CP violation in decays $K^{\pm} \to 3\pi$.

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We know the present results of the search for CP-odd difference in energy distribution of "odd" pions in decays

$$K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp} \qquad (\tau\text{-decay})$$

$$K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm} \qquad (\tau'\text{-decay})$$

What is expected in SM for a difference between the parameters g_{τ}^{+} and g_{τ}^{-} and $g_{\tau'}^{+}$ and $g_{\tau'}^{-}$ defined by the relations

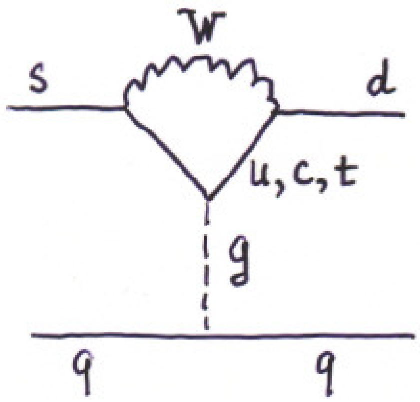
$$\left| M\left(K^{\pm}(k) \to \pi^{\pm}(p_1)\pi^{\pm}(p_2)\pi^{\mp}(p_3)\right)\right|^2 \sim 1 + g_{\tau}^{\pm} Y + \dots$$

$$\left| M\left(K^{\pm}(k) \to \pi^{0}(p_1)\pi^{0}(p_2)\pi^{\pm}(p_3)\right)\right|^2 \sim 1 + g_{\tau'}^{\pm} Y + \dots$$

where

$$Y = \frac{s_3 - s_0}{m_{\pi}^2}, \quad s_3 = (k - p_3)^2, \quad s_0 = \frac{1}{3} m_K^2 + m_{\pi}^2 .$$

The difference is generated in SM by the complex coupling constants figurating in so-called penguin diagrams:

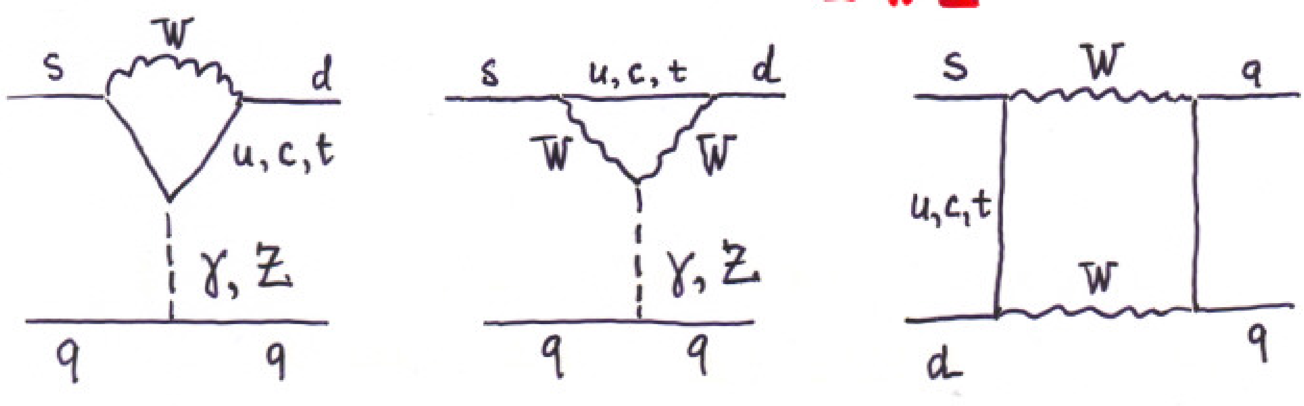


QCD penguin diagram

QCDP

Electroweak penguin diagrams

EWP



For some observables, in particular, for ε' in $K^0 \to 2\pi$ decays, the QCDP and EWP contributions are of opposite signs that leads to considerable decrease of this parameter.

