Perturbative QCD and the Energy Dependence of total cross-sections

Rohini, Godbole
CTS, IISc,
Bangalore, India.

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Les Houches TeV Collider Workshop.

- What gives the energy dependence of total cross-sections?
- A look at $pp, p\bar{p}, \gamma p, \gamma\gamma \to hadrons$
- The role played by e.m. form factors in descriptions of the total cross-section
- Towards a QCD Description of the decrease and the increase of total cross-sections through Soft Gluon Summation (Bloch-Nordsieck Model) and Mini-jets
- Predictions for hadronic backgrounds at future linear colliders.

With G. Pancheri and A. Corsetti, Eikonal Minijet Model for $pp, \gamma p$ and $\gamma\gamma$. PLB **435** (1998) 441, Eur.Phys.J.C19:129-136,2001

With A. de Roeck, A. Grau and G. Pancheri, Testing of models at future Linear colliders JHEP **0306**, 061 (2003) [arXiv:hep-ph/0305071]. 1/x in σ_{jet} drives the rise.

With A. Grau, G. Pancheri and Y. N. Srivastava Soft Gluon Resummation tames the rise. arXiv:hep-ph/0408355.

With A. Grau, G. Pancheri and Y. N. Srivastava Cross talk between HERA, LC and LHC arXiv:hep-ph/0412189.

Some associated work:

M. Drees and R.M. Godbole, Zeit. Phys. **C59** (1993) 591.
Hadronic backgrounds due to photon structure at Linear Colldiers

M. Block, E. Gregores, F. Halzen and G. Pancheri for the Aspen Model Phys.Rev.**D60** (1999) 054024
FACTORIZATION

A. Grau, G. Pancheri and Y. N. Srivastava for the Bloch-Nordsieck Model PR **D60** (1999) 114020
$\alpha_s(k_t \to 0)$ tames the rise

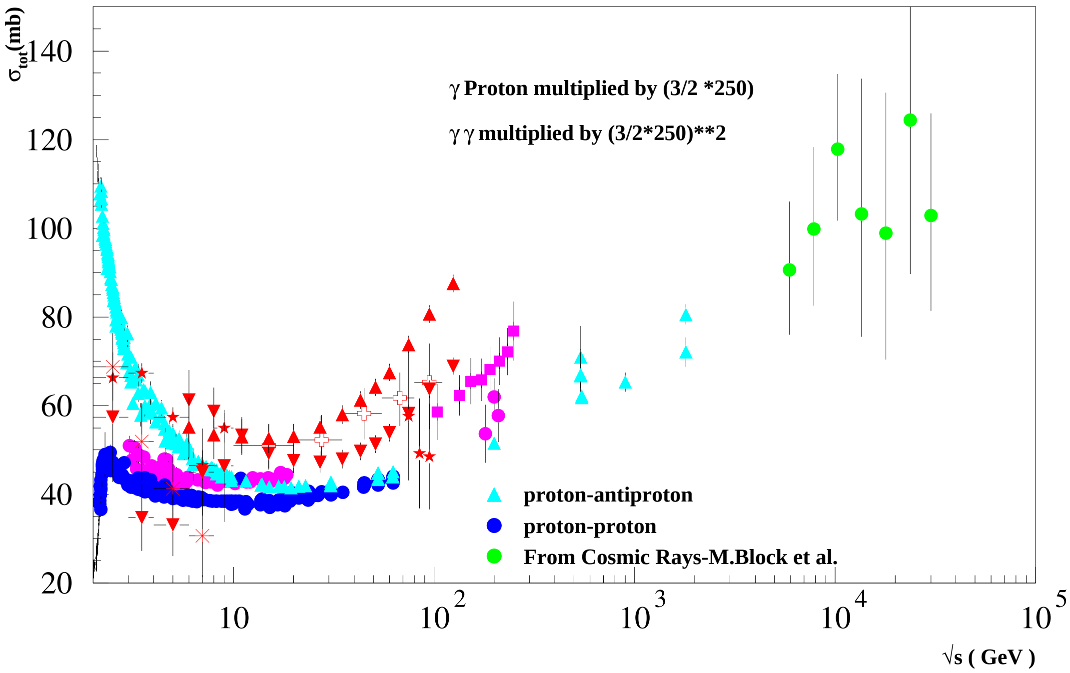
M. Block and K. Kang, hep-ph/0302146, Factorisation and Unitarity

- The energy dependence shown by the data on total cross-sections for proton and photon induced processes.
- Predictions for total cross-sections within unified models, embedding QCD processes, using information on proton and photon structure functions as well as those from the model independent extrapolations to higher energies.
- Taming of the high energy rise with the soft gluon resummation in the eikonalised minijet model (EMM).
- Connections between the energy dependence and the behavior of the strong coupling constant in the infrared regime.
- Possibilities for distinguishing between different models, all of which try to describe the energy dependence of the total cross-section, at the future $e^+e^-/\gamma\gamma$ colliders and implications of this energy dependence for cosmic ray energies.

Scale Factor: From VMD and Quark counting:

$$\sigma_{\gamma p} = \frac{2}{3}\mathcal{P}_{VMD}\sigma_{pp};\ \sigma_{\gamma\gamma} = \frac{2}{3}\mathcal{P}_{VMD}\sigma_{\gamma p}$$

where $\mathcal{P}_{VMD} = \Sigma \frac{4\pi\alpha}{f_V^2} \simeq \frac{1}{250}$



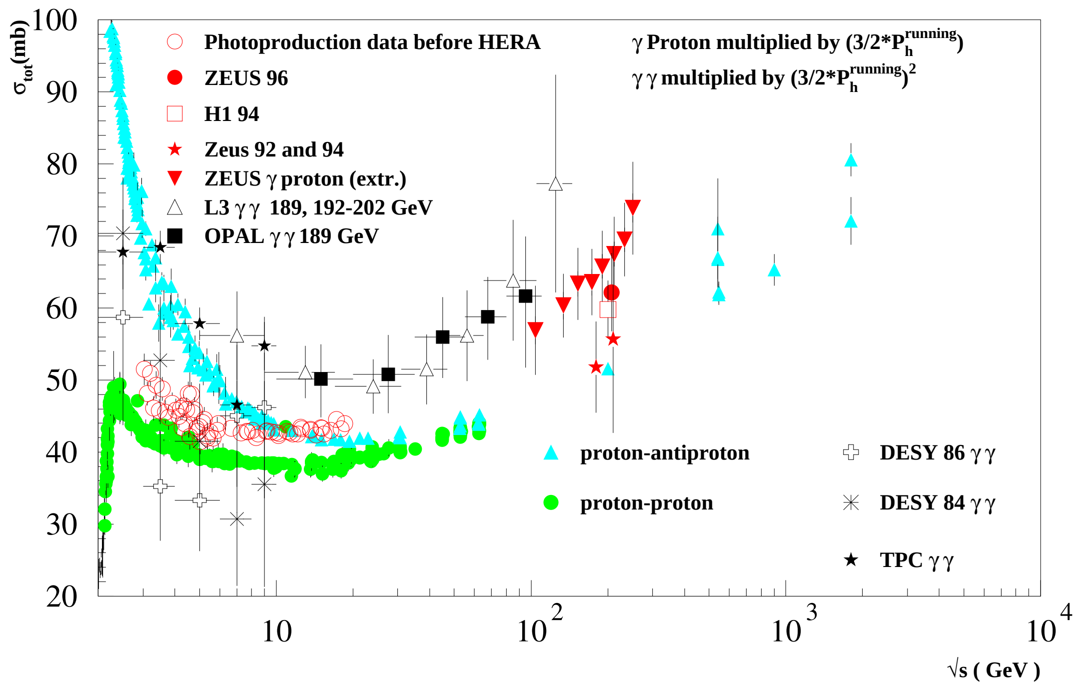
- σ_{tot} for processes involving photons seem to rise faster with energy.

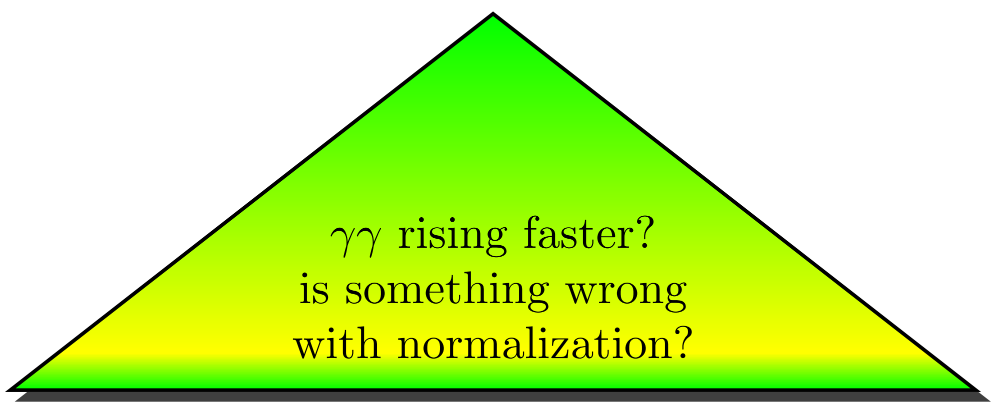
Words of caution:

The knowledge of the $\gamma p/\gamma\gamma$ cross-sections obtained from ep/e^+e^- recations involving unfolding.

