Some ideas on radiative $K_{\ell 3}$ decays

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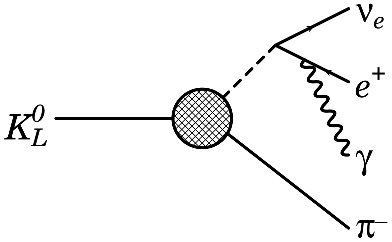
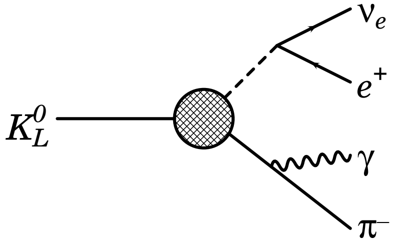
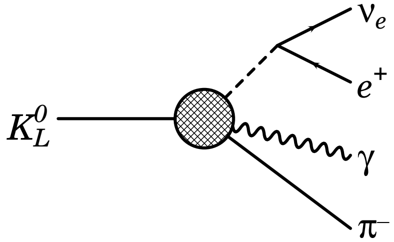
Outline

- Summary on $K^0_{e3\gamma}$ [see last year's workshop]
- What happens for $K^+_{e3\gamma}$?
- And what about $K_{\mu3\gamma}$?
- T-odd correlations in $K_{\ell3\gamma}$
- Conclusions

Summary on $K^0_{e3\gamma}$

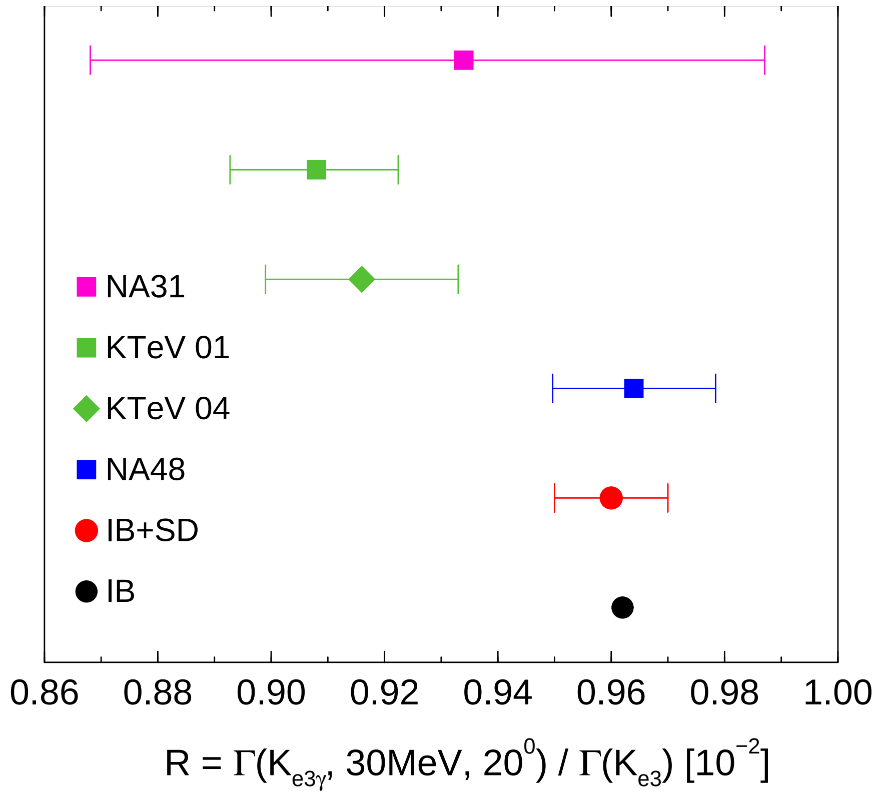
(inner) bremsstrahlung [IB]	structure dependent [SD]
infrared divergent, Low's theorem	Chiral Perturbation Theory
 	
f_+, f_- (or f_0) [$K_{\ell 3}$ form factors]	$V_{1/2}$, V_3, V_4 (vector)
	$A_{1/2}$, A_3, A_4 (axial anomaly)
f_- suppressed by m_e^2	V_3, A_3 suppressed by m_e^2
	V_4, A_4 chirally suppressed

