Observation of a “cusp” in the decay $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$

Preliminary analysis of 25.8 x 10^6 fully reconstructed $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ decays (~ 100 times more than the largest sample from any previous experiment)

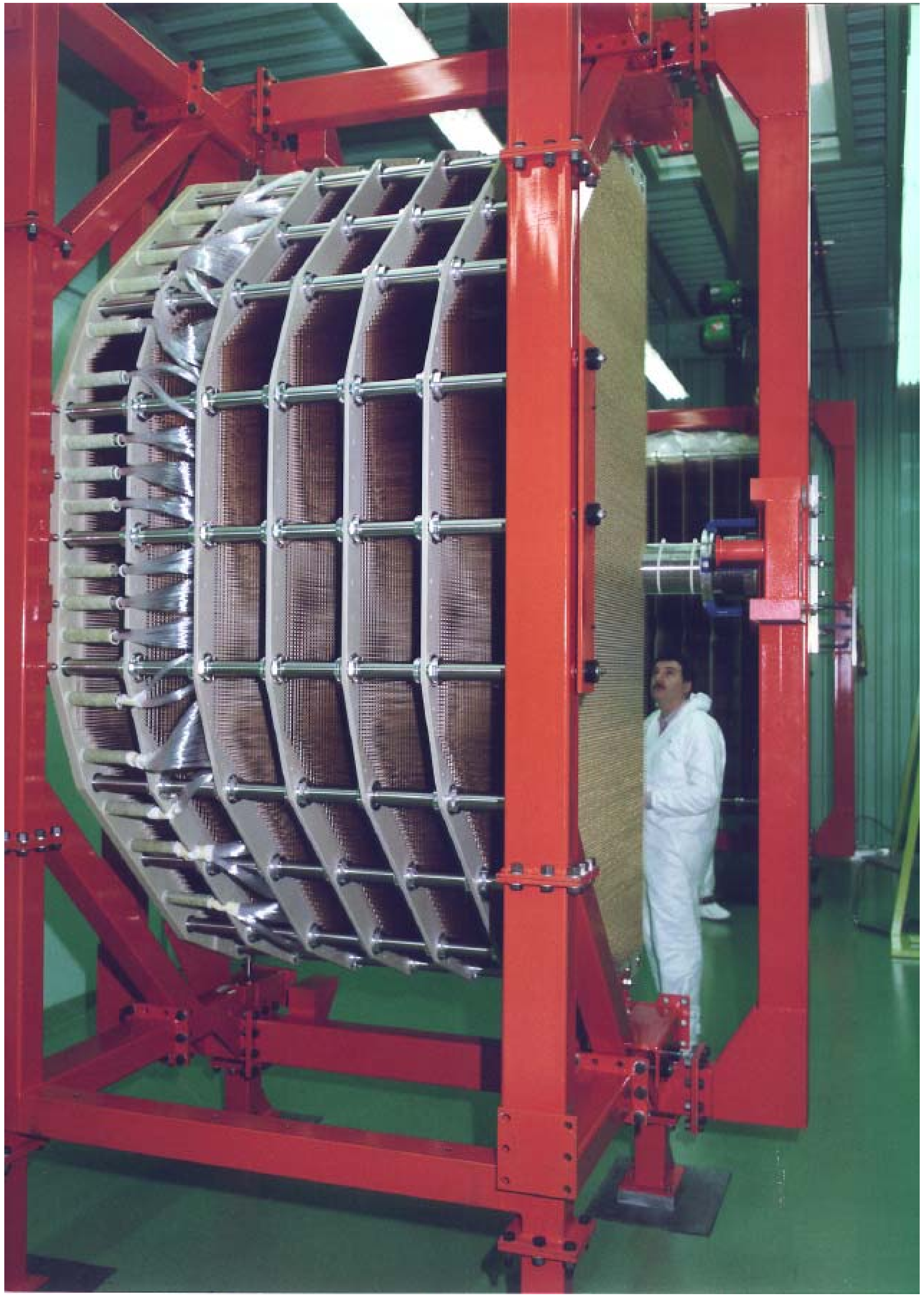
NA 48/2 has very good resolution on the $\pi^{\circ}\pi^{\circ}$ invariant mass

- **Event selection and reconstruction**
- **$\pi^{\circ}\pi^{\circ}$ invariant mass resolution**
- **$\pi^{\circ}\pi^{\circ}$ invariant mass distribution**
- **Interpretation and fits**

L. DiLella, 2 May 2005

Event selection

- **At least one charged particle with momentum $p > 5$ GeV/c**
- **At least 4 photons with $E_\gamma > 3$ GeV detected in the Liquid Krypton (LKr) calorimeter**
- **Geometrical cuts to eliminate detector edge effects (near beam tube and near outer edges of drift chambers and LKr calorimeter)**
- **Distance between photons at LKr > 10 cm**
- **Distance between photons and charged particle at LKr > 15 cm**



Liquid Krypton electromagnetic calorimeter

13248 projective cells, 2 x 2 cm^2

Energy resolution:

$$\frac{\sigma(E)}{E} = \frac{0.032}{\sqrt{E}} \oplus \frac{0.09}{E} \oplus 0.0042$$

$\sigma(E) \approx 142$ MeV for $E = 10$ GeV

Space resolution:

$$\sigma_x = \sigma_y = \frac{0.42}{\sqrt{E}} \oplus 0.06\,\text{cm}$$

$\sigma_x = \sigma_y \approx 1.5$ mm for $E = 10$ GeV

(E in GeV)

Reconstruction of the $\pi^\circ\pi^\circ$ pair

For each photon pair (*i*,*k*) reconstruct common vertex along beam axis under the assumption of $\pi^\circ \rightarrow \gamma\gamma$ decay

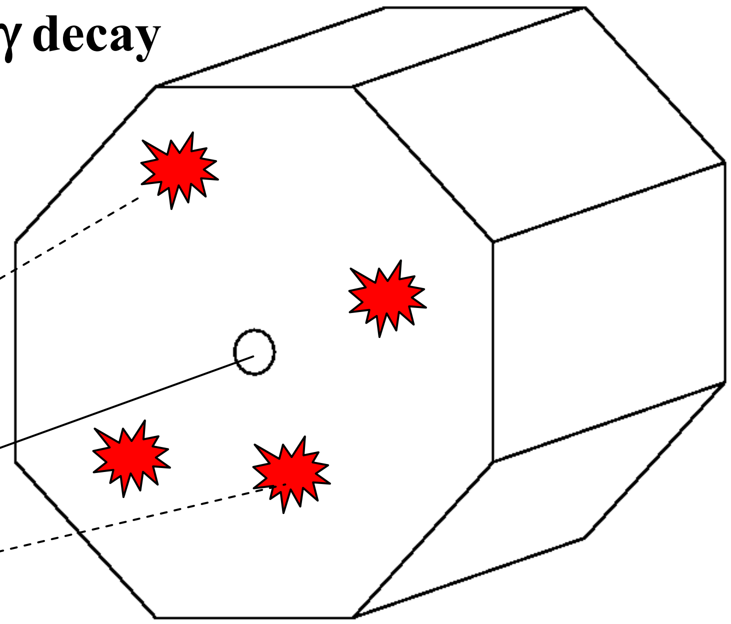
$$m_0^{\ 2} = 2E_iE_k(1-\cos\alpha) \approx E_iE_k\alpha^2 = E_iE_k\left(\frac{D_{ik}}{z_{ik}}\right)^2$$

m_0: π° mass

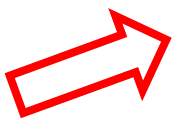
E_i, E_k : photon energies (measured in LKr)