Can we understand these data with QCD?

Using VMD with running α_{QED}





M. Block (Talk at photon 2003) Good simultaneous fits to proton and photon induced cross-sections in a model with factorisation ONLY if $\gamma\gamma$ data are renormalised by 10 %.

Look to the talk by A. de Roeck here.

The task of describing the energy behaviour of total cross-sections can be broken down into three parts:

- the rise
- the initial decrease
- the normalization

The tools:

- Bounds from Analyticity and Unitarity.
- Regge Pomeron exchange.
- The Eikonal Approximation.
- The Eikonal Minijet Model: EMM.
- Bloch-Nordsieck Resummation for the EMM.
- Want an unified description for $pp, \bar{p}p, \gamma p$ and $\gamma\gamma$.

Factorisation based approach: e.g. Block et al

Use only Unitarity, analyticity, crossing symmetry. Treat γ like a proton. Fit functions for protons and make predictions for photons. But the problem of obtaining the functions for protons from first principle remains. The $\gamma\gamma$ data need to be renormalised by 10 %.

QCD Based approach:

Use perturbative QCD as well as measured str. fns. of p and γ. I.e. in terms of quarks and gluons in p and γ .

Starting point: Optical Theorem

$$\sigma_{tot} = \frac{4\pi}{k}\Im(f_{el}(\theta = 0))$$

All the measured cross-sections increase, starting around 10–20 GeV.

Is the increase Unbounded?

Answer known for a long time: NO!.

Froissart Bound

$$\sigma_{tot}(s) < constant \times (lnS)^2$$

Based on:

- Optical Theorem
- Rather weak assumption on the scattering amplitude A(s,t) from a field theory with finite range interactions.

Unitarity and Analytcity $\Rightarrow$ predictions from the Regge - Pomeron approach.

Crossing gives A (s,t) $\Rightarrow f(t)S^{Re\alpha(t)}$ as $s \to \infty$.

This mean that

$\sigma_{tot} \sim s^{\alpha(0)-1}$.

- $\alpha_\rho(o) \simeq \frac{1}{2}$

Gives the decrease with energy initially.

Pomeron trajectory dominates asymptotically.

- $\alpha_{\P}(o) \simeq 1$.

Thus will give constant cross-section at High Energies.

Regge-Pomeron Exchange (Donnachie and Landshoff)

$$\sigma_{tot}(s) = X s^{\epsilon} + Y s^{-\eta}$$

$\eta = 1 - \alpha_R(0) \simeq 0.5$ and $\epsilon = \alpha_{¶} - 1 \simeq$ small.

Factorisation tells:

for $a + b \to a + b$; X, Y are given by

$$X = \beta_{Paa}\beta_{Pbb}, Y = \beta_{Raa}\beta_{Rbb}$$

- Very successful and useful phenomenological parametrisation.

But

- Violates the Froissart Bound asymptotically.
- η and ϵ not Universal (Post 2000)

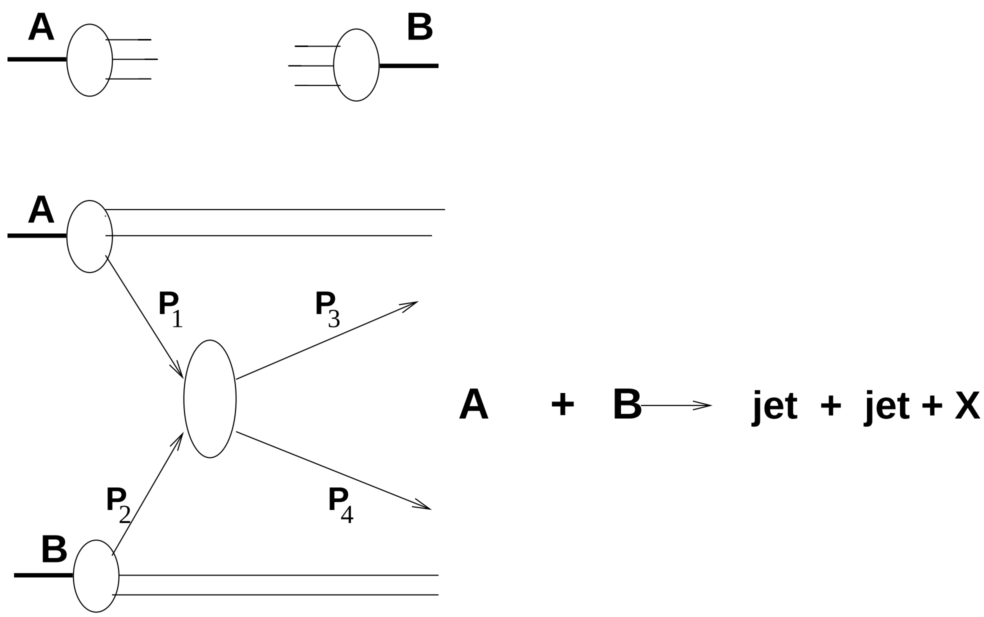
$\epsilon_{pp} = 0.08$;

$\epsilon_{\gamma\gamma} = 0.15 - 0.22$ (Talk by A. de Roeck)

- Where is QCD?

Basic philosophy:

Try to explain the rise and the initial fall in terms of partons in the colliding hadrons using experimentally determined parton densities and basic QCD interactions among partons.



Increasing beam energy ⇒ increase in # and energy of collding partons.

$\sigma_{jet} = \sigma(A + B \rightarrow jet + jet + X)$

calculated in pQCD rises with increasing $\sqrt{s}$.

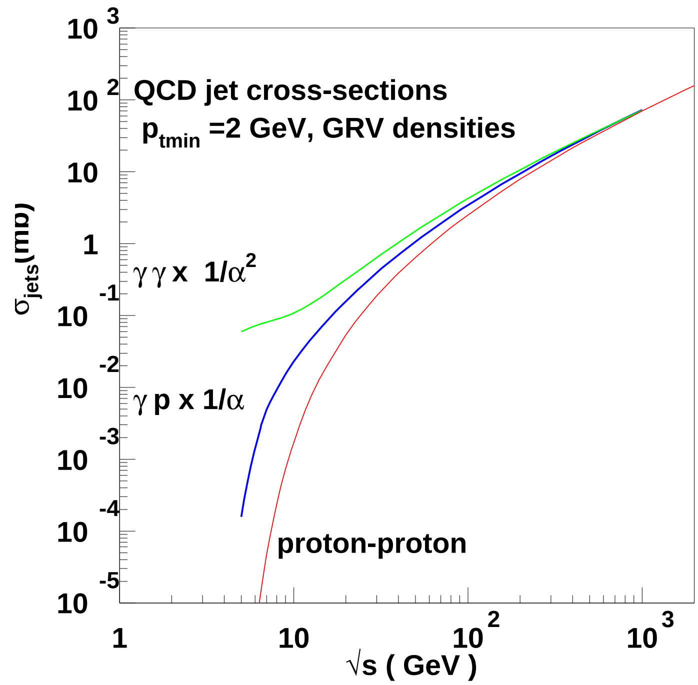
Energy rise in σ_{tot} driven by the rise of σ_{jet}.

Minijet Model Halzen and Cline (1985)

Integrated jet cross-sections

$$\sigma_{jet} = \int_{p_t min} \frac{d^2\sigma_{jet}}{d^2\vec{p}_t} d^2\vec{p}_t =$$

$$= \sum_{partons} \int_{p_t min} d^2\vec{p}_t \int f(x_1)dx_1 \int f(x_2)dx_2 \frac{d^2\sigma^{partons}}{d^2\vec{p}_t}$$



Minijet cross-sections dominated by gluons and similar for $pp, \gamma p$ and $\gamma\gamma$ at high energies when appropriately scaled by $1/\alpha_{em}$

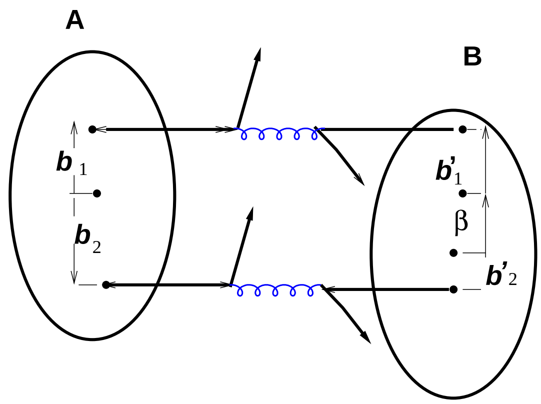
σ_{jet} depend on the densities and very dramatically on p_{tmin} the transverse momentum cut-off

• σ_{jet} rises with s as a power in violation Frossiart Bound too fast towards σ_{tot}.

