Progress toward pp → tt + Jet in next-to-leading order QCD

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Contents:

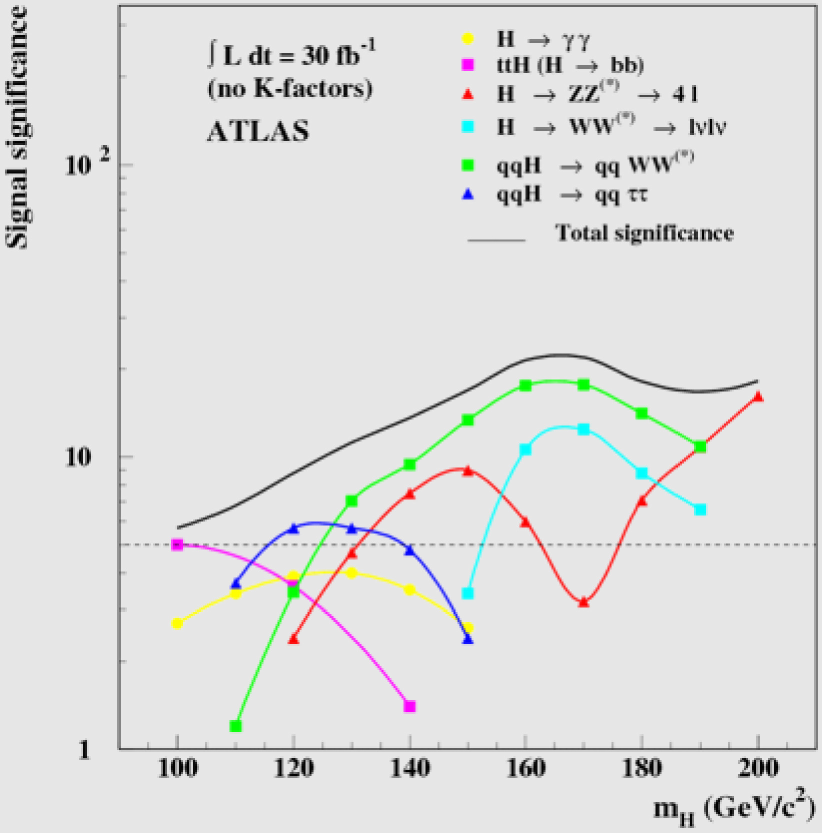
- Why is top quark physics interesting / important ?
- Where do we stand ?
- Status of tt + 1 jet @ NLO
- Conclusion

Why is top quark physics important?

- Interesting in itself
  - Precise study of top properties, search for new physics,…
- Suitable reaction for “commissioning”
  - b-tagging efficiency,…
- Important background for Higgs searches
  - Higgs production via WBF,…

→ Precise theoretical understanding is necessary!

Why is top quark physics important?

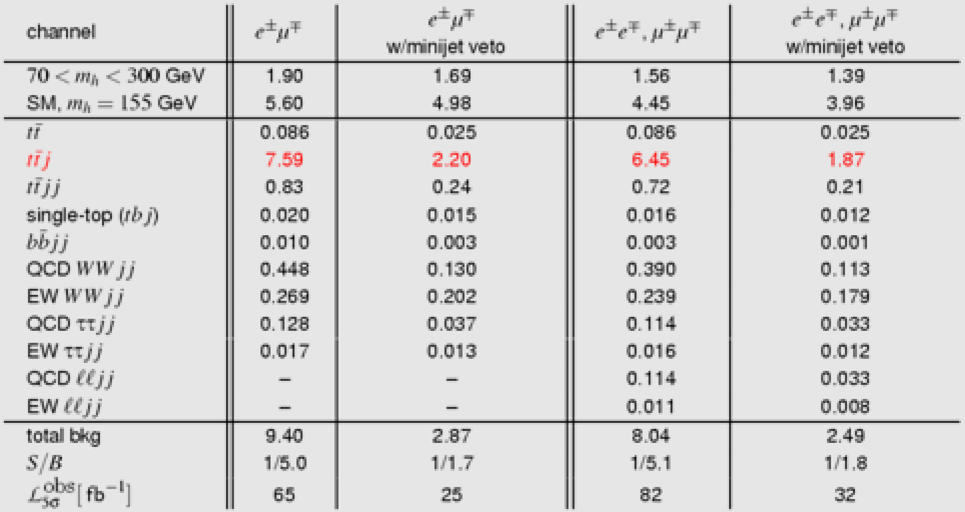


The WBF process

$$qq \to WWqq \to qqH \to WW$$

is important over a wide Higgs mass range

Important backgrounds:



channel	$e^\pm\mu^\mp$	$e^\pm\mu^\mp$ w/minijet veto	$e^\pm e^\mp, \mu^\pm\mu^\mp$	$e^\pm e^\mp, \mu^\pm\mu^\mp$ w/minijet veto
$70 < m_h < 300$ GeV	1.90	1.69	1.56	1.39
SM, $m_h = 155$ GeV	5.60	4.98	4.45	3.96
$t\bar{t}$	0.086	0.025	0.086	0.025
$t\bar{t}j$	7.59	2.20	6.45	1.87
$t\bar{t}jj$	0.83	0.24	0.72	0.21
single-top (tbj)	0.020	0.015	0.016	0.012
$b\bar{b}jj$	0.010	0.003	0.003	0.001
QCD $WWjj$	0.448	0.130	0.390	0.113
EW $WWjj$	0.269	0.202	0.239	0.179
QCD $\tau\tau jj$	0.128	0.037	0.114	0.033
EW $\tau\tau jj$	0.017	0.013	0.016	0.012
QCD $\ell\ell jj$	–	–	0.114	0.033
EW $\ell\ell jj$	–	–	0.011	0.008
total bkg	9.40	2.87	8.04	2.49
S/B	1/5.0	1/1.7	1/5.1	1/1.8
$\mathcal{L}_{5\sigma}^{\text{obs}}$ [fb^{-1}]	65	25	82	32

[Alves, Eboli, Plehn, Rainwater '04]

→ Precise predictions for pp → tt + jet are necessary

Where do we stand?

Important progress over the last years as far as tt production is concerned

- NLO corrections including spin-correlations are known
- Resummation of large log’s
- Combination of NLO fixed order with parton shower MC

[MC@NLO, Frixione, Webber]

As far as tt is concerned theory is in good shape, given today’s technology even 2-loop might be possible

Where do we stand?

Beyond top quark pair production, i.e. tt+ n Jets, tt + γ,… most of the cross sections are only known at LO

→ Easy to use and well tested programs like Alpgen or Madgraph available

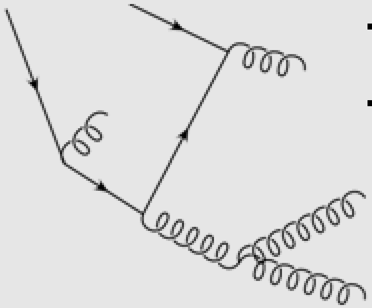
The NLO corrections are only known for a few processes

→ State of the art: 5-point one-loop amplitudes with massive particles

Although conceptionally solved one-loop calculations for more than 4 legs are still non-trivial

Status of pp → tt + 1 Jet @ NLO

Real corrections: Amplitudes



QGRAF Diagram 20

- Calculation of matrix elements straight forward
- Two methods used:

1. Feynman diagram approach + four dimensional helicity scheme (FDH)
2. Recurrence relations à la Berends, Giele

As a check:

comparison with Madgraph

Phase space integration yields IR/coll. singularities

To extract soft and mass singularities we use the dipole subtraction method!

[Catani, Seymour ´96, Nason, Oleari ´98 Phaf,Weinzierl ´01, Catani, Dittmaier, Seymour, Tocsanyi ´02]

Alternatives: Phase space slicing [Giele, Glover ´92, Giele, Glover, Kosower ´93]

Real corrections: Subtraction Method

Basic idea:

Add and subtract a contribution which:

- matches pointwise the singularities
- is easy enough to be integrated analytically over the one-particle unresolved phase space

$$\sigma_{\text{NLO}} = \underbrace{\int_{m+1} \left[\sigma_{\text{real}} - \sigma_{\text{sub}}\right]}_{\text{finite}} + \underbrace{\int_m \left[\sigma_{\text{virt.}} + \bar{\sigma}^1_{\text{sub}}\right]}_{\text{finite}} + \underbrace{\int dx \int_m \left[\sigma_{\text{fact.}}(x) + \bar{\sigma}_{\text{sub}}(x)\right]}_{\text{finite}}$$