J. Gasser, B. Kubis, N. Paver, M. Verbeni, Eur. Phys. J. C40 (2005) 205

Result for $R = \Gamma(K^0_{e3\gamma})/\Gamma(K^0_{e3})$

$$R = (0.96 \pm 0.01) \times 10^{-2}$$

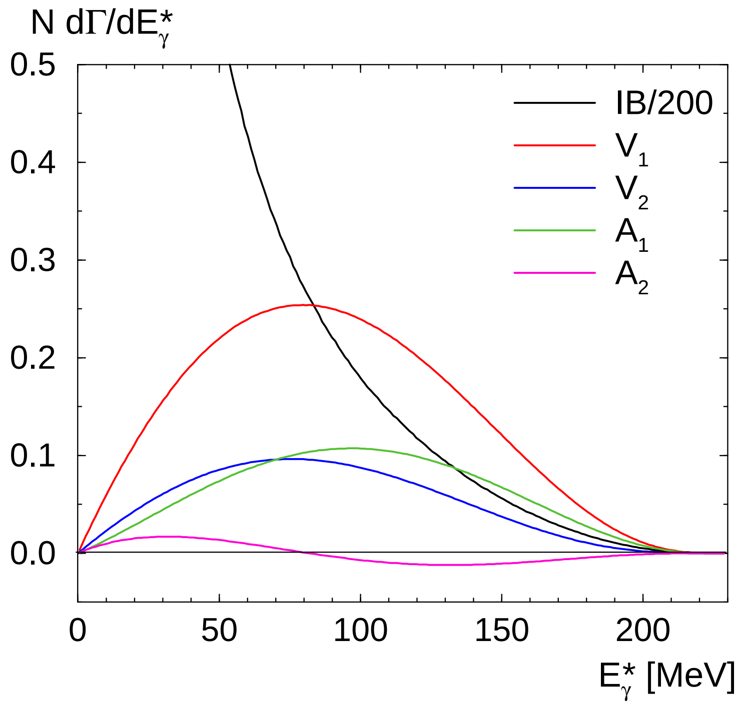
- dependence on coupling constants (G_F, V_{us}) cancels
- f_+ dependence extremely suppressed
- SD terms very small
- error dominated by radiative corrections



J. Gasser, B. Kubis, N. Paver, M. Verbeni, Eur. Phys. J. C40 (2005) 205

Structure dependent terms in $d\Gamma/dE_\gamma^*$

$$\frac{d\Gamma}{dE_\gamma^*} \approx \frac{d\Gamma_{\rm IB}}{dE_\gamma^*} + \sum_{i=1}^{4}\left(\langle V_i\rangle \frac{d\Gamma_{V_i}}{dE_\gamma^*} + \langle A_i\rangle \frac{d\Gamma_{A_i}}{dE_\gamma^*}\right) \approx \frac{d\Gamma_{\rm IB}}{dE_\gamma^*} + \langle X\rangle f(E_\gamma)$$