• Unitarization essential. Done using eikonal formalism

• The steep rise of σ_{jet} with s is NOT reflected in the energy rise of $\sigma_{tot}, \sigma_{inel}$.

With increasing energy the probability of multiple parton scattering (MPS) in a given hard scatter increases



Transverse Overlap of the hadrons

$\sigma_{AB}^{jet}(s) =< n_{pair}^{jet} > (s)\sigma_{AB}^{inel}(s)$

Rising MPS $\Rightarrow$ rising jet pair multiplicity

Need to calculate the s dependence of $< n_{pair}^{jet} >$.

Perhaps need to go beyond pQCD.

s dependence related to that of the MPS probability.

This in turn decided by the overlap of the partons in the transverse plane.

$A_{AB}(\beta) = \int d^2 b_1 \rho_A(\vec{b_1})\rho_B(\vec{\beta} - \vec{b_1})$

Governing quantity # of collisions:

$$n(b, s) = A_{AB}(b, s)\sigma(s) = 2\chi_I(b, s)$$

$\chi(b, s)$ :: EIKONAL function.

Calculate then $\sigma^{\rm inel}$ for **for example** $A, B = p, \bar{p}$,

$$\sigma^{\rm inel}_{pp(\bar{p})} = 2\int d^2\vec{b}[1 - e^{-n(b,s)}]$$

Build n(b,s) for σ^{inel} and use it for

$$\sigma^{\rm tot}_{pp(\bar{p})} = 2\int d^2\vec{b}[1 - e^{-n(b,s)/2} cos(\chi_R)],\ \chi_R = 0 \text{ in EMM}$$

b is impact parameter $\Longrightarrow$ transverse momentum of partons in hadrons

Approximations

- separate Pert. Vs Nonpert. terms
  $\rightarrow n(b,s) = n_{NP}(b,s) + n_P(b,s)$
- Further factorize b vs. s behaviour
  $\rightarrow n(b,s) \approx A(b)\sigma(s)$

simplest model $n(b,s) = A(b)[\sigma_{soft} + \sigma_{jet}]$

$\Uparrow$

matter distribution

- Model for A(b).
- σ_{soft} parametrized
- σ_{jet} LO QCD jet x-sections
- Eikonal model not restricted to calculate ONLY c.sections also used to calculate properties of hadronic events. pioneering: T. Sjostrand , More recent : M. Seymore + Borozan JHEP (2002).

At low energies and small σ^{jet}

$\sigma_{AB}^{\text{inel}} = 2\int d^2\vec{b}[1 - e^{-n(b,s)}] \simeq \sigma_{AB}^{soft} + \sigma_{AB}^{jet}$

At high energies, the eikonalisation softens the energy rise of σ^{inel} compared to that of σ^{jet}.

- Eikonal $\chi(b, s)$ contains information on the energy and the transverse space distribution of the partons in the hadrons.
- σ^{jet} depends on the parton densities $f_{q/A}(x_1), f_{q/B}(x_2)$ x_i the longitudinal mmtm fraction
- Overlap function on the transverse space (momentum) distribution.

Thus simplest formulation with minijets to drive the rise and eikonalization to ensure unitarity :

$2\chi_I(b, s) \equiv n(b, s) = A(b)[\sigma_{soft} + \sigma_{jet}]$

The normalization depends both on σ_{soft} and on the b-distribution.

How to calculate the transverse overlap function in terms of 'measured' quantities?

The simplest hypothesis, is

$$A_{ab}(b) \equiv A(b; k_a, k_b) = \frac{1}{(2\pi)^2} \int d^2\vec{q} e^{iq\cdot b} \mathcal{F}_a(q, k_a) \mathcal{F}_b(q, k_b) \tag{1}$$

$\mathcal{F}_i(q, k_i)$ are the e.m. form factors

- How to generalise this for photons?
  - γ has to 'hadronise'. Treatment of MPS for photons has to be different (Collins and Ladinsky.
  - One choice for $\mathcal{F}$ is to use form factor of π.
  - Corsetti, Pancheri and RG: Use for $\mathcal{F}$ Fourier Transform of the transverse momentum distribution of partonic photons measured by ZEUS. Functional form similar to using $\mathcal{F}_\pi$ but with a different value of the parameter.

To calculate σ^{tot} for photon-induced processes,

$\sigma^{\rm tot}_{\gamma p)} = \mathcal{P}_{had} 2 \int d^2\vec{b}[1 - e^{-n(b,s)/2}$

where

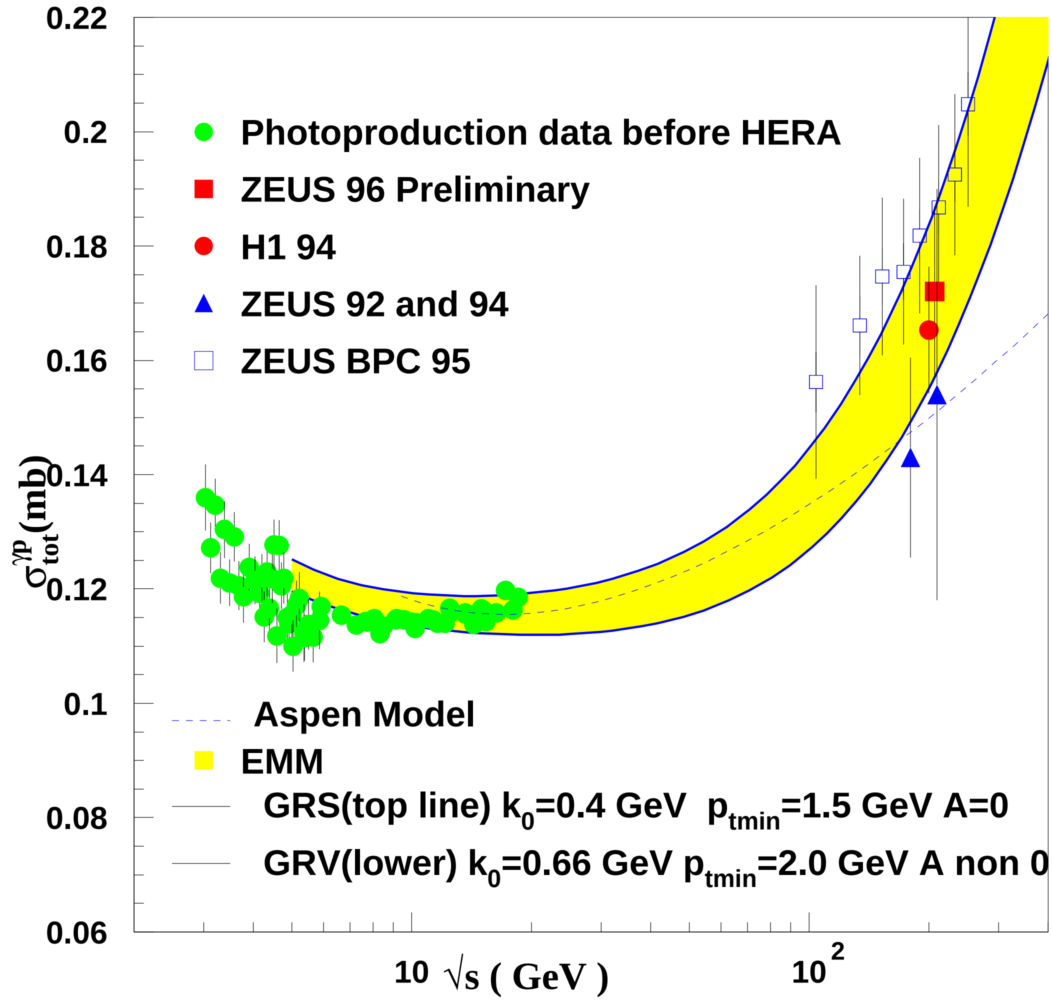
$n(b, s) = A(b)[\sigma^{soft} + \frac{1}{\mathcal{P}_{had}} \sigma^{jet}(s, p_{Tmin}]$

with $\mathcal{P}_{had} = \mathcal{P}_{VMD}$.

For $\gamma\gamma$:

- $\sigma^{\rm tot}_{\gamma\gamma} = 2 P^{\gamma\gamma}_{had} \int d^2\vec{b}[1 - e^{-n(b,s)/2}]$
- $n(b, s) = 2/3 n^{\gamma p}_{soft} + A(b)_{FF} \sigma^{\gamma\gamma}_{jet}(s)/P^{\gamma\gamma}_{had}$
- $P^{\gamma\gamma}_{had} = [P_{had}]^2$

Photo-production and extrapolated datas from DIS can be described through the Eikonal Minijet Model with Form Factors and QCD densities : low energy scaled from proton proceses.

