Hadronic Vacuum Polarization Contributions to $(g-2)_\mu$

Gilberto Colangelo

UNIVERSITÄT BERN

CERN, November 9, 2005

Outline

Introduction

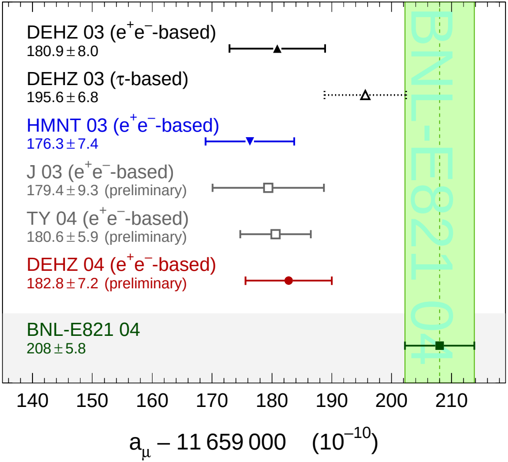
- Status of a_μ as of summer 2004

New data – Summer 2005

Can theory help?

Summary

Status of a_μ as of summer 2004



J = Jegerlehner, DEHZ = Davier, Eidelman, Höcker, Zhang

Figure from Höcker ICHEP (04)

HMNT = Hagiwara, Martin, Nomura, Teubner, TY = de Tróconiz, Ynduráin

Status of a_μ as of summer 2004

Breakdown of $a_\mu^{\rm hvp}$ in contributions of different energy regions

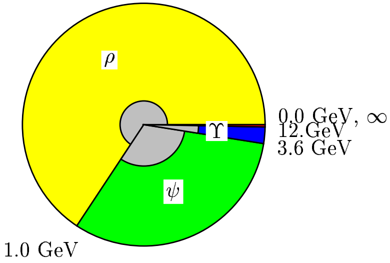


Figure from F. Jegerlehner

$$a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\ell^2}^{\infty} ds\, \frac{\hat{K}(s)}{s^2} R_h(s)$$

The region below 1 GeV is the most important

Status of a_μ as of summer 2004

- isospin relation among e^+e^- and τ not fully understood
  $\Rightarrow$ use e^+e^- data

Status of a_μ as of summer 2004

- isospin relation among e^+e^- and τ not fully understood
  $\Rightarrow$ use e^+e^- data
- the discrepancy between theory and experiment is of about 2 to 3 σ's (depending on the evaluation of $a_\mu^{\rm hvp}$)

Status of a_μ as of summer 2004

- isospin relation among e^+e^- and τ not fully understood
  $\Rightarrow$ use e^+e^- data
- the discrepancy between theory and experiment is of about 2 to 3 σ's (depending on the evaluation of $a_\mu^{\rm hvp}$)
- the discrepancy is about 3% of $a_\mu^{\rm hvp}$

Status of a_μ as of summer 2004

- isospin relation among e^+e^- and τ not fully understood $\Rightarrow$ use e^+e^- data
- the discrepancy between theory and experiment is of about 2 to 3 σ's (depending on the evaluation of a_μ^{hvp})
- the discrepancy is about 3% of a_μ^{hvp}
- the current experimental error is about 0.9% of a_μ^{hvp}

Status of a_μ as of summer 2004

- isospin relation among e^+e^- and τ not fully understood
  $\Rightarrow$ use e^+e^- data
- the discrepancy between theory and experiment is of about 2 to 3 σ's (depending on the evaluation of $a_\mu^{\rm hvp}$)
- the discrepancy is about 3% of $a_\mu^{\rm hvp}$
- the current experimental error is about 0.9% of $a_\mu^{\rm hvp}$
- the challenge is the evaluation of $a_\mu^{\rm hvp}$ to 1% or better

Status of a_μ as of summer 2004

- isospin relation among e^+e^- and τ not fully understood $\Rightarrow$ use e^+e^- data
- the discrepancy between theory and experiment is of about 2 to 3 σ's (depending on the evaluation of a_μ^{hvp})
- the discrepancy is about 3% of a_μ^{hvp}
- the current experimental error is about 0.9% of a_μ^{hvp}
- the challenge is the evaluation of a_μ^{hvp} to 1% or better
- the evaluation of the hadronic contribution at order α^3 is also nontrivial (e.g. hadronic light-by-light) but its size is of the order of the current experimental error