D_{ik} : distance between the two photons on the LKr face

z_{ik} : distance between LKr and π° decay vertex



Liquid Krypton electromagnetic calorimeter

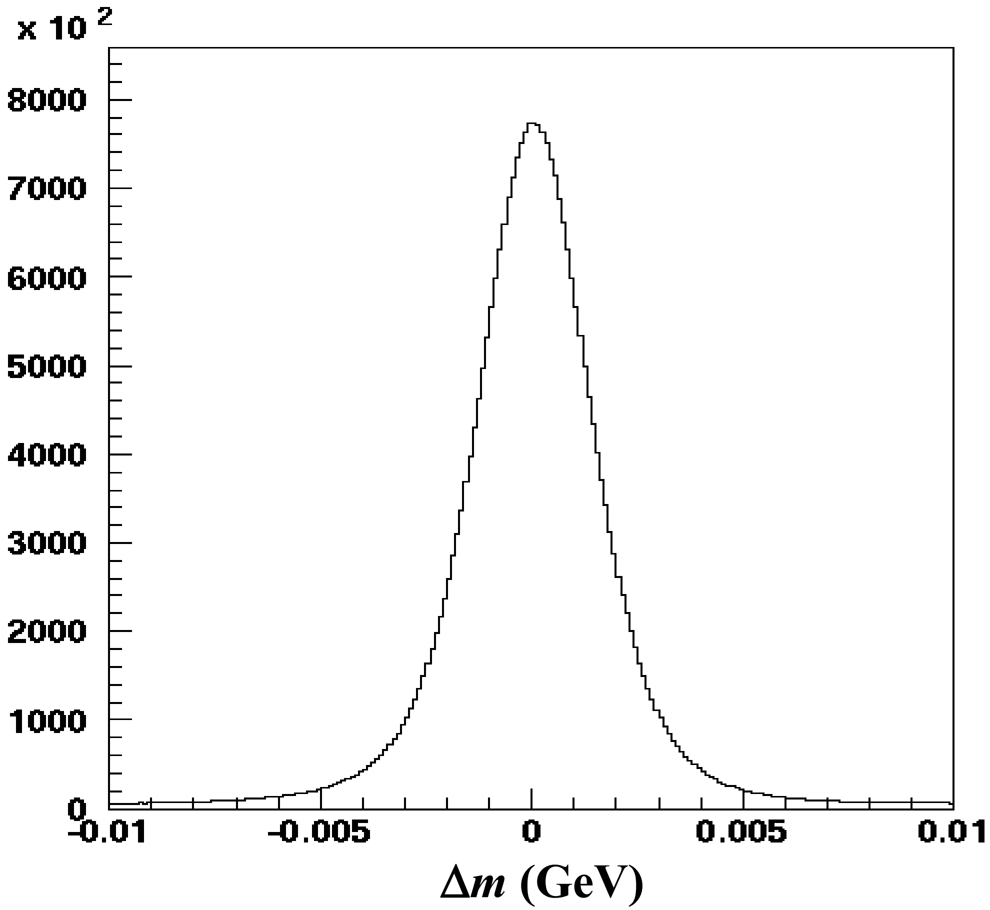


60 GeV beam

Among all possible $\pi^\circ\pi^\circ$ pairs select the pair with minimum difference $|\Delta z| = |z_{ik} - z_{lm}| < 500\ \text{cm} \quad (i\,,k \neq l\,,m)$

Take middle point between the two *z* coordinates as the common origin of the two π° (this choice gives the best $\pi^\circ\pi^\circ$ invariant mass resolution)

Difference Δm between $\pi^{\pm}\pi^{\circ}\pi^{\circ}$ invariant mass and PDG K mass value m_K



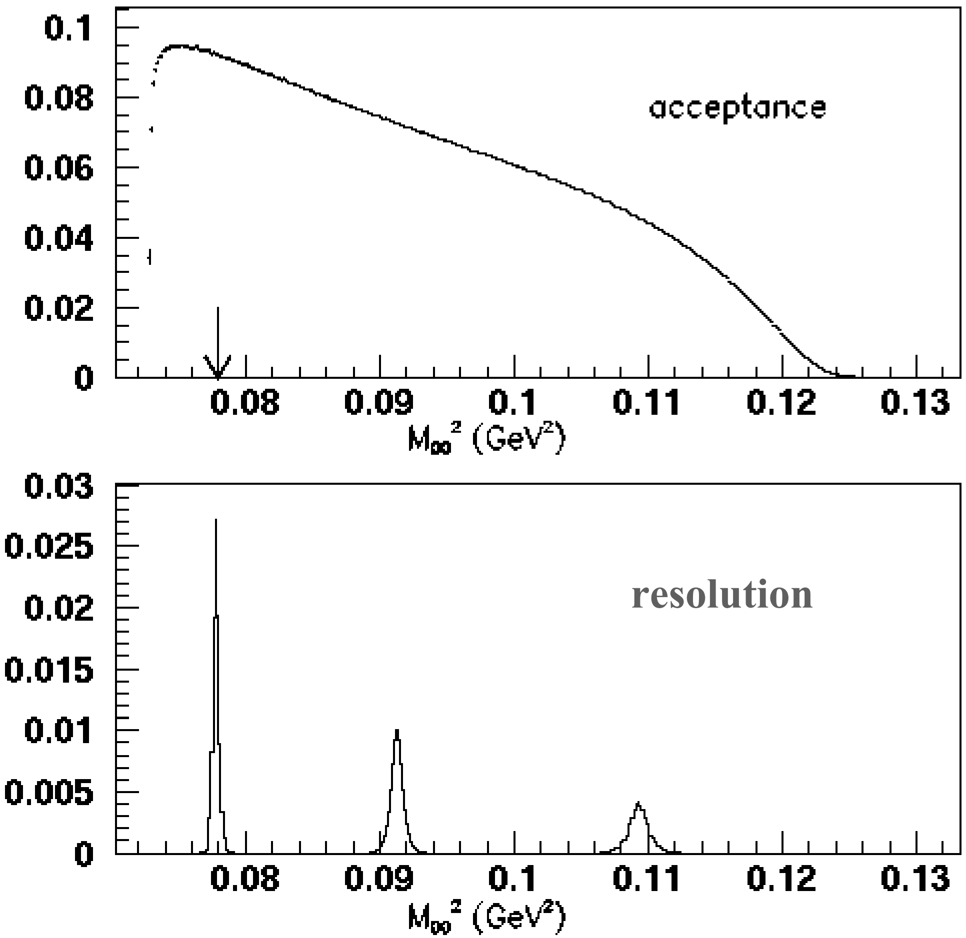
Origin of the tails in the Δm distribution:
$\pi^{\pm} \to \mu^{\pm}$ decay in flight

Select events with $|\Delta m| < 0.005$ GeV

Fraction of events with wrong photon pairings ~ 0.25%
(as estimated from MonteCarlo simulation)

Event acceptance and $\pi°\pi°$ invariant mass resolution

(from MonteCarlo simulation)



