

The performance prediction from a single strand to large size conductors

Why predicting rather than testing

Single strand performance issues

Basic of the n index

Scaling law of strand performance

Large size conductor issues

Transverse load on Nb_3Sn

Self field effect

Where we are, what we wish



The issue of prediction - Cultural aspect

The ability to predict is a measure of the achieved understanding

In Physics, laws and predictions are fundamental. In Technology, the functionality prevails on understanding.

In quick developing technologies, the pressure for progress is large

Many prototypes and small series are built in parallel, “natural” selection by failure is the broadly accepted way to progress

Stagnating technologies drift into pure science (mis)-behavior

“Theories” are elaborated to support “design”, arguing among individual and groups crystallizes. Academic recognition rather than reliable function becomes the measure of success



The issue of prediction - Practical aspect

Superconducting strands have a market and are readily available

Large size conductors are expensive. The lack of a strong market and the large cost of prototypes justifies the effort for “good” prediction, rather than straight testing

The operating conditions of large conductors are not easy to reproduce

Even when a short length prototype exists, it is not obvious to test it at the field, current, temperature, cooling and strain relevant for a magnet. A scaling tool is essential

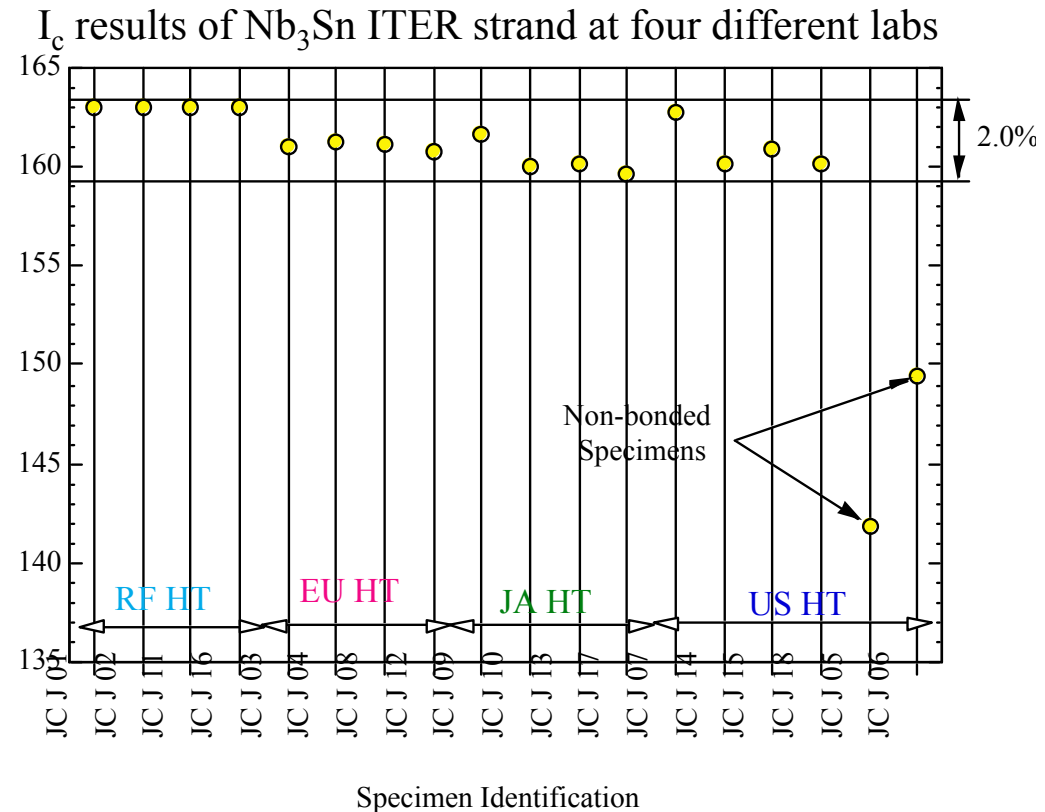


The starting point: strand performance

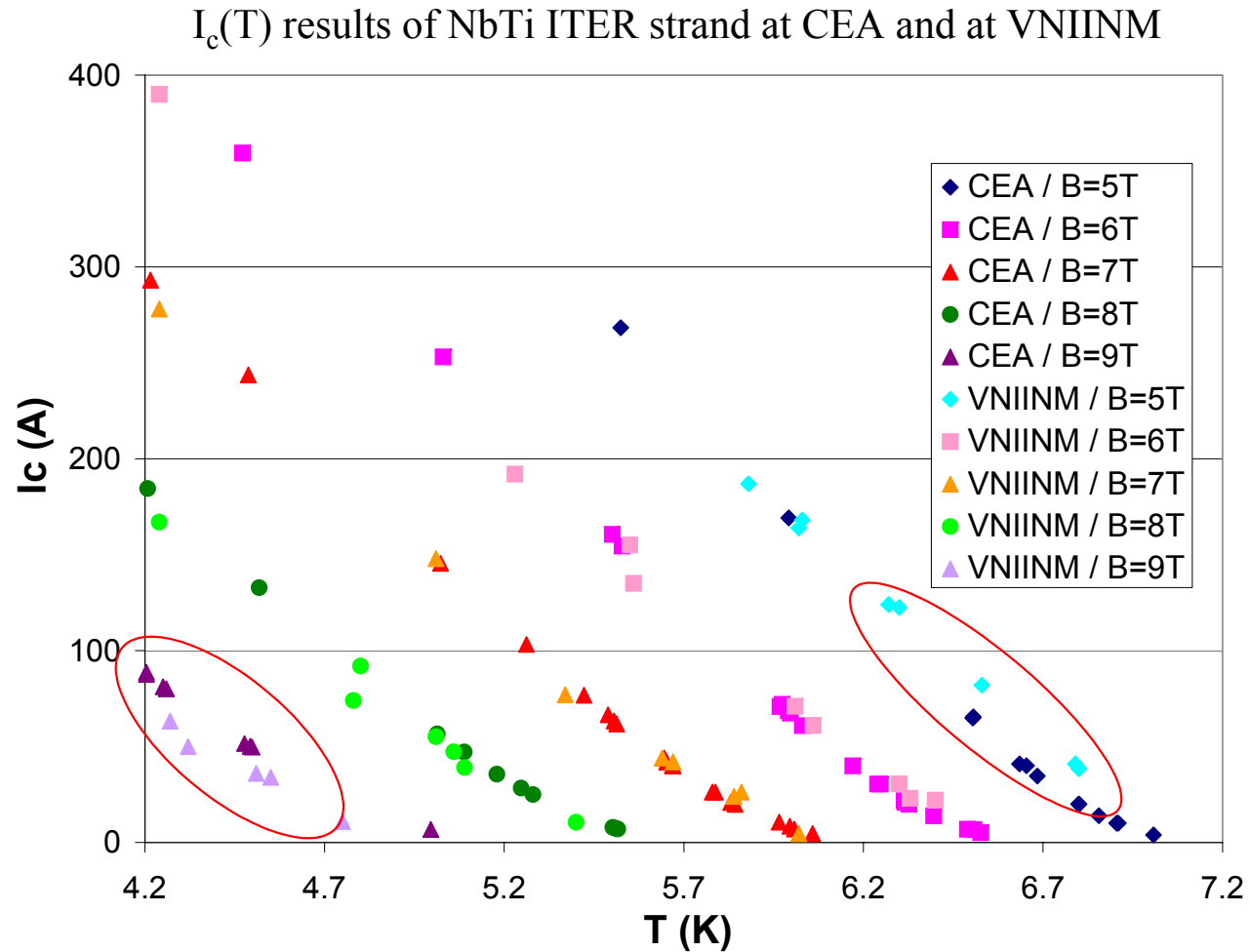
A solid database for strand is the basis for a reliable performance prediction on large conductors