Comparison KLOE–CMD-2

- Until summer 2004, e^+e^- data in the ρ-region were dominated by CMD-2

Comparison KLOE–CMD-2

- Until summer 2004, e^+e^- data in the ρ-region were dominated by CMD-2
- Summer 04: the KLOE data “confirmed the CMD-2 data”

Comparison KLOE–CMD-2

- Until summer 2004, e^+e^- data in the ρ-region were dominated by CMD-2
- Summer 04: the KLOE data “confirmed the CMD-2 data”
- Conclusion: use only e^+e^- for a_μ.
  The discrepancy with τ data (though disturbing)
  can be ignored as far as a_μ is concerned

cf. Höcker, ICHEP 04

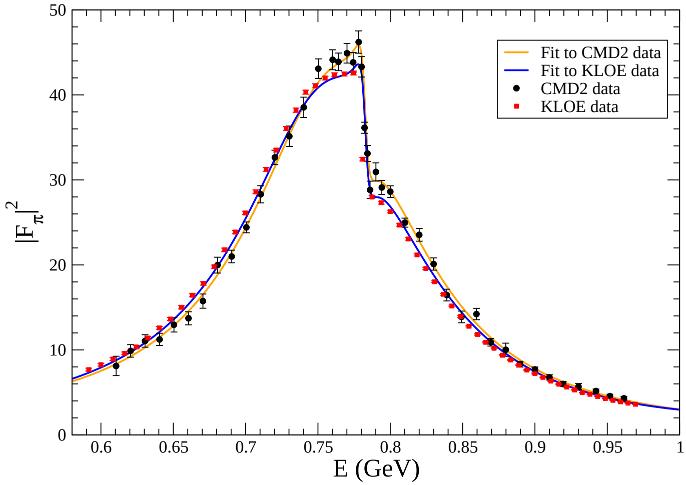
Comparison KLOE–CMD-2

- Until summer 2004, e^+e^- data in the ρ-region were dominated by CMD-2
- Summer 04: the KLOE data "confirmed the CMD-2 data"
- Conclusion: use only e^+e^- for a_μ.
  The discrepancy with τ data (though disturbing) can be ignored as far as a_μ is concerned

  cf. Höcker, ICHEP 04

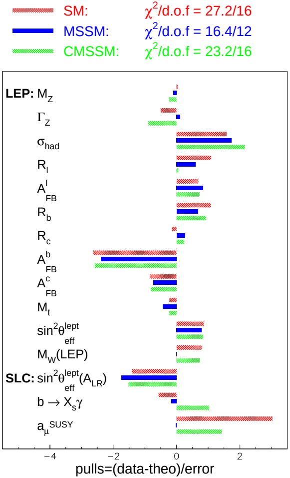
- However, while the integrals evaluated with CMD-2 or KLOE data agree, the two data sets disagree with each other locally

Comparison KLOE–CMD-2



"Harbinger of new physics"?

No error rescaling



Errors rescaled according to PDG

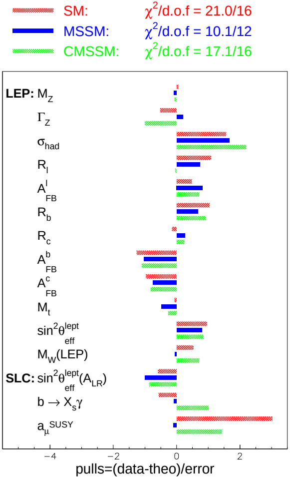


Figure from de Boer and Sander PLB (04)

"Harbinger of new physics"?

- the SUSY corrections are of the right size to explain the (possible) discrepancy between theory and experiment

"Harbinger of new physics"?

- the SUSY corrections are of the right size to explain the (possible) discrepancy between theory and experiment
- the sign of the (possible) discrepancy implies $\mu > 0$
  this is also favoured by other data ($b \to s\gamma$)

“Harbinger of new physics”?