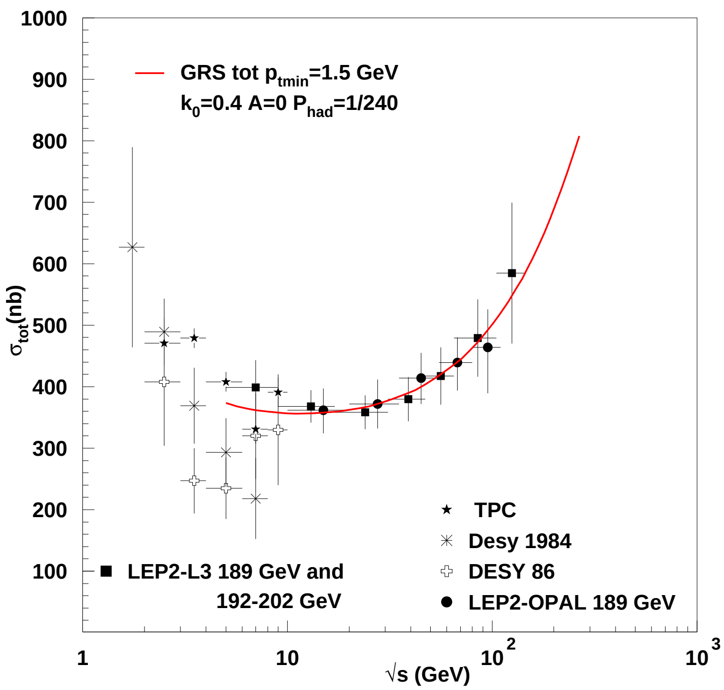


The band is corresponds to $k_0 = 0.66 \pm 0.22$ GeV (ZEUS measurement)

Then

- Using **EMM**, with VMD and Quark Counting at low energy, and same set of parameters which fit γp
- adjusting the overall normalization 10% upwards,
- $k_0 = 0.4$ corresponds to the upper edge in the γp band.

one gets a very good fit to the present data



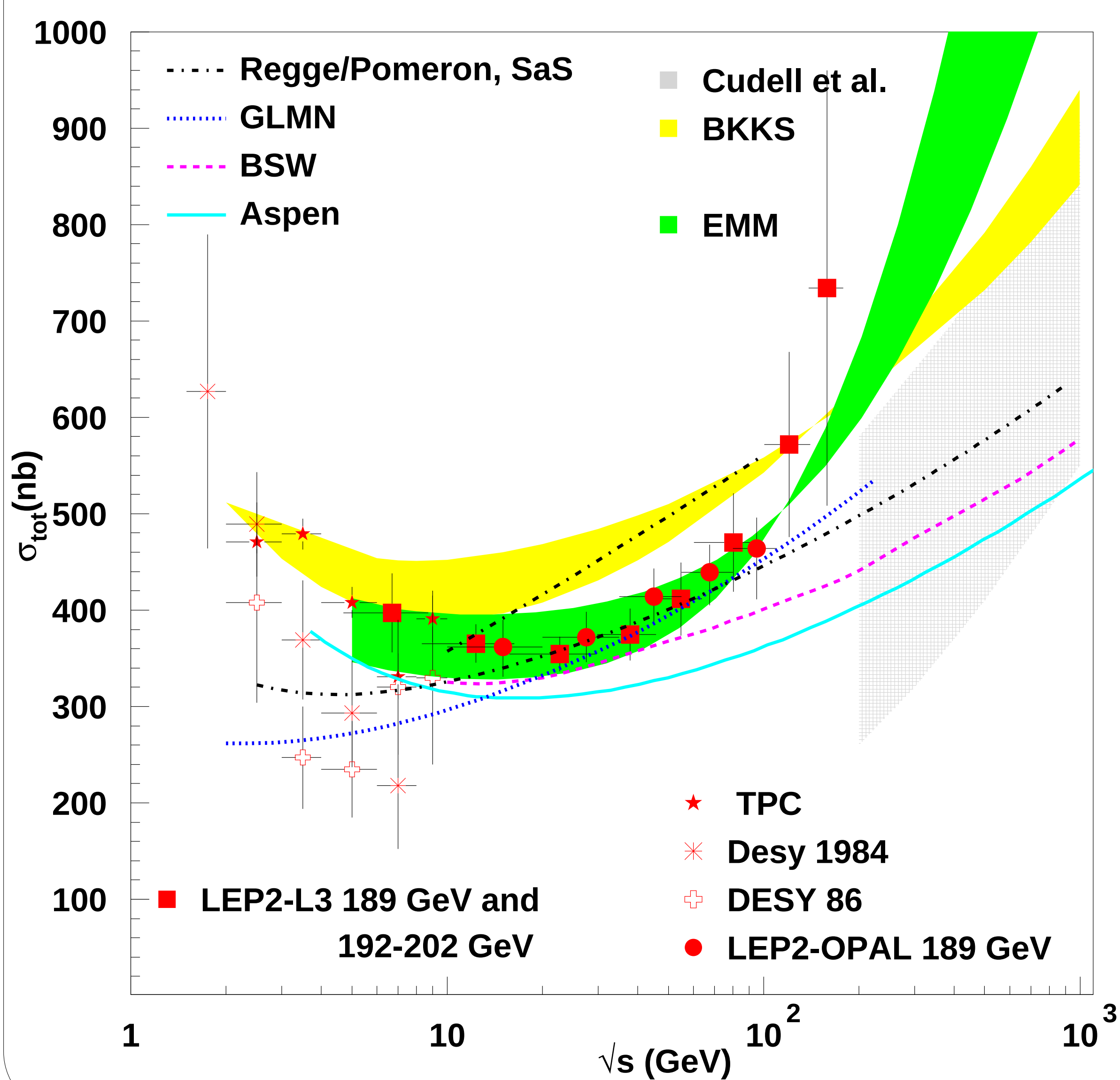
⇓

Data for $\gamma\gamma$ total x-sections show a fast rise which can be reproduced with EMM

Use of 'measured' properties of the γ, p and factorisaation, simple quark counting rule to connect γp parameters to $\gamma\gamma$ case.

Normalization here is 10% off what you get from γp

Already at $\sqrt{s} = 500\ GeV$
predictions differ by a factor 3



❖ for $p\ p$ and $p\bar{p}$

M. Block, R. Fletcher, F. Halzen, B. Margolis PRD41(1990)978

$$\sigma_{tot} = 2\int^{2} \vec{b}[1 - e^{-\chi_I} cos\chi_R]$$

with

$$\chi/2 = P_{gg} + P_{gq} + P_{qq}$$

and

- $P_{ij} = W_{ij}(b, \mu_{ij})\sigma_{ij}(s)$
- $W(b, \mu) = \int \frac{d^2\vec{q}}{(2\pi)^2} e^{ib\cdot q}[\mathcal{F}(q)]^2$
- $\mathcal{F}(q) = (\frac{\mu^2}{\mu^2+q^2})^2$ Dipole Form Factor

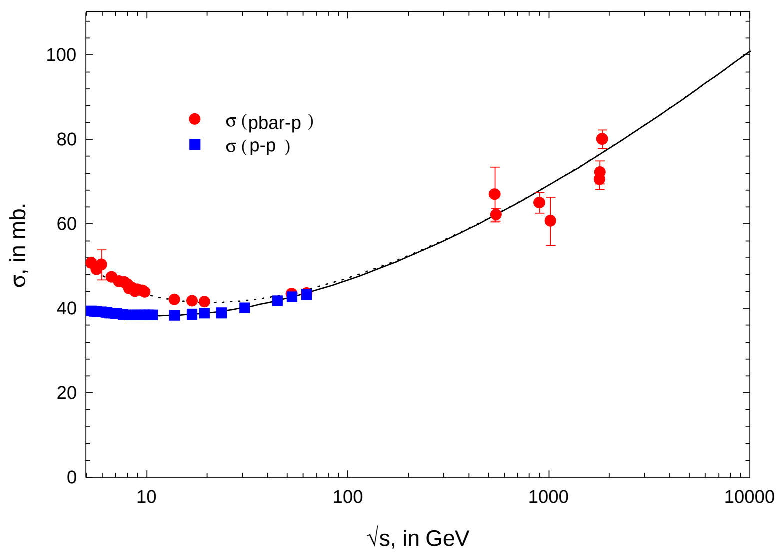
$\mu_{qq},\ \mu_{gq}\ \mu_{gg}$ for each P

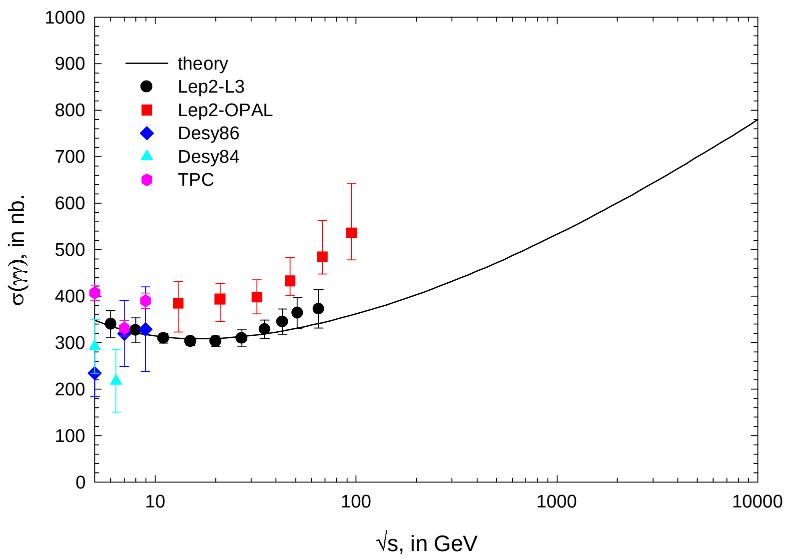
- $\sigma_{ij} = QCD$ *inspired* and parametrized using pp and $p\bar{p}$ data on elastic and total cros-sections.

