Automatic calculation of NLO-QCD cross sections with GRACE

- GRACE status -

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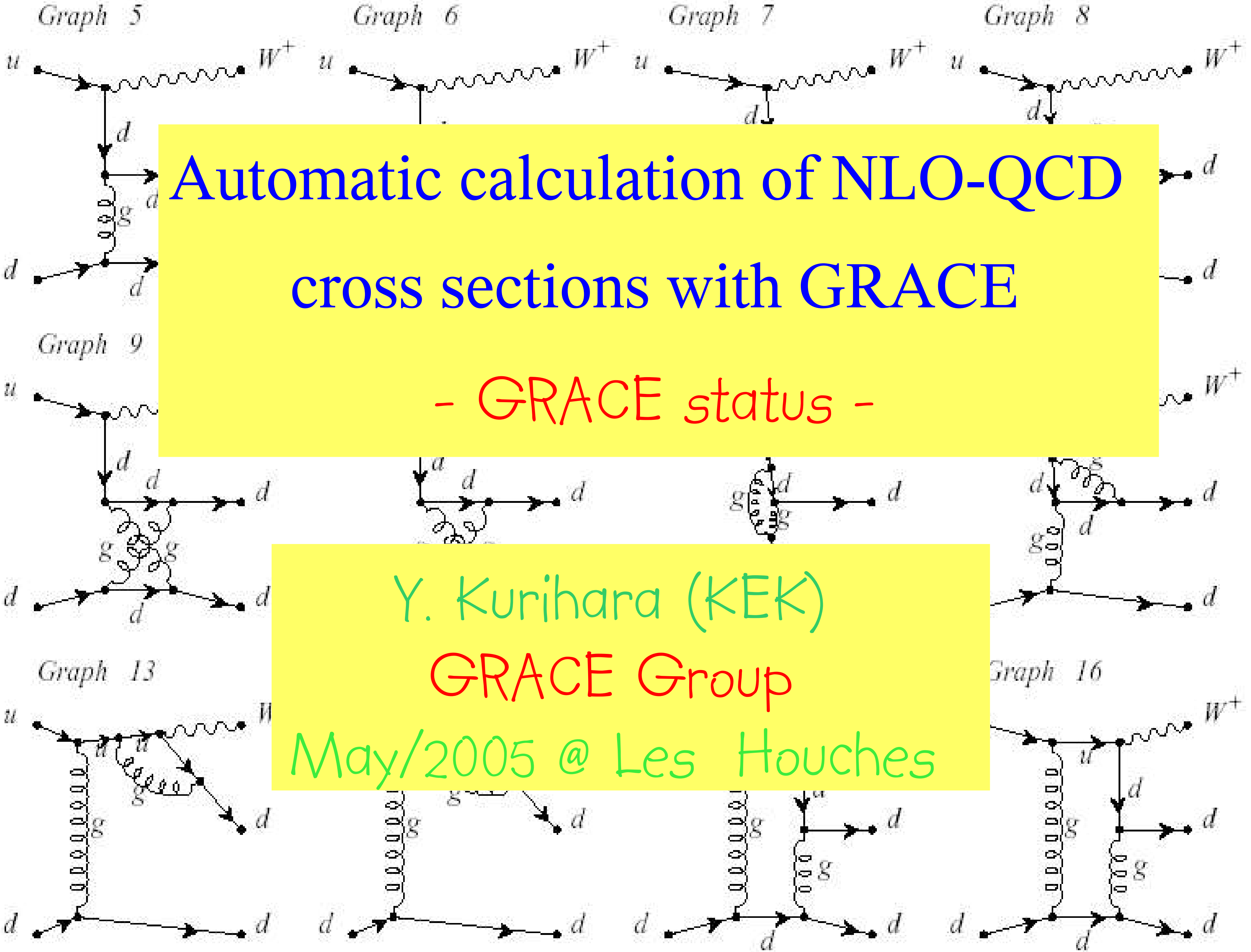
GRACE Group

May/2005 @ Les Houches

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Physics in LHC

- LHC Experimental requirement

  New Particle Search/Precision Measurements

LO-QCD Event generator+K-factor

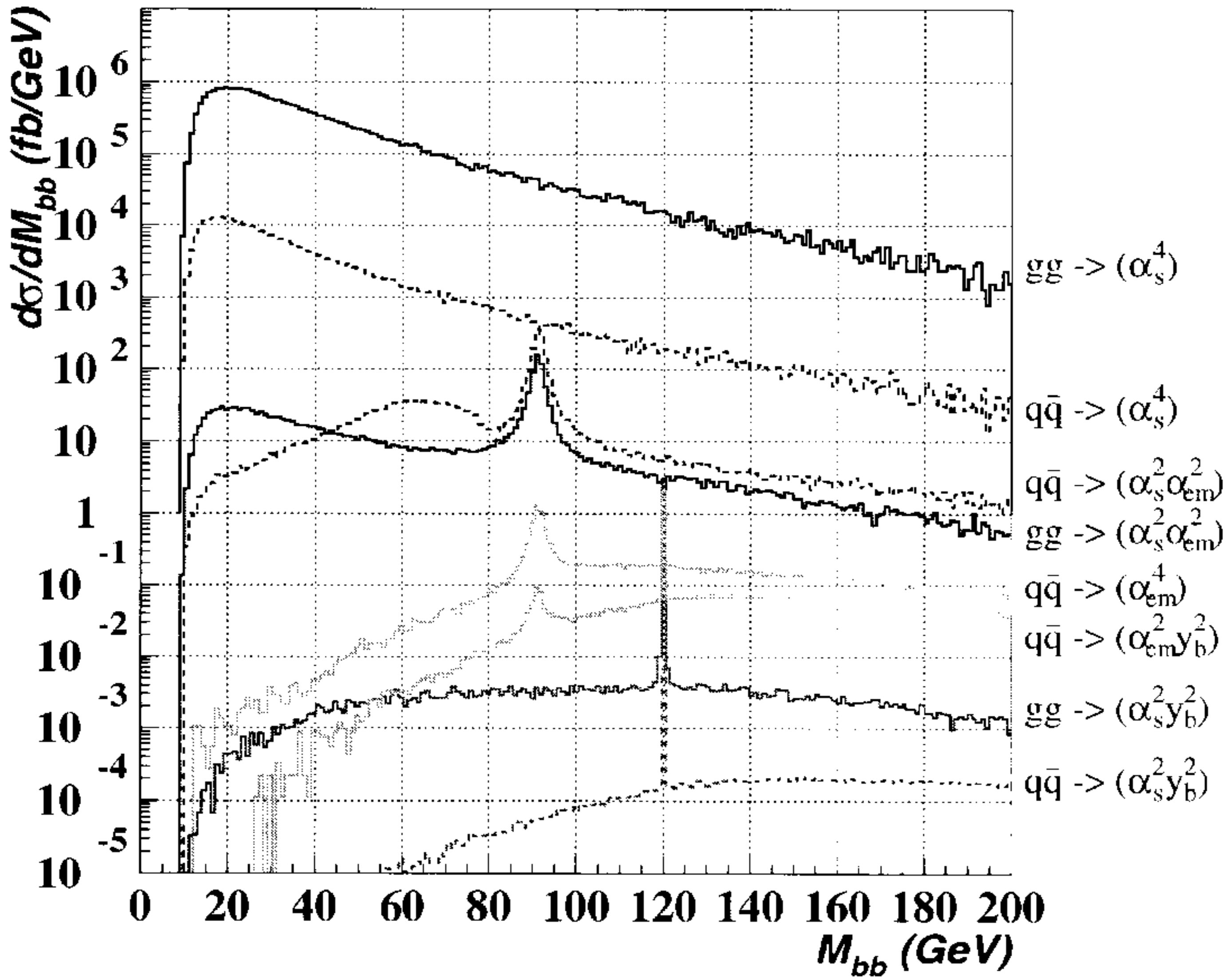
↓

Obviously not enough!

We need
NLO Event generator!

