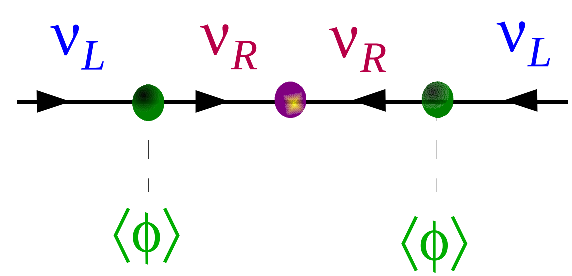


$$M_\nu = U^* \times \mathrm{diag}(m_1, m_2, m_3) \times U^+$$

→ Pontecorvo-Maki- Nakagawa-Sakata matrix

The present knowledge of the observables entries of the neutrino mass matrix is not as good as in the quark sector, but the structure starts to be highly constrained

$$U_{PMNS} = \begin{pmatrix} c\, e^{i\alpha_1/2} & s\, e^{i\alpha_2/2} & s_{13}\, e^{-i\delta} \\ -s\, e^{i\alpha_1/2}/\sqrt{2} & c\, e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \\ s\, e^{i\alpha_1/2}/\sqrt{2} & -c\, e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

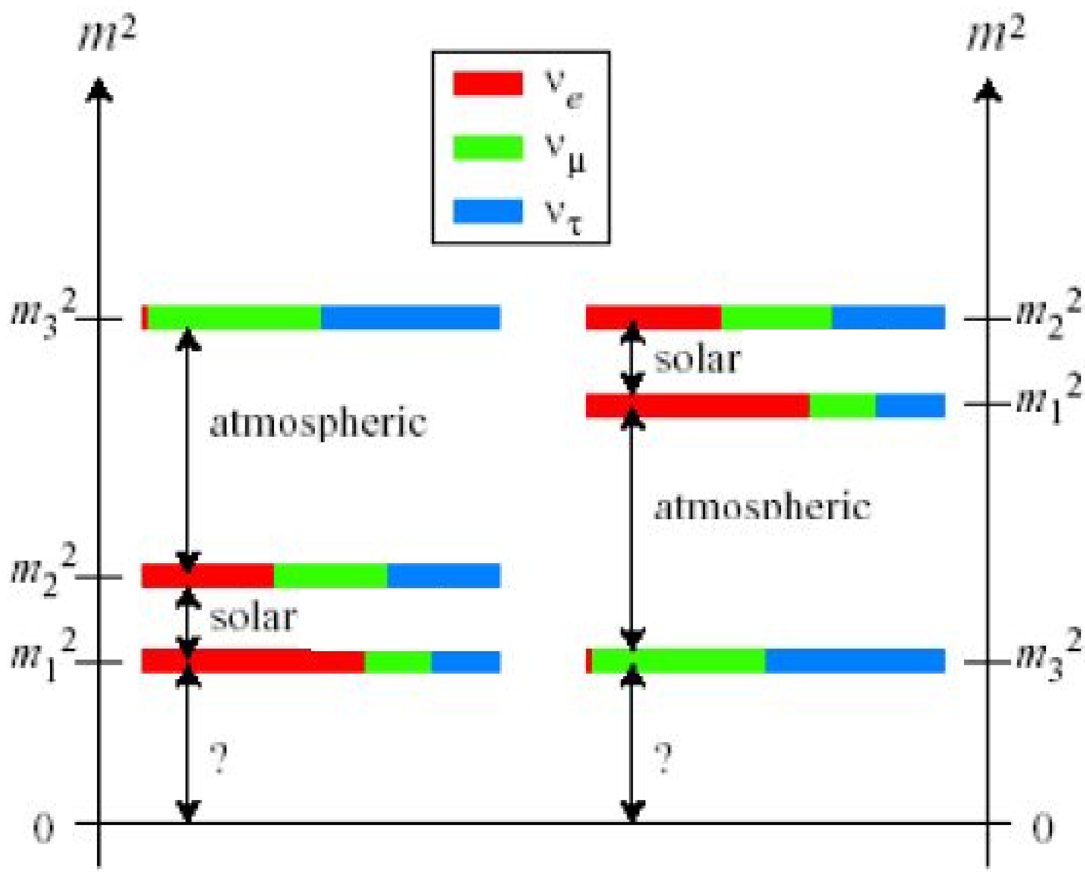
$s = \cos\theta_{sol}$

$s = \sin\theta_{sol}$

$\theta_{sol} \approx 33°$

$\sigma_{13} < 0.25$

$0 < \delta < 2\pi$



Δm^2_{sol}	Δm^2_{atm}
8.0×10^{-5} eV^2	2.5×10^{-3} eV^2

The flavour problem

$$\mathscr{L}_{SM} = \mathscr{L}_{gauge}(A_i, \psi_i) + \mathscr{L}_{Higgs}(\phi_i, A_i, \psi_i ; v) + \sum_{d \geq 5} \frac{c_n}{\Lambda^{d-4}} O_n^{\,d}(\phi_i, A_i, \psi_i)$$