- the SUSY corrections are of the right size to explain the (possible) discrepancy between theory and experiment
- the sign of the (possible) discrepancy implies $\mu > 0$ this is also favoured by other data ($b \to s\gamma$)
- a_μ plays an important role among other precision observables as a test of the SM or estensions thereof

“Harbinger of new physics”?

- the SUSY corrections are of the right size to explain the (possible) discrepancy between theory and experiment
- the sign of the (possible) discrepancy implies $\mu > 0$ this is also favoured by other data ($b \to s\gamma$)
- a_μ plays an important role among other precision observables as a test of the SM or estensions thereof
- if the discrepancy will disappear in the future a_μ will still provide strong constraints on the MSSM parameter space

"Harbinger of new physics"?

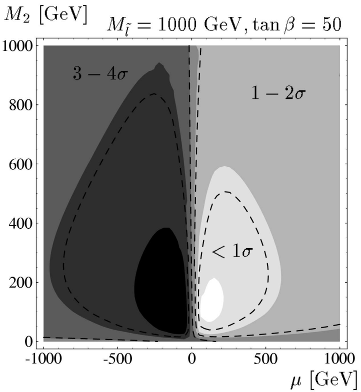
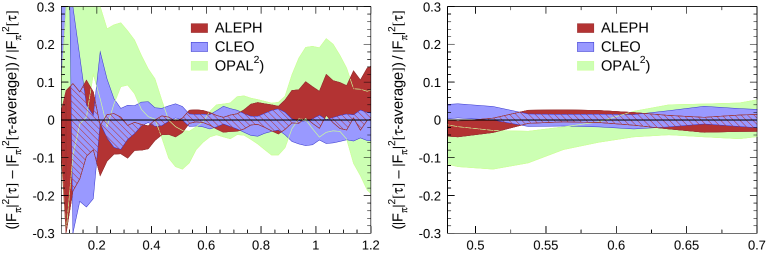
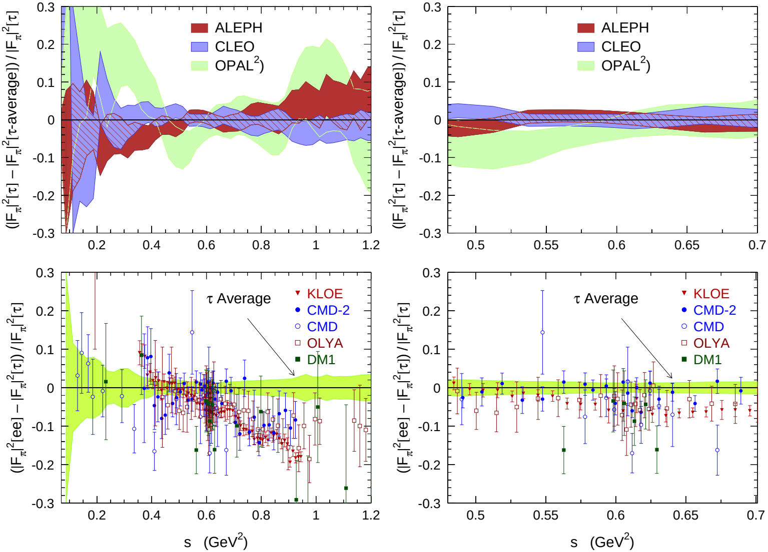


Figure from Heinemeyer, Stöckinger and Weiglein (04)

News, Summer 2005 – τ-data



News, Summer 2005 – τ-data



Figures from ALEPH Coll. hep-ph/0506072

News, Summer 2005 – e^+e^-–data

New data from the SND Coll. (Novosibirsk)