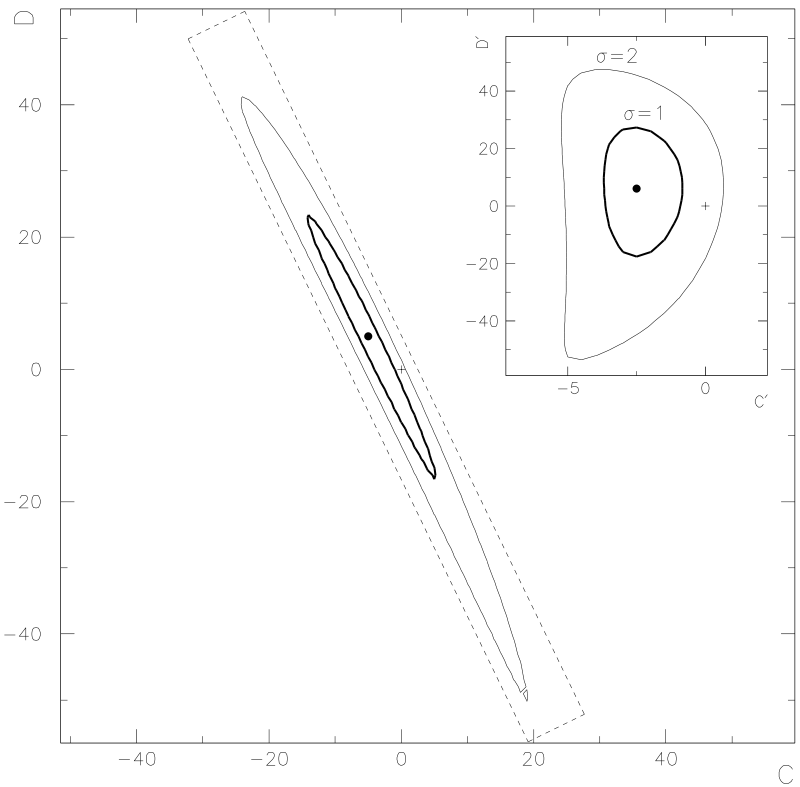


- $\langle V_i\rangle$, $\langle A_i\rangle$ phase-space averages of structure functions
- $d\Gamma_{V_i}/dE_\gamma^*$, $d\Gamma_{A_i}/dE_\gamma^*$ kinematical functions (figure!)

$\Rightarrow$ $d\Gamma/dE_\gamma^*$ is essentially sensitive to *one* effective coupling

$$\langle X\rangle \approx \langle V_1\rangle + 0.4\,\langle V_2\rangle + 0.4\,\langle A_1\rangle$$

Compare KTeV result to ChPT prediction:



KTeV, Phys. Rev. D64 (2001) 112004

- KTeV measures $C' \cong \langle X \rangle$
- KTeV: $C' = -2.5^{+1.5}_{-1.0} \pm 1.5$
- ChPT: $C' = -1.6 \pm 0.4$

$\Rightarrow$ good numerical agreement within 1-σ error

$\Rightarrow$ serious constraints on SD terms feasible !

Some necessary comments on this comparison

KTeV analysis relies on certain theory conventions ($\neq$ ours):

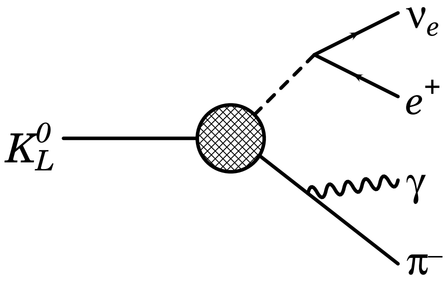
H.W. Fearing, E. Fischbach, J. Smith, Phys. Rev. D2 (1970) 542

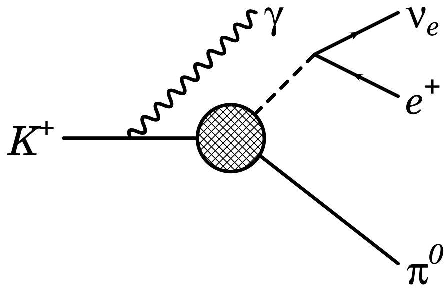
- IB–SD separation leads to singular SD terms $\propto (qW)^2/p_\pi q$
- their basis for SD does not single out m_e^2 suppressed terms
  $\Rightarrow$ simultaneous shifts in all structure functions not measurable
  $\Rightarrow$ C' strictly speaking not measurable
- "soft kaon approximation" leads to a wrong interpretation of the effective coupling $\langle X \rangle$

$\Rightarrow$ should be done better!

What happens for $K^+_{e3\gamma}$?