The dim-5 operator responsible for neutrino masses is quite special since it violates *lepton number*

$$\frac{1}{\Lambda_{LN}} (L_L{}^T)^i \, g_{LL}{}^{ik} \, L_L{}^k \, \phi^T \phi$$

↓

- We can keep $\Lambda_{LN} \gg v$ without fine-tuning problems in the e.w. sector
- If $\Lambda_{LN} \gg v$ some of the $g_{LL}{}^{ik}$ couplings can be O(1)

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- If $\Lambda_{LN} \gg v$ some of the $g_{LL}^{\;ik}$ couplings can be O(1)

On the contrary - because of the stabilization of the Higgs sector - we would expect $\Lambda \sim$ TeV for all the operators which preserves SM symmetries

↓

flavour problem

if $c_n \sim O(1)$ rare processes already imply bounds on the effective scale of new physics well above 1 TeV

The flavour problem

Less severe, but still non-negligible problems exist also in the lepton sector:

E.g.:

$$\mathcal{L} = \frac{e\,\Delta_{\mu e}}{\Lambda^2}\, m_\mu \bar{e}_R \sigma^{\mu\nu} \mu_L\, F_{\mu\nu}$$

↓

$$BR(\mu \to e\gamma) = 0.31 \left(\frac{\mathrm{TeV}}{\Lambda}\right)^4 |\Delta_{\mu e}|^2$$

$BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ ➔ $\Lambda > \sqrt{\Delta_{\mu e}} \times 400\ \mathrm{TeV}$

Two possible solutions:

- *pessimistic* [quite unnatural]: Λ >100 TeV [the nightmare of direct searches...]

  ⇒ rare processes not necessarily sensitive to NP, but a few of them may be more interesting than direct searches

- *natural*: $\Lambda \sim 1$ TeV + flavor-mixing protected by additional symmetries

  ⇒ still a lot to learn from flavour physics and particularly rare decays

[we need to determine the underlying flavour symmetry in a model-independent way]

most restrictive possibility ⇓ [strongly suggested at least in the quark secor]

Minimal Flavour Violation (MFV) hypothesis:

The breaking of the flavour symmetry occurs at very high scales and is mediated at low energies only by terms proportional to SM Yukawa coupl.

- Natural implementation in many consistent scenarios [SUSY, technicolour, extra dimensions,...]

- Possible to build a predictive low-energy EFT model-independent approach

The EFT approach to Minimal Flavour Violation

I. Definition of the Flavour Group

The maximal group of unitary field transf. allowed by $\mathscr{L}^{\text{SM}}_{\text{gauge}}$ is:

$$G_F = U(3)^5 = \underbrace{SU(3)_l^2 \times SU(3)_q^3 \times U(1)_{PQ} \times U(1)_{e_R}}_{\text{subgroup broken by } \mathscr{L}^{\text{SM}}_{\text{Yukawa}}} \times U(1)_B \times U(1)_L \times U(1)_Y$$

$$\mathscr{L}_{\text{Yukawa}} = \bar{Q}_L Y_D D_R H + \bar{Q}_L Y_U U_R (H)_c + \bar{L}_L Y_L e_R H + \text{h.c.}$$

$$SU(3)_l^2 = SU(3)_{L_L} \times SU(3)_{e_R}$$

$$SU(3)_q^3 = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$$

subgroup responsible for quark mixing [Yukawa structure, CKM matrix]

$U(1)_{PQ}$: glob. phase of D_R & e_R

$U(1)_{E_R}$: glob. phase of e_R

groups relevant in multi-Higgs models [overall Yukawa norm.]

II. Definition of the symmetry-breaking terms

Since G_F is already broken within the SM, it is not consistent to impose it as an exact symmetry beyond the SM.