❖ for γp and $\gamma\gamma$

M. Block, E. Gregores and F. Halzen, Phys.Rev.**D60** (1999) 054024, also M. Block,Kang.

use factorization and VMD to get the P_{ij} N.B. $f^{i/\gamma}(Q^2, x)$ not used!

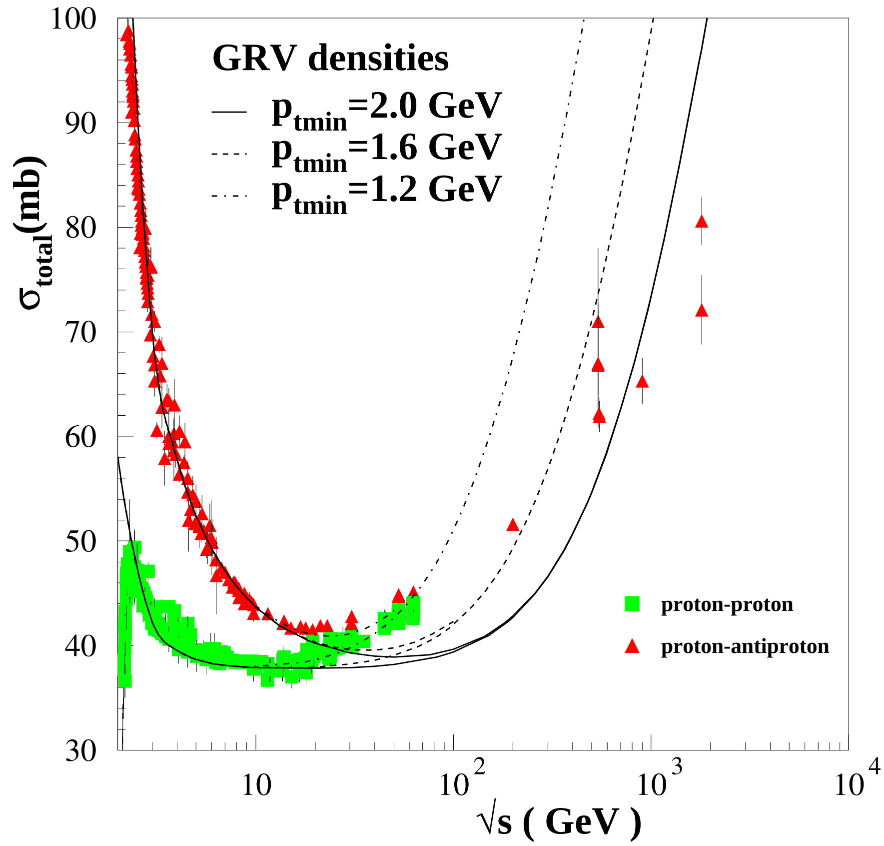




Fit the pp and $p\bar{p}$ data and calculate $\gamma\gamma$ (for example) using the Eional obtained using factorisation and quark counting.

Form Factors + minijets in $pp(\bar{p})$

It is possible to describe the early rise, which takes place around $10 \div 30$ GeV for proton-proton and proton-antiproton scattering, using GRV densities and a $p_{tmin} \simeq 1$ GeV, but then the cross-sections start rising too rapidly, whereas a $p_{tmin} \approx 2$ GeV can reproduce the Tevatron points but it misses the early rise.



- The rise for $pp/\bar{p}p$ is too rapid for $p_{Tmin} \simeq 1$ GeV and miss early rise if $p_{Tmin} \simeq 2$ GeV.
- The best fit to the $\gamma\gamma$ data require 10 % upward normalisation relative to γp data .
- No explanation for the initial decrease.

EMM model does O.K. qualitatively but is certainly not the whole story.

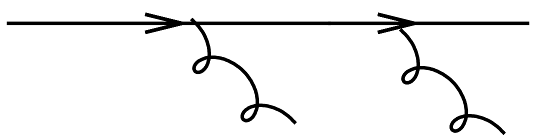
Improve the model by removing the approximations used.

Recall **assumed** $n(b,s) = A(b)[\sigma_{soft} + \sigma_{jet}]$.

• The separation between s and b dependence only an approximation.

• Writing the overlap function as a $\mathcal{F}.\mathcal{T}.$ of measured distributions does not allow for a s dependence of A

Pancheri and Collab. developed a model based on semi-classical method to calculate the impact parameter space distribution of partons in a hadron using resummation of soft gluon emissions.



$A(b,s) = A(b, M(s))$.

Here $M =< q_{max}(s) >$ is the average of the 'maximum' energy allowed for single soft gluon emission.

EMM needs further refinements, including full LLO resummation to tame the rise

$$n(b,s) = n_{soft}(b,s) + A_{PQCD}(b,s)\sigma_{jet}^{LO}$$

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Soft gluons can tame the rise

$A(b) \Longrightarrow$

$A(b,s) \simeq \int d^2\vec{K}_t e^{i\vec{K}_t\cdot\vec{b}}\ \Pi(K_t\ from\ initial\ partons\ )$

Bloch Nordsieck Resummation

$$A_{PQCD}(b,s) \equiv \frac{e^{-h(b,s)}}{\int d^2\vec{b}\ e^{-h(b,s)}}$$

- $h(b,s) = \int_{k_{min}}^{k_{max}} d^3\bar{n}_{gluons}(k)\ [1 - e^{ik_t\cdot b}]$
- $k_{max} \Longrightarrow$ *average over densities* $\Uparrow$ *as* $\sqrt{s}$ $\Uparrow$
- $k_{min} = 0$ in principle but one needs a model for

$\alpha_s(k_t)$ as $k_t \to 0$

The Bloch Nordsieck model

- is like EMM model with σ_{jet}^{QCD} driving the rise

  and in addition

  Soft Gluon Emission from Initial State Valence Quarks in k_t-space to give impact parameter space distribution of colliding partons

- introduces energy dependence in the **b-distribution** of partons in the hadrons $\Longrightarrow$ which depends on

  1. p_{tmin}
  2. parton densities

Two main results :

1. softening effect
2. dependence of hard scattering parameters is reduced

The softening effect happens

- as $\sqrt{s} \Uparrow$ the phase space available for soft gluon emission also $\Uparrow$
- the transverse momentum of the initial colliding pair due to soft gluon emission $\Uparrow$
- more straggling of initial partons $\Rightarrow$ less probability for the collision

The energy dependence which ultimately will soften the rise due to mini-jets comes from the

maximum transverse momentum allowed to a single gluon.

$$q_{max}(\hat{s}) = \frac{\sqrt{\hat{s}}}{2}(1 - \frac{\hat{s}_{jet}}{\hat{s}})$$

with integration to be done over

- $\hat{s}$ the energy of the initial parton-parton subprocess
- the jet-jet invariant mass $\sqrt{\hat{s}_{jet}}$,

Averaging over densities

$$< q_{max}(s) >=$$

$$= \frac{\sqrt{s}}{2} \frac{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \sqrt{x_1 x_2} \int dz(1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \int (dz)}$$

with the lower limit of integration in the variable z given by $z_{min} = 4p_{tmin}^2/(sx_1x_2)$.