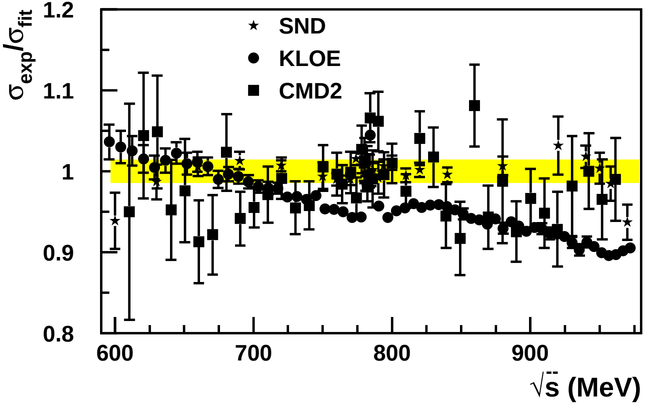
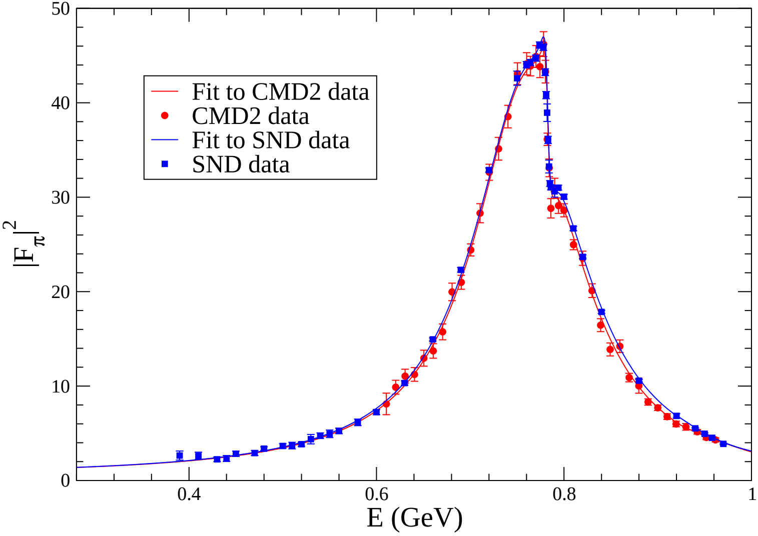


Figure from SND Coll. hep-ph/0506076

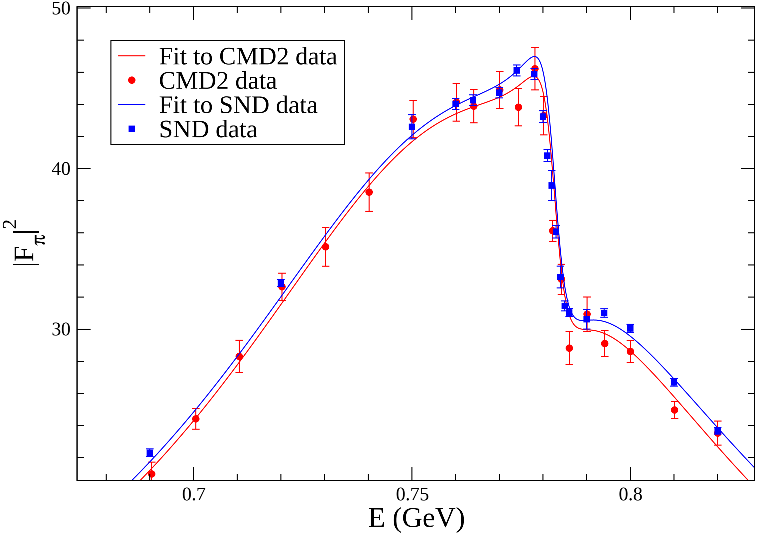
News, Summer 2005 – e^+e^-–data

New data from the SND Coll. (Novosibirsk)



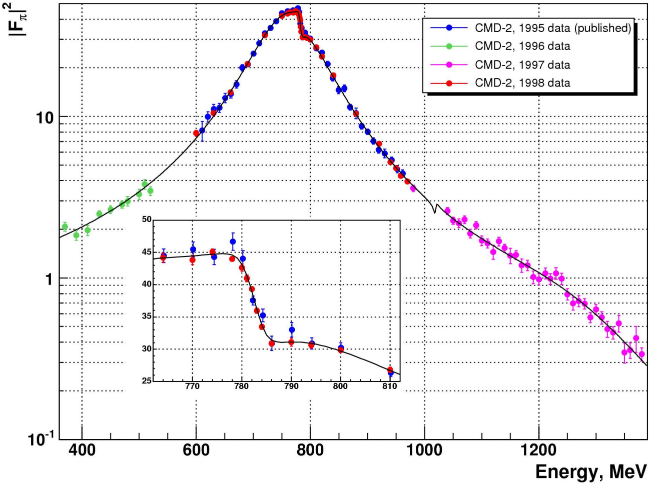
News, Summer 2005 – e^+e^-–data

New data from the SND Coll. (Novosibirsk)