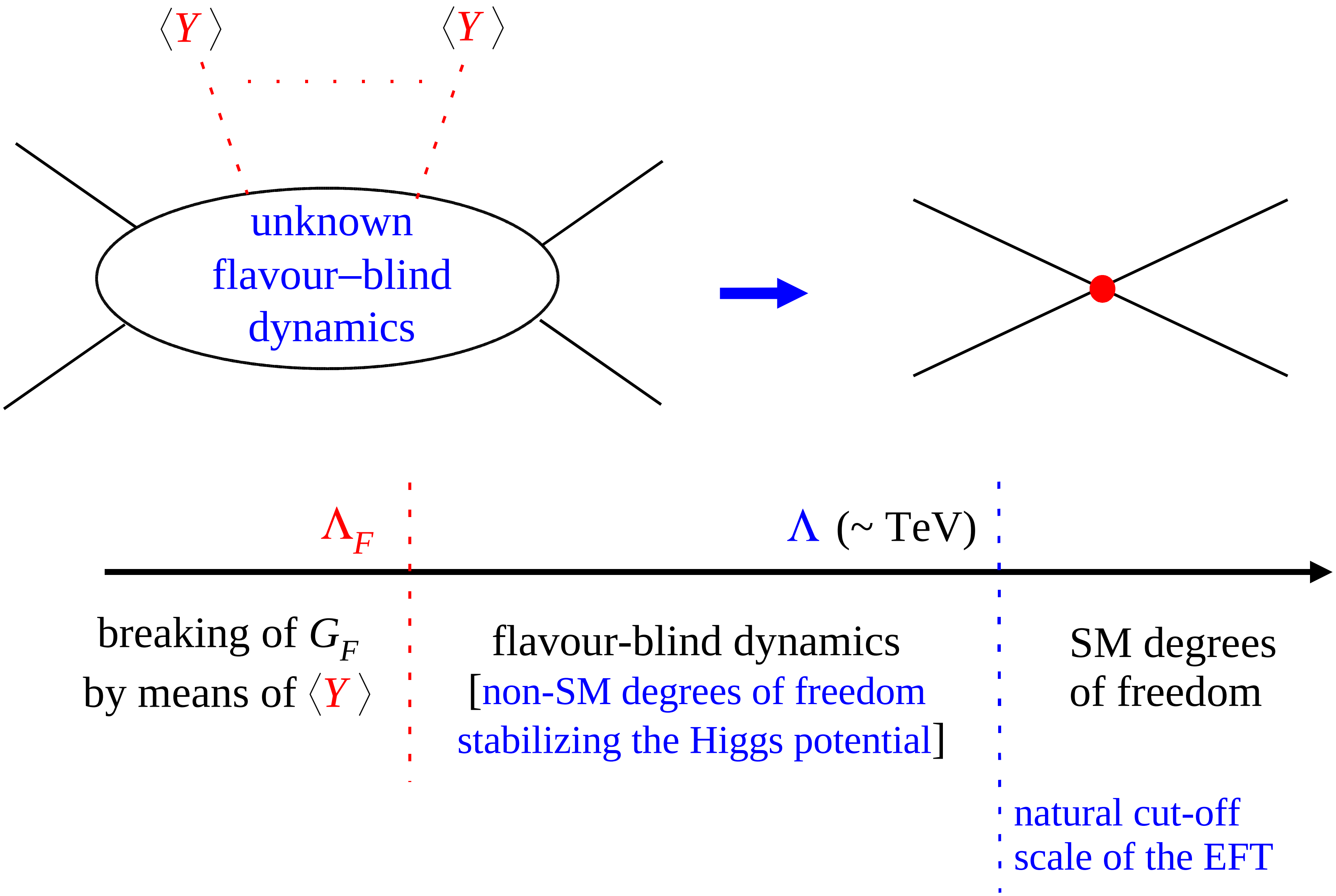
However, we can (formally) promote G_F to be an exact symmetry, assuming the Yukawa matrices are the vacuum expectation values of appropriate auxiliary fields:

$$\text{E.g.:} \quad Y_D \sim (3,1,\bar{3}) \quad \& \quad Y_U \sim (3,\bar{3},1) \quad \text{under} \quad SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$$