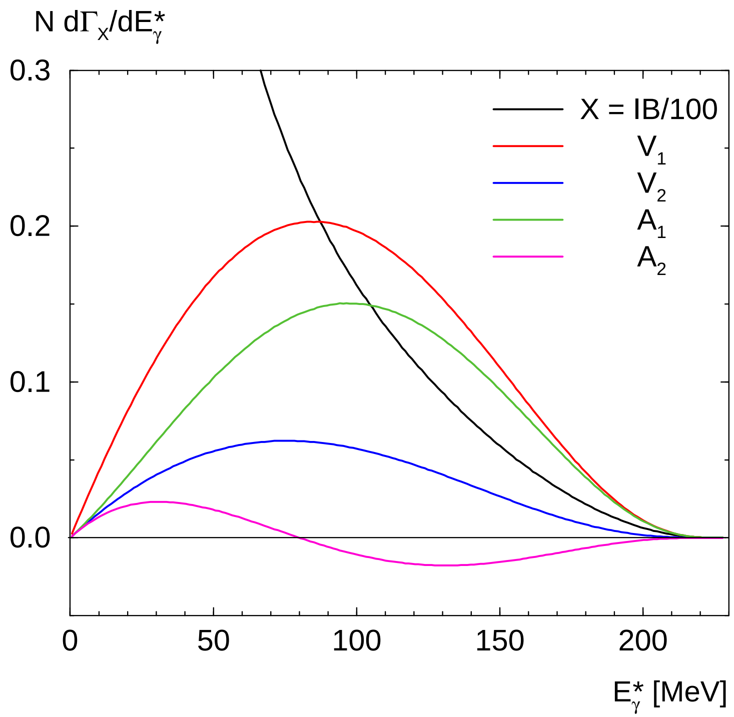
- compare:





- still similar structure: f_+, $V_{1/2}$, $A_{1/2}$ important;
  f_-, V_3, A_3 suppressed by m_e^2, V_4, A_4 chirally suppressed
- similar precision for $\Gamma(K^+_{e3\gamma})/\Gamma(K^+_{e3})$ as for $\Gamma(K^0_{e3\gamma})/\Gamma(K^0_{e3})$:
  f_+–dependence suppressed, SD terms small
  $\Rightarrow$ reduce theoretical uncertainty to 1% level

Changes in the structure dependent terms



$d\Gamma/dE^*_\gamma$ sensitive to

$$\langle X\rangle^+ = \langle V_1\rangle^+ + 0.3\langle V_2\rangle^+ + 0.7\langle A_1\rangle^+$$

$$\left[\langle X\rangle^0 = \langle V_1\rangle^0 + 0.4\langle V_2\rangle^0 + 0.4\langle A_1\rangle^0\right]$$

$K^+_{e3\gamma}$	$K^0_{e3\gamma}$
$\langle V_1\rangle^+ = -1.3$	$\langle V_1\rangle^0 = -1.2$
$\langle V_2\rangle^+ = -0.2$	$\langle V_2\rangle^0 = +0.1$
$\langle A_1\rangle^+ = -1.2$	$\langle A_1\rangle^0 = 0$
$\langle A_2\rangle^+ = +0.3$	$\langle A_2\rangle^0 = -0.3$

$\Rightarrow$ much higher sensitivity to axial anomaly effects in $K^+_{e3\gamma}$

$\Rightarrow$ V_1 and A_1 difficult to disentangle experimentally though

And what about $K_{\mu 3\gamma}$?

- bremsstrahlung has no near-collinear singularity
- structure dependent terms potentially more important?