Event generator @ Tree-level

pp→bbbb, TEVATRON/LHC, GR@PPA_4b, S. Tsuno



S. Tsuno et al., Comput.Phys.Commun.151(2003)216

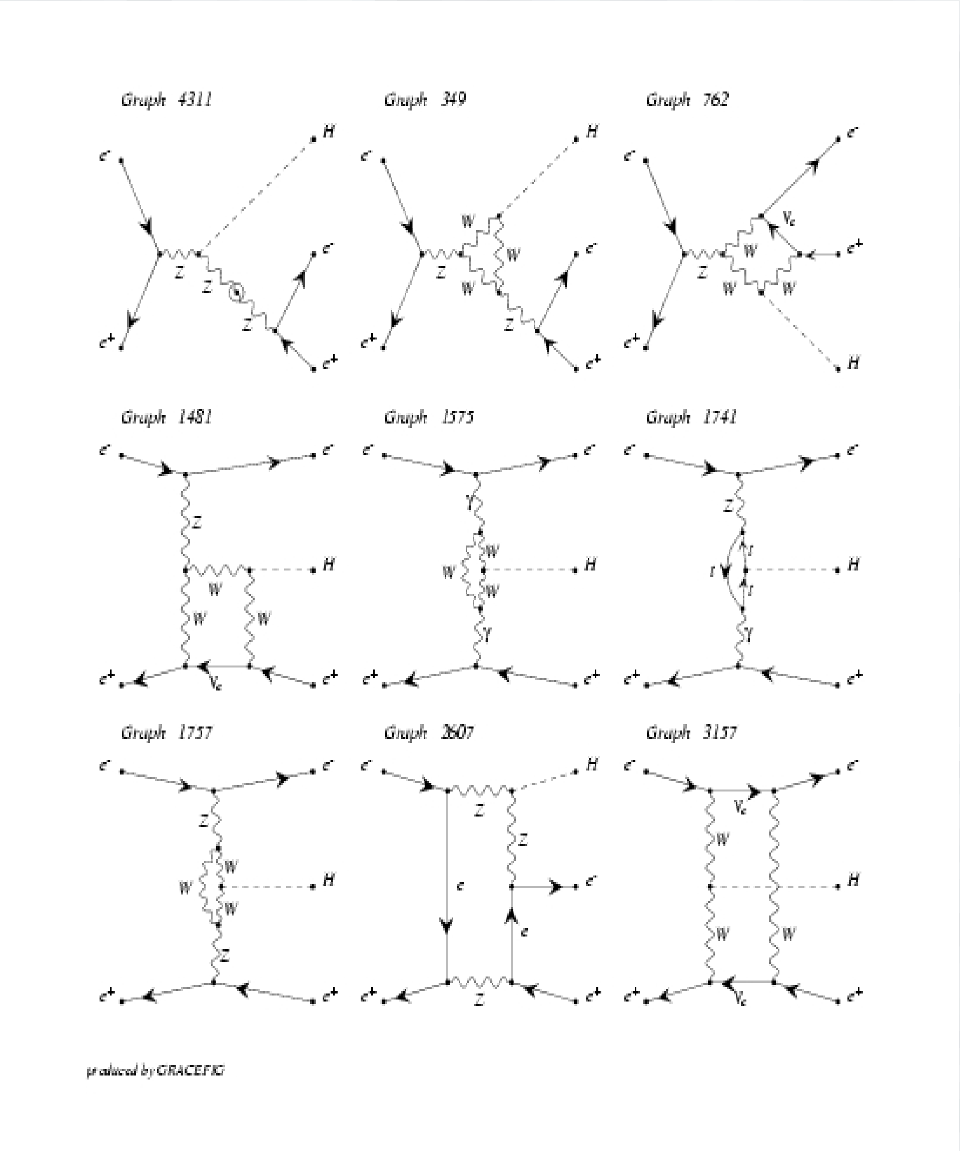
pp → many, TEVATRON/LHC, GR@PPA_ALL,
S. Tsuno, 2004

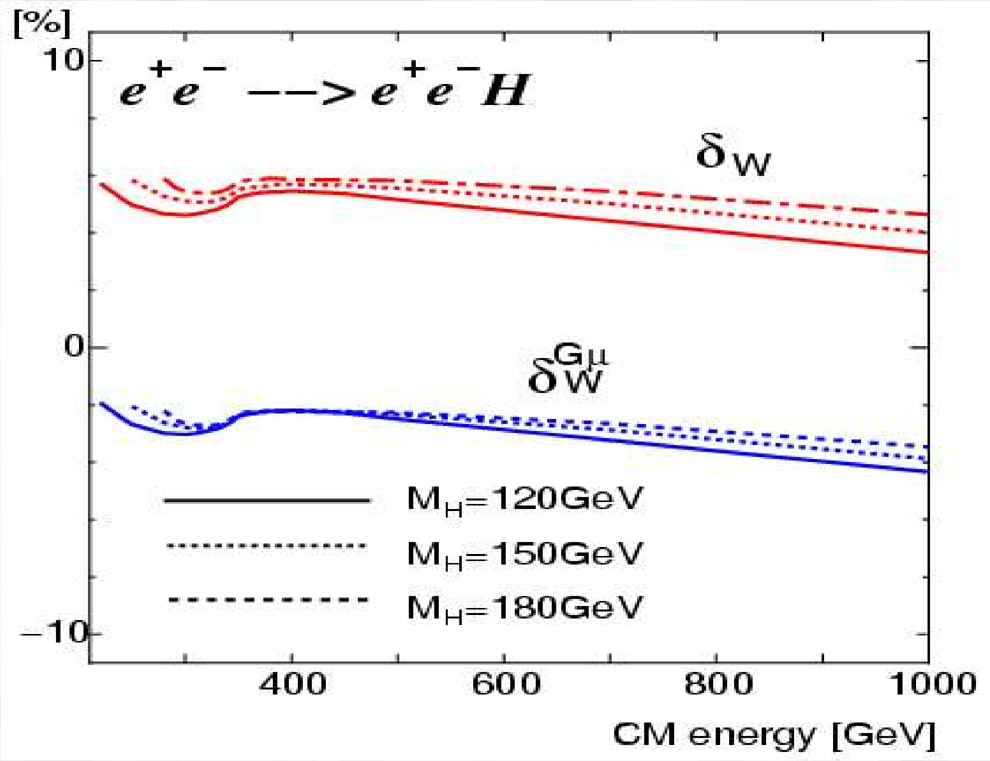
- W + jets (up to 4 jets) with the subsequent W decay to a fermion pair,
- Z + jets (up to 4 jets) with the subsequent Z decay to a fermion pair,
- Four bottom quarks via Z and Higgs-boson mediated processes as well as those from pure QCD interactions (same as GR@PPA_4b),
- top-quark pair with the subsequent decay to W and b, and the W decay to a fermion pair,
- di-boson (WW, WZ and ZZ) with the subsequent W/Z decay to a fermion pair.

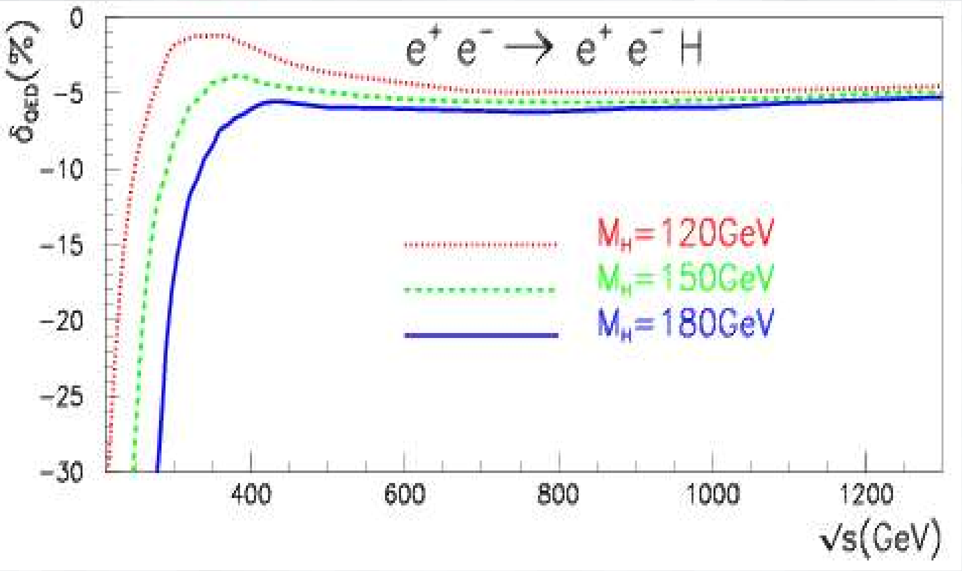
Loop Calc. by GRACE-loop in ELWK proc. (Higgs production)

- ***Single Higgs production***
  - $e^+e^- \to$ ZH (full number of graphs = 341)
  - $e^+e^- \to$ $\nu\ \nu\ H$ (1,350) Phys.Lett. B559 (2003) 252-262
    A.Denner et.al. PLB 560(2003)196, NPB 660(2003)289
  - $e^+e^- \to$ $e^+e^-\ H$ (4,470) Phys.Lett.B600 (2004) 65-76
- ***top Yukawa***
  - $e^+e^- \to$ $t\ t\ H$ (2,327) Phys.Lett. B571 (2003) 163-172
    Y.You et.al.PLB 571(2003)85
    A.Denner et.al. PLB 575(2003)290, NPB 680 (2004)85
- ***Multi Higgs production***
  - $e^+e^- \to$ ZHH (5,417) Phys.Lett. B576 (2003) 152-164
    R.Zhang et.al.PLB(2004)349
  - $e^+e^- \to$ $\nu_e \nu_e\ HH$ (19,638) $\Rightarrow$ **Preliminary**