$M_{oo} \equiv \pi°\pi°$ invariant mass

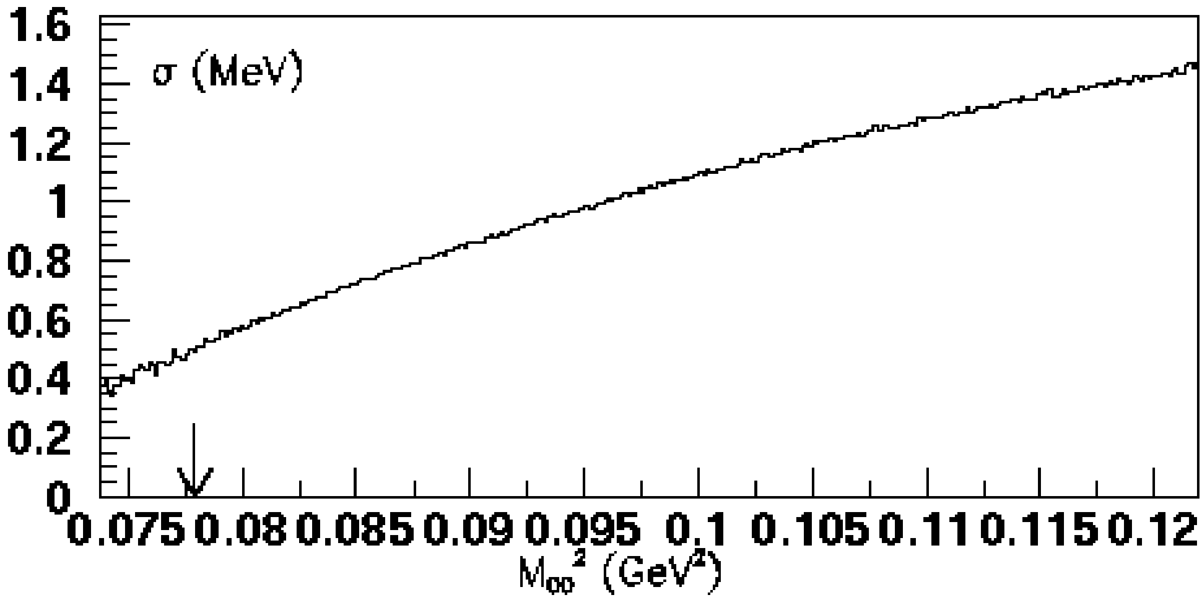
Arrow: $M_{oo} = 2m_+$

$\boldsymbol{m_+}$: π^+ mass

Expected M_{oo}^2 distributions for three generated values of M_{oo}

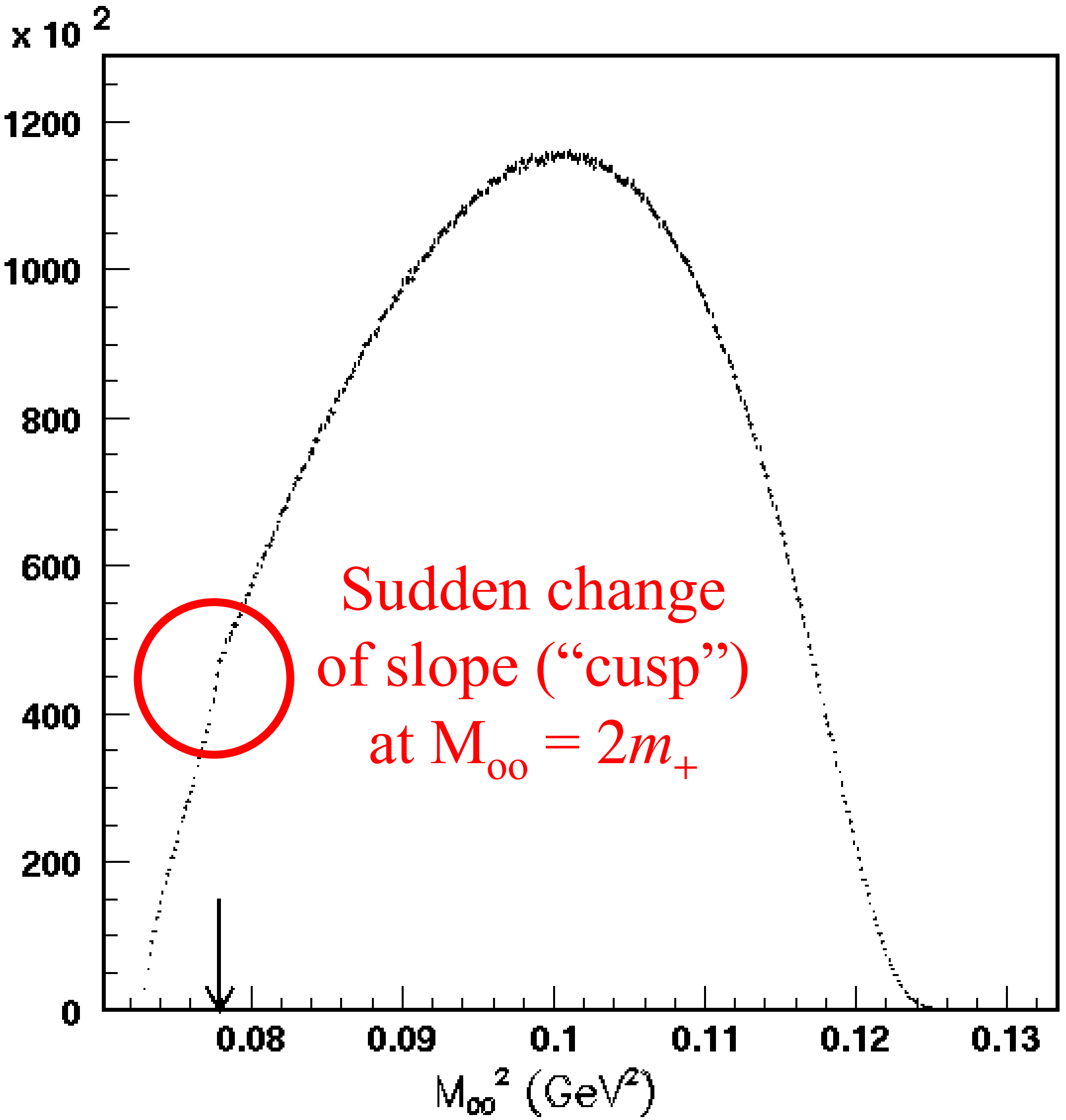
$\pi^\circ\pi^\circ$ invariant mass resolution (σ) versus M_{oo}^2

(from MonteCarlo simulation)

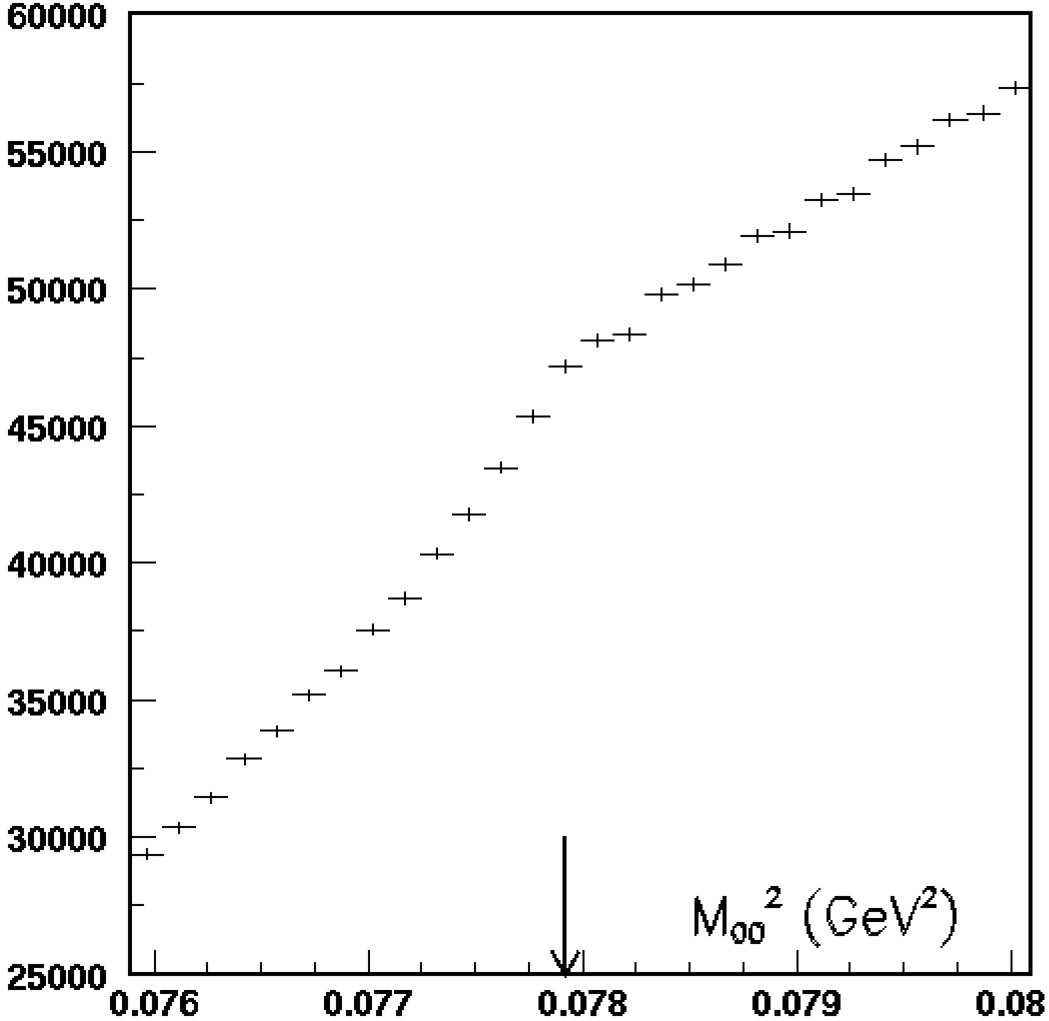


$\sigma \approx 0.5$ **MeV at** $M_{oo} = 2m_+$

Experimental $M_{oo}{}^2$ distribution for 25.82×10^6 $K^\pm \to \pi^\pm \pi^\circ \pi^\circ$ decays