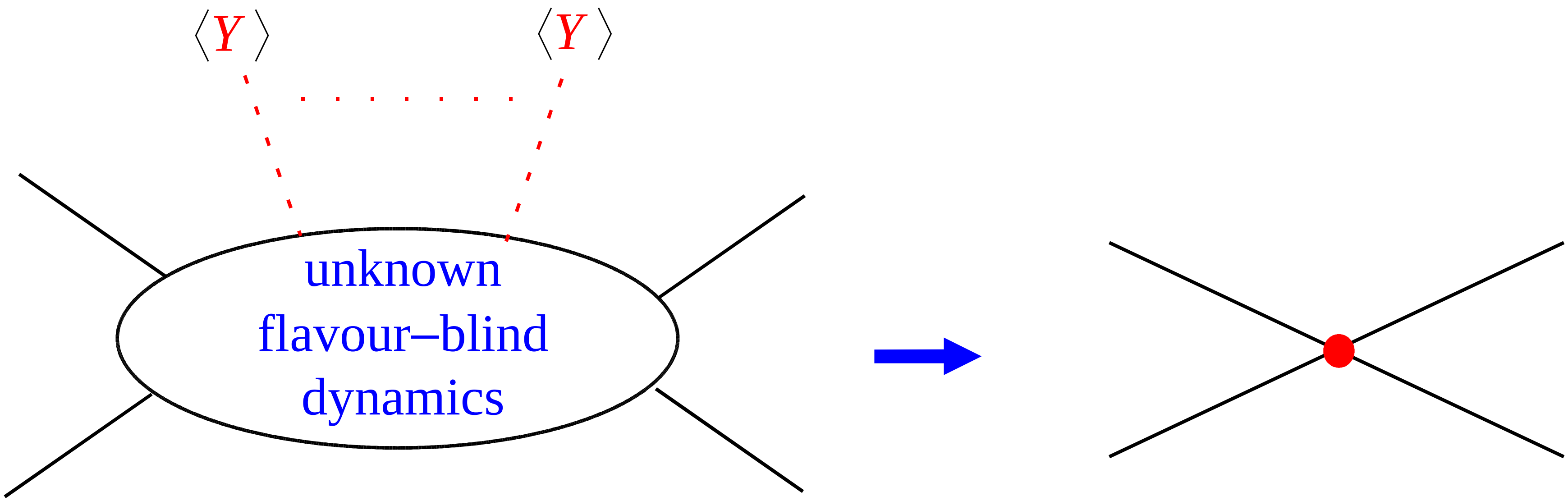
$$\mathscr{L}_{\text{Yukawa}} = \bar{Q}_L Y_D D_R H + \bar{Q}_L Y_U U_R (H)_c + \bar{L}_L Y_L e_R H + \text{h.c.}$$

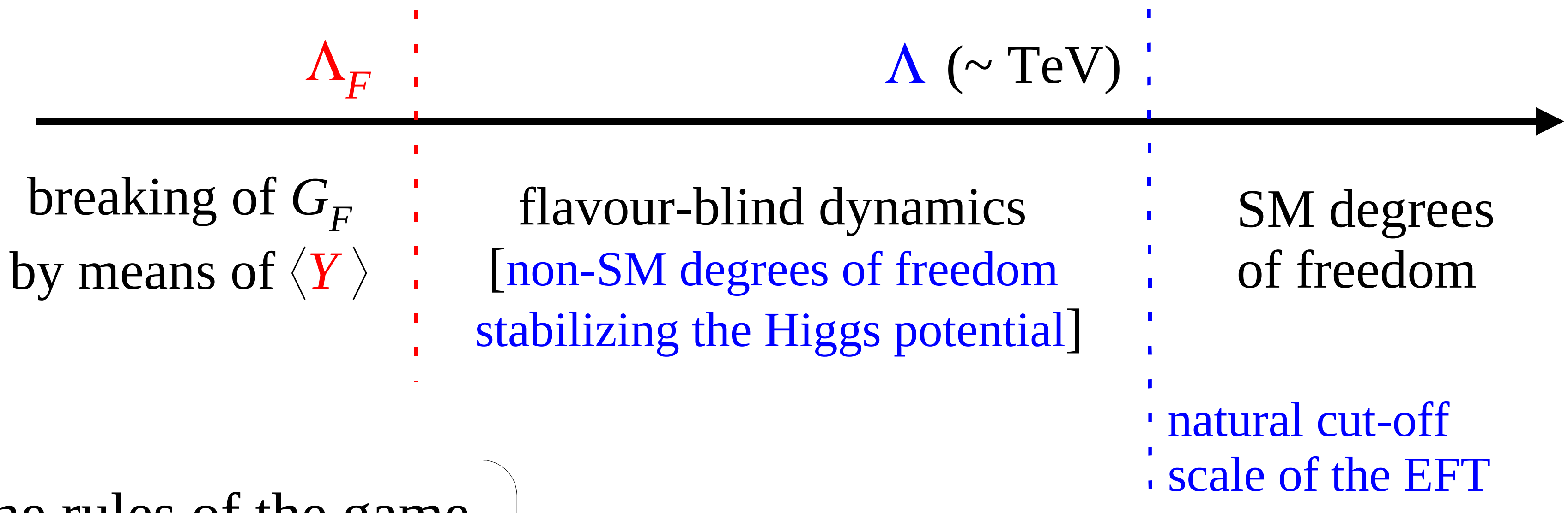
$(\bar{3},1,1)$ $\quad$ $(3,1,\bar{3})$ $\quad$ $(1,1,3)$

$(3,1,\bar{3})$ ⬇ $(1,1,1)$



⟨Y⟩
⟨Y⟩
unknown
flavour–blind
dynamics
Λ_F
Λ (~ TeV)
breaking of G_F
by means of ⟨Y⟩
flavour-blind dynamics
[non-SM degrees of freedom
stabilizing the Higgs potential]
SM degrees
of freedom
natural cut-off
scale of the EFT