$e^+e^- \rightarrow \quad e^+e^- H$

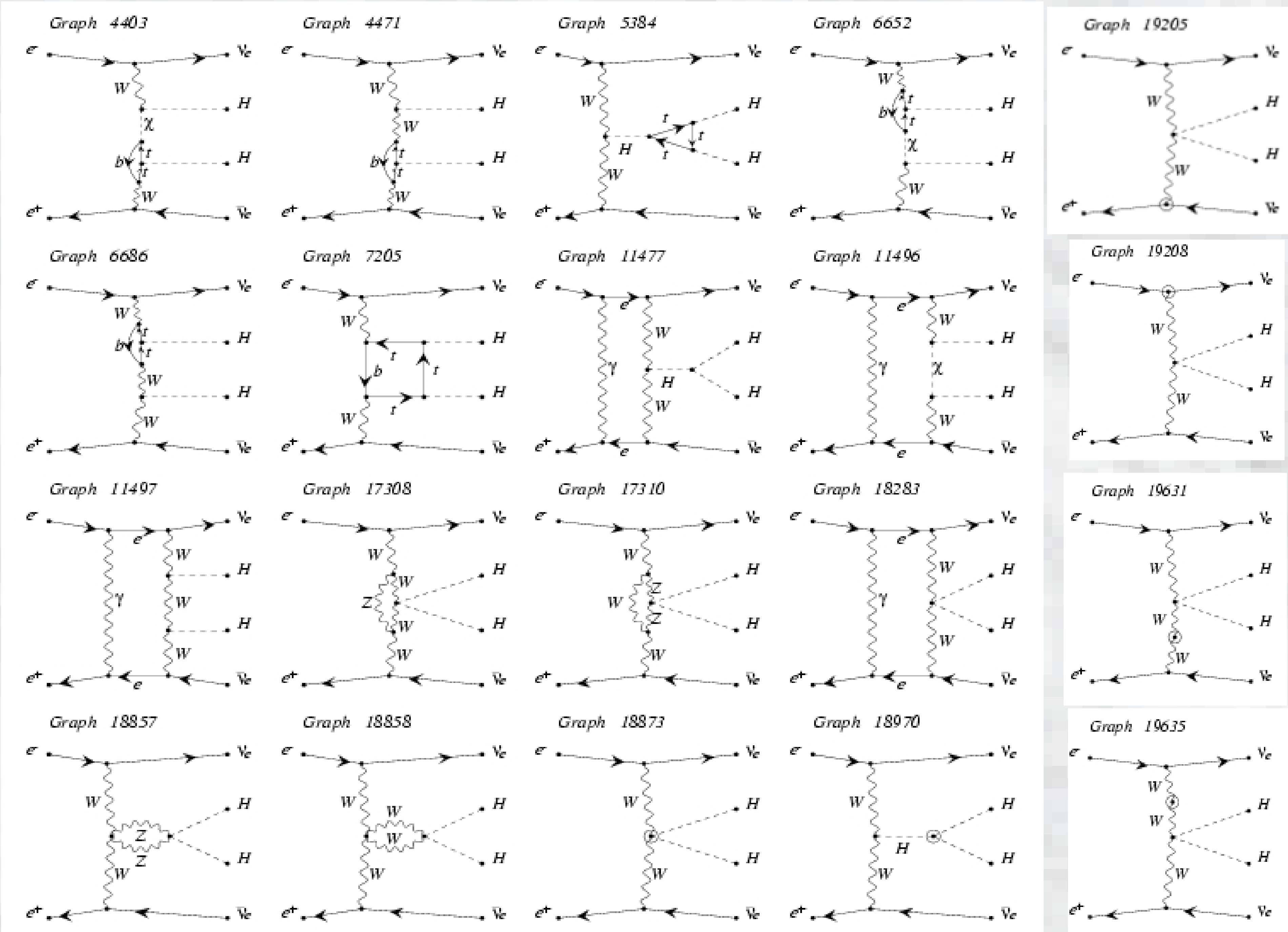




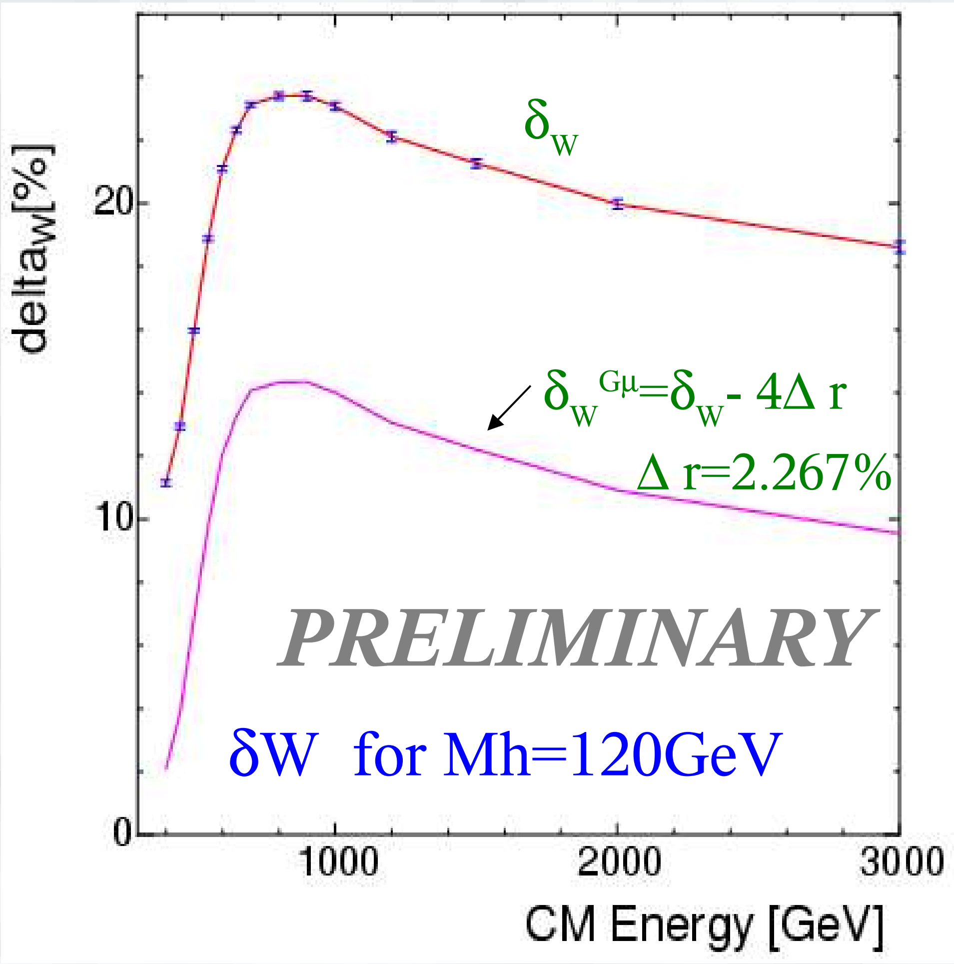


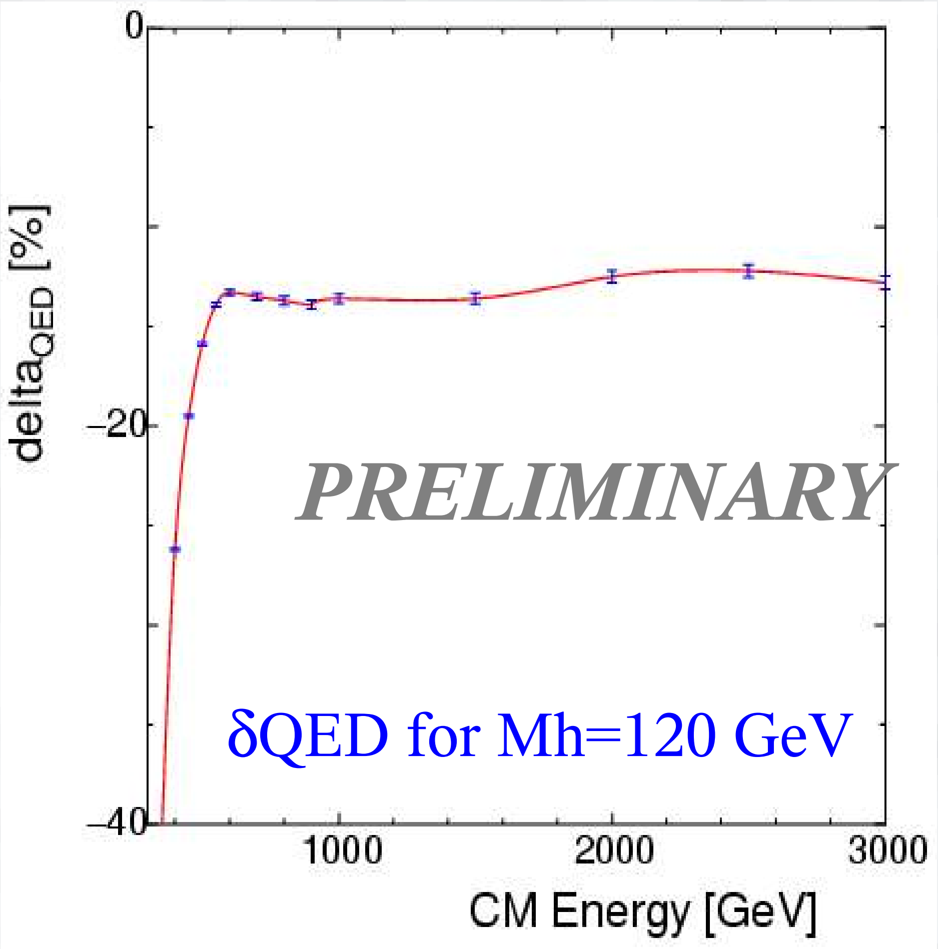
GRACE *Phys.Lett. B559 (2003) 252-262*

$e^+e^- \to \nu_e \nu_e HH$ (final 4-body process)



$\delta = \sigma(O(\alpha))/\sigma(\text{tree}) - 1$





KEK IBM RS600 (Power 3 :375MHz)
IDRIS IBM (Power 4: 1.3/1.7GHz)

Internal consistency check

For EW part:

System passes succesfully the usual checks:

- ultraviolet finiteness (better than 20 digits)
- infrared finitenesss (better than 20 digits)
- NLG dependence (better than 20 digits)
- k_c dependence consistent with MonteCarlo statistical error (0.02%)

1. One random phase space point
2. Full set of diagrams
3. Quadruple precision

New feature of NLO-QCD issues in GRACE

QCD-tree : OK
ELWK 1-loop : OK

- PDF/PS ↔ Real emission Double counting
- Dimensional regularization in loop integrals for IR (fictitious mass in photon in ELWK)
- IR (soft/collinear) approximation terms

Double Counting

No IR-divergence

Double counting

$$\sigma_{NLO} = [\sigma_{tree}(1 + \delta_V + \delta_{s/c}) + \sigma_{vis}] \otimes \text{PDF/PS}$$

