

# NLO/NNLO group

- Introduction VDD
- A complete formalism for one-loop multileg amplitudes G. Heinrich
- A method for the numerical evaluation of one-loop integrals Z. Kunszt
- Status of GRACE Y. Kurihara
- NLO corrections to (pseudoscalar)  $H t \bar{t}$  S. Dittmaier
- Progress on NNLO subtraction VDD
- More progress on NNLO subtraction M. Grazzini

# Precision QCD

Precise determination of

- strong coupling constant  $\alpha_s$
- parton distributions
- electroweak parameters
- LHC parton luminosity

Precise prediction for

- Higgs production
- new physics processes
- their backgrounds

# Desired **NLO** cross sections

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

# NLO history

- $e^+e^- \rightarrow 3 \text{ jets}$  K. Ellis, D. Ross, A. Terrano 1981
- $e^+e^- \rightarrow 4 \text{ jets}$  Z. Bern et al., N. Glover et al., Z. Nagy Z. Trocsanyi 1996-97
- $pp \rightarrow 1, 2 \text{ jets}$  K. Ellis J. Sexton 1986, W. Giele N. Glover D. Kosower 1993
- $pp \rightarrow 3 \text{ jets}$  Z. Bern et al., Z. Kunszt et al. 1993-1995, Z. Nagy 2001
- $pp \rightarrow V + 1 \text{ jet}$  W. Giele N. Glover & D. Kosower 1993
- $pp \rightarrow V + 2 \text{ jet}$  Bern et al., Glover et al. 1996-97, K. Ellis & Campbell 2003
- $pp \rightarrow V b \bar{b}$  K. Ellis & J. Campbell 2003
- $pp \rightarrow V b \bar{b} + 1 \text{ jet}$  ??
- $pp \rightarrow VV$  Ohnemus & Owens, Baur et al. 1991-96, Dixon et al. 2000
- $pp \rightarrow VV + 1 \text{ jet}$  ??
- $pp \rightarrow \gamma\gamma$  B. Bailey et al 1992, T. Binoth et al 1999
- $pp \rightarrow \gamma\gamma + 1 \text{ jet}$  Z. Bern et al. 1994, V. Del Duca et al. 2003
- $pp \rightarrow Q\bar{Q}$  Dawson K. Ellis Nason 1989, Mangano Nason Ridolfi 1992
- $pp \rightarrow Q\bar{Q} + 1 \text{ jet}$  A. Brandenburg et al. 2005 ?

# NLO production rates

Process-independent procedure devised in the 90's

- 👉 slicing Giele Glover & Kosower
- 👉 subtraction Frixione Kunszt & Signer; Nagy & Trocsanyi
  - 👉 dipole Catani & Seymour
  - 👉 antenna Kosower; Campbell Cullen & Glover

$$\sigma = \sigma^{\text{LO}} + \sigma^{\text{NLO}} = \int_m d\sigma_m^B J_m + \sigma^{\text{NLO}}$$

$$\sigma^{\text{NLO}} = \int_{m+1} d\sigma_{m+1}^R J_{m+1} + \int_m d\sigma_m^V J_m$$

the 2 terms on the rhs are divergent in  $d=4$

use **universal IR** structure to subtract divergences

$$\sigma^{\text{NLO}} = \int_{m+1} \left[ d\sigma_{m+1}^R J_{m+1} - d\sigma_{m+1}^{\text{R,A}} J_m \right] + \int_m \left[ d\sigma_m^V + \int_1 d\sigma_{m+1}^{\text{R,A}} \right] J_m$$