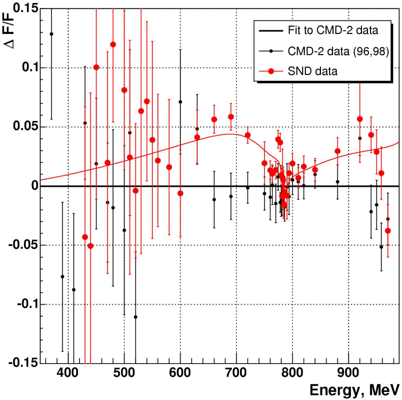
News, Summer 2005 – e^+e^-–data

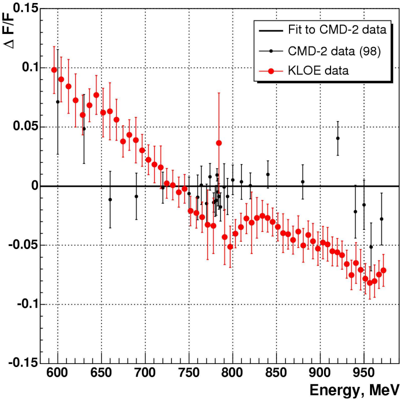
New data from the CMD-2 Coll. (Novosibirsk)



Talk by Logashenko, HEP Conference, Lisbon 2005

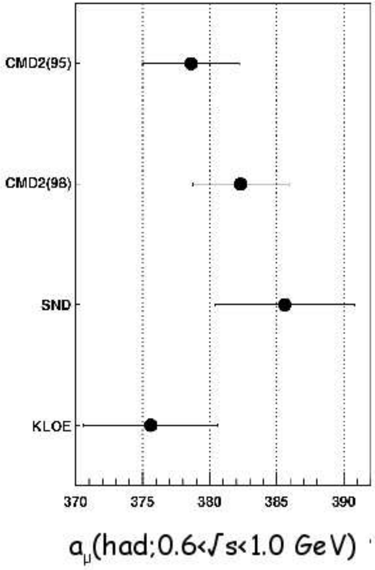
News, Summer 2005 – e^+e^-–data





Talk by Logashenko, HEP Conference, Lisbon 2005

News, Summer 2005 – e^+e^-–data



Very preliminary evaluation of a_μ^{hvp} from the various data sets, as presented by Logashenko (CMD-2) in Lisbon 2005

Can theory help?

- QCD test of the spectral function [Maltman 05]

- Use unitarity, analyticity and chiral symmetry in order to construct an explicit representation of the vector form factor

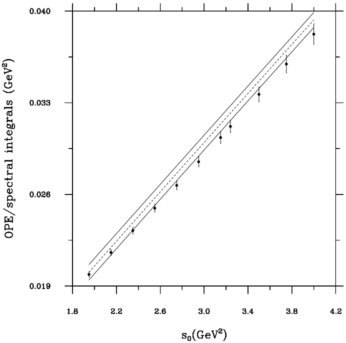
[Heyn and Lang 81]

[de Trocóniz and Ynduráin 02]

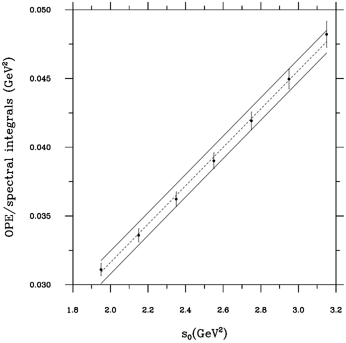
[Caprini, GC and Leutwyler work in progr.]