  $\Rightarrow$ investigate!
- but: no m_ℓ^2 suppression of certain structures any more:

  f_- (resp. f_0) play a role

  $\Rightarrow$ more potential uncertainty in predicting R

  A_3 with certain K–pole contributions important

  $\Rightarrow$ analysis of SD terms more messy

T-odd correlations in $K_{\ell 3\gamma}$

- Aim: access *direct* CP-violation in charged K-decays
- simplest CP-violating observable: charge asymmetry of decay widths

$$\Gamma(K^+ \to f) - \Gamma(K^- \to \bar{f})$$

$\Rightarrow$ only for two weak amplitudes with different final-state phases

- alternative: T-odd correlations

1. $K^+_{\mu 3}$: $P^T_\mu = \langle \boldsymbol{\sigma}_\mu \cdot (\mathbf{p}_\mu \times \mathbf{p}_\pi)/|\mathbf{p}_\mu \times \mathbf{p}_\pi|\rangle$
   transverse μ polarisation
2. $K^+_{\ell 3\gamma}$: $\xi = \mathbf{q} \cdot (\mathbf{p}_\pi \times \mathbf{p}_\ell)/M_K^3$
   (similar in $K_{\ell 4}$)

T-odd observables:

- define partial decay width with respect to ξ

$$\frac{d\Gamma}{d\xi} = f_{\text{even}}(\xi) + f_{\text{odd}}(\xi)$$

- introduce asymmetry

$$A_\xi = \frac{N_+ - N_-}{N_+ + N_-}$$

where $N_\pm = \#$ events with $\xi \gtrless 0 \Rightarrow A_\xi$ depends only on $f_{\text{odd}}(\xi)$

- two sources for contributions to $f_{\text{odd}}(\xi)$:
    1. final state interaction phases
    2. new (beyond SM) interactions with complex couplings

- note: 1. can be eliminated by combining K^+ and K^- asymmetries if not, calculated SM background to constrain new physics

T-odd observables in the Standard Model:

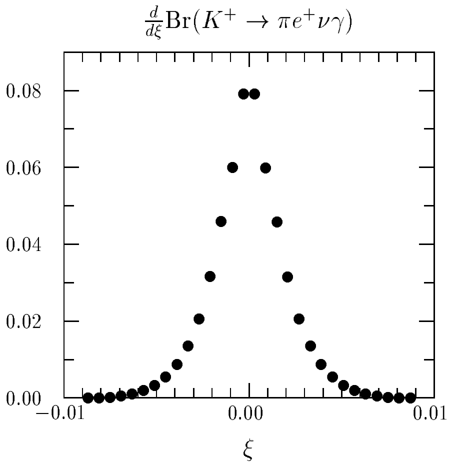
- amplitude squared

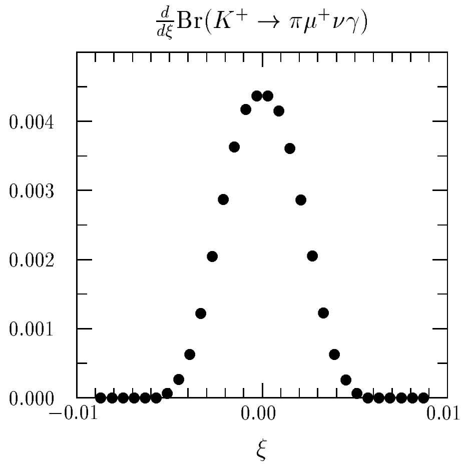
$$\sum_{\text{spins}} |T|^2 = \big[\xi\text{−even part}\big] + \xi \sum_{i=1}^{4} \big(d_i \, \mathrm{Im} V_i + d_i^5 \, \mathrm{Im} A_i\big) f_+ + \dots$$

- in ChPT at $\mathcal{O}(p^4)$: $\quad \mathrm{Im} V_i = \mathrm{Im} A_i = 0$

J. Bijnens, G. Ecker, J. Gasser, Nucl. Phys. B396 (1993) 81

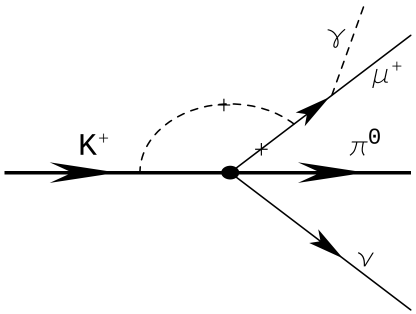
$\Rightarrow$ at this order $f_{\text{odd}}(\xi) = 0$, $d\Gamma/d\xi = f_{\text{even}}(\xi)$:

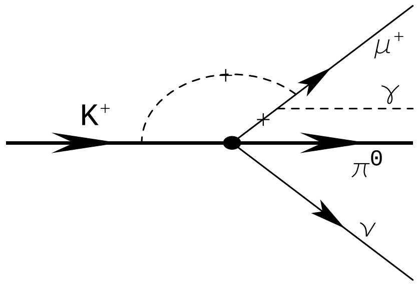




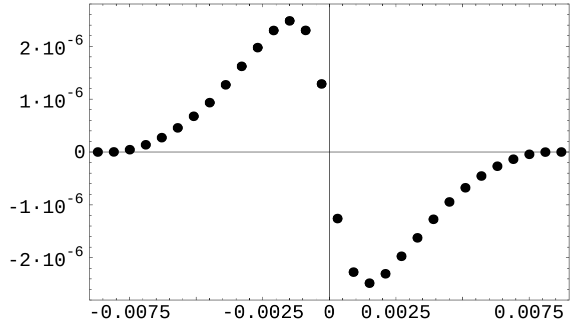
V.V. Braguta et al., Phys. Rev. D65 (2002) 054038

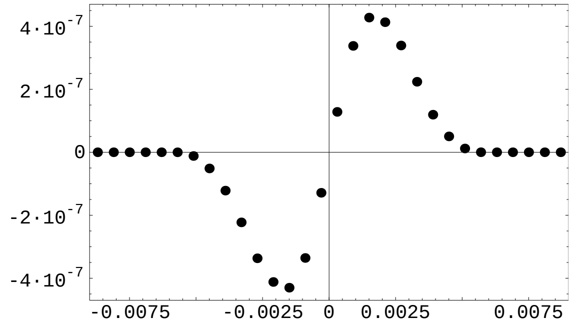
- only contributions from imaginary parts of photon loops





- $f_{\text{odd}}(\xi)$ (for electron and muon channels resp.):



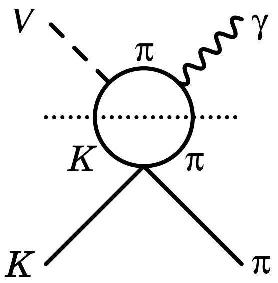


- asymmetries:

$$A_\xi(K^+_{e3\gamma}) = -0.59 \times 10^{-4} \; , \quad A_\xi(K^+_{\mu 3\gamma}) = +1.14 \times 10^{-4}$$

V.V. Braguta et al., Phys. Rev. D65 (2002) 054038

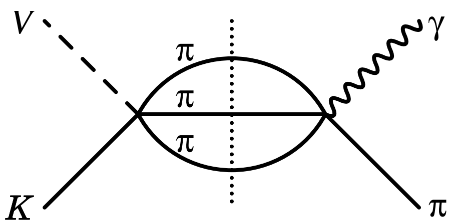
“Forgotten”: strong phases at $\mathcal{O}(p^6)$!



$\mathcal{O}(p^4)$: only t-channel cut in V_i

for $t = (p_K - p_\pi)^2 > (M_K + M_\pi)^2$

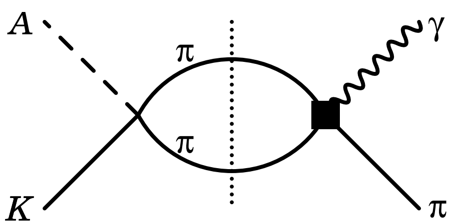
far outside physical region $\Rightarrow$ real!



$\mathcal{O}(p^6)$: “physical” s-channel cut in V_i

for $s = (p_\pi + q)^2 > 9M_\pi^2$

$\hat{=}$ intermediate 3π state $\Rightarrow$ two–loop



$\mathcal{O}(p^6)$: “physical” s-channel cut in A_i

for $s = (p_\pi + q)^2 > 4M_\pi^2$

$\hat{=}$ intermediate 2π state $\Rightarrow$ one–loop + anomalous rescattering

So are the SM asymmetries much larger than assumed?

not necessarily:

photon loops	pion loops
$\alpha_{\text{QED}} \times$ IB	SD $\approx$ 1% IB
whole phase space	part of phase space:
	$s > 4M_\pi^2$

$\Rightarrow$ perform complete analysis in order to re-assess SM asymmetries!