The test accuracy

Various round robin test initiatives for I_c and hysteresis loss (VAMAS, ITER bench mark) showed limited agreement, even for identical specimens assembled under tight specification. About 1% of the scattering is believed to be due to equipment accuracy (field setting, current metering, operating temperature). For Nb_3Sn strands, the high sensitivity to handling is an additional issue

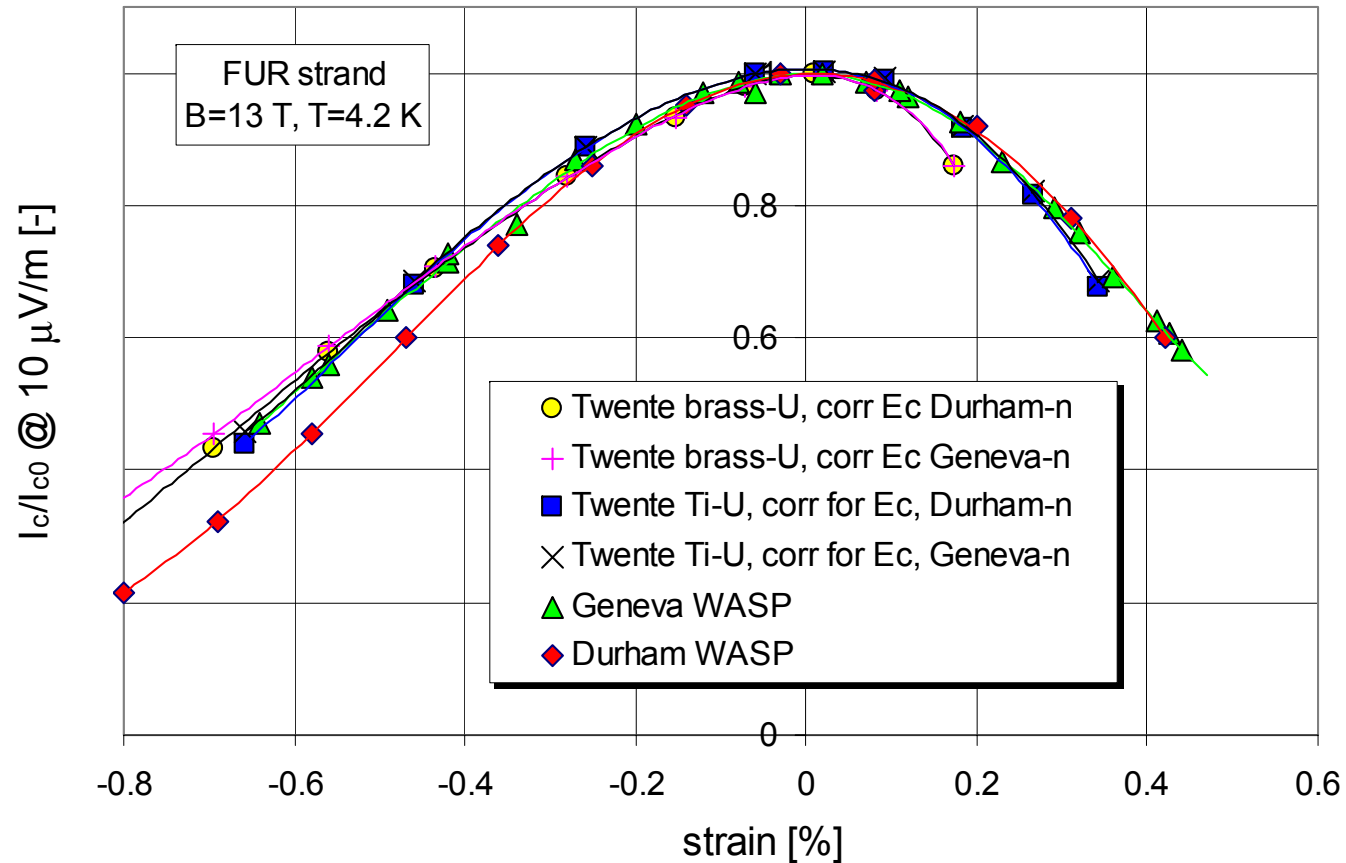


For I_c test at $T > 4.2\text{K}$, the scattering among labs increases because of the error bar in the assessment of the operating temperature (in vacuum) and the large $\partial I_c / \partial T$



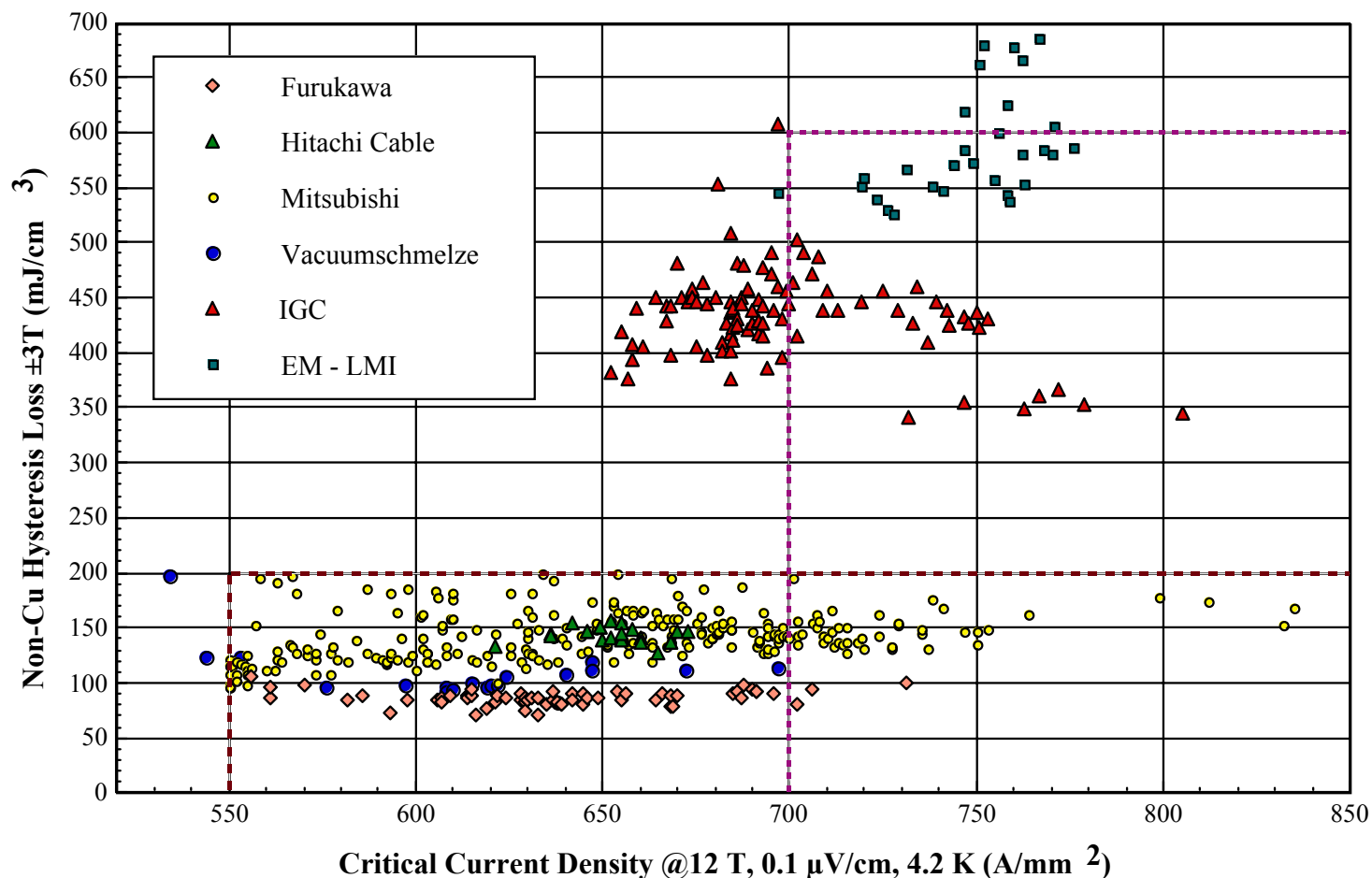
I_c test versus applied strain in Nb_3Sn strands are also difficult to reproduce better than 5%