FESR test of the e^+e^- and τ spectral functions

e^+e^- data



τ data



Maltman, hep-ph/0504201

An improved representation of the form factor

- Omnés representation (57)

$$F_V(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s)}\right] \equiv \Omega(s)$$

An improved representation of the form factor

- Omnés representation (57)

$$F_V(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s)}\right] \equiv \Omega(s)$$

- Split elastic from inelastic contributions

$$\delta = \delta_{\pi\pi} + \delta_{\rm in} \quad \Rightarrow \quad F_V(s) = \Omega_{\pi\pi}(s)\Omega_{\rm in}(s)$$

Eidelman-Lukaszuk: unitarity bound on $\delta_{\rm in}$

$$\sin^2\delta_{\rm in} \le \frac{1}{2}\left(1-\sqrt{1-r^2}\right) \qquad r = \frac{\sigma^{I=1}_{e^+e^-\to\neq 2\pi}}{\sigma_{e^+e^-\to 2\pi}}$$

$$\Rightarrow \quad {\rm Im}\Omega_{\rm in}(s) \simeq 0 \qquad s \le (M_\pi + M_\omega)^2$$

An improved representation of the form factor

- Omnés representation (57)

$$F_V(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s)}\right] \equiv \Omega(s)$$

- Split elastic from inelastic contributions

$$\delta = \delta_{\pi\pi} + \delta_{\rm in} \quad \Rightarrow \quad F_V(s) = \Omega_{\pi\pi}(s)\Omega_{\rm in}(s)$$

Eidelman-Lukaszuk: unitarity bound on $\delta_{\rm in}$

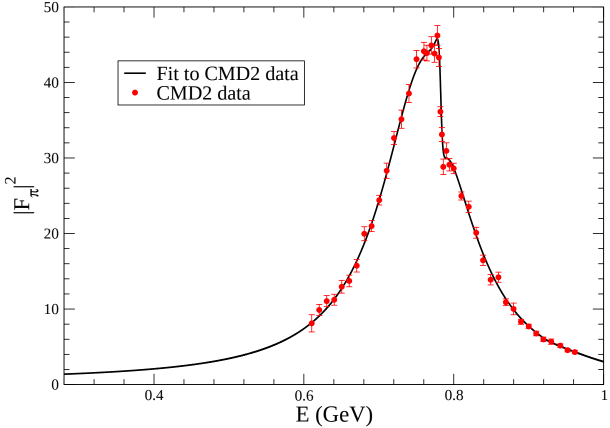
$$\sin^2\delta_{\rm in} \le \frac{1}{2}\left(1-\sqrt{1-r^2}\right) \qquad r = \frac{\sigma^{I=1}_{e^+e^-\to\neq 2\pi}}{\sigma_{e^+e^-\to 2\pi}}$$

$$\Rightarrow \quad {\rm Im}\Omega_{\rm in}(s) \simeq 0 \qquad s \le (M_\pi + M_\omega)^2$$

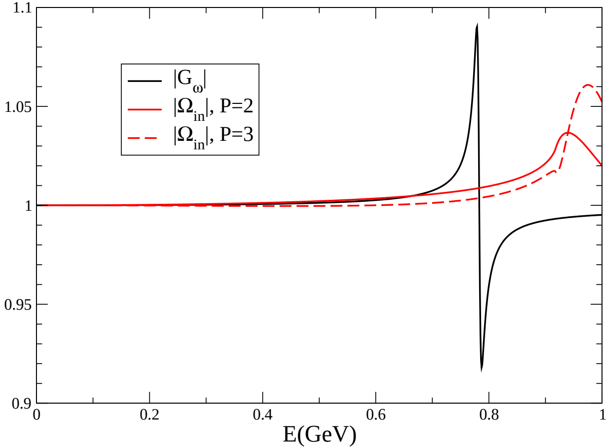
- $\rho - \omega$–mixing must also be explicitly taken into account