$1/\varepsilon^2_{IR}, 1/\varepsilon_{IR}$ cancellation

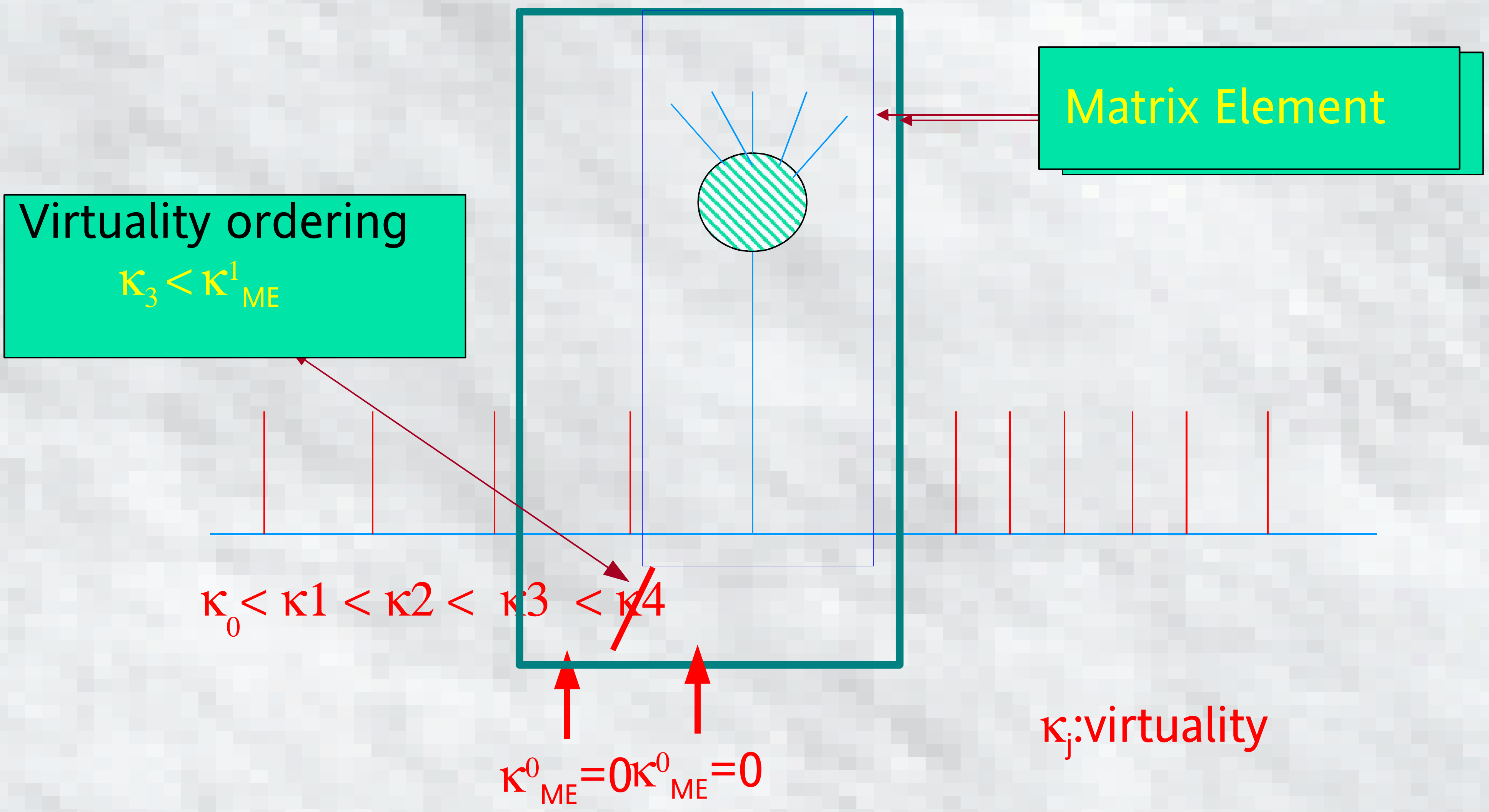
PDF/Parton Shower

$$\frac{1}{\varepsilon_{IR}} f_c \frac{\alpha_s}{2\pi} \mathrm{P}(x)$$

P(x) : Splitting function

Space/time dimension : $d=4+2\varepsilon_{IR}$

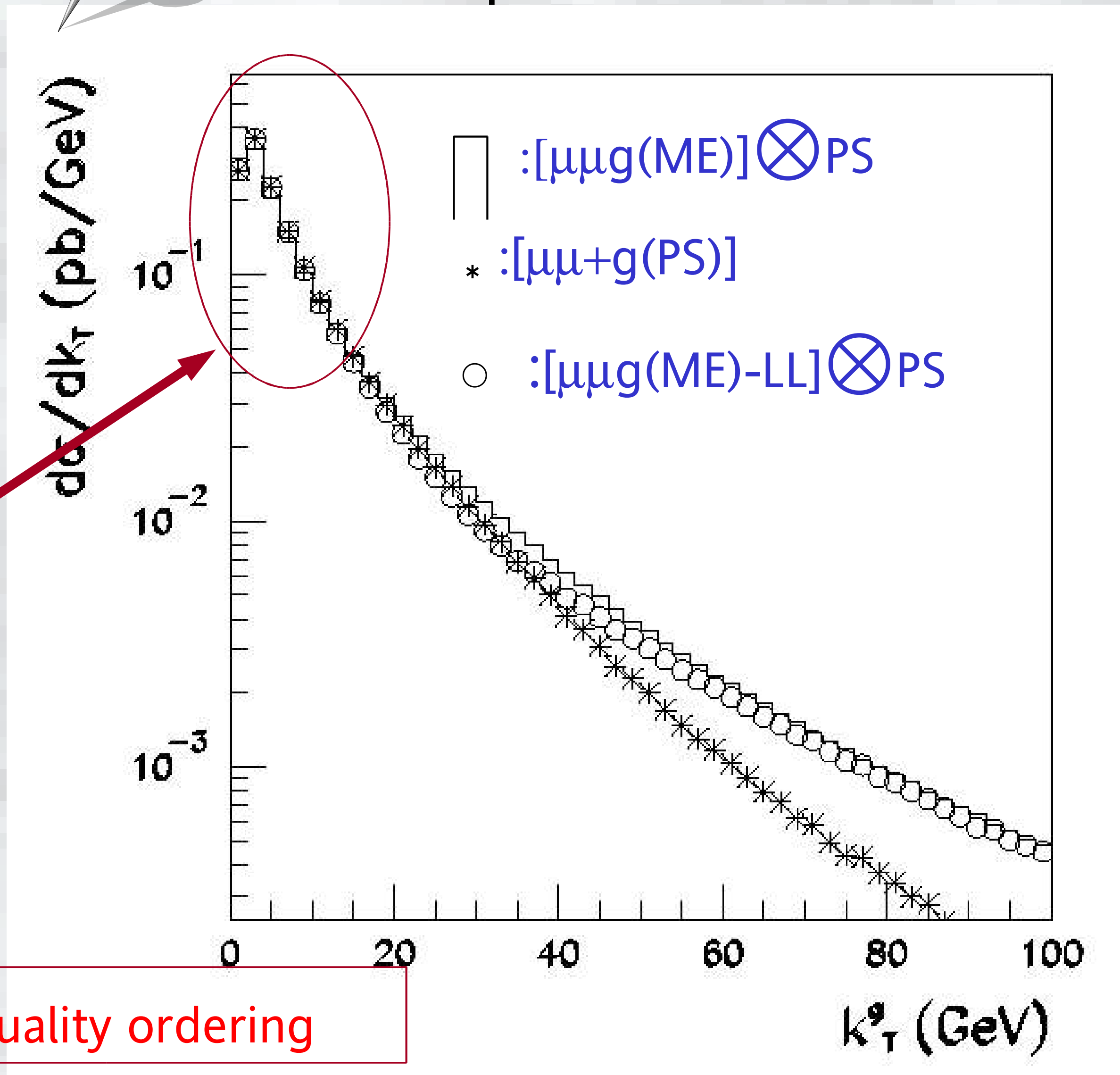
Double Counting Rejection

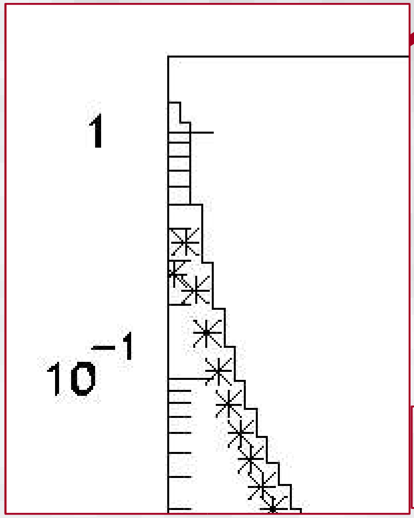


Drell-Yan process

- Process :
  $u\bar{u} \to \mu^+\mu^-$(+gluon)
  in $p\bar{p}$ collision
- Cuts:
  $\sqrt{s_{\mu\mu}}$>40GeV
  k_T^g> 1 GeV