$I_c(\epsilon)$ results, normalized to $I_{c,max}$, of Nb_3Sn ITER strand at three labs



The performance scattering over a large series production also affects the reliability of the strand data base for predictions/extrapolations

J_c vs Loss Performance according to the Interface Documents (no Cross Check by Bench Mark)

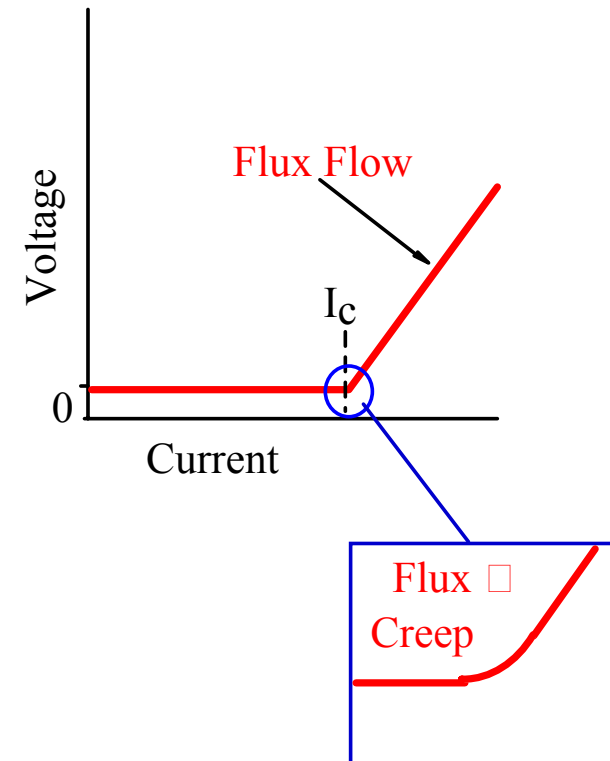


The V-I characteristic of superconductor: what we expect and what we got

In the assumption of perfect, homogeneous structure of pinning centers, above I_c the flux quanta are free to move. The viscous flow of flux is described by a resistivity ρ_f

At very low electric field, $< 10^{-12}$ V/m, a non-linear regime is caused by thermally activated flux movement (flux creep) $R_0 \exp(\alpha J - U/kT)$

In reality, the non-linear range of the Volt-Ampere Characteristic (VAC) in bulk and filamentary superconductors extends *much beyond the flux creep* region. The flux flow linear range is mostly not observed, as it is preempted by the thermal runaway



A simple model to understand the non-linearity of the VAC

In 1967, *Baixeras&Fournet* and *Jones&Rhoderick&Rose-Innes* proposed a model for non-homogeneous critical current (i.e. pinning forces), with distribution $f(i_c)$ such that

$$\int_{i_c^{\min}}^{i_c^{\max}} f(i_c) di_c = 1$$

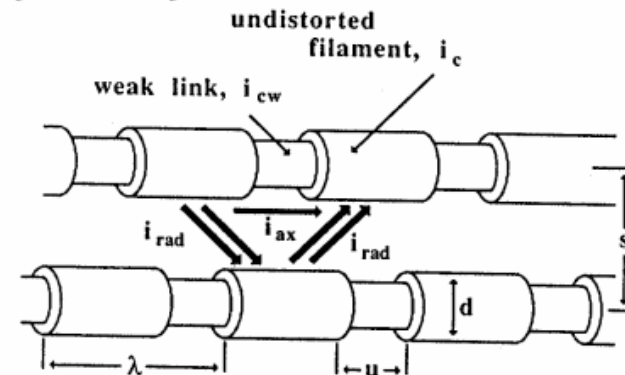
Assuming that all the segments with different i_c and same ρ_f are connected in series, the local voltage generated by flux flow at the current i is given by

$$dV = \frac{\rho_f}{A} x di = \frac{\rho_f}{A} \left[\int_{i_c^{\min}}^i f(i_c) di_c \right] di$$

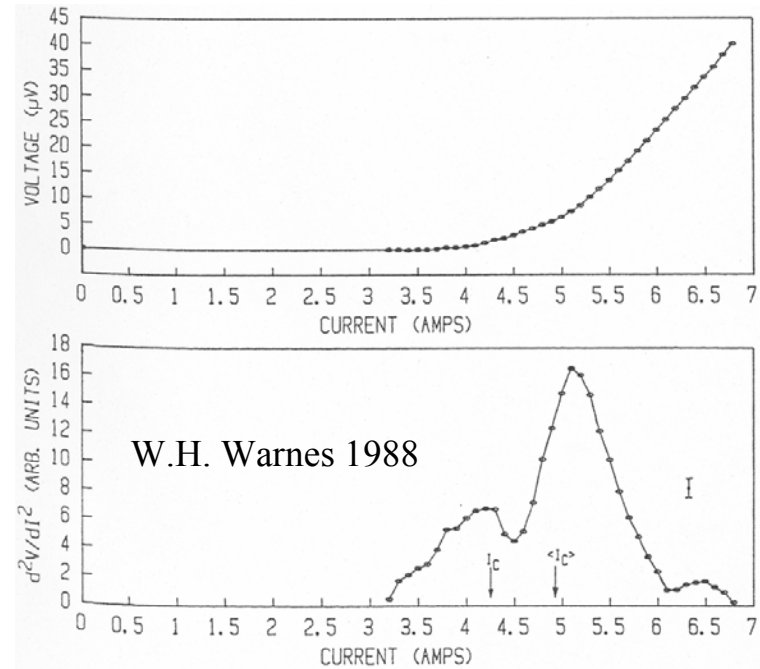
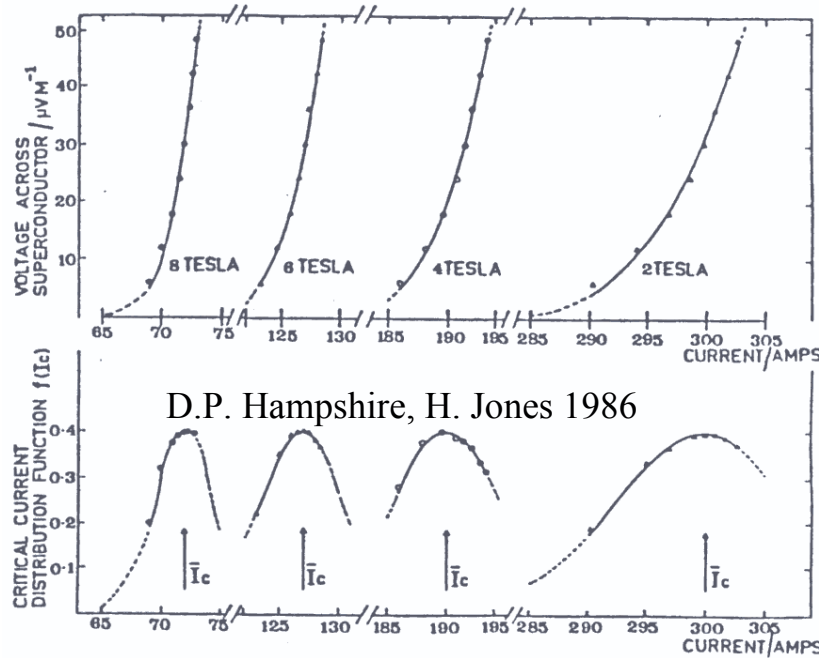
Due to partial flux flow, the VAC is non-linear between i_c^{\min} and i_c^{\max} and becomes linear (flux flow) above i_c^{\max} .