Flynn, Randall '89
Buchalla et al '90
Pashos, Wu '91

There were many attemps to calculate QCDP and EWP contributions, but they gave ε' with uncertainty of order ten

$$\text{few} \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq \text{few} \cdot 10^{-3} .$$

Now, when ε' is measured and known with 15% accuracy

$$\varepsilon'/\varepsilon = (1.67 \pm 0.26)\, 10^{-3}$$

it becomes to be possible to clear up the individul role of QCDP and EWP in **direct** CP violation

It may be done measuring the quantities

$$\Delta g_\tau = \frac{g_\tau^+ - g_\tau^-}{g_\tau^+ + g_\tau^-} \quad \text{and} \quad \Delta g_{\tau'} = \frac{g_{\tau'}^+ - g_{\tau'}^-}{g_{\tau'}^+ + g_{\tau'}^-}$$

in $K^\pm \to 3\pi$ decays.

It will be shown that EWP increases Δg_τ and decreases $\Delta g_{\tau'}$ <u>and what's more</u>, $\Delta g_{\tau'}$ turns out to be proportional practically to the same combination of QCDP and EWP contributions <u>as for ε'</u>!

Then, $\Delta g_{\tau'}$ becomes fixed and the separate contributions of QCDP and EWP may be evaluated from $\Delta g_\tau / \Delta g_{\tau'}$.

Our analysis is based on employment of the effective $\Delta S = 1$ non-leptonic lagrangian in the form proposed by SVZ'77 (JETP $\underline{72}$, 1275)

$$L(\Delta S=1) = \sqrt{2}\, G_F \sin\theta_C \cos\theta_C \sum c_i O_i$$

where (for example)

$$O_1 = \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L - \bar{s}_L \gamma_\mu u_L \cdot \bar{u}_L \gamma_\mu d_L \quad (\{8_f\}, \Delta I = 1/2)$$

....

....

$$O_4 = \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \cdot \bar{u}_L \gamma_\mu d_L - \bar{s}_L \gamma_\mu d_L \cdot \bar{d}_L \gamma_\mu d_L \quad (\{27\}, \Delta I = 3/2)$$

$$O_5 = \bar{s}_L \gamma_\mu \lambda^a d_L \Big(\sum_{q=u,d,s} \bar{q}_R \gamma_\mu \lambda^a q_R \Big) \quad (\{8\}, \Delta I = 1/2)$$

....

$$O_7 = \frac{3}{2}\, \bar{s} \gamma_\mu (1+\gamma_5) d \Big(\sum_{q=u,d,s} e_q\, \bar{q} \gamma_\mu (1-\gamma_5) q \Big) \quad (\Delta I = 1/2, 3/2)$$

....

The bosonization:

Bardeen, Buras Gerard'87

$$\bar{q}_j (1+\gamma_5) q_k = -\frac{1}{\sqrt{2}} F_\pi r \Big(U - \frac{1}{\Lambda^2} \partial^2 U \Big)_{kj}$$

$$\bar{q}_j \gamma_\mu (1+\gamma_5) q_k = i \left[\partial_\mu U \cdot U^+ - U \cdot \partial_\mu U^+ \right]_{kj} - \frac{r F_\pi}{\sqrt{2} \Lambda^2} \left[m\, \partial_\mu U^+ - \partial_\mu U\, m \right]_{kj}$$

$r = \frac{2 m_\pi^2}{m_u + m_d}$, $\Lambda \approx 1$ GeV, $F_\pi = 93$ MeV, $m = \mathrm{diag}\{m_u, m_d, m_s\}$

$U = \frac{F_\pi}{\sqrt{2}}\, e^{i \frac{\sqrt{2}\hat{\pi}}{F_\pi}}$, $\hat{\pi}$ is 3×3 matrix of pseudoscalar fields