III. The rules of the game

A low-energy EFT (including only SM fields) satisfies the criterion of MFV if all higher-dimensional operators, constructed from SM and Y fields, are (formally) invariant under G_F

D'Ambrosio, Giudice, G.I., Strumia '02

MFV in the quark sector

A low-energy EFT (including only SM fields) satisfies the criterion of MFV if all higher-dimensional operators, constructed from SM and Y fields, are (formally) invariant under G_F

We can always choose a quark basis where:

$$Y_D = \mathrm{diag}(y_d, y_s, y_b) \qquad Y_U = V^+ \times \mathrm{diag}(y_u, y_c, y_t)$$

V^+ → CKM matrix (= the only source of quark mix.)

Typical FCNC dim.-6 operator: $\bar{Q}_L^{\,i} (Y_U Y_U^{+})_{ij} Q_L^{\,j} \times \bar{L}_L L_L$

Y_U: $(3,\bar{3},1)$ $\qquad$ Y_U^+: $(\bar{3},3,1)$

↓

$(1,1,1)$

$SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$

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$$(Y_U Y_U^{+})_{ij} \approx y_t^2\, V_{3i} V_{3j}^{*}$$

↓

$$V^{+} \times \mathrm{diag}(y_u^2, y_c^2, y_t^2) \times V$$

$$\approx V^{+} \times \mathrm{diag}(0, 0, y_t^2) \times V$$

same CKM - Yukawa structure of the SM contribution !

↓

equivalent -in practice- to the "pragmatic" definition by Buras *et al.* '00-'03

Bounds on the scale of New Physics within MFVmodels:

Minimally flavour violating dimension six operator	main observables	Λ [TeV] $-$	$+$
$\mathcal{O}_0 = \frac{1}{2}(\bar{Q}_L \lambda_{FC} \gamma_\mu Q_L)^2$	ϵ_K, Δm_{B_d}	2.6	4.9
$\mathcal{O}_{F1} = e\, H^\dagger (\bar{D}_R \lambda_d \lambda_{FC} \sigma_{\mu\nu} Q_L)\, F_{\mu\nu}$	$B \to X_s \gamma$	6.0	6.1
$\mathcal{O}_{G1} = g\, H^\dagger (\bar{D}_R \lambda_d \lambda_{FC} \sigma_{\mu\nu} T^a Q_L)\, G^a_{\mu\nu}$	$B \to X_s \gamma$	2.3	3.8
$\mathcal{O}_{\ell 1} = (\bar{Q}_L \lambda_{FC} \gamma_\mu Q_L)(\bar{L}_L \gamma_\mu L_L)$	$B \to (X)\ell\bar{\ell}$, $K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	4.2	3.8
$\mathcal{O}_{\ell 2} = (\bar{Q}_L \lambda_{FC} \gamma_\mu \tau^a Q_L)(\bar{L}_L \gamma_\mu \tau^a L_L)$	$B \to (X)\ell\bar{\ell}$, $K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	4.5	4.0
$\mathcal{O}_{H1} = (\bar{Q}_L \lambda_{FC} \gamma_\mu Q_L)(H^\dagger i D_\mu H)$	$B \to (X)\ell\bar{\ell}$, $K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	2.5	2.5
$\mathcal{O}_{q5} = (\bar{Q}_L \lambda_{FC} \gamma_\mu Q_L)(\bar{D}_R \gamma_\mu D_R)$	$B \to K\pi$, $\epsilon'/\epsilon, \ldots$	~ 1	

Recent update
M. Bona *et al.* '05

⇒ *The MFV hypothesis is very powerful (restrictive) !*

within this framework the flavour problem is basically solved

The bounds are typically weaker (or at most as stringent as) those obtained from flavour -conserving e.w. dynamics [~ 5-10 TeV]

⇒ *We are slowly entering in the interesting region...*

▶ *MFV in the lepton sector*

The situation of the lepton sector is more interesting but also more complicated

⇒ identification of the minimal sources of flavour-symmetry breaking not straightforward [Yukawa not enough to generate ν masses]

MFV in the lepton sector

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$\Rightarrow$ identification of the minimal sources of flavour-symmetry breaking not straightforward [Yukawa not enough to generate ν masses]

Basic assumptions:

- Decoupling of $U(1)_L$ breaking [Lepton Number] - associated to some high scale [$\Lambda_{LN} \gg \Lambda \sim$ TeV] and the SU(3) breaking $\Rightarrow$ small neutrino masses with *natural* [$g_\nu \sim O(1)$] flavour-violating couplings

$$\frac{1}{\Lambda_{LN}} (L_L^T)^i \, g_\nu^{ik} \, L_L^k \, \phi^T \phi$$

V.Cirigliano, B.Grinstein, G.I., M.Wise, '05

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- The effective neutrino couplings (masses + mixing) allow to determine completely the flavour-breaking structures:

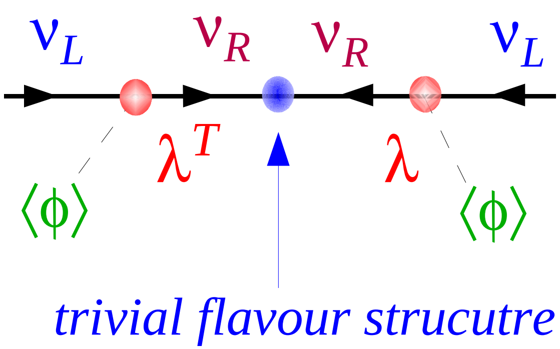
V.Cirigliano, B.Grinstein, G.I., M.Wise, '05

$$\frac{1}{\Lambda_{LN}} (L_L^T)^i \, g_\nu^{ik} \, L_L^k \, \phi^T \phi$$

irreducible

reducible [see-saw type]

$$g_\nu \sim \lambda^T \lambda$$



MFV in the lepton sector

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G.I.,M.Wise, '05

Two independent formulations:

- Minimal field content [L_L, e_R $\rightarrow$ $SU(3)_{L_L} \times SU(3)_{e_R}$]

Non-Yukawa mass terms: $\frac{1}{\Lambda_{LN}} L_L^T g_\nu L_L \phi^T \phi$

Neutrino mass matrix: $M_\nu = \frac{g_\nu v^2}{\Lambda_{LN}} = U^* m_\nu^{diag} U^+$

Basic coupling relevant for FCNC processes:

$$\Delta = g_\nu^+ g_\nu = \left(\frac{\Lambda_{LN}}{v^2}\right)^2 U (m_\nu^{diag})^2 U^+$$

E.g.: $\bar{L}_L^i (\Delta)_{ij} L_L^j \times \bar{e}_R e_R$

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- Extended field content $[L_L, e_R, \nu_R \rightarrow SU(3)_{L_L} \times SU(3)_{e_R} \times O(3)_{\nu_R}]$

Non-Yukawa mass terms: $\nu_R^T \lambda_\nu L_L \phi^T + M_\nu \nu_R^T \nu_R$

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not the same !

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The construction of the EFT proceeds as in the quark sector [higher-dimensional operators formally invariant under the symmetry group] $\Longrightarrow$ link between neutrino masses (& mixing) and lepton-flavour violating rare decays

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E.g.:

$$\bar{L}_L^i\,(\Delta)_{ij}\,L_L^j \times \bar{e}_R\, e_R \longrightarrow \tau \to \mu e e$$

$$\bar{L}_L^i\,(\Delta)_{ij}\,Y_L\,\sigma^{\mu\nu}\,e_R^j\,F^{\mu\nu} \longrightarrow \mu \to e\gamma$$

Much more freedom/uncertainty with respect to the quark case because of

- the overall scale
- the value of s_{13}, δ and the spectrum ordering
- the two scenarios

$$\Delta = g_\nu^+ g_\nu = \left(\frac{\Lambda_{LN}}{v^2}\right)^2 U\,(m_\nu^{diag})^2\,U^+$$

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$$\Delta_{\mu e} = \frac{\Lambda_{\rm LN}^2}{v^4}\frac{1}{\sqrt{2}}\left(s\,c\,\Delta m^2_{\rm sol} \pm s_{13}\,e^{i\delta}\,\Delta m^2_{\rm atm}\right)$$

$$\Delta_{\tau e} = \frac{\Lambda_{\rm LN}^2}{v^4}\frac{1}{\sqrt{2}}\left(-s\,c\,\Delta m^2_{\rm sol} \pm s_{13}\,e^{i\delta}\,\Delta m^2_{\rm atm}\right)$$

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$$\Delta = \lambda_\nu^{+}\lambda_\nu = \frac{M_\nu}{v^2}\,U\,m_\nu^{diag}\,U^{+}$$

$$\begin{aligned}
\Delta_{\mu e} &= \frac{M_\nu}{v^2}\frac{1}{\sqrt{2}}\left[s\,c\,(m_{\nu_2}-m_{\nu_1}) \pm s_{13}\,(m_{\nu_3}-m_{\nu_1})\right] \\
\Delta_{\tau e} &= \frac{M_\nu}{v^2}\frac{1}{\sqrt{2}}\left[-s\,c\,(m_{\nu_2}-m_{\nu_1}) \pm s_{13}\,(m_{\nu_3}-m_{\nu_1})\right] \\
\Delta_{\tau\mu} &= \frac{M_\nu}{v^2}\frac{1}{2}\left[-c^2\,(m_{\nu_2}-m_{\nu_1}) + (m_{\nu_3}-m_{\nu_1})\right]
\end{aligned}$$