The model, developed for **bulk superconductors**, was further developed in the eighty, adding the **parallel resistance of the stabilizer** and the **interfilament current sharing**

W.H.Warnes 1988



Deconvolution of VAC for composite



The distribution of the local critical current, $f(i)$ is derived by deconvolution of

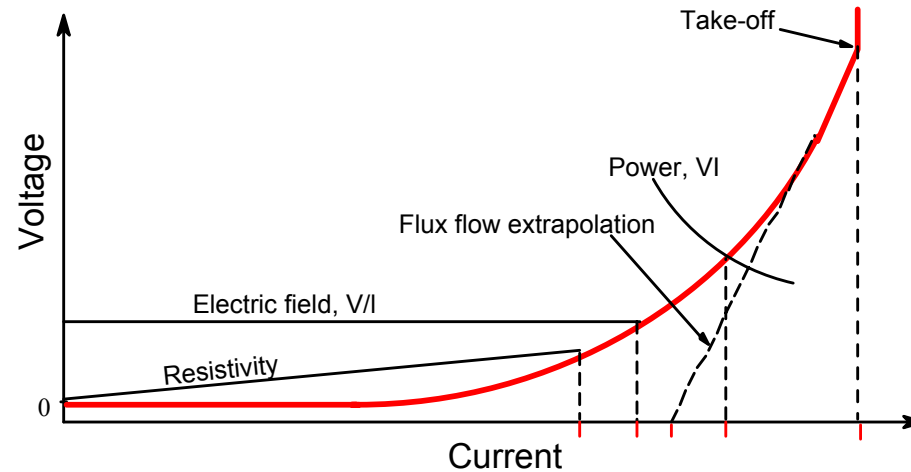
$$dV = \frac{\rho_f}{A} x di = \frac{\rho_f}{A} \left[\int_{i_c^{\min}}^i f(i_c) di_c \right] di \quad \text{into} \quad \frac{\partial^2 E}{\partial I^2} = \rho_f f(I_c)$$

The presence of a single peak in $f(i)$ indicates a “clean” homogeneous material where the scattering of I_c is only due to the “**intrinsic**” microstructure of the pinning centers. Multiple peaks in $f(i)$ suggest an I_c limitation due to “**extrinsic**” factors, like breakage, filament necking, macroscopic defects, scattering of filament size, etc.

A realistic Volt-Ampere Characteristic in a composite

The critical current of a practical superconductor can be arbitrarily defined on the base of criteria:

- Resistivity onset
- Axial electric field
- Power dissipation
- Flux flow extrapolation
- Thermal runaway (take-off)



The I_c criteria cannot be made equivalent over the whole range of field and temperature. Some criteria depends on the heat removal capability of the experimental arrangement.

$E_c = 10 \mu\text{V/m}$ is practically retained in most applications

Empirical Fits: the most popular empirical fit for VAC is the power law

$$E = E_c \left(\frac{I}{I_c} \right)^n$$

Over a limited range of electric field, n is constant through the VAC. However, in case of non isothermal conditions, n tends to increase at higher dissipated power.

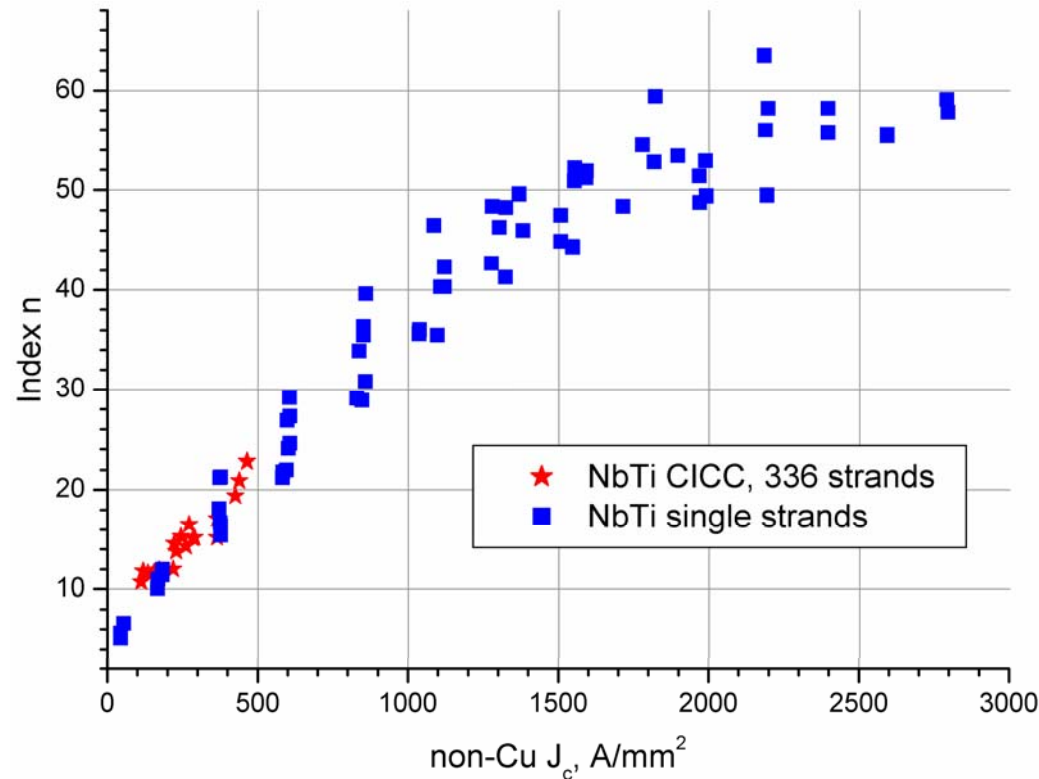
When is n good or bad?

- *A low n index (<20), weakly changing as a function of B and T suggests extrinsic limitations. Individual filament breakage and Cu:non-Cu ratio marginally affect n . Extended necking, filament bridging, filament size < 3 μm , long heat treatments (causing non-homogeneity of J_c in Nb_3Sn) depress n . Strain in Nb_3Sn further decreases n , possibly due to lower B_{c2} phases*
- *The n index is not an absolute **quality** index. The strand manufacturers use n to monitor the production **stability***
- *In non-permanent magnets, low n can be accepted, but it is an evidence that the potential of the superconductor is not properly exploited*



In “good” strands, the index n is not constant

The index n decreases at either increasing field or temperature, i.e. approaching the $T_c(B)$ curve. The variations of n can be conveniently summarized in a plot of $n(j_c)$. Single strand and cables of the same strand follow approximately the same $n(j_c)$ curve



Scaling laws commonly in use for J_c in strands

NbTi

$$J_c(B, T) = C_0(\varepsilon) \frac{b^\alpha (1-t^{n'})^\gamma (1-b)^\beta}{B}$$

where

$$b = \frac{B}{B_{c2}(T)}$$

$$B_{c2}(T) = B_{c20}(1-t^{n'})$$

$$t = \frac{T}{T_{c0}}$$

*The parameters are obtained by fitting of strand results over a limited range. They do not have a physical meaning. The scaling laws can be used to **interpolate** within the range of test results, not to **extrapolate** beyond it.*

Nb₃Sn

$$J_c(B, T, \varepsilon) = C(\varepsilon) \frac{[(1-t^2)(1-b^2)]^2}{\sqrt{b} B_{c2}(T, \varepsilon)}$$

where

$$b = \frac{B}{B_{c2}(T, \varepsilon)}$$

$$B_{c2}(T, \varepsilon) = B_{c20}(\varepsilon)(1-t^2)[1 - 0.31t(1 - 1.77 \ln t)]$$

$$t = \frac{T}{T_{c0}(\varepsilon)}$$

$$T_{c0}(\varepsilon) = T_{c0m} \sqrt[3]{(1 - a|\varepsilon|^{1.7})}$$

$$B_{c20}(\varepsilon) = B_{c20m} (1 - a|\varepsilon|^{1.7})$$

$$C(\varepsilon) = C_0 \sqrt{(1 - a|\varepsilon|^{1.7})}$$

$$a = 900 \quad \text{for } \varepsilon < 0, \quad a = 1250 \quad \text{for } \varepsilon > 0$$

Enrico Fermi

*“I remember my friend Johnny von Neumann used to say, with **four** parameters I can fit an elephant, and with **five** I can make him wiggle his trunk.”*

[Dyson, Nature, 427, 297 (2004)].