Requirements:

$$0 = -\int_{m+1} \sigma_{\text{sub}} + \int_m \bar{\sigma}^1_{\text{sub}} + \int dx \int_m \bar{\sigma}_{\text{sub}}(x)$$

$\sigma_{\text{sub}} \approx \sigma_{\text{real}}$ in all single-unresolved regions

Real corrections: Subtraction terms

Due to universal structure:

$$\sigma_{\mathsf{sub}} = \sum_{\mathsf{dipoles}} \mathcal{D}_{ij,k}(p_i, p_j, p_k)$$

Generic form of individual dipole:

Leading-order amplitudes
Vector in color space

$$\mathcal{D}_{ij;k} = -\frac{1}{(p_i+p_j) - m_{ij}^2}\langle \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots \Big| \underbrace{\frac{\mathsf{T}_a \cdot \mathsf{T}_{ij}}{\mathsf{T}_{ij}} V_{ij,k}}_{\text{universal}} \Big| \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots \rangle$$

Color charge operators,
induce color correlation

Spin dependent part,
induces spin correlation

Real corrections: Status

For gg → ttg the real corrections are finished

- Matrix element checked point wise
- Subtraction term checked in singular regions
- Two different programs for the phase space integration
- Integration is stable

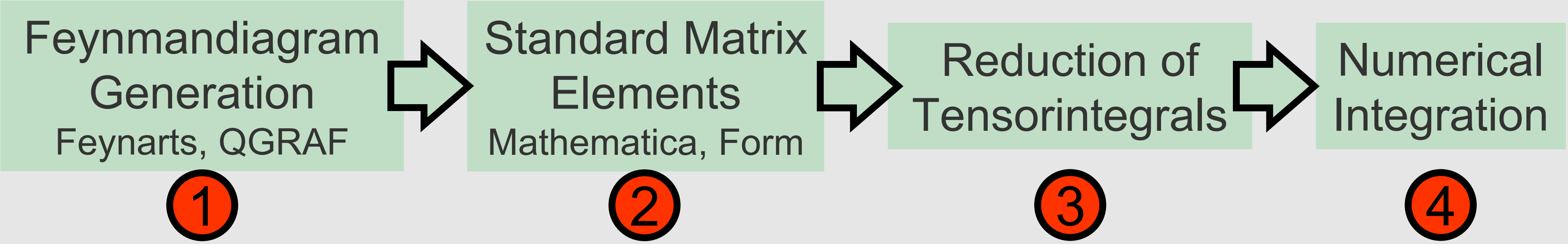
Note:

Remaining processes (qq→ttg, qg→ttq,…) are much easier, many things can be reused…

Virtual corrections

Calculation similar to pp → ttH @ NLO

[Beenakker, Dittmaier, Krämer, Plümper, Spira, Zerwas 03, Dawson, Jackson, Orr, Reina, Wackeroth 03]



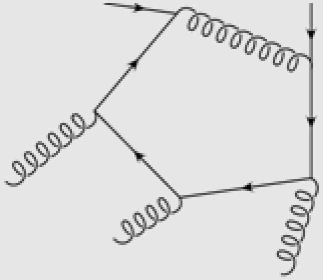
Feynmandiagram Generation
Feynarts, QGRAF
①

Standard Matrix Elements
Mathematica, Form
②

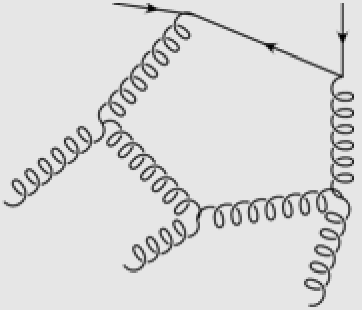
Reduction of Tensorintegrals
③

Numerical Integration
④

Example diagrams:



QGRAF Diagram 265



QGRAF Diagram 266

Steps 1, 2, 4 more or less standard

Step 3 involves non-trivial complications

→ 5-point integrals

How do we obtain a fast and numerical stable reduction of the tensor integrals?