B. Kubis, E. Müller, work in progress

T-odd correlations beyond the Standard Model

- More general current-current Lagrangian...

$$\mathcal{L}_{4-\mathrm{f}} = \frac{G_F}{\sqrt{2}} \sin\theta_c \Big\{ \bar{s}\gamma^\mu \big((1+g_v) - (1-g_a)\gamma_5\big) u \times \bar{\nu}\gamma_\mu(1-\gamma_5)\ell + \bar{s}\big(g_s + g_p\gamma_5\big) u \times \bar{\nu}(1-\gamma_5)\ell + g_t\, \bar{s}\sigma^{\mu\nu} u \times \bar{\nu}\sigma_{\mu\nu}\ell \Big\}$$

- ...leads to a more general asymmetry:

$$\sum_{\text{spins}} |T|^2 = \big[\xi-\text{even part}\big] + \xi \Big[C_{v+a}\mathrm{Im}(g_v + g_a) + C_s\mathrm{Im}(g_s) + C_p\mathrm{Im}(g_p) + C_t\mathrm{Im}(g_t) \Big]$$

V.V. Braguta et al., Phys. Rev. 68 (2003) 094008

G. Colangelo, M. Gerber, unpublished

- asymmetries numerically:

$$A_\xi(K^+_{e3\gamma}) = -\Big[3.0 \times 10^{-3}\,\mathrm{Im}(g_v + g_a) + 2.9 \times 10^{-6}\,\mathrm{Im}(g_s) + 3.7 \times 10^{-5}\,\mathrm{Im}(g_p) + 4.3 \times 10^{-6}\,\mathrm{Im}(g_t)\Big]$$

$\Rightarrow$ g_s and g_a suppressed by m_e^2

$$A_\xi(K^+_{\mu3\gamma}) = -\Big[1.0 \times 10^{-2}\,\mathrm{Im}(g_v + g_a) + 3.6 \times 10^{-3}\,\mathrm{Im}(g_s) + 1.2 \times 10^{-2}\,\mathrm{Im}(g_p) + 2.5 \times 10^{-4}\,\mathrm{Im}(g_t)\Big]$$

$\Rightarrow$ can be used to constrain models for new physics

Consider specific new physics models with richer CP-violation:

- LR-models with $\mathrm{Im}(g_v) = \mathrm{Im}(g_a) \neq 0$
- other low-energy data imply $|\mathrm{Im}(g_v)| = |\mathrm{Im}(g_a)| < 1.3 \times 10^{-2}$
  $\Rightarrow$ asymmetries

$$|A_\xi(K^+_{e3\gamma})| < 0.8 \times 10^{-4} \ , \quad |A_\xi(K^+_{\mu 3\gamma})| < 2.6 \times 10^{-4}$$

- compare to "electromagnetic" asymmetries

$$A_\xi(K^+_{e3\gamma}) = -0.59 \times 10^{-4} \ , \quad A_\xi(K^+_{\mu 3\gamma}) = +1.14 \times 10^{-4}$$

$\Rightarrow$ measurement at level of "electromagnetic" asymmetries would further constrain model parameters

V.V. Braguta et al., Phys. Rev. 68 (2003) 094008

Conclusions

- consistent theory predictions for structure dependent terms in $K_{e3\gamma}$
- pioneering KTeV measurement in $K^0_{e3\gamma}$ should be improved upon
- photon energy distribution in $K^+_{e3\gamma}$ gives access to axial anomaly
- T-odd correlations in $K^+_{\ell 3\gamma}$ as a window to direct CP-violation:
  1. electromagnetic SM background known
  2. will be supplemented by strong background soon
  3. first theoretical hints at how to constrain new physics