Experimental M_{oo}^2 distribution
"Zoom" on the cusp region



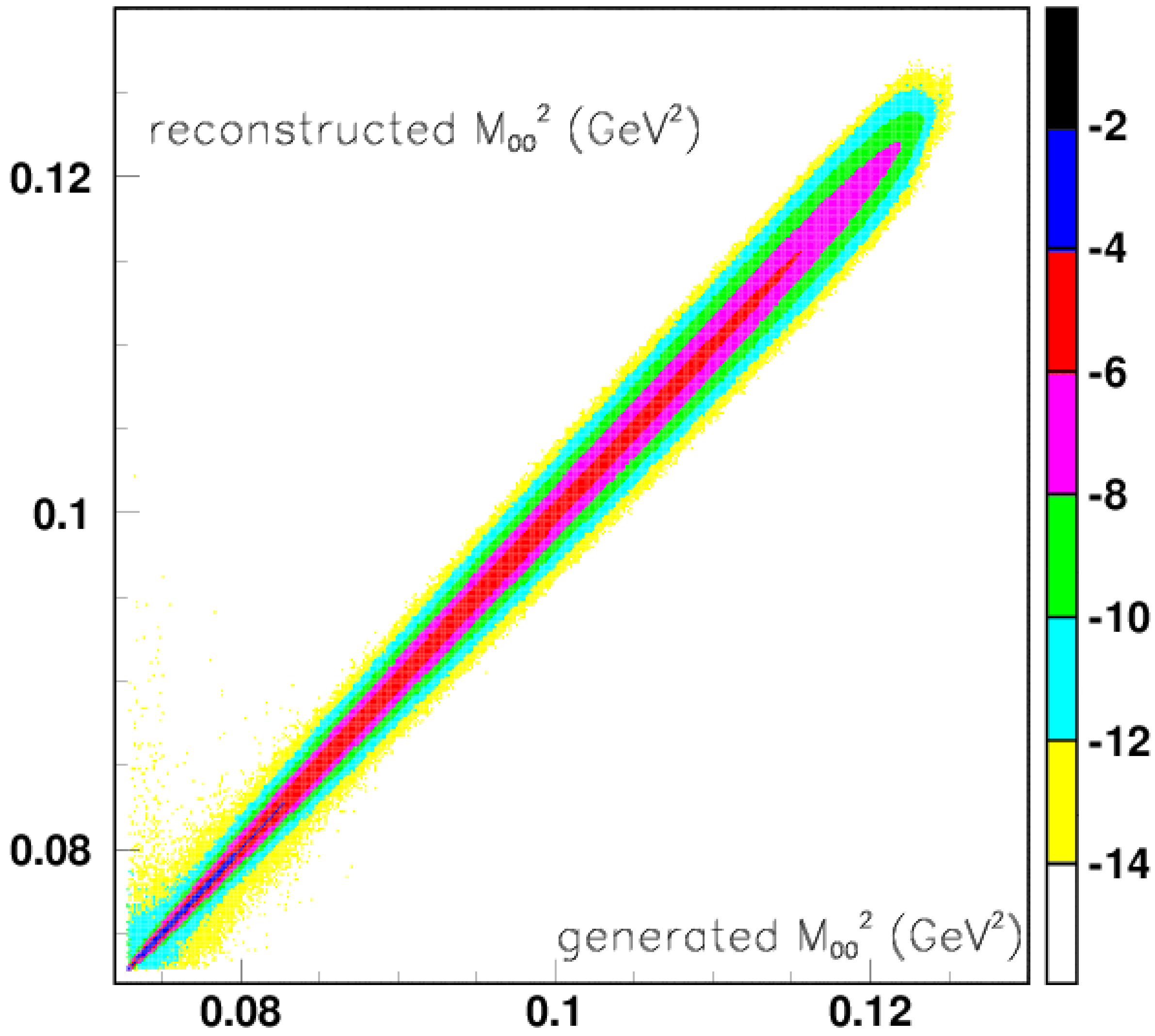
Fits to the experimental M_{oo}^2 distribution METHOD

- Generate theoretical M_{oo}^2 distribution G_i (420 bins of 0.00015 GeV^2)
- From MonteCarlo simulation derive 420 x 420 matrix T_{ik}
  T_{ik} = probability that an event generated with M_{oo} in bin ***i*** is detected and measured in bin ***k*** (T_{ik} includes both acceptance and resolution)
- Produce "reconstructed" M_{oo}^2 distribution R_k :

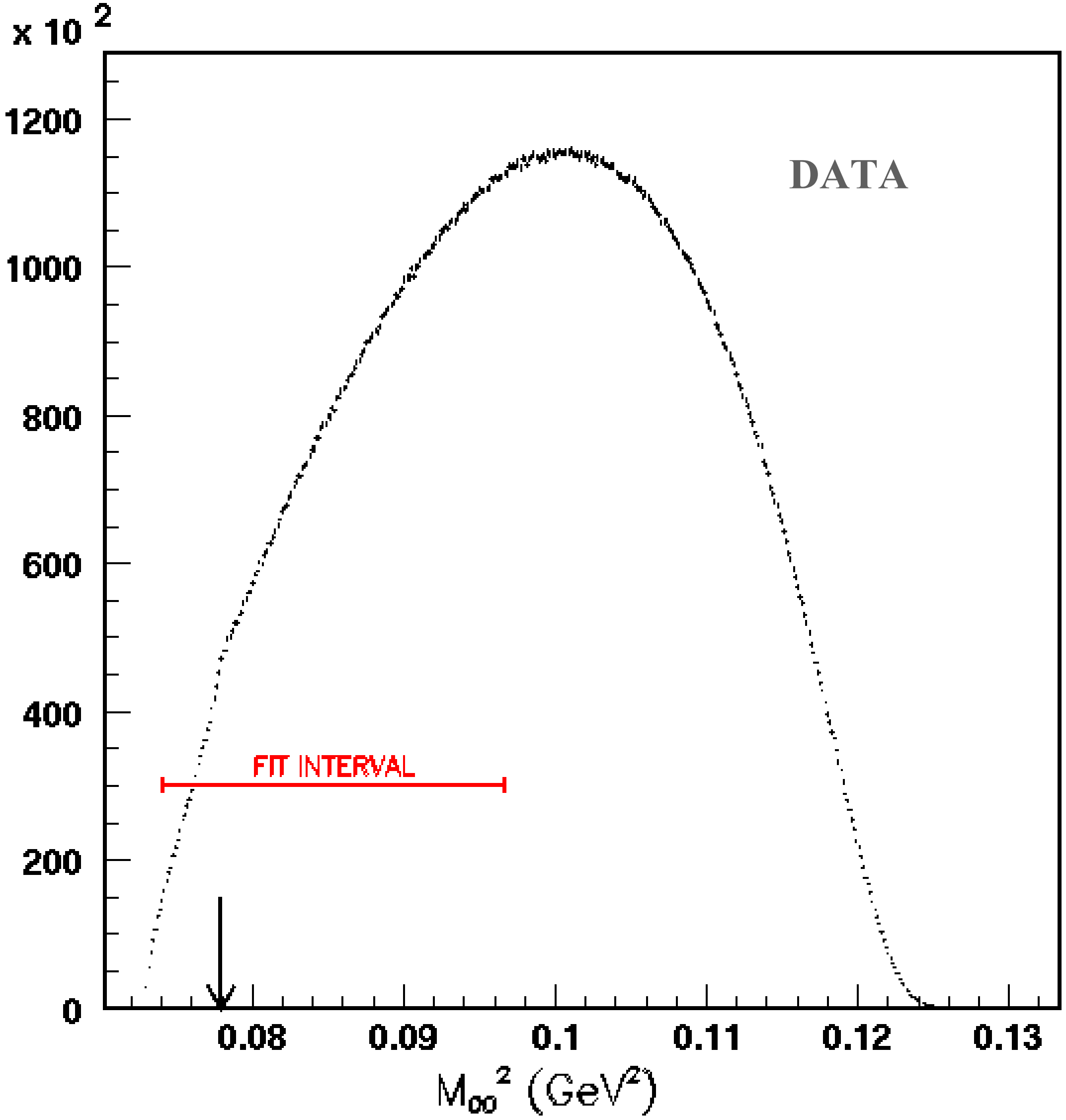
$$\mathbf{R}_k = \sum_i \mathbf{T}_{ik} \mathbf{G}_i$$