From strand to large conductor

Even before considering the large conductors, we already know that any prediction will be affected by errors because:

- Reliability of the test results on strand
- Statistical variation of strand performance over large batch
- Variation of index n with operating current
- Accuracy of scaling law fitting (for interpolation)

Two further issues, characteristic of large conductors, dramatically amplify the error on the performance prediction

Self Field (NbTi and Nb₃Sn)

Actual operating load on Nb₃Sn



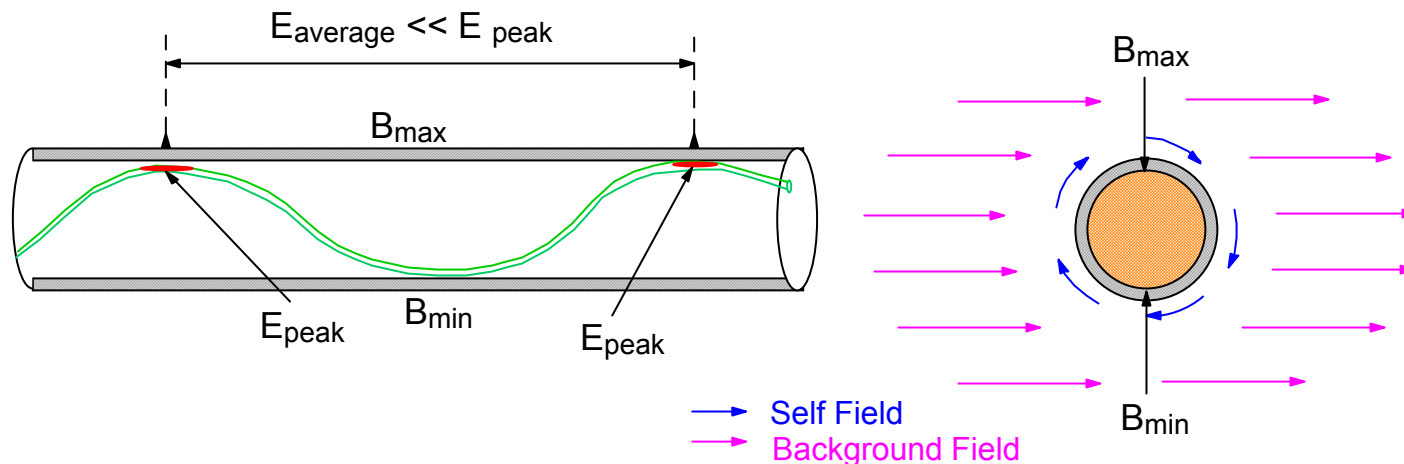
Self Field

Any current carrying conductor with finite size generates a self field which superimposes to the background field, with a field gradient across the conductor.

$$B_{sf} = \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 J R}{2}$$

In transposed (twisted) superconductors, the filaments (of a strand) or the strands (of a cable) travel continuously from the low field to the high field.

In multifilament composites, the amplitude of self-field is only few tens of mT and does not affect the assessment of $I_c(B)$. In otherwise perfect composite, with very low matrix resistivity, the self-field is the limiting factor the n index (inter-filament current sharing)



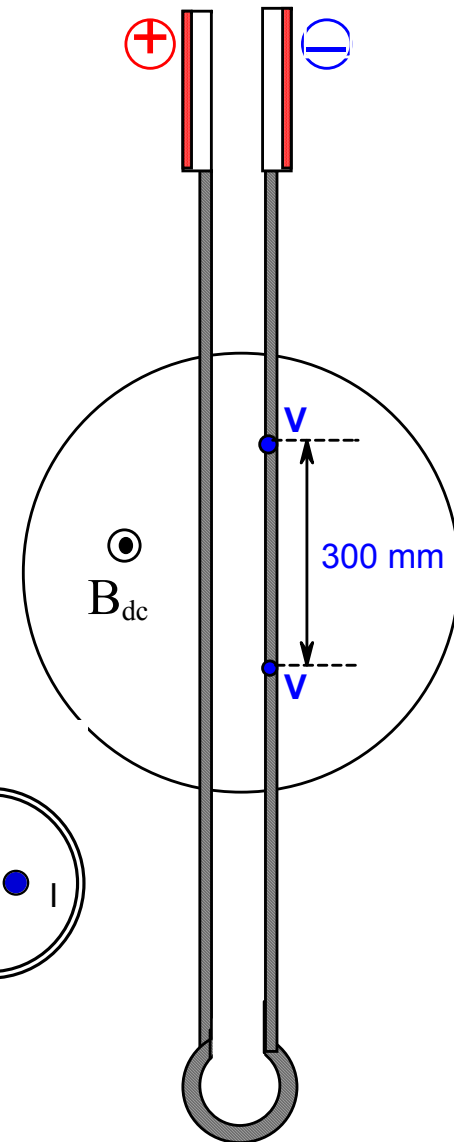
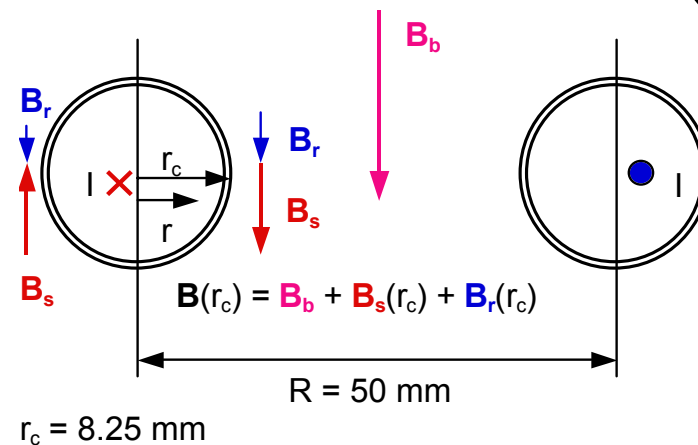
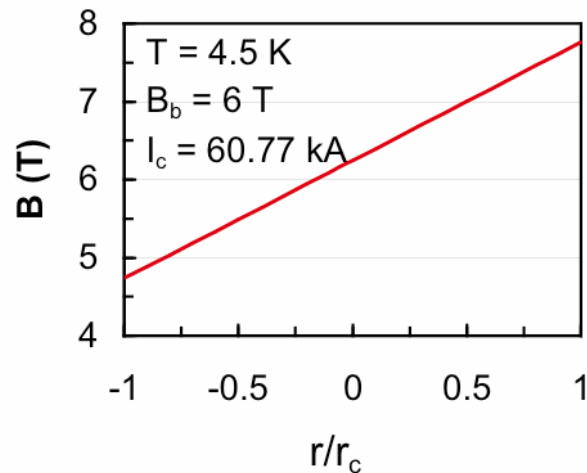
Self Field in large conductor test

In a medium/large conductor, the field gradient over the cable can easily exceeds two Tesla at I_c