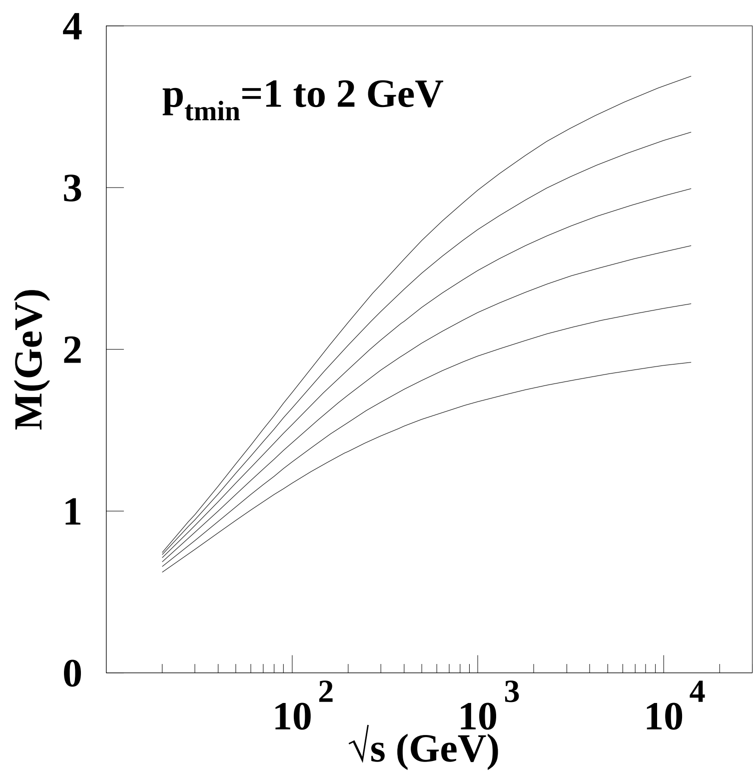
Maximum energy
for single soft gluon emission

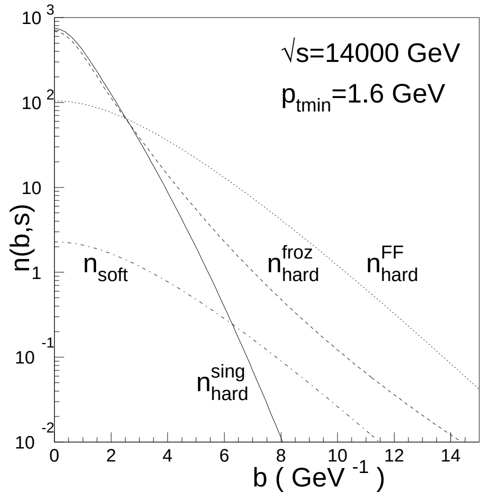
$$M \equiv < q_{max}(s) >=$$

$$\frac{\sqrt{s}}{2} \frac{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \sqrt{x_1 x_2} \int dz(1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \int (dz)}$$

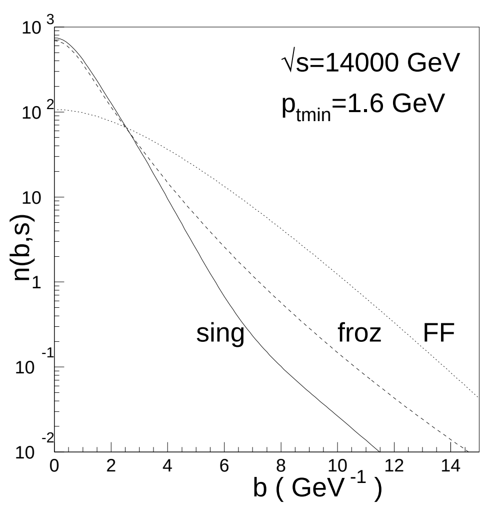
with $z_{min} = 4p_{tmin}^2/(sx_1x_2)$, one can plot the quantity M as a function of s for different values of p_{tmin}.

With GRV94 densities





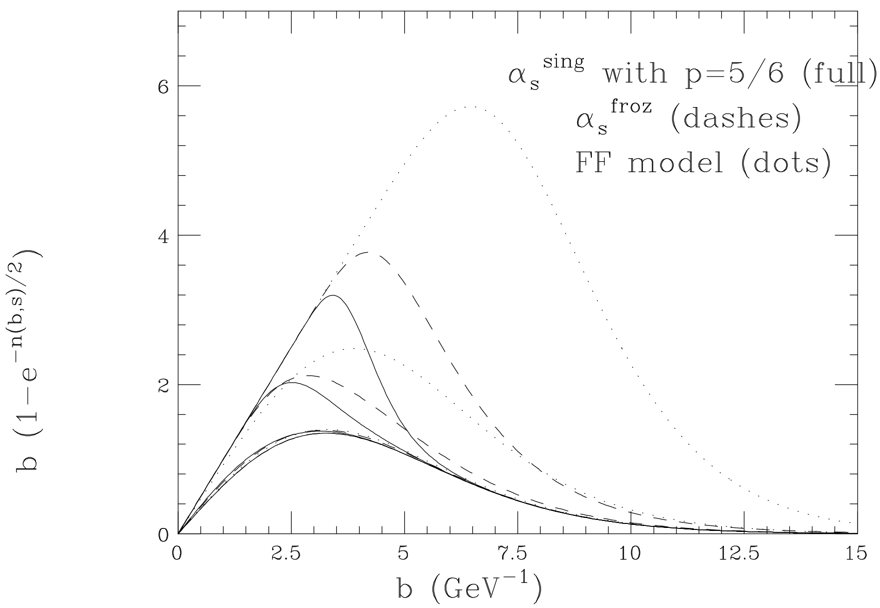
Soft and hard component of n(b,s) in the three models



The average number of collisions in the form factor model and the Bloch Nordsieck model, at LHC energy

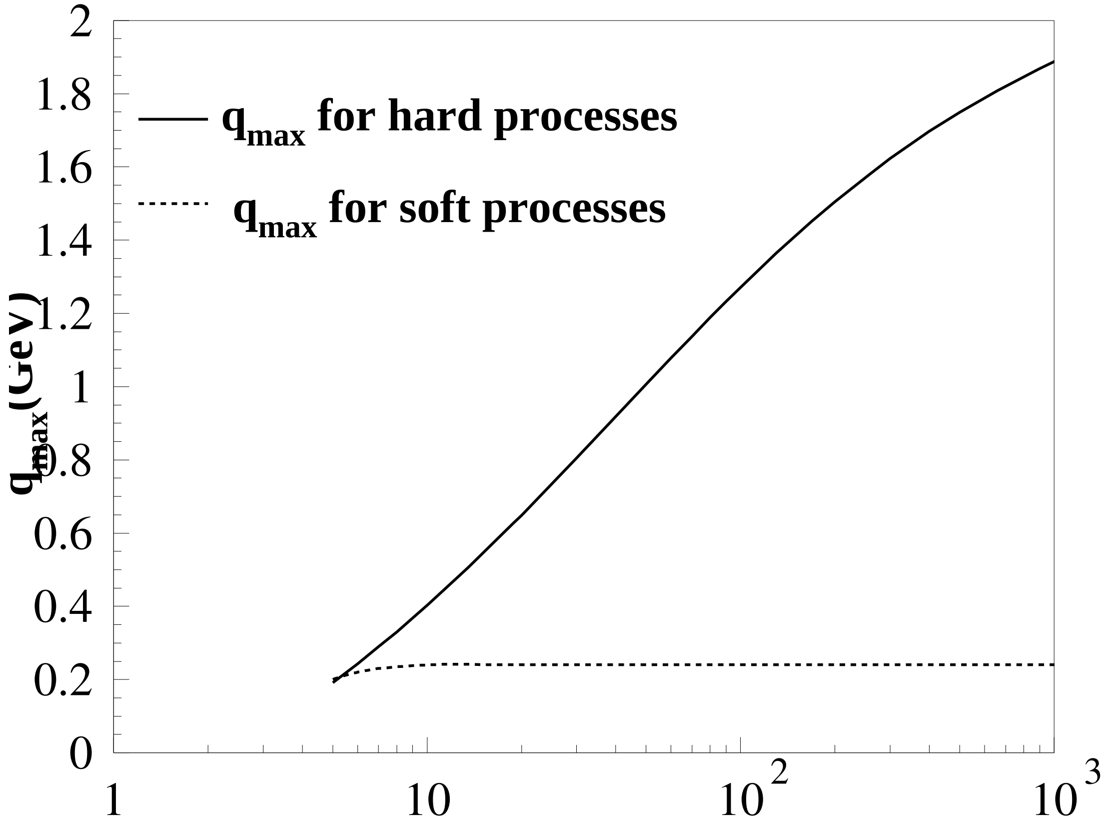
The Effect of the Soft Gluon Summation model can be observed in the

Integrand of the eikonal formulation for σ_{tot} in the three different models



The integrand is peaked at different b-values as the energy increases, but also as the model for $A(b)$ changes.

The rise with energy of the area under the curve, i.e. the cross-section, at the same energy shrinks for the more singular α_s behaviour.



Same functional form for A_{BN} as obtained from pQCD, except different q_{max}.