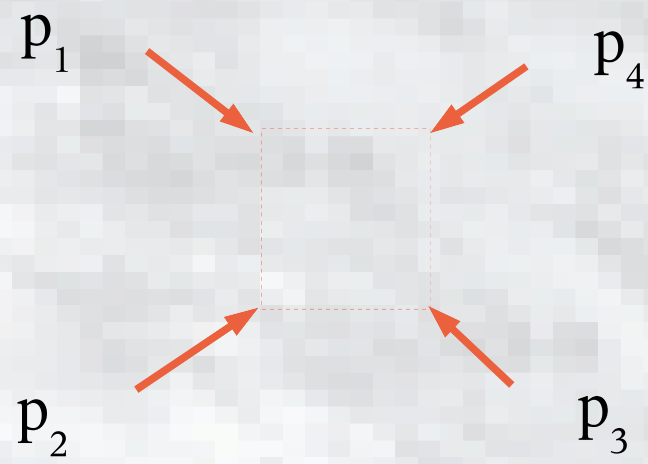
k^g_T Test





w/o virtuality ordering

Box Integral



$$J_{(4)}(s,t;p_1^2,p_2^2,p_3^2,p_4^2;n_x,n_y,n_z) = \frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{x^{n_x} y^{n_y} z^{n_z}}{D^{2-\varepsilon_{IR}}},$$

$$\begin{aligned}
D &= -s\,xz - t\,yw - p_1^2\,xy - p_2^2\,yz - p_3^2\,zw - p_4^2\,xw - i0,\\
w &= 1-x-y-z,\\
s &= (p_1+p_2)^2,\\
t &= (p_1+p_4)^2.
\end{aligned}$$

YK hep-ph/0504251

All on-shell (massless) external legs

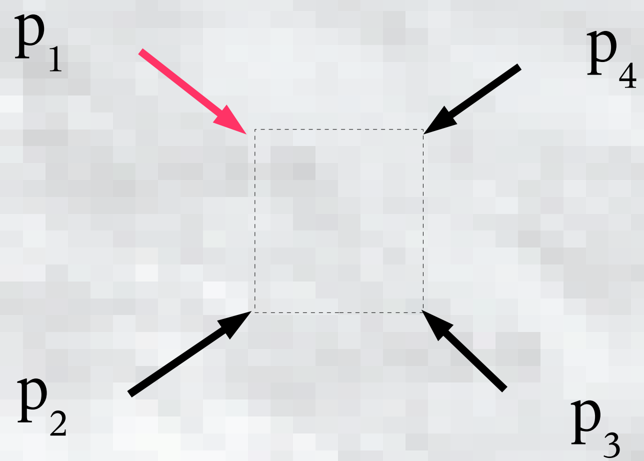
$$J_4(s,t;0,0,0,0;n_x,n_y,n_z) = \frac{1}{(4\pi)^2 s\, t} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})$$
$$\times \left[\left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \left(\frac{-t}{s}\right)^{n_x} \frac{B(1+n_z, n_x+n_y+\varepsilon_{IR})}{\Gamma(n_x+\varepsilon_{IR})} \right.$$
$$\times \; {}_2F_1\left(1+n_x, n_x+n_y+\varepsilon_{IR}, 1+n_x+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right)$$
$$+ \left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \sum_{l=0}^{n_x} \left(\frac{-s}{t}\right)^l \frac{(-1)^l}{\Gamma(l+\varepsilon_{IR})(n_x-l)!} B(1+n_y, l+n_z+\varepsilon_{IR})$$
$$\left. \times \; {}_2F_1\left(1+l, l+n_z+\varepsilon_{IR}, 1+l+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right) \right].$$

Scalar Integral

$$J_{(4)}(s,t;0,0,0,0;0,0,0) = \frac{1}{(4\pi)^2 s\, t} \frac{B(\varepsilon_{IR},\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})}{\varepsilon_{IR}}$$
$$\times \left[\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{t}}\right) + \left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{s}}\right) \right]$$

This result is compared with G. Duplanžić, B. Nižić, Eur. Phys. J. C **20**, 357 (2001)

One off-shell box integral



$$J_4(s,t;p_1^2,0,0,0;n_x,n_y,n_z) =$$

$$\frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz\frac{x^{n_x}y^{n_y}z^{n_z}}{\left(-xzs-y(1-x-y-z)t-p_1^2xy-i0\right)^{2-\varepsilon_{IR}}}$$

$$\begin{aligned}
= \quad & \frac{1}{(4\pi)^2 s\ t}B(n_x+\varepsilon_{IR},n_y+n_z+\varepsilon_{IR})n_x!\Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR}) \\
\times \quad & \left[\left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}}\left(\frac{-t}{s}\right)^{n_x}\frac{B(1+n_z,n_x+n_y+\varepsilon_{IR})}{\Gamma(n_x+\varepsilon_{IR})}\mathcal{I}^{(1)}\right. \\
+ \quad & \left.\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}}\sum_{l=0}^{n_x}\frac{(-1)^l B(1+n_y,l+n_z+\varepsilon_{IR})}{\Gamma(l+\varepsilon_{IR})(n_x-l)!}\mathcal{I}_l^{(2)}\right]
\end{aligned}$$

$$\mathcal{I}^{(1)} = B(1+n_z, n_x+n_y+\varepsilon_{IR})\ {}_2F_1\left(1+n_x, n_x+n_y+\varepsilon_{IR}, 1+n_x+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right)$$

$$\begin{aligned}
\mathcal{I}_l^{(2)} &= \sum_{k_1=0}^{n_z} {}_{n_z}C_{k_1} \left(\frac{s}{p_1^2 - s}\right)^{n_y+k_1} \sum_{k_2=0}^{n_y+k_1} {}_{n_y+k_1}C_k\ (-1)^{n_y+k_2} \left(\frac{-t}{s}\right) \\
&\times \int_0^1 dw \left(1+\frac{\tilde{u}}{\tilde{s}}w\right)^{-(l+1)} \left(1+\frac{\tilde{t}+\tilde{u}}{\tilde{s}}w\right)^{k_2+l-1+\varepsilon_{IR}} \\
&= \sum_{k_1=0}^{n_z} \sum_{k_2=0}^{n_y+k_1} {}_{n_z}C_{k_1}\ {}_{n_y+k_1}C_k\ (-1)^{k_1+k_2} \left(\frac{s}{p_1^2 - s}\right)^{n_y+k_1} \frac{1}{l+k_2+\varepsilon_{IR}} \left(1+\frac{u}{t}\right)^l \\
&\times \Bigg[{}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right) \\
&- \left(\frac{\tilde{p_1}^2}{\tilde{s}}\right)^{l+k_2+\varepsilon_{IR}} {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}\tilde{p_1}^2}{\tilde{t}\tilde{s}}\right)\Bigg],
\end{aligned}$$

Scalar Integral

$$J_4(s,t;p_1^2,0,0,0;0,0,0) = \frac{1}{(4\pi)^2 s\, t}\frac{B(\varepsilon_{IR},\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})}{\varepsilon_{IR}}$$

$$\times \left[\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{t}}\right) + \left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{s}}\right)\right.$$

$$\left. - \left(\frac{-\tilde{p_1}^2}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}\tilde{p_1}^2}{\tilde{t}\tilde{s}}\right)\right],$$

This result is compared with G. Duplančić, B. Nižić, Eur. Phys. J. C **20**, 357 (2001)

Numerical calculation (IR finite case)

For $l, m, n \in N$,

$$
\begin{aligned}
&{}_2F_1(l, m+1, n+m+2; z) \\
&= \frac{1}{B(m+1, n+1)} \int_0^1 \tau^m (1-\tau)^n (1-z\tau)^{-l} d\tau \\
&= \sum_{k_1=0}^{n} \sum_{k_2=0}^{m+k_1} (-1)^{k_1+k_2} \frac{{}_nC_{k_1}\, {}_{m+k_1}C_{k_2}}{B(m+1, n+1)} \frac{1}{z^{m+k_1}} \int_0^1 (1-z\tau)^{-l+k_2} d\tau.
\end{aligned}
$$

$$
\int_0^1 (1-z\tau)^{-l+k_2} d\tau = \begin{cases} -\frac{\ln(1-z)}{z} & k_2 - l + 1 = 0, \\ \frac{1}{k_2-l+1} \frac{(1-z)^{k_2-l+1}-1}{-z} & k_2 - l + 1 \neq 0. \end{cases}
$$

Numerical check: mathematica ↔ our program

more than ten digit agreement

Numerical calculation (IR divergent case)

$$
\begin{aligned}
\mathcal{I}_{l,m,n} &\equiv \int_0^1 \tau^{l+n-1+\varepsilon_{IR}}(1-\tau)^m(1-z\tau)^{-(l+1)}d\tau, \\
&= B(1+m, l+n+\varepsilon_{IR})\ {}_2F_1\left(1+l, l+n+\varepsilon_{IR}, 1+l+n+m+\varepsilon_{IR}, z\right) \\
&= \sum_{j=-1}^{\infty} \mathcal{F}^{(j)}_{l,m,n}(z)\varepsilon^j_{IR},
\end{aligned}
$$

Expansion w.r.t. ε_{IR}

$$
\tilde{F}^{(n)}_{j_1,j_2}(z) \equiv \frac{(-1)^n}{n!}\int_0^1 d\tau\ \tau^{j_1-1}(1-z\tau)^{-(j_2+1)}\ln^n\tau,
$$

When n=1

$$\tilde{F}^{(1)}_{1,0}(z) = \frac{\mathrm{Li}_2(z)}{z},$$

$$\tilde{F}^{(1)}_{1,1}(z) = -\frac{\ln(1-z)}{z}$$

$$\tilde{F}^{(1)}_{1,j_2+1}(z) = \frac{j_2}{j_2+1}\tilde{F}^{(1)}_{1,j_2} + \frac{(1-z)^{-j_2}-1}{j_2(j_2+1)z}$$

$$\tilde{F}^{(1)}_{j_1,j_2}(z) = \frac{1}{z^{j_1-1}}\sum_{k=0}^{j_1-1}(-1)^k\ {}_{j_1-1}C_k\ \tilde{F}^{(1)}_{1,j_2-k}(z)$$