Reduction of tensor integrals

Four and lower-point tensor integrals:

Reduction à la Passarino-Veltman,
with special reduction formulae in singular regions

Five-point tensor integrals: [Beenakker et al ´03]

1. Form *regularization scheme independent* quantities by dressing propagators with a small mass
2. Split into finite and divergent pieces
3. Apply 4-dimensional reduction scheme, 5-point tensor integrals are reduced to 4-point tensor integrals

→ No dangerous Gram determinants!

In 4 dim. 5-point integrals can be reduced to 4-point integrals [Melrose ´65]

Reduction of finite 5-point integrals

[Melrose ´65, van Neerven, Vermaseren ´84]

Consider:

$$E^4 = \frac{1}{i\pi^2}\int d^4\ell \prod_{j=0}^{4} \frac{1}{(\ell+p_j)^2 - m_j^2 + i\varepsilon} = \frac{1}{i\pi^2}\int \frac{d^4\ell}{N_0 N_1 N_2 N_3 N_4}$$

$$p_0 = 0$$

In 4 dimensions the loop-momentum can be expressed in p_1–p_4:

⇓

$$0 = \det\begin{pmatrix} 2\ell^2 & 2\ell\cdot p_1 & \dots & 2\ell\cdot p_4 \\ 2p_1\cdot\ell & 2p_1\cdot p_1 & \dots & 2p_1\cdot p_4 \\ \vdots & \vdots & \ddots & \vdots \\ 2p_4\cdot\ell & 2p_4\cdot p_1 & \dots & 2p_4\cdot p_4 \end{pmatrix}$$

Plug in the determinant into the integral, we obtain…

Virtual corrections: Evaluation of 5-point integrals

$$0 = \frac{1}{i\pi} \int d^4\ell \frac{1}{N_0 \ldots N_4} \, \mathrm{det}(\ldots)$$

Rewriting the scalar products in terms of propagators, i.e.:

$$\begin{aligned} 2p_1 \cdot \ell &= \ell^2 + 2p_1\ell - m_1^2 - \ell^2 + m_0^2 - m_0^2 + m_1^2 \\ &= N_1 - N_0 + m_0^2 - m_1^2 \end{aligned}$$

We obtain reduction of 5-point integral in terms of lower-point integrals

Method only applicable to finite integrals…

Reduction of singular 5-point integrals

Dress all the mass less propagators with small mass λ:

$$E^d \to E^{\lambda,d}$$

[Dittmaier ´03]

Consider:

$$E^{\lambda,d} = E^{\lambda,d}_{\mathrm{sing.}} + \underbrace{(E^{\lambda,d} - E^{\lambda,d}_{\mathrm{sing.}})}_{\text{regularization scheme indep.}}$$

For λ→0, $E^{\lambda,d}_{\mathrm{sing.}}$ reproduces the singular behaviour of E^d

$E^{\lambda,d}_{\mathrm{sing.}}$ obtained from soft and collinear limits of $E^{\lambda,d}$

→ simple combination of 3-point integrals

$$E^d = E^{\lambda=0,d}_{\mathrm{sing.}} + (E^{\lambda,\underset{\uparrow}{d=4}} - E^{\lambda,d=4}_{\mathrm{sing.}})$$

can now be reduced to lower-point integrals

Virtual corrections: Evaluation of 5-point Tensorintegrals

Direct reduction, same trick as in the scalar case:

[Denner, Dittmaier ´03]

To regularize spurious UV singularities in individual terms

$$0 = \frac{1}{i\pi^2}\int d^4\ell \frac{\ell_{\mu_1}\cdots\ell_{\mu_P}}{N_0 N_1 \ldots N_4}\frac{-\Lambda^2}{\ell^2-\Lambda^2}\det\begin{pmatrix} 2\ell^2 & 2\ell\cdot p_1 & \ldots & 2\ell{\cdot}4 \\ 2p_1\cdot\ell & 2p_1\cdot p_1 & \ldots & 2p_1\cdot p_4 \\ \vdots & \vdots & \ddots & \vdots \\ 2p_4\cdot\ell & 2p_4\cdot p_1 & \ldots & 2p_4\cdot p_4 \end{pmatrix}$$

with $N_i = (\ell+p_i)^2 - m_i^2 + i\epsilon), p_0 = 0$

Can be expressed in terms of N_i

Reduction of $E_{\mu_1\mu_2\ldots}$ in terms of $D_{\mu_1\mu_2\ldots}$

No dangerous Gram-Determinants appear in the denominator!