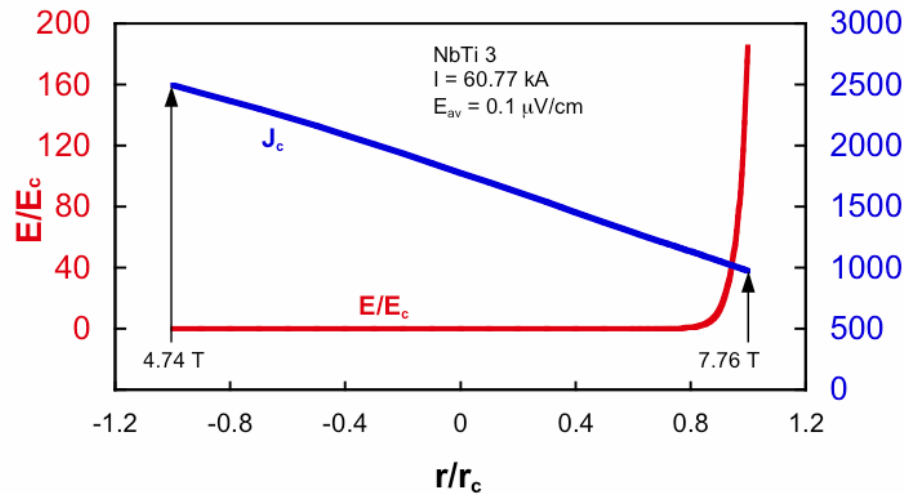
The question arises: which field must be retained for the I_c assessment when the self field is large?

Average field (background field) ?

Peak field ?



*Due to the magnetic field gradient, a large electric field builds up at B_{max} before any voltage appears at B_{min} . However, the voltage sensed by voltage taps on the conductor jacket gives the **average** electric field, not the **peak** electric field.*



The local electric field $E(r) = E_c (J/J_c(r))^n$ may vary substantially over few mm length. For uniform current distribution and no interstrand current transfer (insulated strands), the average, measurable electric field over a length longer than a transposition length is estimated from the integral over the cross section, i.e. over all the statistically possible location of the strand

$$\frac{E}{E_c} = \int_{-r_c}^{r_c} 2 \frac{(r_c^2 - r^2)^{1/2}}{\pi r_c^2} \left(\frac{J}{J_c(B)} \right)^{n(J_c(B))} dr$$

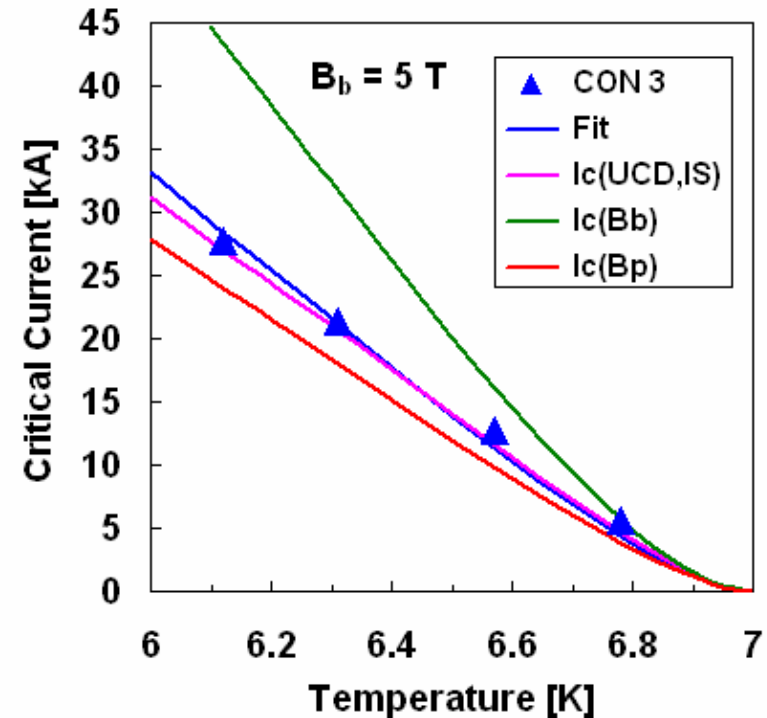
The “correct” assessment of operating field for I_c

As $E_{av} < E_{peak}$, it would be “unfair” to retain B_{peak} for the assessment of the conductor I_c

The model with insulated strands and uniform current density (UCD, IS) leads to a performance prediction higher than for peak field.

A performance better than UCD-IS means that the some minor current sharing occurs among strands at high field, smoothing the voltage increase.

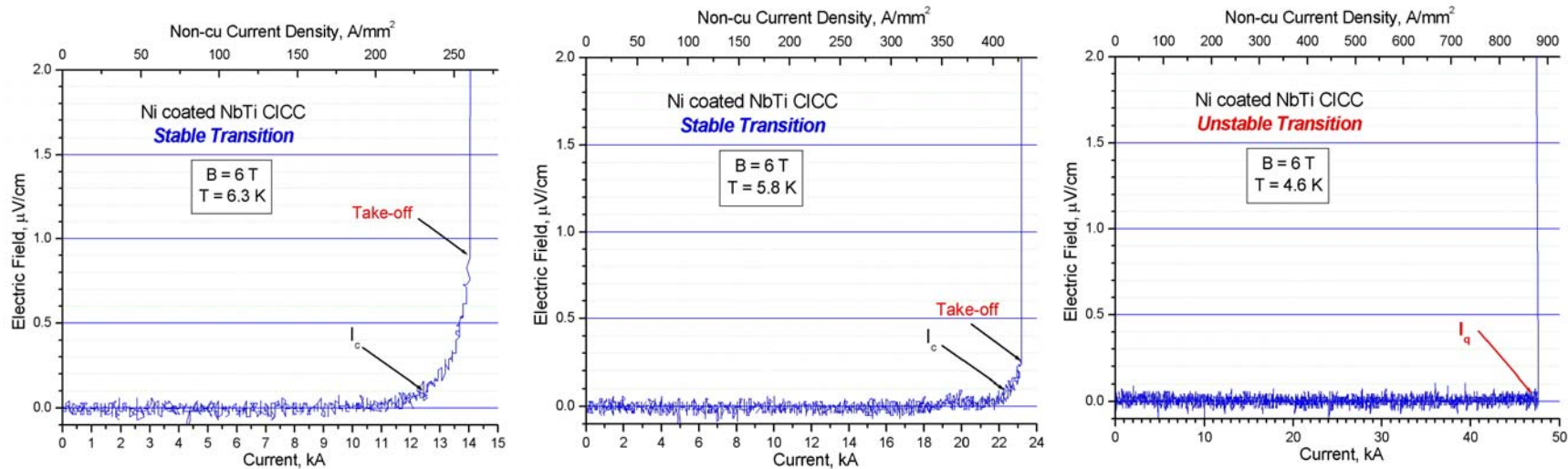
A performance worse than UCD-IS or even worse than the peak field prediction is an evidence of current unbalance which cannot be re-distributed



No measurable I_c ?

In conductors with large self field and large n , the criterion for I_c ($0.1 \mu\text{V}/\text{cm}$) is expected to occur when the peak electric field is well above $15\text{-}20 \mu\text{V}/\text{cm}$.