When n=2

$$\tilde{F}^{(2)}_{1,0}(z) = \frac{\mathrm{Li}_3(z)}{z}$$

$$\tilde{F}^{(2)}_{1,1}(z) = \frac{\mathrm{Li}_2(z)}{z}$$

$$(j_2+1)\tilde{F}^{(2)}_{1,j_2+1}(z) - j_2\tilde{F}^{(2)}_{1,j_2}(z) - \tilde{F}^{(1)}_{1,j_2}(z) = 0$$

$$\tilde{F}^{(2)}_{j_1,j_2}(z) = \frac{1}{z^{j_1-1}} \sum_{k=0}^{j_1-1} (-1)^k \; {}_{j_1-1}C_k \; \tilde{F}^{(2)}_{1,j_2-k}(z)$$

Infrared finite BOX integral

$$J_4(s,t;p_1^2,0,0,0;n_x,n_y,n_z) = \frac{1}{(4\pi)^2 s\, t} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})$$

$$\times \left[\left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \left(\frac{-t}{s}\right)^{n_x} \frac{1}{\Gamma(n_x+\varepsilon_{IR})} \mathcal{I}^{(1)} + \left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \sum_{l=0}^{n_x} \frac{(-1)^l}{\Gamma(l+\varepsilon_{IR})(n_x-l)!} \mathcal{I}_l^{(2)} \right],$$

$$\mathcal{I}^{(1)} = B(1+n_z, n_x+n_y+\varepsilon_{IR})\ {}_2F_1\left(1+n_x, n_x+n_y+\varepsilon_{IR}, 1+n_x+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right), \qquad \text{(B.79)}$$

$$\mathcal{I}_l^{(2)} = \sum_{k_1=0}^{n_z} \sum_{k_2=0}^{n_y+k_1} {}_{n_z}C_{k_1}\ {}_{n_y+k_1}C_{k_2}\ (-1)^{k_1+k_2} \left(\frac{s}{s-p_1^2}\right)^{n_y+k_1} \frac{1}{l+k_2+\varepsilon_{IR}} \left(1+\frac{u}{t}\right)^l$$

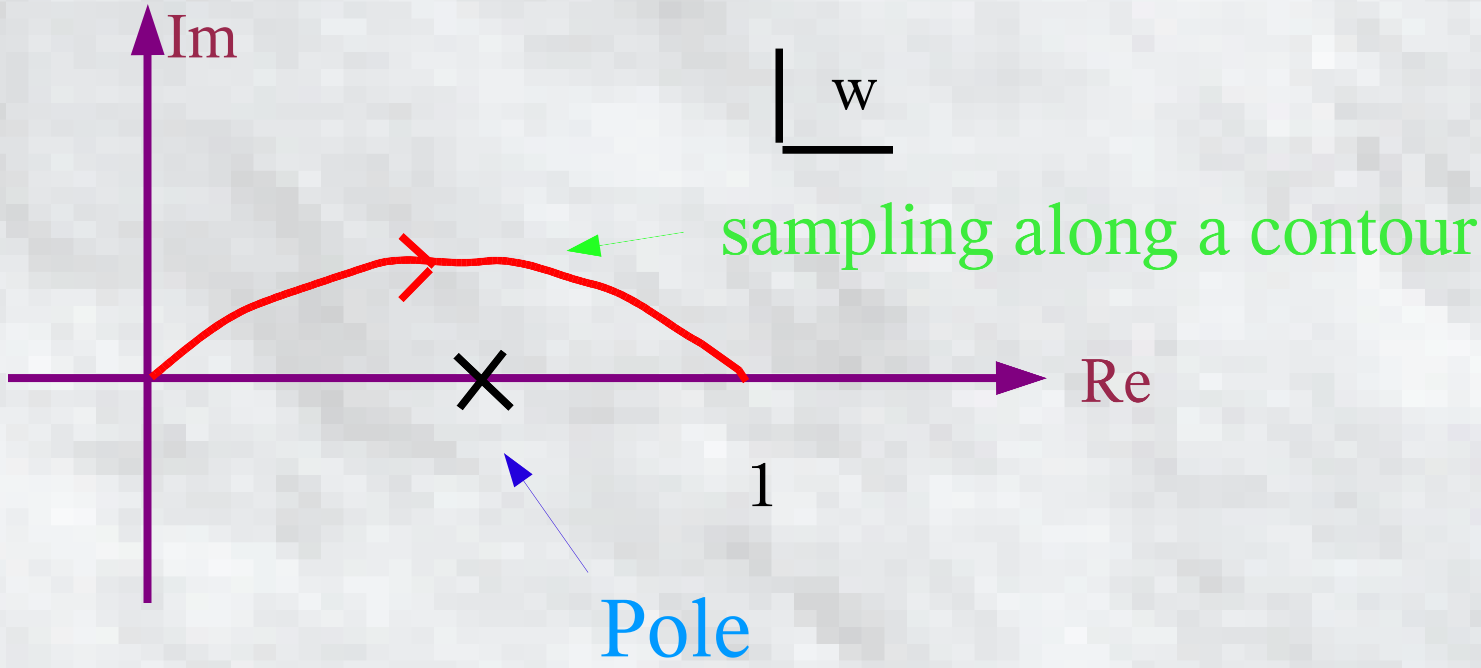
$$\times \left[{}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right) \right.$$

$$\left. - \left(\frac{\tilde{p_1}^2}{\tilde{s}}\right)^{l+k_2+\varepsilon_{IR}} {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}\tilde{p_1}^2}{\tilde{t}\tilde{s}}\right) \right], \qquad \text{(B.80)}$$

How can we test them?

Numerical contour integral

$$J_4 = \left(4\pi\mu^2\right)^{-\varepsilon_{IR}} \int_0^1 dr\ r^{-1+n_x+\varepsilon_{IR}}(1-r)^{-1+n_y+n_z+\varepsilon_{IR}}$$
$$\times \int_0^1 dv \int_0^1 dw \frac{w^{n_y}(1-w)^{n_z} v^{n_x}}{(-s\ v(1-w) - t\ (1-v)w - i0)^{2-\varepsilon_{IR}}}$$
$$= \left(4\pi\mu^2\right)^{-\varepsilon_{IR}} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR})$$
$$\times \int_0^1 dv \int_0^1 dw \frac{w^{n_y}(1-w)^{n_z} v^{n_x}}{(-s\ v(1-w) - t\ (1-v)w - i0)^{2-\varepsilon_{IR}}}$$



YK, T. Kaneko hep-ph/0503003

$J_4(s, t; 0, 0, 0, 0; n_x, n_y, n_z)$

n_x	n_y	n_z	real/imag.	analytic	NCI
1	2	3	real	-2.15298×10^{-9}	$-2,15297 \times 10^{-9}$
			imag.	-2.78647×10^{-9}	-2.78650×10^{-9}
2	0	2	real	9.74570×10^{-9}	8.74572×10^{-9}
			imag.	-3.22229×10^{-8}	-3.22230×10^{-8}

$J_4(s, t; p_1^2, 0, 0, 0; n_x, n_y, n_z)$

n_x	n_y	n_z	real/imag.	analytic	NCI
1	2	3	real	-7.88683×10^{-10}	-7.88689×10^{-10}
			imag.	-1.95176×10^{-9}	-1.95176×10^{-9}
2	0	2	real	1.48133×10^{-8}	1.48133×10^{-8}
			imag.	-2.04318×10^{-8}	-2.04318×10^{-8}

s=123, t=-200, p_1^2=80

IR cancellation test

ex. Prompt photon production

@ one phase point

$\delta = a_2/\varepsilon_{IR}^2 + a_1/\varepsilon_{IR} + a_0$

BOX

a_1 = -18601.9993715016

a_2 = -3793.95539013131

S/C+vertex-(terms included in PDF)

a_1 = 18601.9993714494

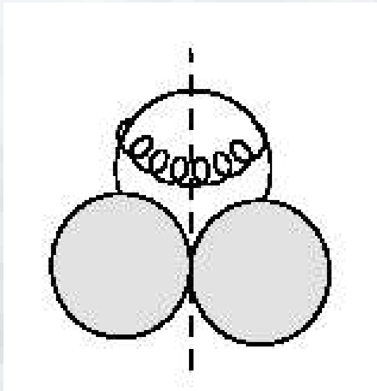
a_2 = 3793.95539013130

$1/\varepsilon_{IR}^2$: Soft/Coll. + Loop~$O(10^{-15})$

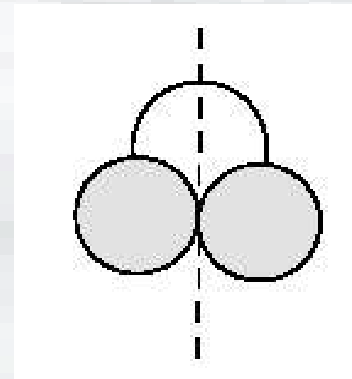
$1/\varepsilon_{IR}$: Soft/Coll. + Loop~$O(10^{-12})$