Bloch-Nordsieck Model for $p - p$ and $\bar{p} - p$

Now make fits to the pp and $p\bar{p}$ in the Bloch-Nordsieck (BN) model , the eikonal is of the form

$$n(b,s) = \sigma_{soft} A_{BN}^{soft} + \sigma_{jet} A_{BN}^{jet}$$

Soft gluon emission has here a twofold effect as the energy increases :

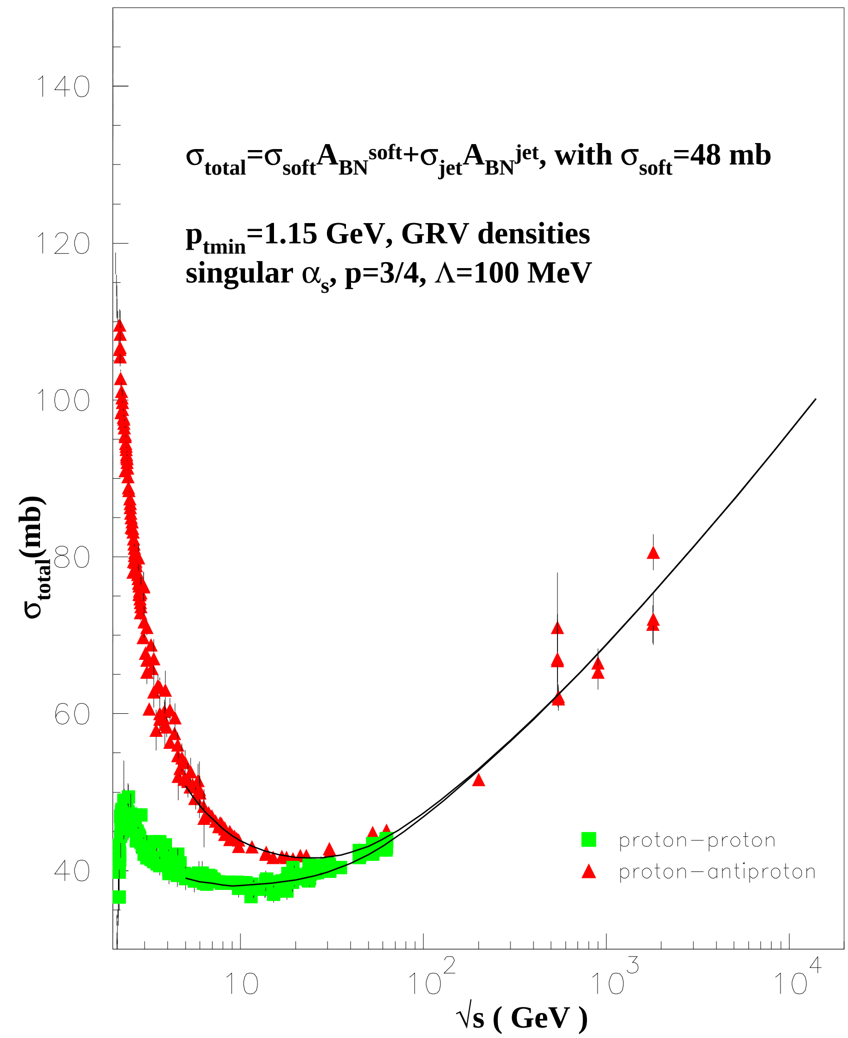
- with σ_{soft} constant or $\Downarrow$ $\quad \sigma_{soft} A_{BN}^{soft} \Downarrow$
- with $\sigma_{jet} \Uparrow$ $\quad \sigma_{jet} A_{BN}^{jet} \Uparrow$ but not as much as without soft gluons

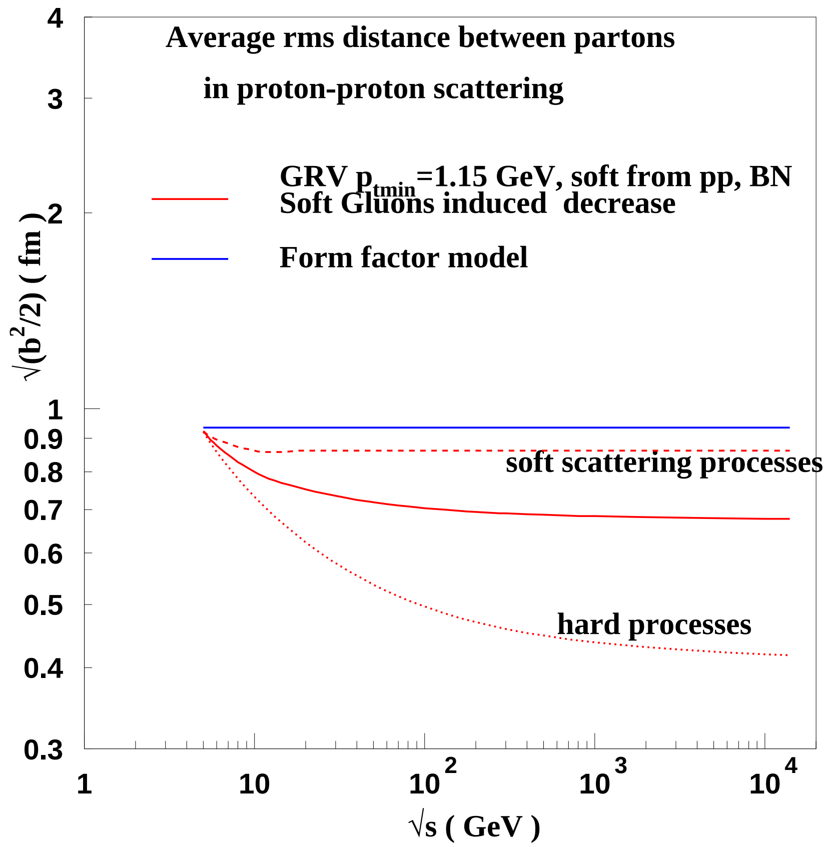
A good description is obtained with a soft part given by

$$\sigma^{pp}_{soft} = \sigma_0 A^{soft}_{BN}(b,s) \qquad \sigma_0 = 48mb$$

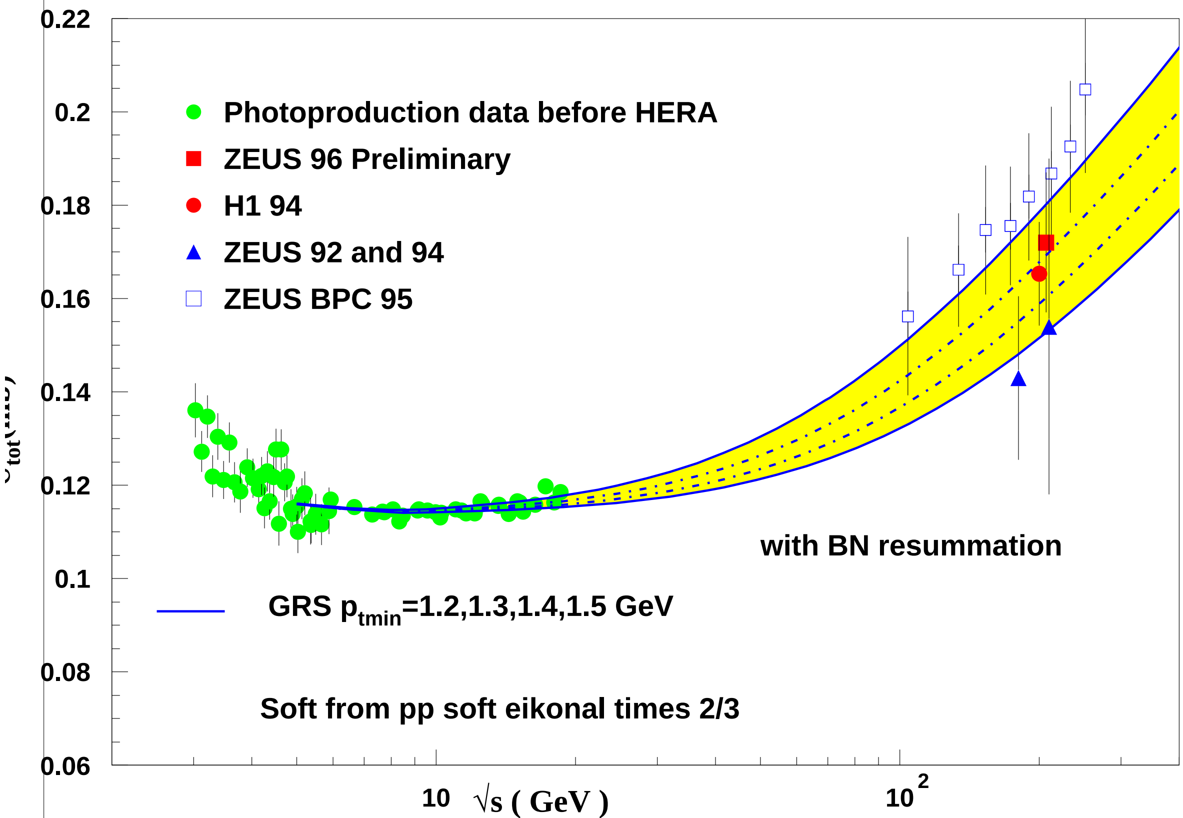
and

$$\sigma^{p\bar{p}}_{soft} = \sigma_0 (1 + \frac{2}{\sqrt{s}}) A^{soft}_{BN}(b,s)$$