- Fit distribution R_k to experimental M_{oo}^2 distribution

$\mathrm{Log}(T_{ik})$
(from MonteCarlo simulation)



Fit interval: $0.0741 < M_{oo}^2 < 0.0967$ GeV^2



- **Fit using modified PDG prescription for decay amplitude:**

$$A_{+00} = 1 + \frac{1}{2} g_o u + \frac{1}{2} h' u^2$$

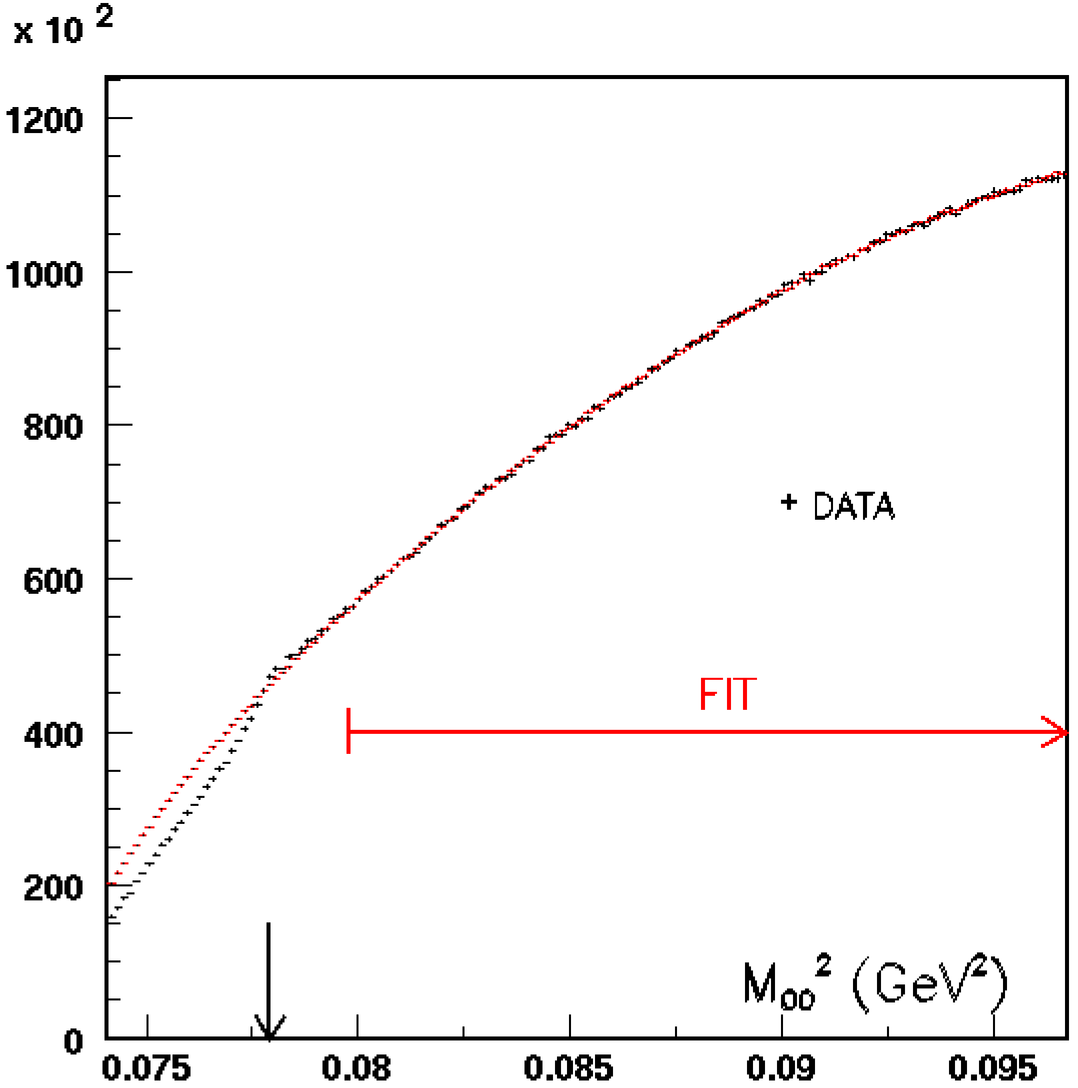
where :

$$u = \frac{\mathrm{M}_{00}{}^2 - s_0}{m_+{}^2} \qquad s_0 = \frac{m_{\mathrm{K}}{}^2 + m_+{}^2 + 2m_0{}^2}{3}$$

Very bad fit: $\chi^2 = 13574$ / 148 d.o.f.

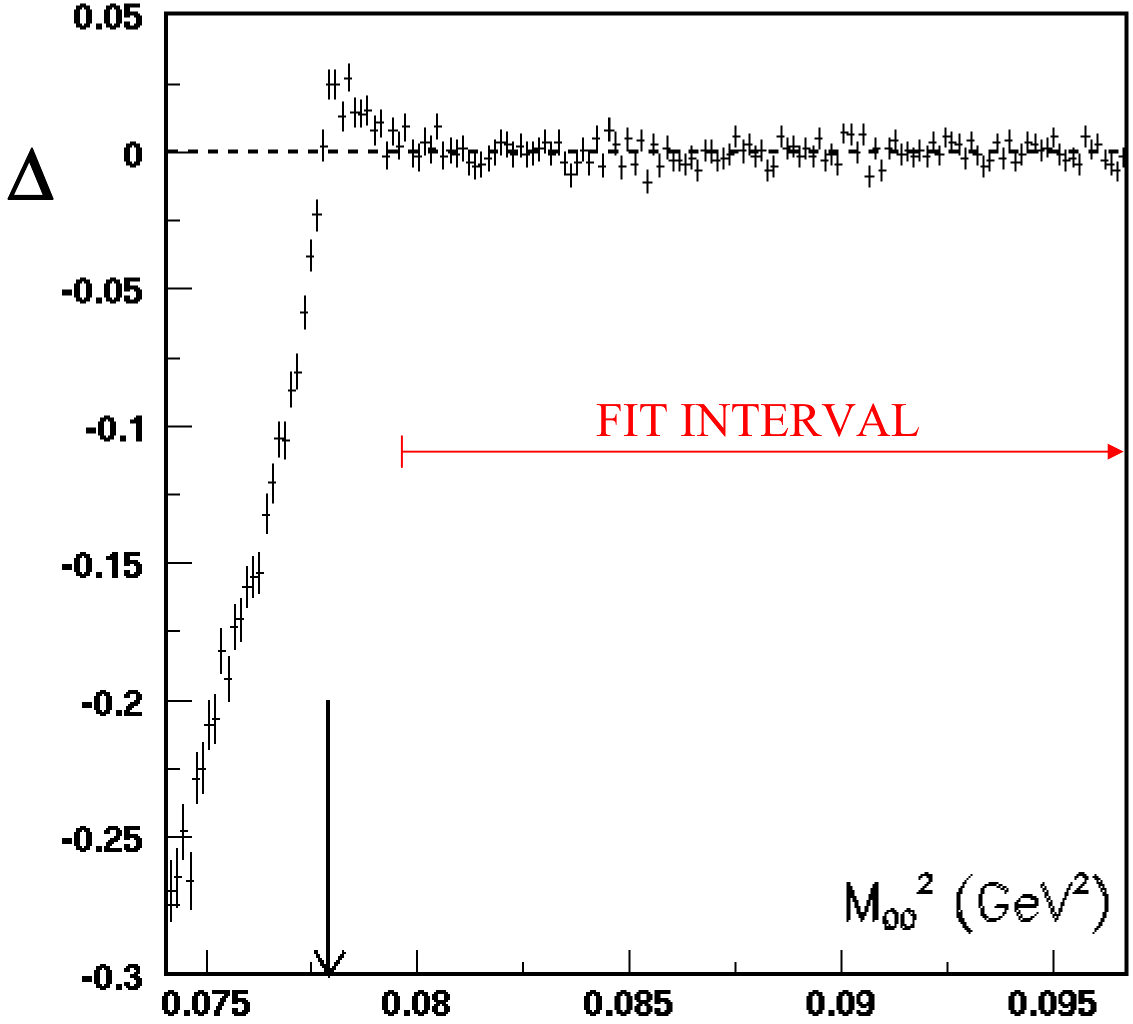
- **Move lower limit of fit interval 13 bins above cusp point**