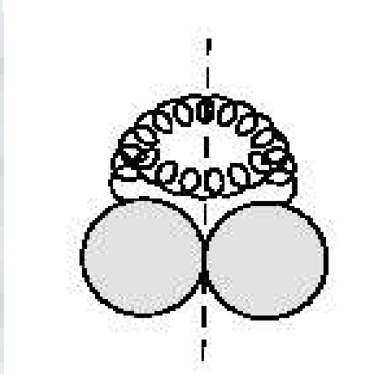
Soft/Collinear Approximation

In axial gauge,

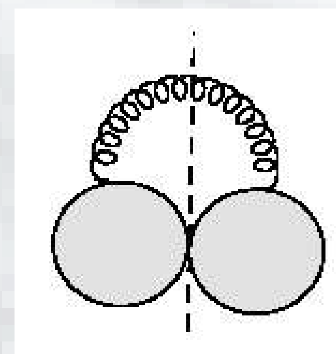


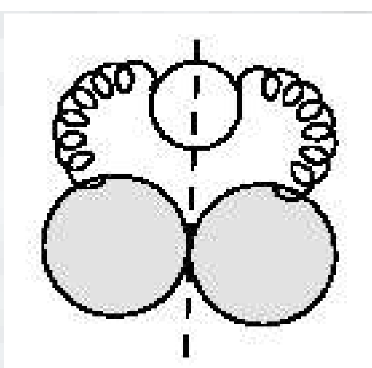
$$= f_{q \to qg} \times$$



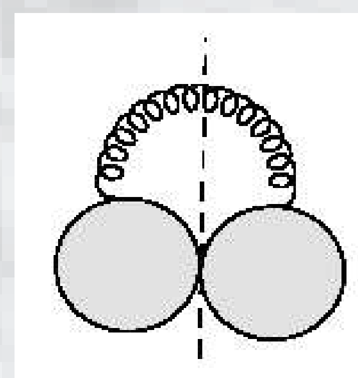


$$= f_{g \to gg} \times$$



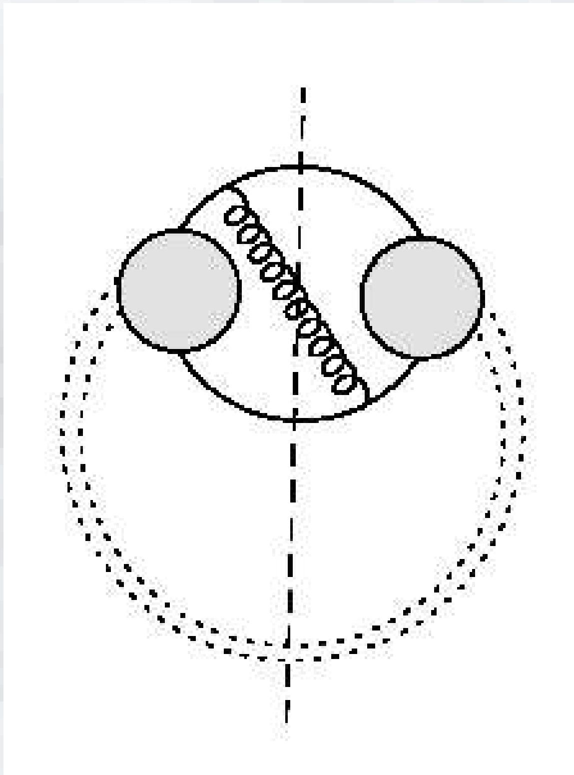


$$= f_{g \to qq} \times$$

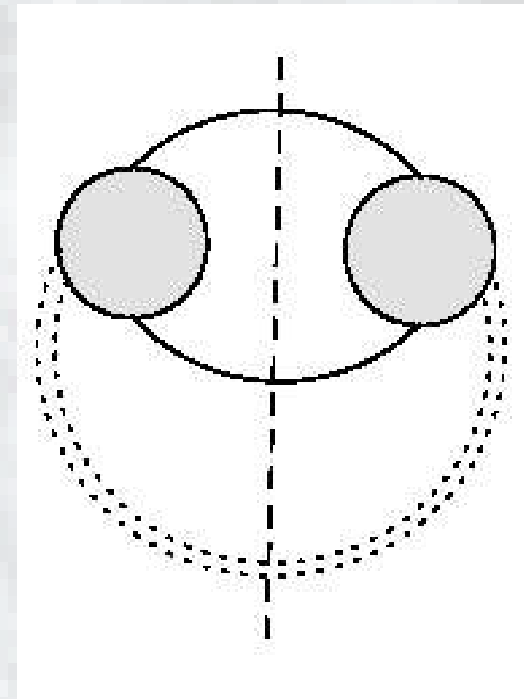


Soft/Collinear Approximation

In axial gauge,



$$= f^{int}_{q\to qg} \times$$



$$f^{int}_{q\to qg} = 0 \quad \text{in collinear region}$$

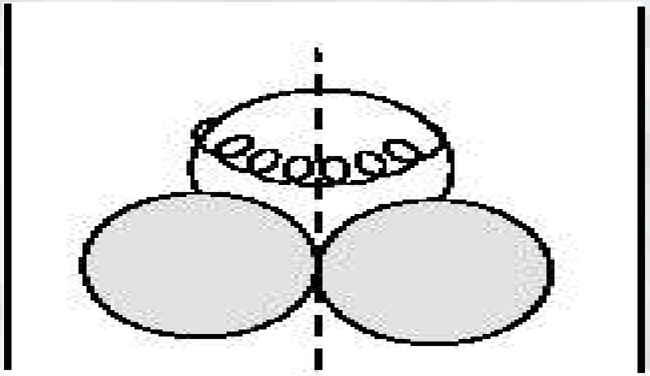
$$\neq 0 \quad \text{in soft region}$$

All f's will be implemented in the system soon.

Soft/Collinear Approximation

$f_{q_{in} \to q_{in} g_{out}}$

$$\sigma_{coll} = \frac{1}{(2p_1^0)(2p_2^0)v_{rel}} \int_{\Omega_{full}} d\Phi_{N+1}^{(d)}$$



$$= \left(\frac{s}{4\pi\mu^2}\right)^{\varepsilon_{IR}} \frac{B(\varepsilon_{IR}, \varepsilon_{IR})}{2\Gamma(1+\varepsilon_{IR})} f_c \frac{\alpha_s}{2\pi} \int_0^1 dx \sigma_0(xs) \mathrm{P}(x) \left(\frac{1-x}{x}\right)$$

$$= \sigma_0(s) \frac{\alpha_s}{2\pi} f_c \left[\frac{1}{\varepsilon_{IR}^2} + \frac{2L-3}{2\varepsilon_{IR}} - \frac{\pi^2}{4} + \frac{L^2}{2}\right]$$

$$+ \int_0^1 dx \sigma_0(xs) \phi(x, \varepsilon_{IR})$$

$$+ f_c \frac{\alpha_s}{2\pi} \int_0^1 dx \sigma_0(xs) \left[L \frac{1+x^2}{(1-x)_+} + \frac{(1+x^2)\ln(1-x)}{(1-x)_+} - \frac{1+x^2}{1-x} \ln x\right]$$

$$\phi(x, \varepsilon_{IR}) = \frac{1}{\varepsilon_{IR}} f_c \frac{\alpha_s}{2\pi} \mathrm{P}(x) = \frac{1}{\varepsilon_{IR}} f_c \frac{\alpha_s}{2\pi} \frac{1+x^2}{(1-x)_+}$$