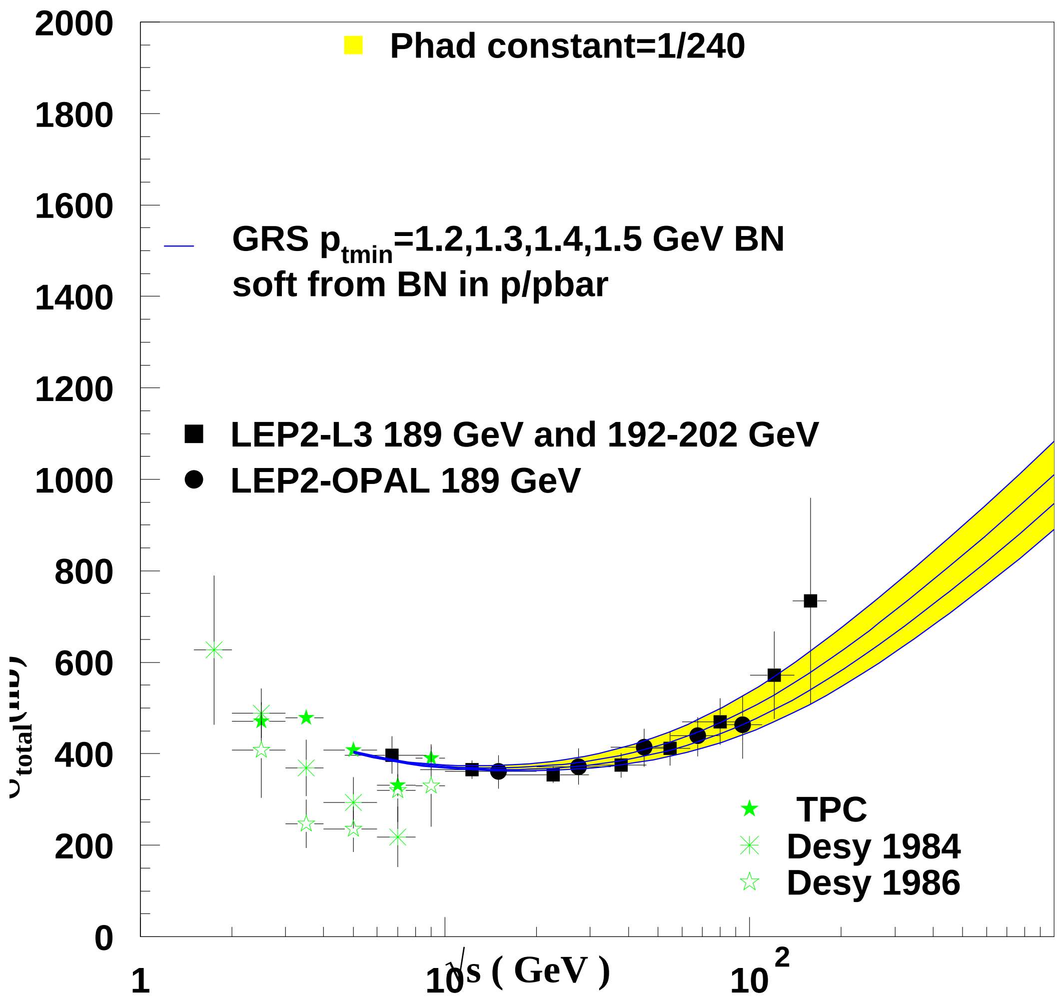


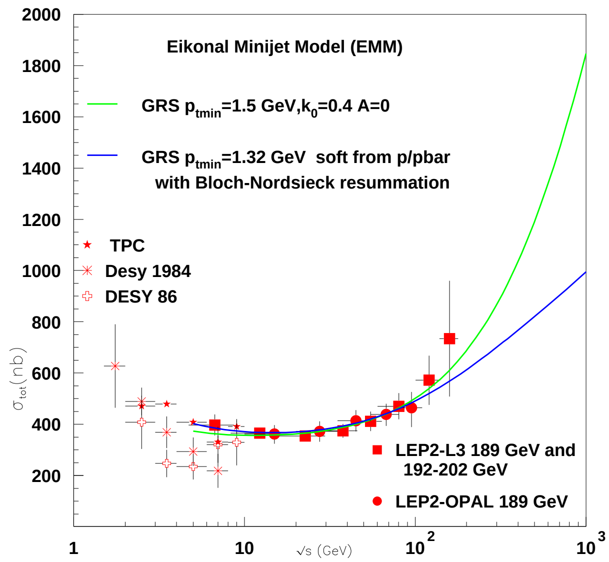
- Indeed the rms distance between the centres of two hadrons decreases with energy causing more shadowing and taming the rise
- Similar observation by M. Seymore and collab. from a study of properties of the events in $p\bar{p}$ data from CDF in an eikonal picture.



for reasonable range of p_{Tmin} decent description of γp data is possible.

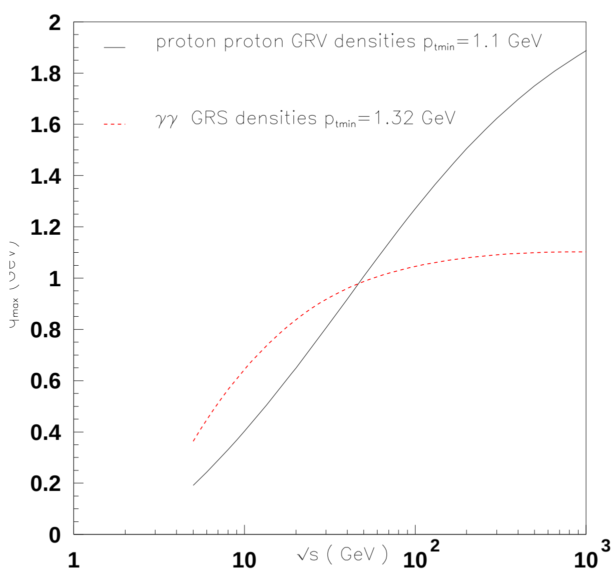
The case for $\gamma\gamma$: GRS





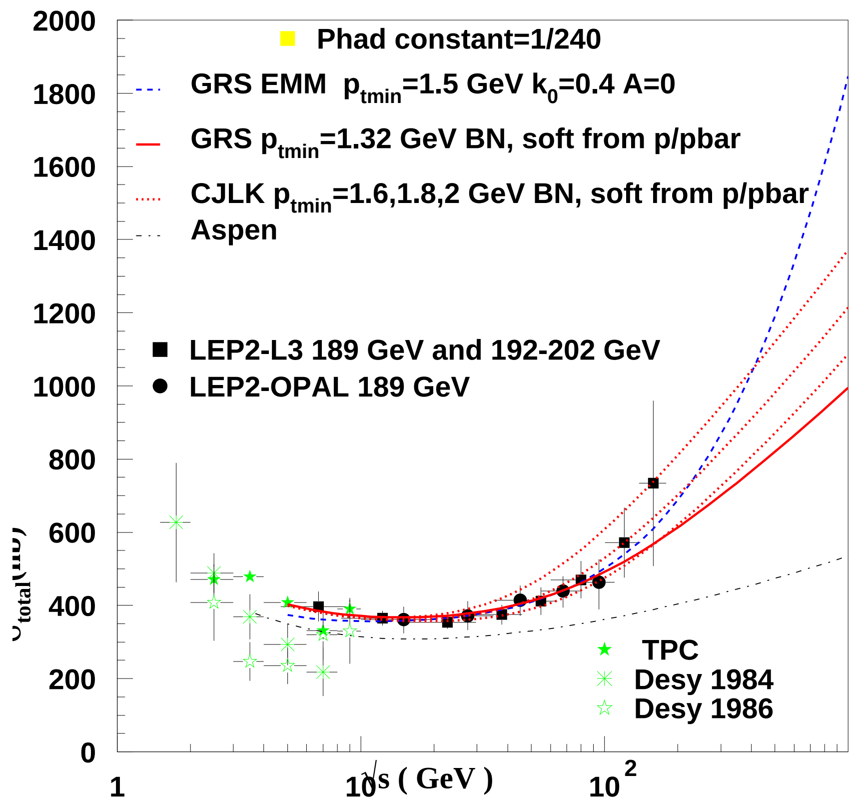
Recall M depended on the parton densities in the hadron.

BN effect stronger for protons.



$\gamma\gamma$ Aspen, EMM and BN

1. Soft part of the eikonal $n(b,s)$ directly from proton and antiproton processes $\rightarrow n^{\gamma\gamma}_{soft}(b,s)$ given by $\frac{4}{9}\frac{n^{pp}_{soft}+n^{p\bar{p}}_{soft}}{2}$ using fit to protons,
2. soft resummation for hard scattering,



- Large differences between EMM in the FF formulation and BN resummed form.
- A^{BN}_{soft} and A^{BN}_{hard} give the early fall and the taming of the fast rise.
- The normalisation and rise seem to be fixed simultaneously.

- σ_{tot} for photon induced processes 'seems' to rise faster than for the pp case. Clarification for $\gamma\gamma$ case and newer measurement for the γp from HERA will be much appreciated.
- QCD based models, using exeprimentally measured quark and gluon densities do predict such a faster rise.
- Plain EMM does need improvement to take into account the energy dependence of the transverse size of hadrons.
- The soft gluon resummation does seem to predict such a reduction with increasing energy which tames the high energy rise. The model produces the initial decrease too.
- For Aspen model, simultaneous fits to all the data, assuming factorisation seems to require 'renormalisation' of the $\gamma\gamma$ data. In BN model there seems to be loss of factorisation in going from pp to γp.
- The transverse overlap function derived in BN model can be confronted with data by using it to make predictions for the hadronic properties of the events in the eikonal model.