Using this technics and representing $M(K \to 2\pi)$ in the form

$$M(K_1^0 \to \pi^+\pi^-) = A_0 e^{i\delta_0} - A_2 e^{i\delta_2}$$

$$M(K_1^0 \to \pi^0\pi^0) = A_0 e^{i\delta_0} + 2A_2 e^{i\delta_2}$$

$$M(K^+ \to \pi^+\pi^0) = -\frac{3}{2} A_2 e^{i\delta_2}$$

we obtain

$$A_0 = \varkappa \left[c_1 - c_2 - c_3 + \frac{32}{9} \beta \left(\mathrm{Re}\, \tilde{c}_5 + i\, \mathrm{Im}\, \tilde{c}_5 \right) \right]$$

$$A_2 = \varkappa \left[c_4 + i \frac{2}{3} \beta \Lambda^2\, \mathrm{Im}\, \tilde{c}_7 \left(m_K^2 - m_\pi^2 \right)^{-1} \right]$$

where

$$\varkappa = G_F F_\pi \sin\theta_C \cos\theta_C \frac{m_K^2 - m_\pi^2}{\sqrt{2}},$$

$$\tilde{c}_5 = c_5 + \frac{3}{16} c_6, \quad \tilde{c}_7 = c_7 + 3c_8$$

$$\beta = \frac{2 m_\pi^4}{\Lambda^2 (m_u + m_d)^2}$$

As $\tilde{c}_7 / \tilde{c}_5 \sim \alpha_{em}$, we have neglected EWP contribution to A_0.

From data on $K \to 2\pi$ rates

$$c_4 = 0.328; \quad c_1 - c_2 - c_3 + \frac{32}{9} \beta\, \mathrm{Re}\, \tilde{c}_5 = -10.13$$

At $c_1 - c_2 - c_3 = -2.89$

$$\frac{32}{9} \beta\, \mathrm{Re}\, \tilde{c}_5 = -7.24$$

Using the general relation

$$\varepsilon' = i\, e^{i(\delta_2-\delta_0)}\left[-\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0} + \frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2}\right]\cdot\left|\frac{A_2}{A_0}\right|$$

and experimental value of ε', we obtain

$$\boxed{\boxed{\beta\,\mathrm{Im}\,\tilde{c}_5\left(1+\frac{24.36}{1-\Omega}\,\frac{\mathrm{Im}\,\tilde{c}_7}{\mathrm{Im}\,\tilde{c}_5}\right)} = \frac{3.32\,(1\pm 0.15)\,10^{-4}}{1-\Omega}} \quad (*)$$

Ω takes into account $K \to \pi^0\eta\,(\eta') \to \pi^0\pi^0$ transit.

Knowledge of magnitude of quantity $(*)$ will allow to fix $\Delta g_{\tau'}$!

$K^{\pm} \to 3\pi$ decays

Applying the same technics and taking into account an appearance of CP-even imaginary parts due to strong $\pi\pi$ rescattering we find in leading p^2 approximation:

$$M\left(K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}(p_3)\right) =$$

$$= \tilde{\varkappa}\left[1 + ia + \tfrac{1}{2} g_{\tau} Y\left(1 + i b^{\tau} \pm i\left(b^{\tau}_{KM} - a^{\tau}_{KM}\right) + \ldots\right]\right.$$

$$M\left(K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}(p_3)\right) =$$

$$= \frac{\tilde{\varkappa}}{2}\left[1 + ia + \tfrac{1}{2} g_{\tau'} Y\left(1 + i b^{\tau'} \pm i\left(b^{\tau'}_{KM} - a^{\tau'}_{KM}\right) + \ldots\right]\right.$$

where

a, b^{τ}, $b^{\tau'}$ are CP-even imaginary parts having the same sign for K^+ and K^- mesons;

$a^{\tau,\tau'}_{KM}$, $b^{\tau,\tau'}_{KM}$ are CP-odd imaginary parts produced by K-M phase and having the opposite signs for K^+ and K^- mesons.