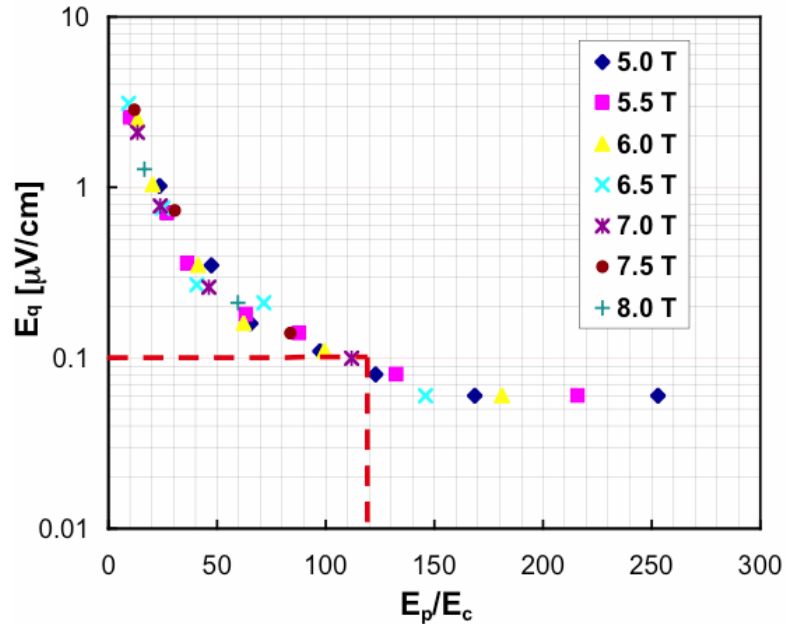
*At such high, local electric field, the balance of locally generated to removed power exceed the stability limits and the **conductor takes off reproducibly even when no “measurable” voltage is detected at the voltage taps***



Pierluigi Bruzzone
The prediction of large conductor performance
CERN Academic Training, June 2nd 2005

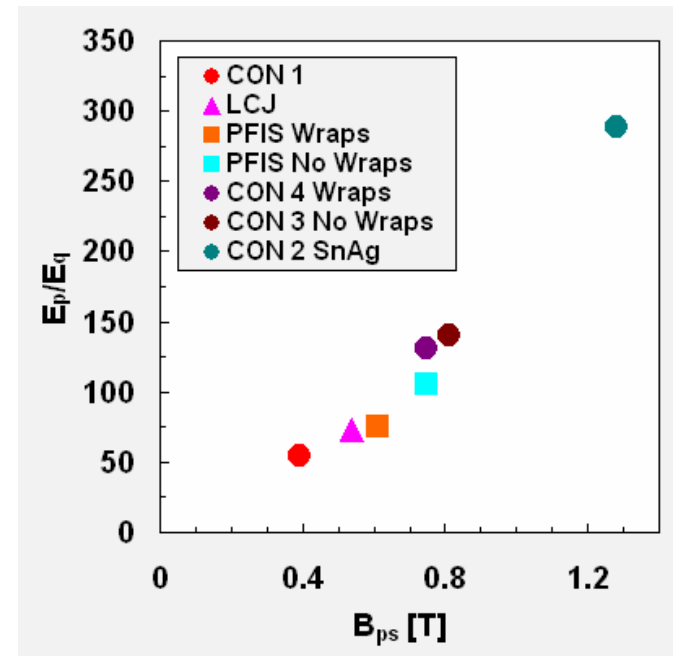


Threshold for self-field induced sudden take-off



For a specific conductor, the take-off electric field, E_q , can be plotted against the calculated ratio of peak to average electric field, to find the threshold for measurable critical current

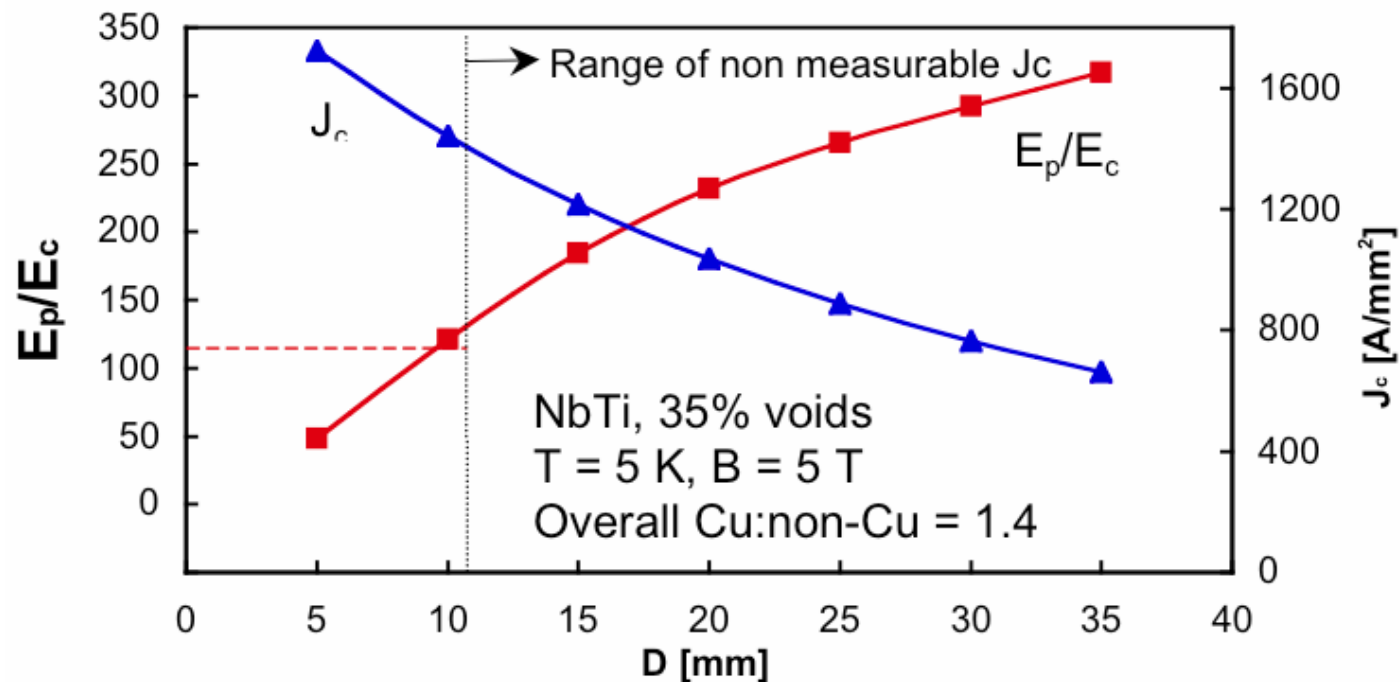
Transition to sudden take-offs in NbTi typically occurs for B_{ps} values of 0.5 to 0.8 T corresponding to E_p/E_q values between 70 and 140.



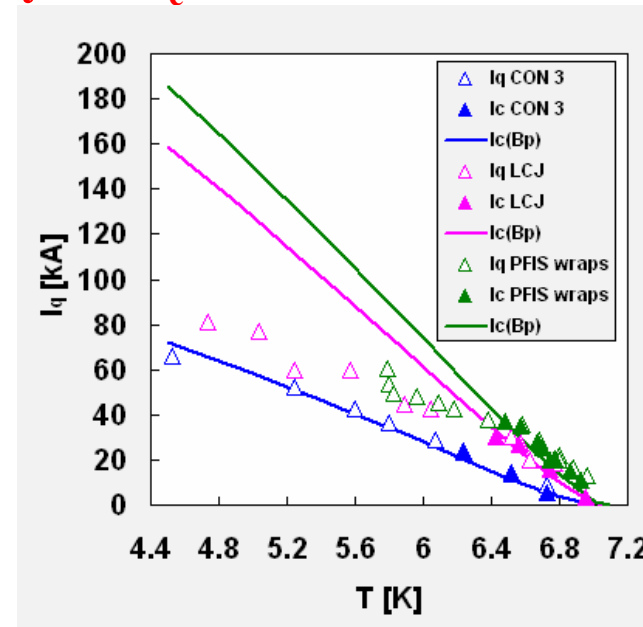
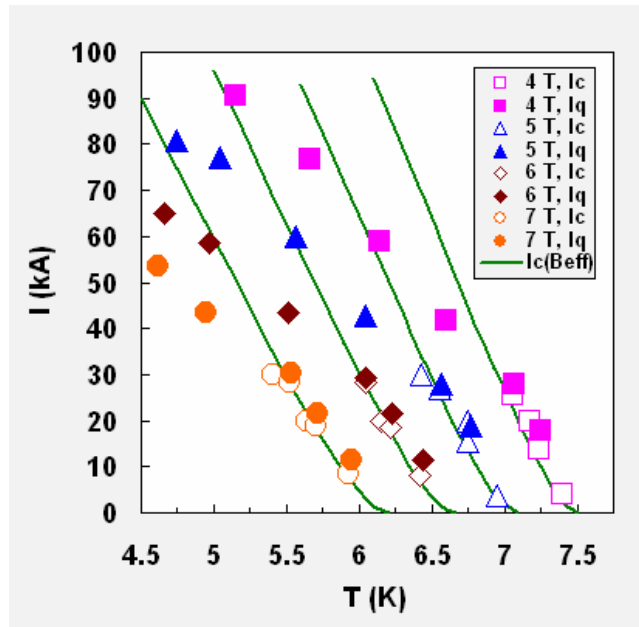
The role of the cable size