$$F_V(s) = \Omega_{\pi\pi}(s)\Omega_{\rm in}(s)G_\omega(s)$$

An improved representation of the form factor

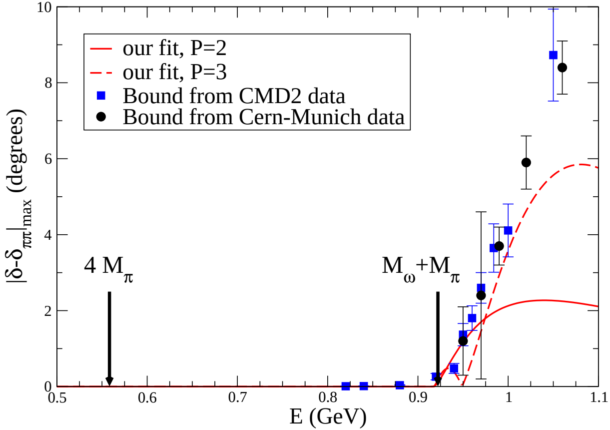


An improved representation of the form factor



$P =$ number of parameters in Ω_{in}

An improved representation of the form factor



P = number of parameters in Ω_{in}

Discussion

- Reduced statistical error in the evaluation of the integral

P	χ^2/d.o.f.	a_ρ	a_{2M_K}
0	84.0/83	420.0 ± 2.1	489.5 ± 2.2
1	75.9/82	423.4 ± 2.4	493.7 ± 2.5
2	75.8/81	423.1 ± 2.6	493.2 ± 2.8
3	73.7/80	422.2 ± 2.7	492.2 ± 2.9

GC SIGHAD (04)

Cf. Jegerlehner (03) (using the trapezoidal rule):

$$a_\rho = 429.02 \pm 4.95 \ \text{(stat.)}$$

Difference in central value mostly due to FS radiation, not included in our analysis

$$10^{-10} a_\rho = a_\mu^{\rm hvp}(\sqrt{s} \leq 0.81 {\rm GeV}) \qquad 10^{-10} a_{2M_K} = a_\mu^{\rm hvp}(\sqrt{s} \leq 2M_K)$$

Discussion

- Reduced statistical error in the evaluation of the integral

$$\Delta a_\rho = 4.95 \rightarrow 2.7$$

- Being able to fit a set of data with this parametrization is quite nontrivial and provides a check on the data

Discussion

- Reduced statistical error in the evaluation of the integral

$$\Delta a_\rho = 4.95 \to 2.7$$

- Being able to fit a set of data with this parametrization is quite nontrivial and provides a check on the data
- None of the analyses so far has taken into account all the information coming from analyticity, unitarity and chiral symmetry

Discussion

- Reduced statistical error in the evaluation of the integral

$$\Delta a_\rho = 4.95 \to 2.7$$

- Being able to fit a set of data with this parametrization is quite nontrivial and provides a check on the data
- None of the analyses so far has taken into account all the information coming from analyticity, unitarity and chiral symmetry

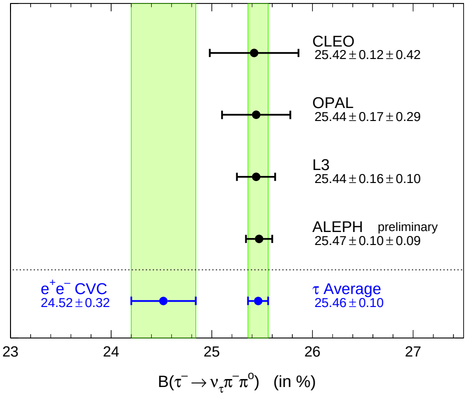
- The analysis is work in progress with I. Caprini, H. Leutwyler and F. Jegerlehner

Summary

- the precision currently achieved in the measurement of $(g-2)_\mu$ implies a thorough test of our current understanding of particle physics
- the experimental uncertainty is at present lower than the expected size of contributions from supersymmetric extensions of the standard model
- in order to disentangle these we must control the contributions of hadronic physics at low energy at the 1% level
- the current experimental situation concerning $e^+e^- \to$ hadrons and the hadronic τ decay is unfortunately still unclear, but changing rapidly
- theory [$\equiv$ analyticity, unitarity and χ-symmetry] can help in the evaluation of the integral

τ vs e^+e^- data

Isospin relation between $e^+e^- \to \pi^+\pi^-$ and $\tau \to \nu\pi\pi^0$ is currently not understood



use of τ data in the evaluation of a_μ^{hvp} is problematic

τ vs e^+e^- data

If we apply our analysis to the (isospin-corrected) τ data we get (for $P = 3$)

$$\begin{aligned} \tau : \quad & a_\rho = 429.9, \quad a_{2M_K} = 504.3 \\ e^+e^- : \quad & a_\rho = 422.2, \quad a_{2M_K} = 492.2 \end{aligned}$$