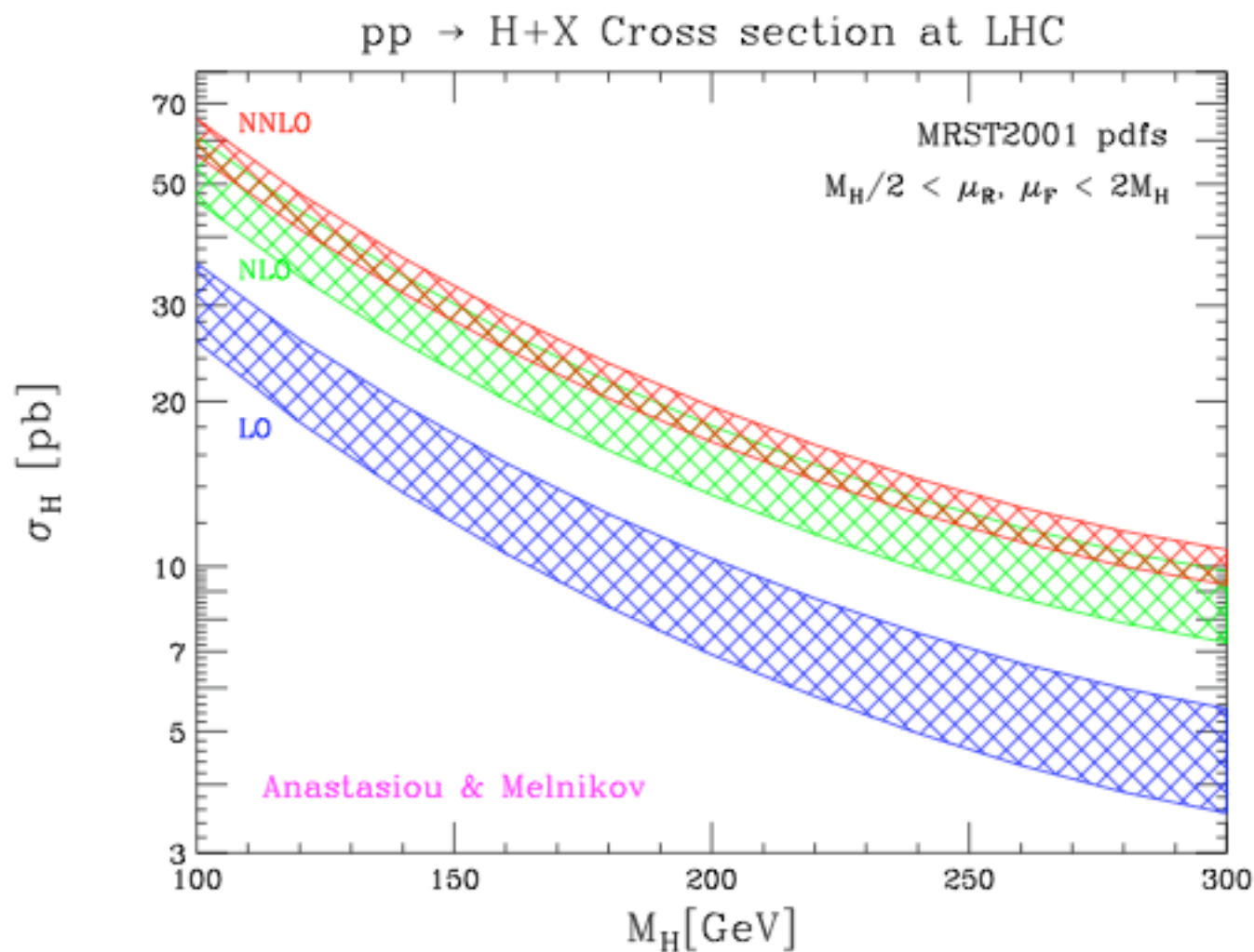
the 2 terms on the rhs are finite in  $d=4$

# NLO complications

- loop integrals are involved and process-dependent
- more particles  $\Rightarrow$  many scales  $\Rightarrow$  lengthy analytic expressions
  - even though it is known how to compute loop integrals with  $2 \rightarrow n$  particles no integrals with  $n > 4$  have been computed (either analytically or numerically)
- no numeric methods yet for hadron collisions
  - counterterms are subtracted analytically

# Is **NLO** enough to describe data ?

## Total cross section for inclusive **Higgs** production at LHC



contour bands are  
lower

$$\mu_R = 2M_H \quad \mu_F = M_H/2$$

upper

$$\mu_R = M_H/2 \quad \mu_F = 2M_H$$

scale uncertainty  
is about 10%

**NNLO** prediction stabilises the perturbative series

# NNLO state of the art

## 🏆 Drell-Yan $W, Z$ production

🏅 total cross section [Hamberg, van Neerven, Matsuura 1990](#)  
[Harlander, Kilgore 2002](#)

🏅 rapidity distribution [Anastasiou et al. 2003](#)

## 🏆 Higgs production

🏅 total cross section [Harlander, Kilgore; Anastasiou, Melnikov 2002](#)

🏅 fully differential cross section

[Anastasiou, Melnikov, Petriello 2004](#)

## 🏆 $e^+e^- \rightarrow 3$ jets

🏅 the  $C_F^2$  term [the Gehrmanns, Glover 2004](#)



# NNLO cross sections

- 🌐 Sector decomposition Denner Roth 1996; Binoth Heinrich 2000  
Anastasiou, Melnikov, Petriello 2003