$$L = \ln(s/\mu^2).$$

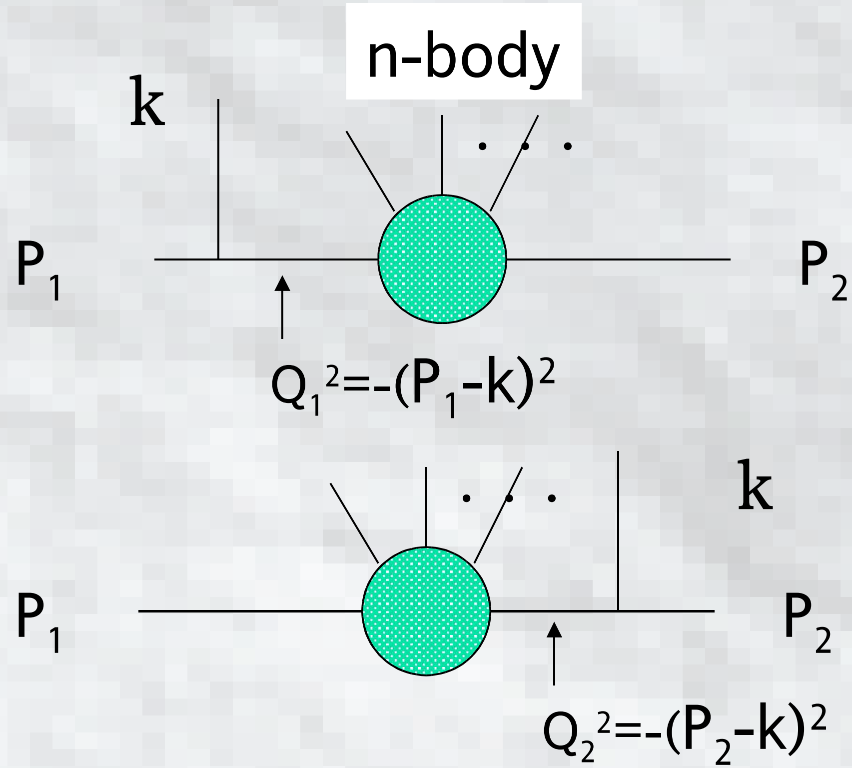
Soft/Collinear Treatment

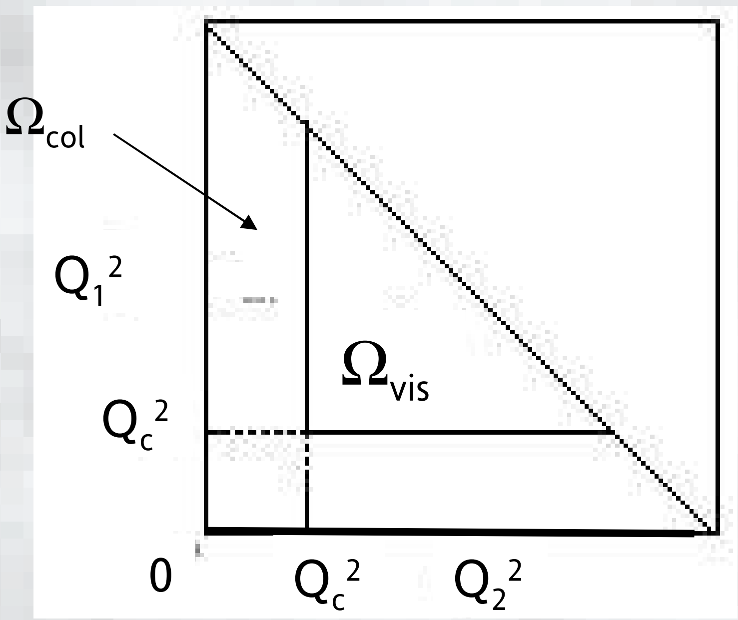
- Subtraction method

S.Catani, M.M. Seymour, hep-ph/9605323

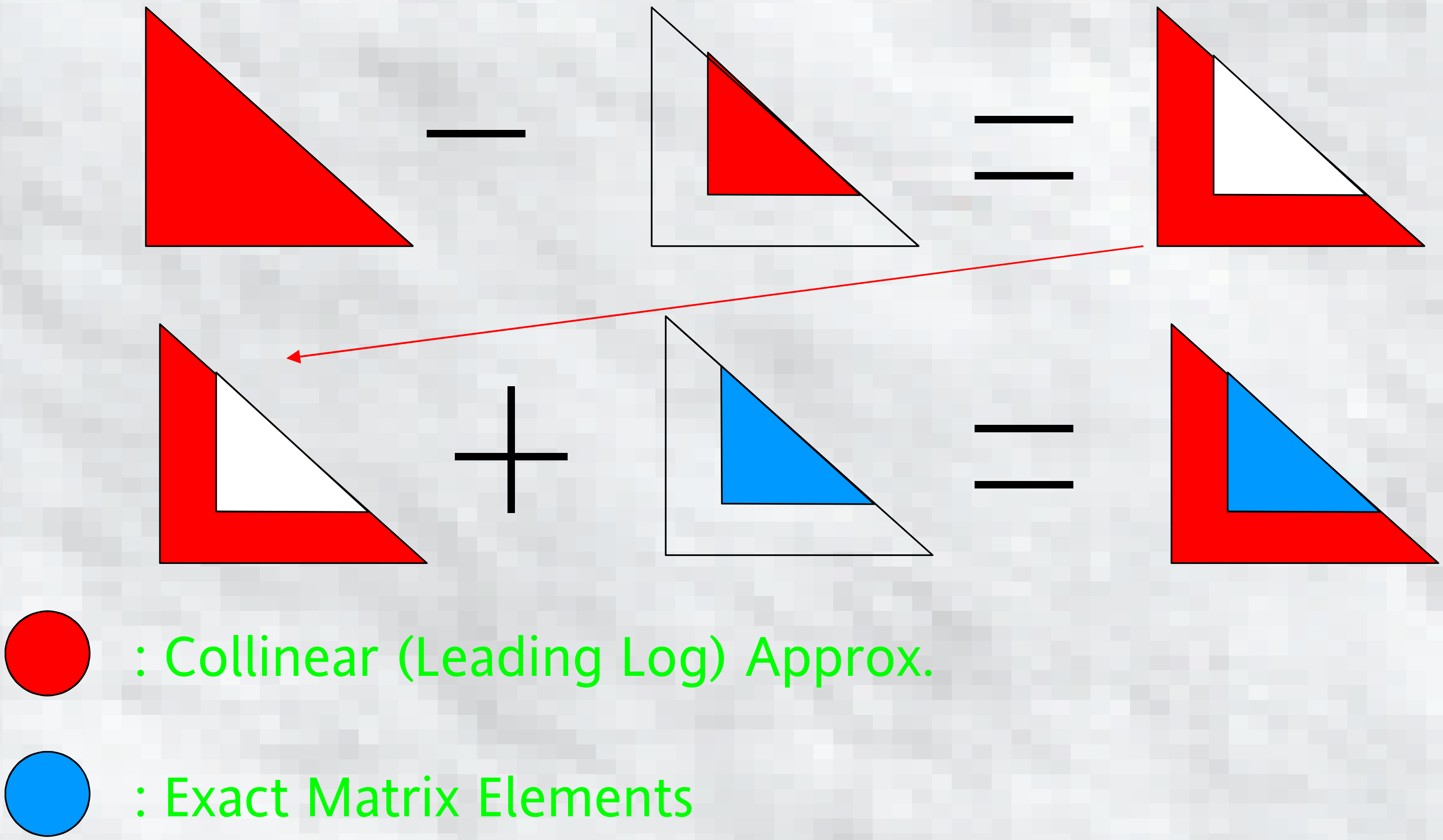
- Phase-space slicing

W.B. Harris, J.F.Owens, Phys. Rev. D65 (2002) 094032





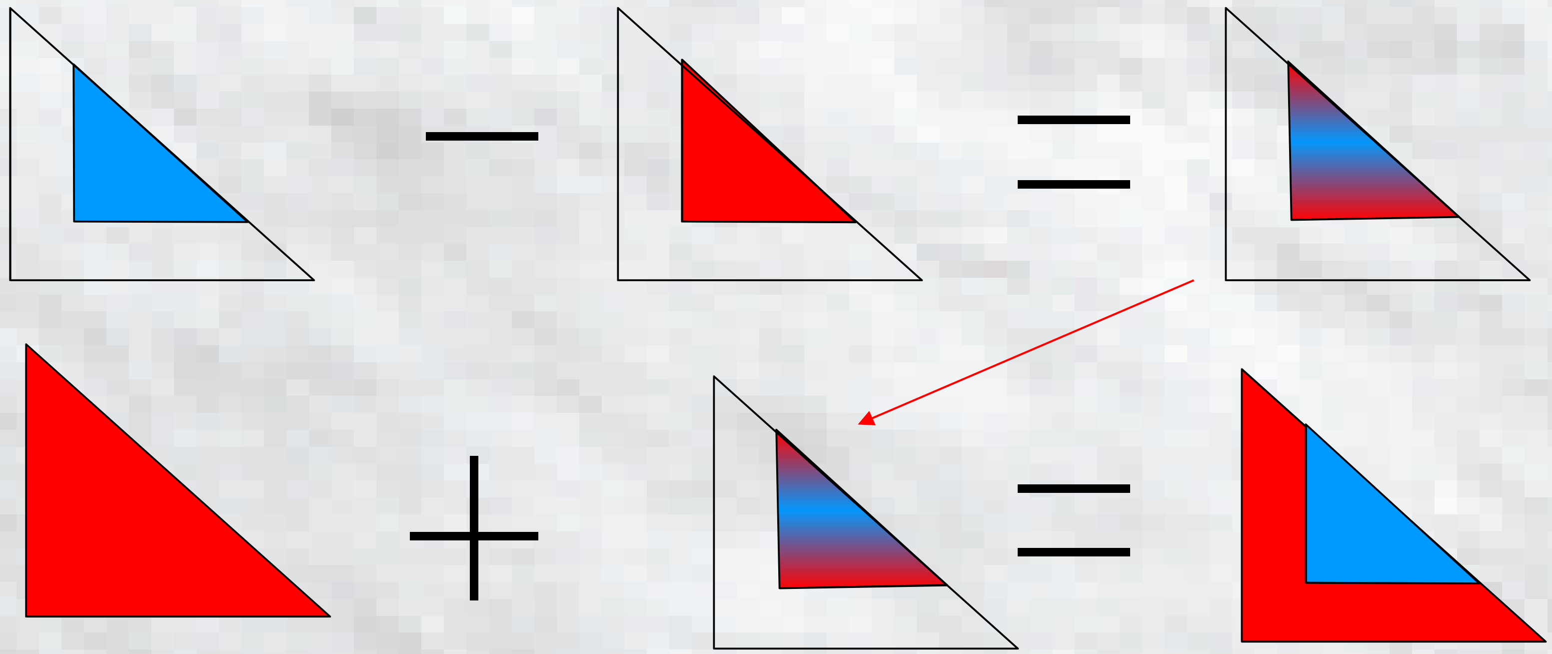
Phase space slicing



: Collinear (Leading Log) Approx.

: Exact Matrix Elements

Leading Log Subtraction



: Collinear (Leading Log) Approx.

: Exact Matrix Elements

Status of GRACE NLO Generator

Already tested

- Drell-Yan process
- W production
- Prompt photon

Under development

- V+1 jet

Future plan

- V+2 jets,.....
- VV+jet,2jets,....

Summary

(1) Automatic loop calculation in ELWK is established.

(2) QCD-NLO Matrix Elements
- ○Automatic generation by GRACE

(3) Loop integral
- ○Numerical loop-integration library

(4) Soft/Collinear treatment
- ○Basic tools of NLL-PS is available (under implementation)
- ○LL-subtraction method
- ○General calculation recipe

(5) Application
- ○Drell-Yan process/w production
- ○ prompt photon
- ○ V+1jet,2jet, VV+1 jet