Reasonable fit: $\chi^2 = 120$ / 110 d.o.f.

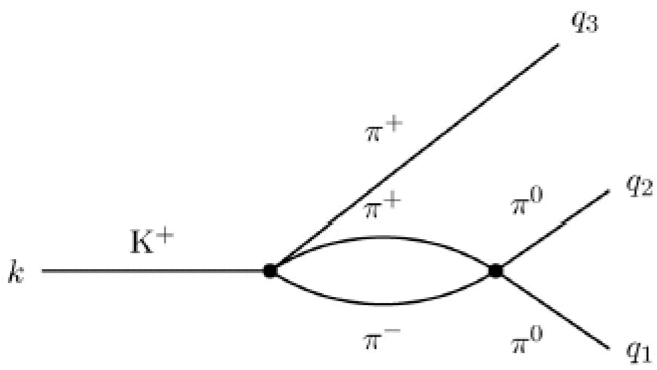


Data – fit comparison shows important "deficit" of events below cusp point

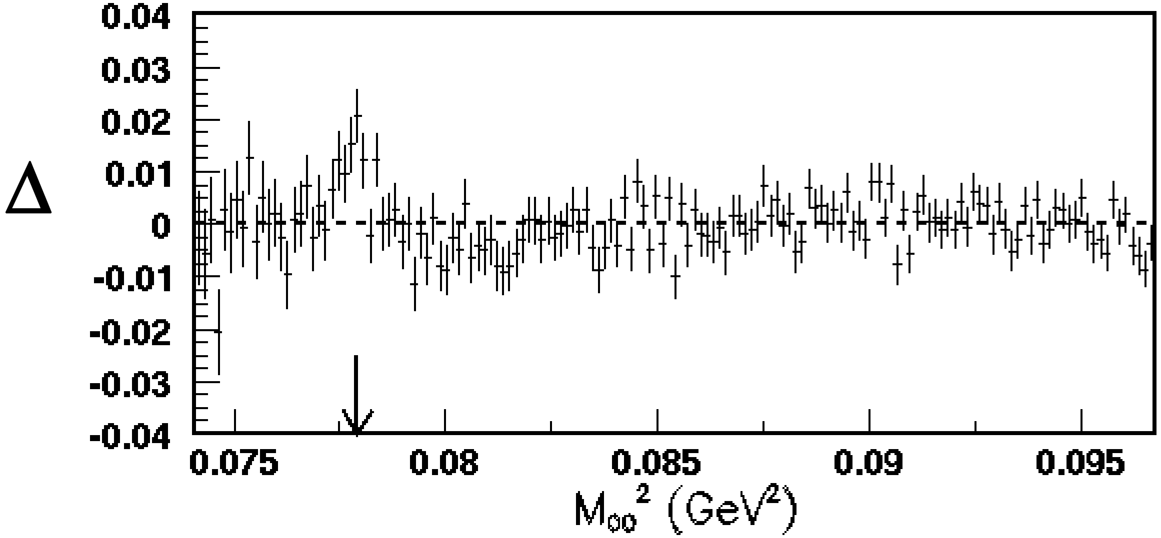
$\Delta \equiv$ (data – fit) /data versus M_{oo}^2



N. Cabibbo
Determination of the a_0–a_2 Pion Scattering Length from $K^+ \to \pi^+\pi^\circ\pi^\circ$ decay
Phys. Rev. Letters 93 (2004) 121801



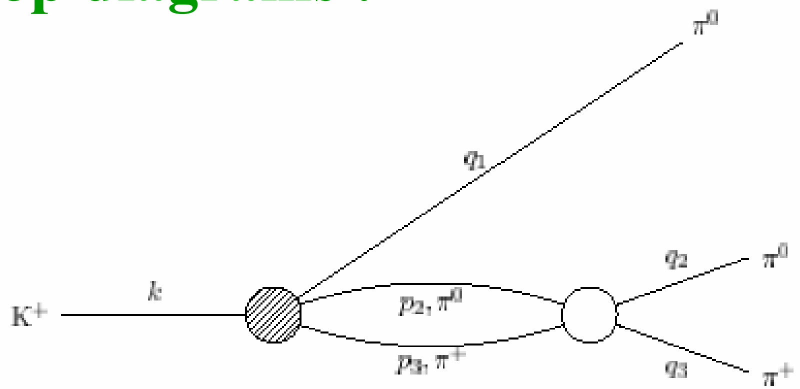
Only one additional free parameter: $(a_0 - a_2)m_+$

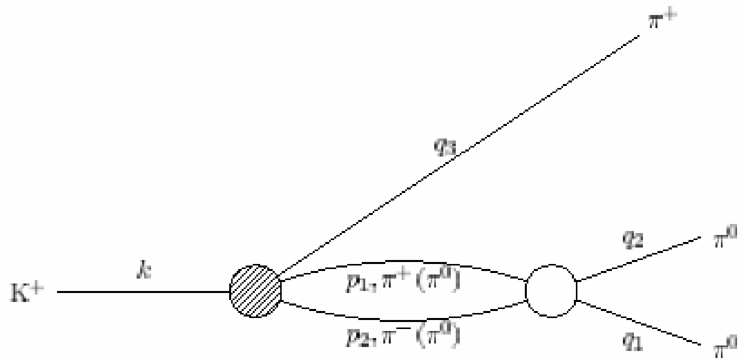


$\chi^2 = 217 / 147$ d.o.f.