... but some general phenomenological conclusions can still be obtained:

I. Visible effects in rare decays possible only with a large hierarchy between the $U(1)_L$ breaking scale (Λ_{LN}) and the low-scale of new physics (Λ):

$$B(\mu \to e\gamma) \sim 10^{-13} \quad \Rightarrow \quad \begin{aligned} \Lambda_{LN} &\sim 10^{12}\,\mathrm{GeV} \times (\Lambda / 1\ \mathrm{TeV}) \\ M_\nu &\sim 10^{10}\,\mathrm{GeV} \times (\Lambda / 1\ \mathrm{TeV})^2 \end{aligned}$$

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$$B(\mu \to e\gamma) \sim 10^{-13} \quad \Rightarrow \quad \Lambda_{LN} \sim 10^{12}\,\text{GeV} \times (\Lambda / 1\ \text{TeV}), \qquad M_\nu \sim 10^{10}\,\text{GeV} \times (\Lambda / 1\ \text{TeV})^2$$

II. Clear pattern $B(\tau \to \mu\gamma) \gg B(\tau \to e\gamma) \sim B(\mu \to e\gamma)$ dictated essentially only by the structure of U_{PMNS} [with $\mu \to e / \tau \to \mu$ suppression increasing as $s_{13} \to 0$]

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III. Despite the $B(\tau \to \mu\gamma) \gg B(\mu \to e\gamma)$ pattern, present experimental limits on $B(\mu \to e\gamma)$ $\Rightarrow$ not possible to observe $\tau \to \mu\gamma$ at B factories [except in fine-tuned configurations]

Examples: A) Minimal field content

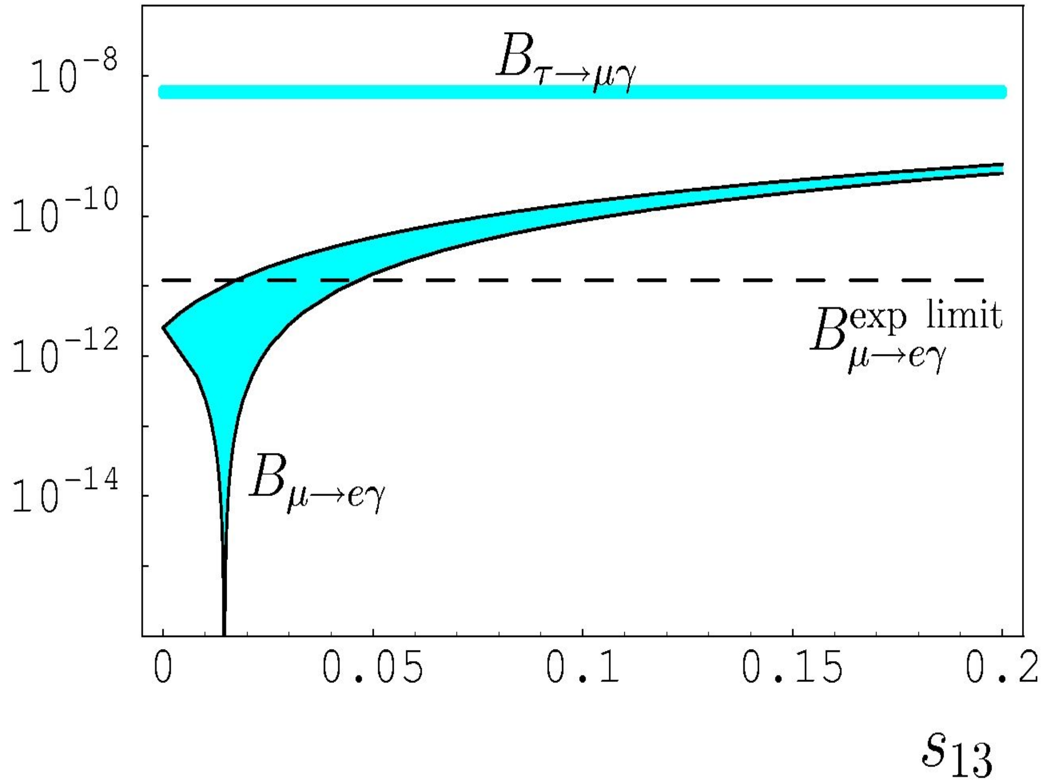

Figure 3: $B_{\tau\to\mu\gamma}$ and $B_{\mu\to e\gamma}$ as a function of s_{13}, for $\Lambda_{\mathrm{LN}}/\Lambda = 10^{10}$
The shading corresponds to different values of the phase δ and the normal/inverted spectrum.