Virtual corrections: Evaluation of 5-point Tensorintegrals

General result:

[Denner, Dittmaier ´03]

Lower point tensor integrals

$$E_{\mu_1\mu_2\ldots\mu_p} = -\sum_{i=0}^{4} \frac{\det(Y_i)}{\det(Y)} D^{(\mathrm{fin})}_{\mu_1\mu_2\ldots\mu_p}(i) + \sum_{i,j=1}^{4} (-1)^{i+j} \frac{\det(\hat{Z}^{(4)}_{ij})}{\det(Y)} 2p_j^{\alpha} \mathcal{D}_{\mu_1\mu_2\ldots\mu_p}(i) + \frac{1}{\det(Y)} U_{\mu_1\mu_2\ldots\mu_p}$$

With: $Y_{ij} = m_i^2 + m_j^2 - (p_i - p_j)^2$

$\hat{Z}^{(4)}_{ij}$ from $Z^{(4)} = \begin{pmatrix} 2p_1 \cdot p_1 & \ldots & 2p_1 \cdot p_4 \\ \vdots & \ddots & \vdots \\ 2p_4 \cdot p_1 & \ldots & 2p_4 \cdot p_4 \end{pmatrix}$

By discarding the *i*th row and *j*th column

$D^{(\mathrm{fin})}_{\mu_1\mu_2\ldots\mu_p}(i)$, $\mathcal{D}_{\mu_1\mu_2\ldots\mu_p}(i)$

Tensor integrals with the *i*th denominator removed

$$U_{\mu_1\mu_2\ldots\mu_p} = \begin{cases} 0 & \text{for } p <= 3, \\ -\frac{1}{48}(g_{\mu_1\mu_2}g_{\mu_1\mu_2} + g_{\mu_1\mu_3}g_{\mu_4\mu_2} + g_{\mu_1\mu_4}g_{\mu_2\mu_3}) & \text{for } p = 4 \end{cases}$$

Checks and status

General philosophie:

Every contribution should be checked independently,
If possible using different methods and different tools

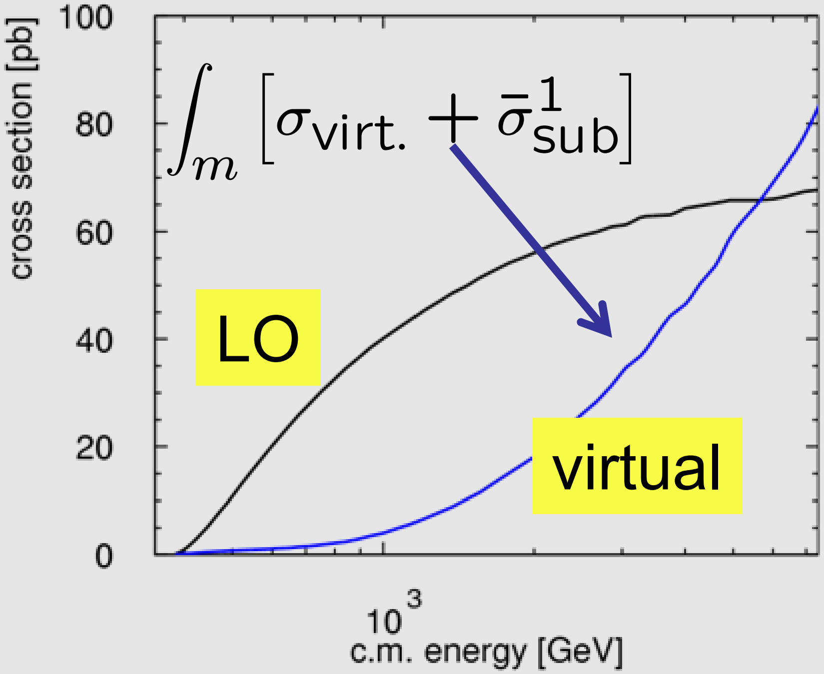
Current status:

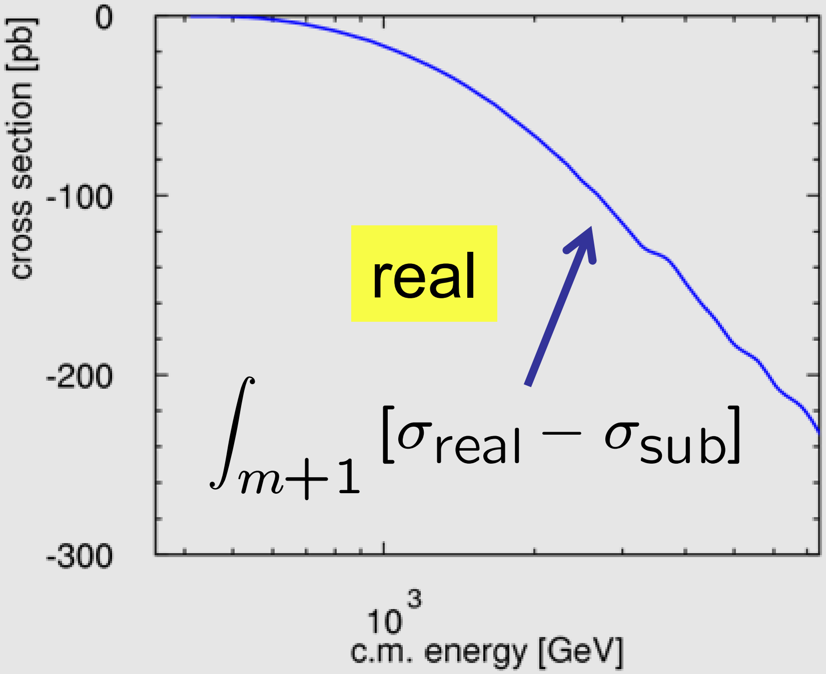
- ✓ All tree amplitudes are checked
- ✓ Cancellation of UV and mass/soft singularities works
- ✓ Virtual corrections are stable
- ✓ Dipols are checked at individual phase space points + in singular regions → numerically stable
- ✓ Integration of real corrections works and is checked
- ➢ Not every thing is cross checked so far

Still missing: full cross check of virtual corrections,
so far only some pieces have been checked

First Results

cross section for $gg \to t\bar{t} + Jet + X, k_\perp >$ 20 GeV





Settings:

$m_t =$ 174 GeV,
$\alpha_s(\mu = m_t,$ 1-loop$) =$ 0.1177,
$\alpha_s(\mu = m_t,$ 2-loop$) =$ 0.1075,
CTEQ6

Note: missing contribution from factorization + relicts:

$$\int dx \int_m \left[\sigma_{\text{fact.}}(x) + \bar{\sigma}_{\text{sub}}(x)\right]$$

But we are ahead of time: “This calculation will never be done in my lifetime”, D. Rainwater

tt+Jet Production: Conclusions

The NLO corrections to pp → tt + jets are important :

- To improve the precision of the background for Higgs searches/measurements in WBF with H → WW
- For precision measurements in the Topquark sector, i.e. anomalous gtt-couplings, charge measurements (ttγ)

Calculation of gg → ttg @ NLO almost complete

5-point techniques work,
direct reduction of tensor integrals numerically stable,
real corrections tested, integration is stable,...

To do:
- Finish missing cross checks
- Remaining processes qq → ttg, qg → ttq ← easier!

Top quark physics at the LHC

Important observables:

- ✓ tt cross section

  Precise determination of top mass, consistency checks with theo. predictions, search for new physics in the tt invariant mass spectrum

- ✓ W-Polarization in top decay

  Test of the V-A structure in top decay

- ✓ ttH cross section

  Measurement of the Yukawa coupling

- ✓ Single top production

  Direct measurement of the CKM matrix element Vtb, top polarization, search for anomalous Wtb couplings

- ✓ Spin correlations

  Weak decay of a \`free' quark, bound on the top width and V_{tb}, search for anomalous couplings

in preparation

- ttγ cross section

  Measurement of the electric charge

- tt+Jet(s) production

  Search for anomalous couplings, important background

- ?

As far as top is concerned theory is not in a bad shape!

Conclusion

- NLO calculations for 5-point amplitudes including masses are still non-trivial
- Further progress on the theory side useful
- NLO calculations are time consuming

  → Experimentalists and theorists should think about what we really need
- Not everything needs to be done in NLO !


CERN