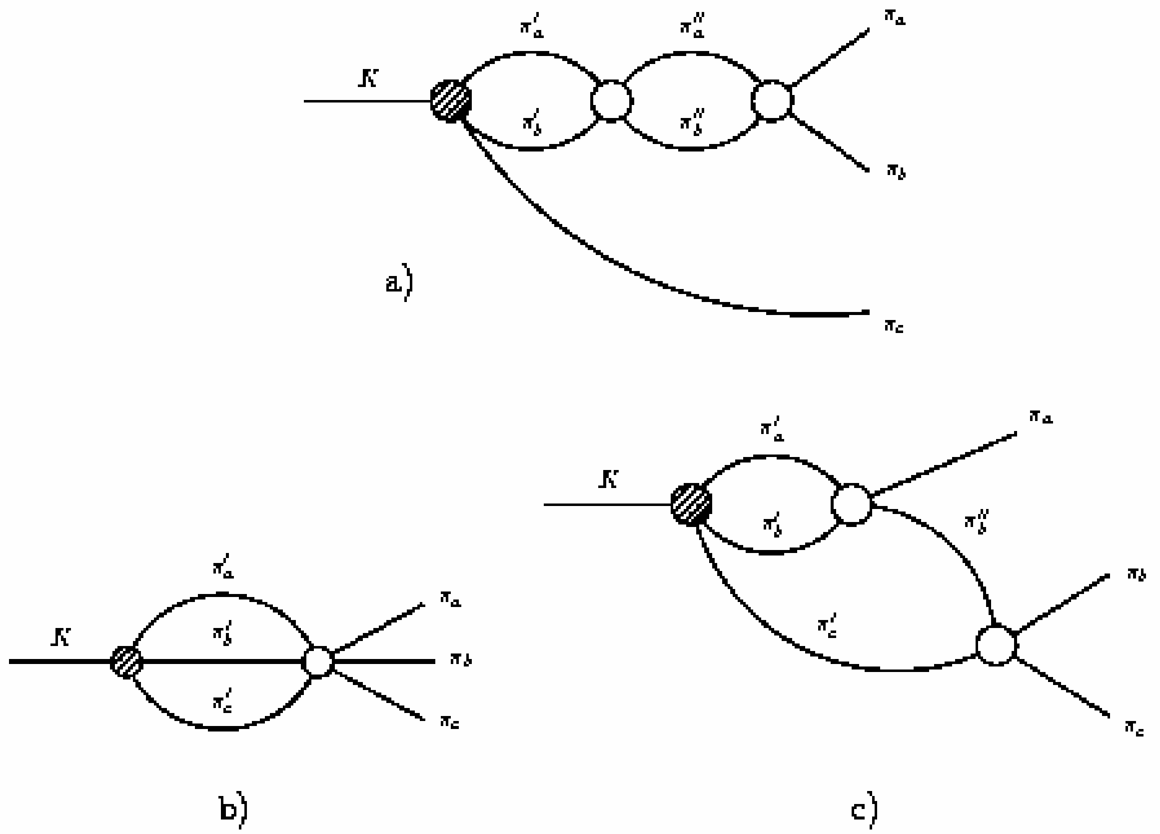
N. Cabibbo and G. Isidori:
Pion – pion scattering and the K $\rightarrow 3\pi$ decay amplitudes
JHEP03 (2005) 021

More one-loop diagrams :



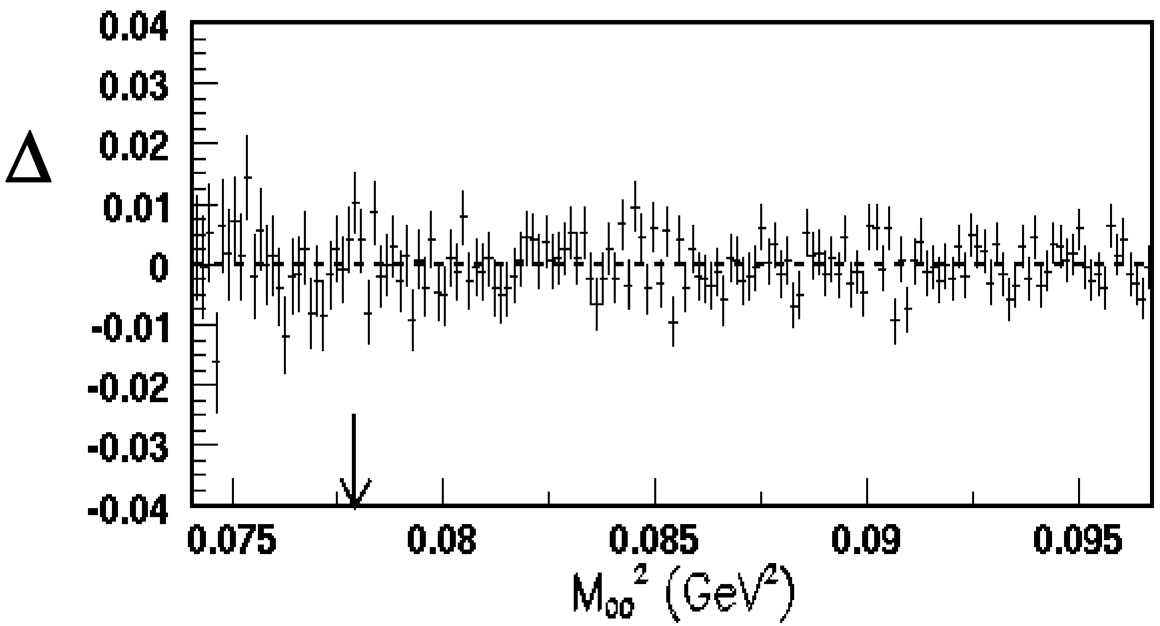


... and also two-loop and three-pion diagrams



One additional free parameter: $a_2 m_+$

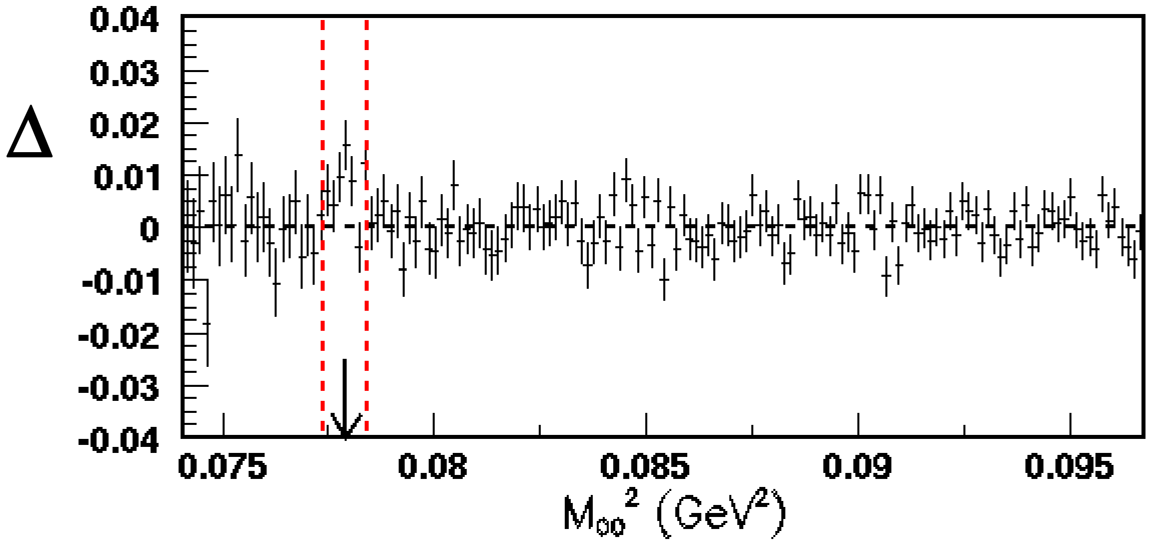
Decay amplitude depends on both Dalitz plot variables –
for each value of M_{oo}^2 set the other variable to its average value



$\chi^2 = 156 / 146$ d.o.f.