↑ the only method which, so far, yields  
useful NNLO cross sections

↑ cancellation of divergences is performed numerically

↓ process dependent

- 🌐 Subtraction

↑ process independent

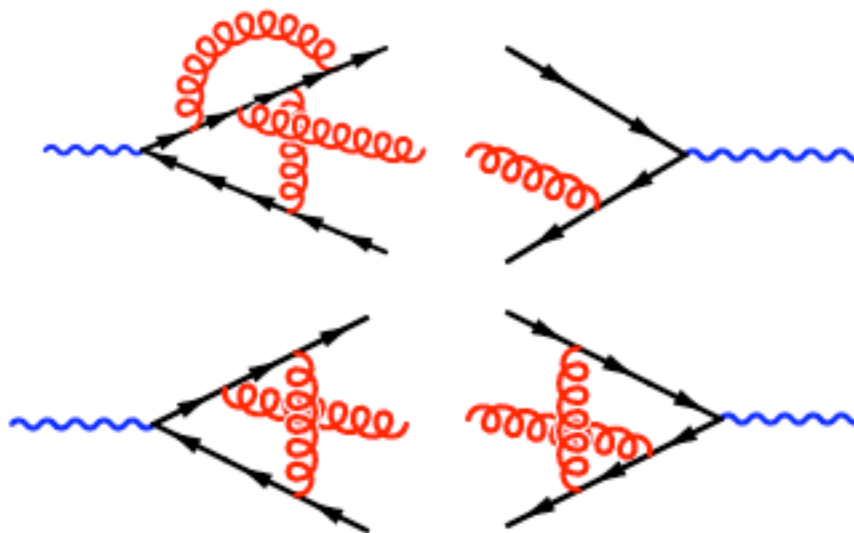
↓ cancellation of divergences is semi-analytic

↓ despite a great deal of efforts, it's not there yet

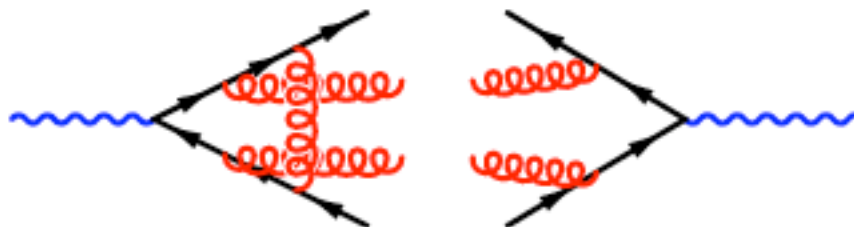
# NNLO assembly kit

$e^+e^- \rightarrow 3 \text{ jets}$

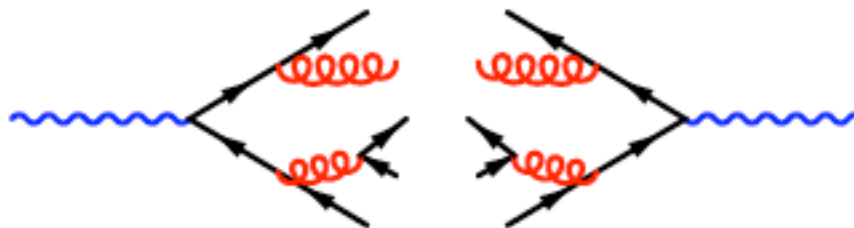
double virtual



real-virtual



double real



# Two-loop matrix elements

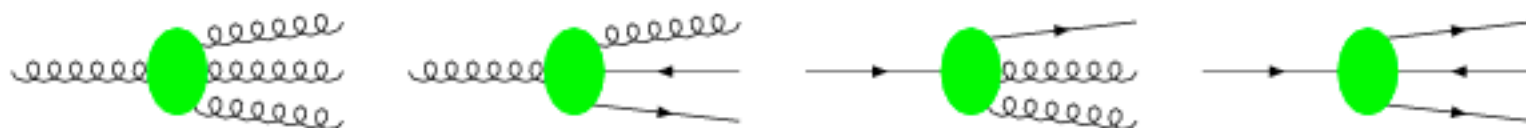
- two-jet production  $qq' \rightarrow qq', q\bar{q} \rightarrow q\bar{q}, q\bar{q} \rightarrow gg, gg \rightarrow gg$   
C. Anastasiou N. Glover C. Oleari M. Tejada-Yeomans 2000-01  
Z. Bern A. De Freitas L. Dixon 2002
- photon-pair production  $q\bar{q} \rightarrow \gamma\gamma, gg \rightarrow \gamma\gamma$   
C. Anastasiou N. Glover M. Tejada-Yeomans 2002  
Z. Bern A. De Freitas L. Dixon 2002
- $e^+e^- \rightarrow 3$  jets  $\gamma^* \rightarrow q\bar{q}g$   
L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002
- $V + 1$  jet production  $q\bar{q} \rightarrow Vg$   
T. Gehrmann E. Remiddi 2002
- Drell-Yan  $V$  production  $q\bar{q} \rightarrow V$   
R. Hamberg W. van Neerven T. Matsuura 1991
- Higgs production  $gg \rightarrow H$  (in the  $m_t \rightarrow \infty$  limit)  
R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

# NNLO cross sections: subtraction

universal IR structure  $\rightarrow$  process-independent procedure

universal collinear and soft currents

3-parton tree splitting functions



J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

2-parton one-loop splitting functions



Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; D. Kosower 2003

universal subtraction counterterms

several ideas and works in progress  
but so far not yet completely figured out

D. Kosower; S. Weinzierl; the Gehrmanns & G. Heinrich 2003  
S. Frixione M. Grazzini 2004; G. Somogyi Z. Trocsanyi VDD 2005  
the Gehrmanns & N. Glover 2004-5