Keeping the same J_{op} , the self-field increases for increasing conductor diameter and so the E_p/E_c

In the example below, it is expected that above $\approx 11\text{mm}$ diameter the I_c cannot be measured anymore



The range beyond I_c



When the high local electric field prevents to achieve I_c , the conductor performance is slightly worse than predicted by the UCD-IS model. The local stability limit must complement the UCD-IS model.

In the case of current unbalance, the lack of significant voltage at the take-off prevents current re-distribution and the quench is driven by the overloaded strand. The deviation from the predicted performance may be very severe.

Self field impact on J_c for NbTi vs. Nb₃Sn

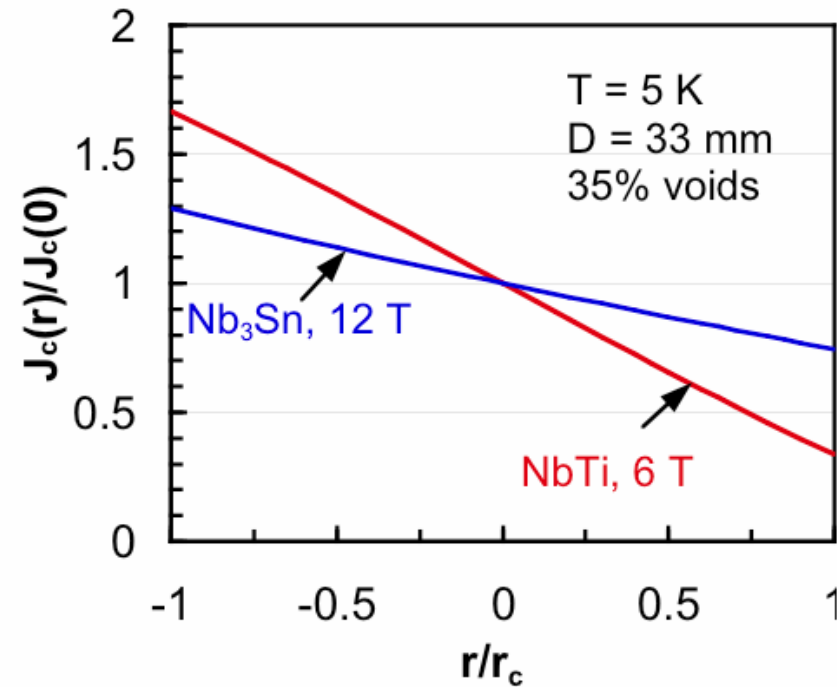
Compare NbTi @ 6 T vs. Nb₃Sn @ 12 T:

For same Cu:non-Cu, void fraction, T_{op} and cable diameter, the I_c is respectively 85 and 116 kA

The J_c variation over the cable cross section is much higher for NbTi than Nb₃Sn

$$\left(\frac{J_{c \max}}{J_{c \min}} \right)^{NbTi} > 5.6$$

$$\left(\frac{J_{c \max}}{J_{c \min}} \right)^{Nb_3Sn} = 1.85$$



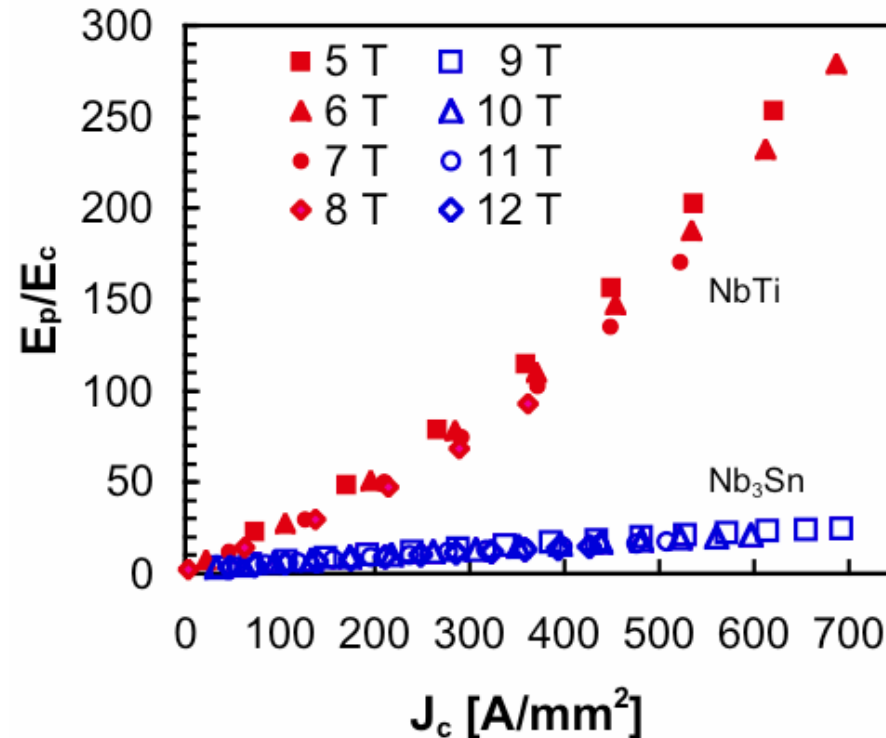
Electrical Peak Field effect on NbTi vs. Nb₃Sn

The E_p/E_c ratio is affected by both

- $J_c(r)/J_c(0)$
- strand n index

The index n is mostly higher in NbTi than in Nb₃Sn strands. We retain here

$$n = 15 \neq f(J) \quad \text{for Nb}_3\text{Sn strand}$$
$$n = f(J) \quad \text{for NbTi strand}$$



At all operating fields, the Nb₃Sn cable maintains a moderate E_p/E_c , below 25

No sudden take-off is expected, i.e. $E_q \gg E_c$

Summary of self-field impact on prediction

- The **UCD-IS model** provide a reasonable assessment of the operating field to be retained for I_c in presence of self-field
- At large local electric field, the quench is driven by **local stability limit** before the criterion for I_c is achieved
- The deviation from the UCD-IS prediction at very large self-field is marginal as long as the current density is uniform
- For unbalanced current, no re-distribution is possible in the range of sudden take-off and the performance drop is proportional to the unbalance (by definition, the current unbalance is not predictable)



Conclusion on prediction

- ✓ *For **NbTi** conductors, the large conductor performance can be realistically predicted, i.e. within the accuracy of strand results and scaling laws, except in case high **self field***
- ✓ *In **Nb₃Sn** conductor, the lack of knowledge of the **actual strain** in operation (combination of thermal and bending strain) largely affects the accuracy of the performance prediction*
- ✓ *A **conductor test** strongly reduces the range of uncertainties, but the performance in a long coil can still deviate from the short length conductor result because of*
 - *Different build-up of the local strain*
 - *Different drive of current re-distribution*
- ✓ *A knowledge of the limits of performance prediction is essential in design*