Search for formation of $\pi^+\pi^-$ atoms ("pionium") in $K^+ \rightarrow \pi^+\pi^+\pi^-$ decay followed by charge exchange $\pi^+\pi^- \rightarrow \pi^\circ\pi^\circ$

Repeat the fit excluding 7 bins centred at $M_{oo} = 2m_+$



$\chi^2 = 141 / 139$ **d.o.f.**

Excess of events in excluded bins $\Rightarrow$ evidence for pionium
Statistical significance ~2.5 σ

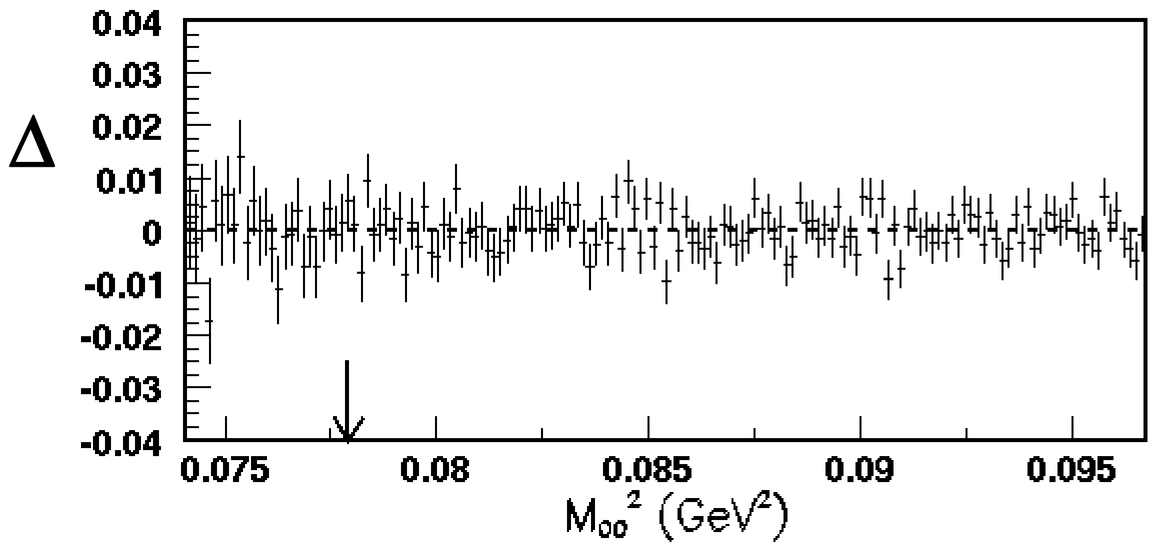
Prediction of pionium formation in $K^+ \to \pi^+\pi^+\pi^-$ decay

(Z.K. Silagadze, hep-ph/9411382 v2 24 Nov 1994)

$$\frac{K^+ \to \pi^+ + pionium}{K^+ \to \pi^+\pi^+\pi^-} \approx 7.4\times10^{-6}$$

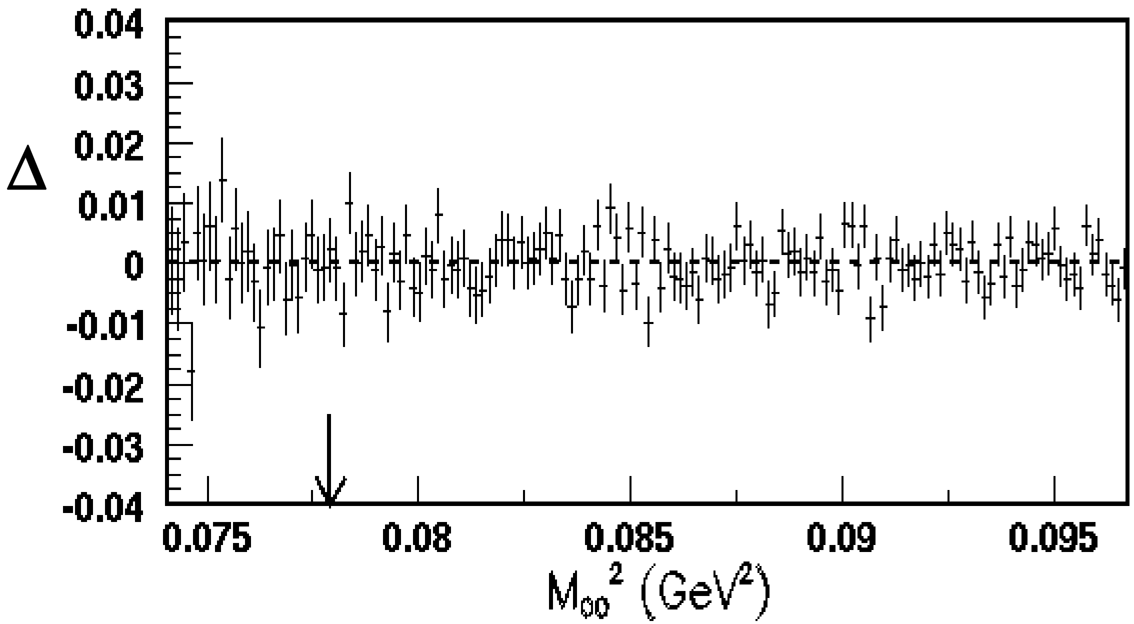
(recalculated by using Silagadze's formulae and more recent $K^+ \to \pi^+\pi^+\pi^-$ data)

Fix pionium contribution at theoretical prediction



$\chi^2 = 150$ / 146 d.o.f.

Pionium contribution as additional free parameter



$\chi^2 = 149 / 145$ **d.o.f.**

Pionium contribution = 1.7 ± 0.6

(Theoretical prediction = 1.0)

Preliminary result presented at seminars and Winter conferences based on fit with pionium contribution = theoretical expectation

$$(a_0 - a_2)m_+ = 0.281 \pm \underset{\text{(stat.)}}{0.007}$$

Preliminary, conservative estimate of systematic uncertainties:

▪ Excluding pionium region from fit interval	0.008
▪ Varying min. distance between photons and charged particle at LKr calorimeter	0.004
▪ From dependence on location of decay vertex along beam axis	0.009
▪ From K^+ / K^- difference	0.006
TOTAL (adding in quadrature)	0.014

acceptance uncertainties

No surprises from the other fitting parameters:

- a_2 **consistent with ChPT prediction**
- g_0, h' **in reasonable agreement with previous experiments**

Statistical errors on the other fitting parameters :

$$\sigma(g_0) = \pm\, 0.004$$

$$\sigma(h') = \pm\, 0.009$$

$$\sigma(a_2 m_+) = \pm\, 0.018$$

Studies of systematic uncertainties on these parameters still to be done $\Rightarrow$ no best fit values presented yet

Systematic uncertainties on $(a_0 - a_2)m_+$ are expected to become comparable to the statistical error, or even smaller, from further analysis

Theoretical uncertainties on $(a_0 - a_2)m_+$

Estimate by Cabibbo and Isidori : ± 0.014 ($\pm 5\%$)
(from missing radiative corrections and higher-order diagrams)
MOST LIKELY THE DOMINANT UNCERTAINTY
AT THE END OF THE DATA ANALYSIS

Are these uncertainties reduced by excluding from the fit the pionium region ?

Note additional uncertainty from ratio of weak decay amplitudes $R = A\,(K^+ \to \pi^+ \pi^+ \pi^-)\ /\ A\,(K^+ \to \pi^+ \pi^\circ \pi^\circ)$

- **From isospin invariance $R = 2$**
- **R can be calculated by integrating PDG matrix elements over phase space and comparing result with ratio of branching ratios: $R = 1.972 \pm 0.023$**
  (this procedure should be modified to take into account NA48/2 results on $K^+ \to \pi^+ \pi^\circ \pi^\circ$)

± 0.03 uncertainty on $R \Rightarrow \pm 0.003$ uncertainty on $(a_0 - a_2)\, m_+$

CONCLUSIONS

- A clear cusp has been observed by NA48 / 2 in the $\pi^\circ\pi^\circ$ invariant mass distribution from $K^\pm \to \pi^\pm\,\pi^\circ\,\pi^\circ$ decay at $M_{oo} = 2\,m_+$
- The new level of precision of the NA48 / 2 data requires a redefinition of the parameters generally used to describe $K^\pm \to \pi^\pm\,\pi^\circ\,\pi^\circ$ decay (e.g., PDG 2004)
- This cusp is the effect of $\pi\pi$ scattering in the final state, dominated by the charge exchange process $\pi^+\pi^- \to \pi^\circ\pi^\circ$.
- The study of the $\pi^\circ\pi^\circ$ invariant mass distribution from $K^\pm \to \pi^\pm\,\pi^\circ\,\pi^\circ$ decay offers a new, potentially precise method to measure $(a_0 - a_2)m_+$ independently of other methods (e.g., measurement of pionium lifetime)
- The final $K^\pm \to \pi^\pm\,\pi^\circ\,\pi^\circ$ decay sample collected in 2003 – 04 will contain $\sim 10^8$ events
- We need theoretical guidance to extract values of the $\pi\pi$ scattering parameters from these data with the best possible precision