$$a = a(s_0)\,, \quad s_0 = \tfrac{1}{3} m_K^2 + m_\pi^2$$

For $K^\pm \to \pi^\pm \pi^\pm \pi^\mp$

$$a^\tau_{KM} = \left[\frac{32}{9}\beta\,\mathrm{Im}\,\tilde{c}_5 + 4\beta\,\mathrm{Im}\,\tilde{c}_7\left(\frac{3\Lambda^2}{2m_K^2}+2\right)\right]\Big/ c_0$$

$$b^\tau_{KM} = \left[\frac{32}{9}\beta\,\mathrm{Im}\,\tilde{c}_5 + 8\beta\,\mathrm{Im}\,\tilde{c}_7\right]\Big/(c_0+9c_4)$$

$$a = 0.12\,;\quad b^\tau = 0.71\,;\quad g_\tau = -\frac{3m_\pi^2}{m_K^2}\left(1+\frac{9c_4}{c_0}\right)$$

$$c_0 = c_1 - c_2 - c_3 - c_4 + \frac{32}{9}\beta\,\mathrm{Re}\,\tilde{c}_5 = -10.46$$

For $K^\pm \to \pi^0 \pi^0 \pi^\pm$

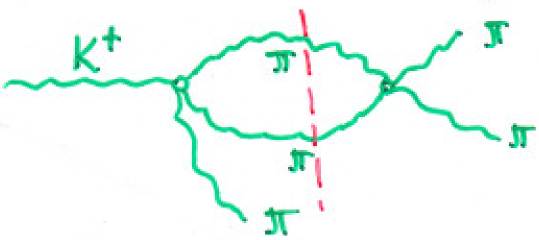
$$a^{\tau'}_{KM} = \left[\frac{32}{9}\beta\,\mathrm{Im}\,\tilde{c}_5 + \frac{6\beta\Lambda^2\,\mathrm{Im}\,\tilde{c}_7}{m_K^2}\right]\Big/ c_0$$

$$b^{\tau'}_{KM} = \left[\frac{32}{9}\beta\,\mathrm{Im}\,\tilde{c}_5 + \frac{3\beta\Lambda^2\,\mathrm{Im}\,\tilde{c}_7}{m_K^2 - m_\pi^2}\right]\Big/\left(c_0 - \frac{9c_4}{2}\right)$$

G. Fäldt and E. Sh' 2005 hep-ph/0503241

$$b^{\tau'} = 0.49\,;\quad g_{\tau'} = \frac{6m_\pi^2}{m_K^2}\left(1-\frac{9c_4}{2c_0}\right)$$

The quantities a, b^τ, $b^{\tau'}$ are obtained by calculations of the diagrams



with different allowed charge compositions of on-mass-shell pions inside the loop.

From our formulae, it follows that

$$\Delta g_{\tau'} = \frac{g^+_{\tau'} - g^-_{\tau'}}{g^+_{\tau'} + g^-_{\tau'}} = \frac{a\left(b^{\tau'}_{KM} - a^{\tau'}_{KM}\right)}{1 + a b^{\tau'}} \, .$$

Replacing τ' by τ, one obtains Δg_τ.

$$b^{\tau'}_{KM} - a^{\tau'}_{KM} = \frac{16 c_4}{c_0\left(c_0 - \frac{9c_4}{2}\right)} \boxed{\beta \operatorname{Im}\tilde{c}_5\left(1 + 27.8 \operatorname{Im}\tilde{c}_7/\operatorname{Im}\tilde{c}_5\right)}$$

The combination inside $\boxed{}$ is very like the combination defining ε':

$$\boxed{\beta \operatorname{Im}\tilde{c}_5\left(1 + \frac{24.36}{1-\Omega} \operatorname{Im}\tilde{c}_7/\operatorname{Im}\tilde{c}_5\right)} = \frac{3.32\,(1 \pm 0.15)\,10^{-4}}{1-\Omega}$$

At $\Omega = 0.124$ these combinations coincide. Then

$$\boxed{\Delta g_{\tau'} = 1.8\,(1 \pm 0.15)\,10^{-6}}$$

At $\Omega = 0.25$

$$\Delta g_{\tau'} = 2.1 \cdot 10^{-6}\left(1 - \frac{4.7 \operatorname{Im}\tilde{c}_7/\operatorname{Im}\tilde{c}_5}{1 + 32.48 \operatorname{Im}\tilde{c}_7/\operatorname{Im}\tilde{c}_5}\right)$$

$$\approx 2.1 \cdot 10^{-6}\,(1 + 0.14)$$

Both values of Ω do not contradict to the estimates figurating in literature:

Cirigliano et al' 2004 $\Rightarrow \Omega = 0.06 \pm 0.08$

Many others' (96-98) $\Rightarrow \Omega = 0.25 \pm 0.08$

For $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$

$$b^{\tau}_{KM} - a^{\tau}_{KM} = -2\,\frac{16c_4}{c_0(c_0+9c_4)}\left(1 - 14.34\,\mathrm{Im}\,\tilde{c}_7/\mathrm{Im}\,\tilde{c}_5\right)$$

Then

$$\boxed{-\frac{\Delta g_\tau}{\Delta g_{\tau'}} = 2\,\frac{c_0 - \frac{9c_4}{2}}{c_0+9c_4}\cdot\frac{1+a\,b^{\tau'}}{1+a\,b^{\tau}}\cdot\frac{1-14.34\,\mathrm{Im}\,\tilde{c}_7/\mathrm{Im}\,\tilde{c}_5}{1+27.8\,\mathrm{Im}\,\tilde{c}_7/\mathrm{Im}\,\tilde{c}_5}}$$

Therefore:

1) If EWP do not play any significant role in direct CP violation, then

$$-\frac{\Delta g_\tau}{\Delta g_{\tau'}} = 3.1 \qquad \text{or} \qquad -\Delta g_\tau > 0.56\cdot 10^{-5}$$

2) If EWP cancel half of QCDP contribution, then

$$-\Delta g_\tau = 7.8\,\Delta g_{\tau'} > 1.4\cdot 10^{-5}$$

If EWP cancel 3/4 of QCDP contribution, then

$$-\Delta g_\tau = 17.2\,\Delta g_{\tau'} > 3.1\cdot 10^{-5}$$

The role of P^4 corrections.

For Δg_τ was studied in [E. Sh., Physics of Atomic Nucl. 68 (2005) 88; hep-ph/0405229].
Such corrections increase Δg_τ by 23%.

For $\Delta g_{\tau'}$ such examination is not fulfilled so far. But one effect is seen immediately:
because $\Delta g_{\tau,\tau'} \sim a$, and $(a)_{p^2+p^4} \approx 1.3\,(a)_{p^2}$, it is expected

$$(\Delta g_{\tau'})_{corr} \approx 2.4 \cdot 10^{-6}.$$

Conclusions.

1) $\Delta g_{\tau'}$ is fixed in the interval $(2 \div 3)10^{-6}$

2) Δg_τ may be one order larger: few $\cdot\, 10^{-5}$

[If EWP cancel half of QCDP contribution:
Hambye, Peris, de Rafael '03
or
more: Donoghue, Golovich '2000]

Δg

Δg_τ (in units 10^{-5})	$\Delta g_{\tau'}$ (in 10^{-5})	Refs.
-700 ± 500	-15 ± 275	Bel'kov et al.' 1989
$\lvert\Delta g_\tau\rvert_{LO} \leq 0.7$	—	D'Ambrosio, Isidori Paver '1991
-0.16	—	Isidori, Maiani Pugliese' 1992
$\lvert\Delta g_\tau\rvert = 38.2$	$\lvert\Delta g_{\tau'}\rvert = 31.5$	Bel'kov et al.' 1993
-0.23 ± 0.06	0.13 ± 0.04	Maiani, Paver ' 1995
$(-4.9 \pm 0.9)\sin\delta$	—	Shabalin ' 2003
-2.4 ± 1.2	1.1 ± 0.7	Scimemi, Gamiz Prades' 2004
$-(3.0 \pm 0.5)x$ $0.5 \leq x \leq 5$	—	Shabalin' 2004
$(-\Delta g_\tau)_{LO} > 0.56\, f(y)$ $(-\Delta g_\tau)_{y=1} = 2.9 \pm 0.6$	0.18 ± 0.02	Fäldt, Shabalin' 2005