Examples: B) Extendend field content

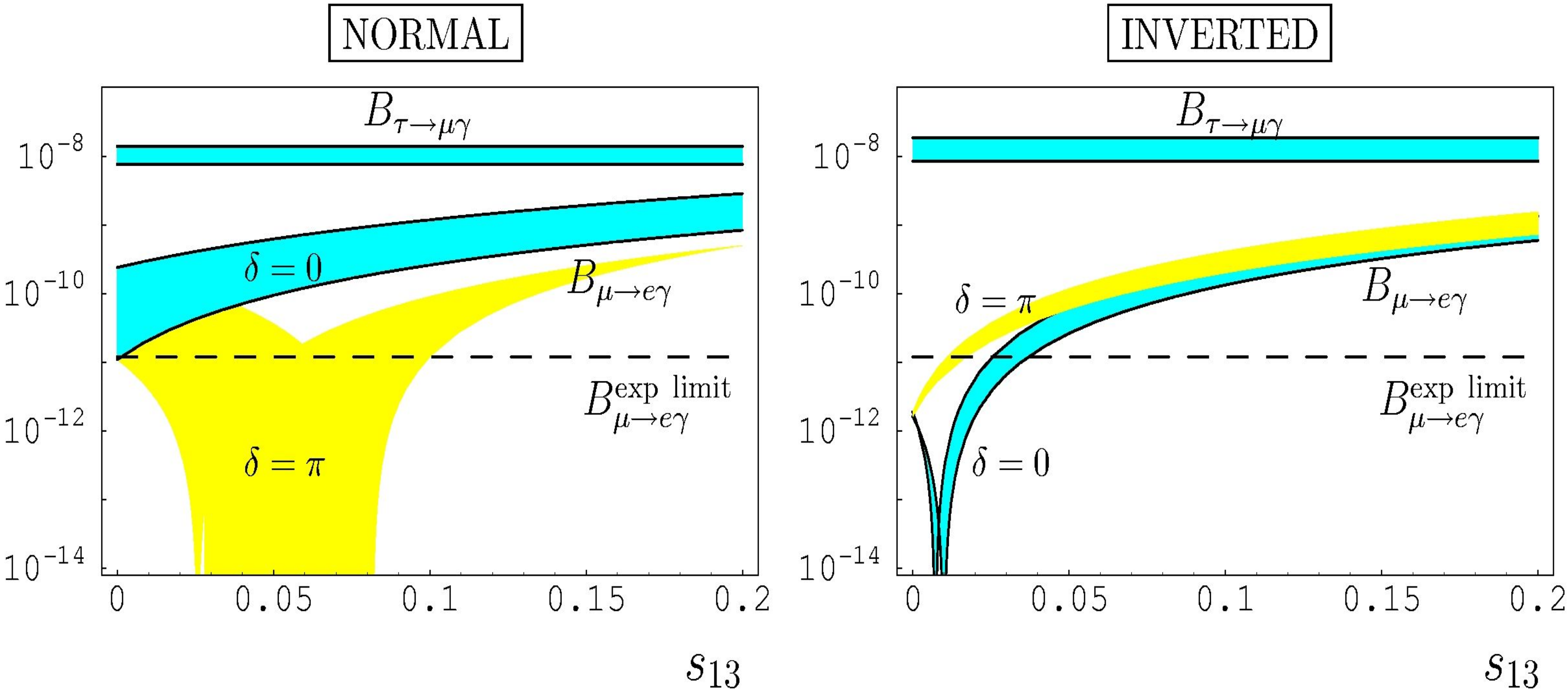

Figure 5: $B_{\tau\to\mu\gamma}$ and $B_{\mu\to e\gamma}$ as a function of s_{13}, for $(vM_\nu)/\Lambda^2 = 5\times10^7$ The shading corresponds to different values of the lightest neutrino mass, ranging between 0 and 0.02 eV.

Conclusions

A top $\rightarrow$ bottom approach to the flavour problem would in principle be preferable, but we still lack of a simple and clear theory to describe the breaking of the flavour symmetry

The general MFV-EFT approach provides a bottom $\rightarrow$ top alternative, particularly useful to analyse present & future precise data on *rare decays*

QUARK SECTOR :

- Natural explanation for the absence of large non-standard effects
- still hope to find some small deviations (~10%) in clean rare decays

LEPTON SECTOR :

- MFV hypothesis not *needed* as in the quark sector (scale ambiguity) but still quite useful as *organising principle* $\Rightarrow$ 2 natural implementations
- The most natural choice of new-physics scales [$\Lambda_{LN} \gg \Lambda$ such that $g_\nu \sim O(1)$] implies that rare FCNC decays of charged leptons should be observed soon [$\mu \rightarrow e\gamma$ best candidate]