Universal Extra Dimensions

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Talk at Les Houches - May 2005

Bosons in compact spatial dimensions

4D flat spacetime $\perp$ one dimension of size πR:

$$\text{Boundary conditions}: \frac{\partial}{\partial y}\phi(x,0) = \frac{\partial}{\partial y}\phi(x,\pi R) = 0$$

$$\text{KK decomposition}: \phi(x,y) = \frac{1}{\sqrt{\pi R}}\left[\phi^0(x) + \sqrt{2}\sum_{j\geq 1}\phi^j(x)\cos\left(\frac{jy}{R}\right)\right]$$

Zero-mode: ϕ^0 - wave function is flat along the extra dimension.

Kaluza-Klein modes, $\phi^j(x)$:

particles with momentum in extra dimensions,

or 4D point of view: a tower of massive particles:

$$m_j^2 = m_0^2 + \frac{j^2}{R^2}$$

Gauge bosons in 5D:

$A_\mu(x^\nu, y)$, $\mu, \nu = 0, 1, 2, 3$, and

$A_y(x^\nu, y)$ – polarization along the extra dimension.

From the point of view of the 4D theory:

$A_y(x^\nu, y)$ is a tower of spinless KK modes.

$$\text{Dirichlet B.C}: \quad A_y(x, 0) = A_y(x, \pi R) = 0$$

$$\text{KK decomposition}: \quad A_y(x, y) = \sqrt{\frac{2}{L}} \sum_{j \geq 1} A_y^j(x) \sin\left(\frac{jy}{R}\right)$$

→ $A_y(x^\nu, y)$ does not have a 0-mode! *(Odd field)*

Fermions in a compact dimension

Lorentz group in 5D $\Rightarrow$ vector-like fermions:

$$\chi = \chi_L + \chi_R$$

Chiral boundary conditions:

$$\begin{aligned}
\chi_L(x^\mu, 0) &= \chi_L(x^\mu, \pi R) = 0 \\
\frac{\partial}{\partial y}\chi_R(x^\mu, 0) &= \frac{\partial}{\partial y}\chi_R(x^\mu, \pi R) = 0
\end{aligned}$$

Kaluza-Klein decomposition:

$$\chi = \frac{1}{\sqrt{\pi R}}\left\{\chi_R^0(x^\mu) + \sqrt{2}\sum_{j\geq 1}\left[\chi_R^j(x^\mu)\cos\left(\frac{\pi j y}{L}\right) + \chi_L^j(x^\mu)\sin\left(\frac{\pi j y}{L}\right)\right]\right\}$$

Universal Extra Dimensions

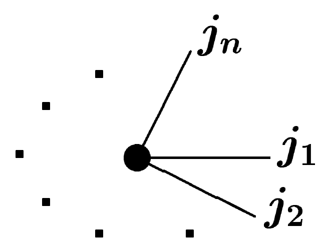
T. Appelquist, H.-C. Cheng, B. Dobrescu, Phys.Rev.D64 (2001)

All Standard Model particles propagate in $D \geq 5$ dimensions.

Kaluza-Klein modes are states of definite momentum along the compact dimensions.

Momentum conservation → KK-number conservation

$$\mathcal{L}_{4D} = \int dy \, \mathcal{L}_{5D}$$



At each interaction vertex:

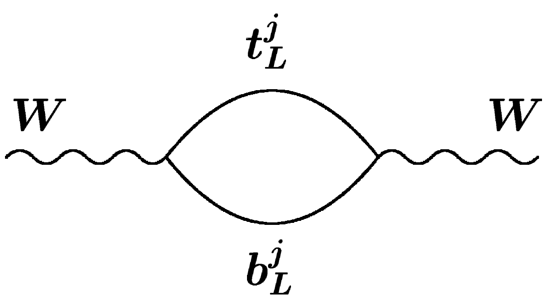
$j_1 \pm j_2 \pm ... \pm j_n = 0$ **for a certain choice of $\pm$**

In particular: $0 \pm \cdots \pm 0 \neq 1$

$\Rightarrow$ tree-level exchange of KK modes does not contribute to currently measurable quantities

$\Rightarrow$ no single KK 1-mode production at colliders

Bounds from one-loop shifts in W and Z masses, and other observables:

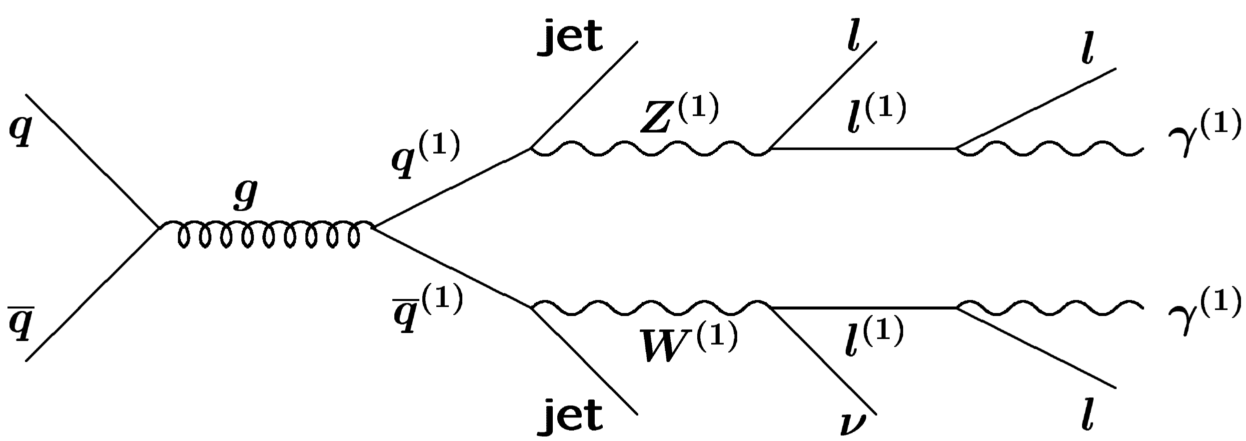


$$\frac{1}{R} \gtrsim 300 \text{ GeV}$$

- Pair production of KK 1-modes at colliders: cascade decays to $4l + \not{E}_T$ (soft leptons). Could be discovered soon!

(Cheng, Matchev, Schmaltz, hep-ph/0205314)

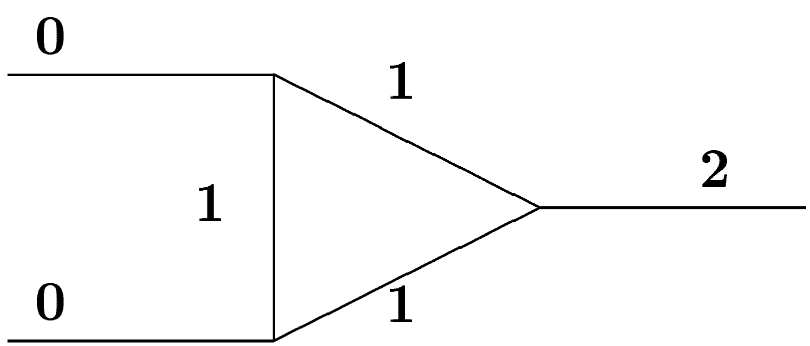
CDF analysis of $3l + \not{E}_T$ (soft leptons):



At one-loop level: $j_1 \pm j_2 \pm ... \pm j_n =$ even

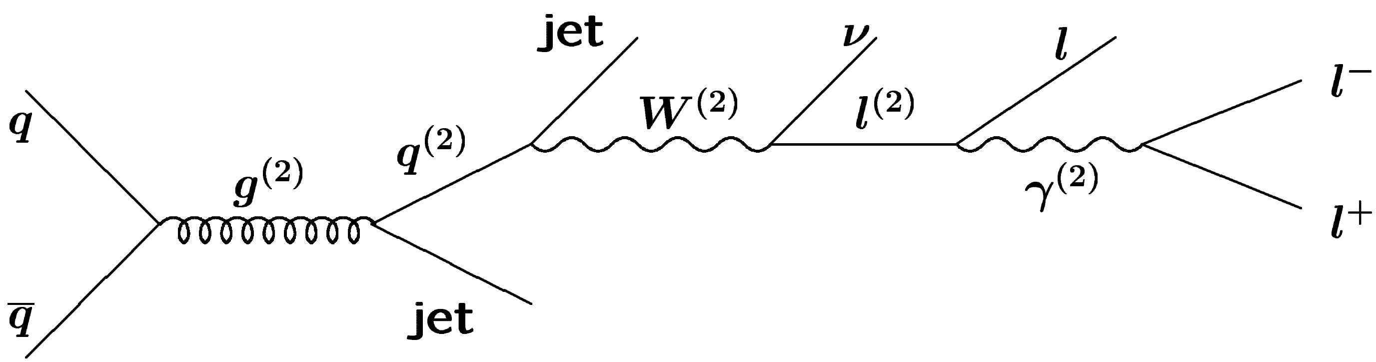
KK parity is conserved: $(-1)^j$

At colliders: s-channel production of the 2-modes



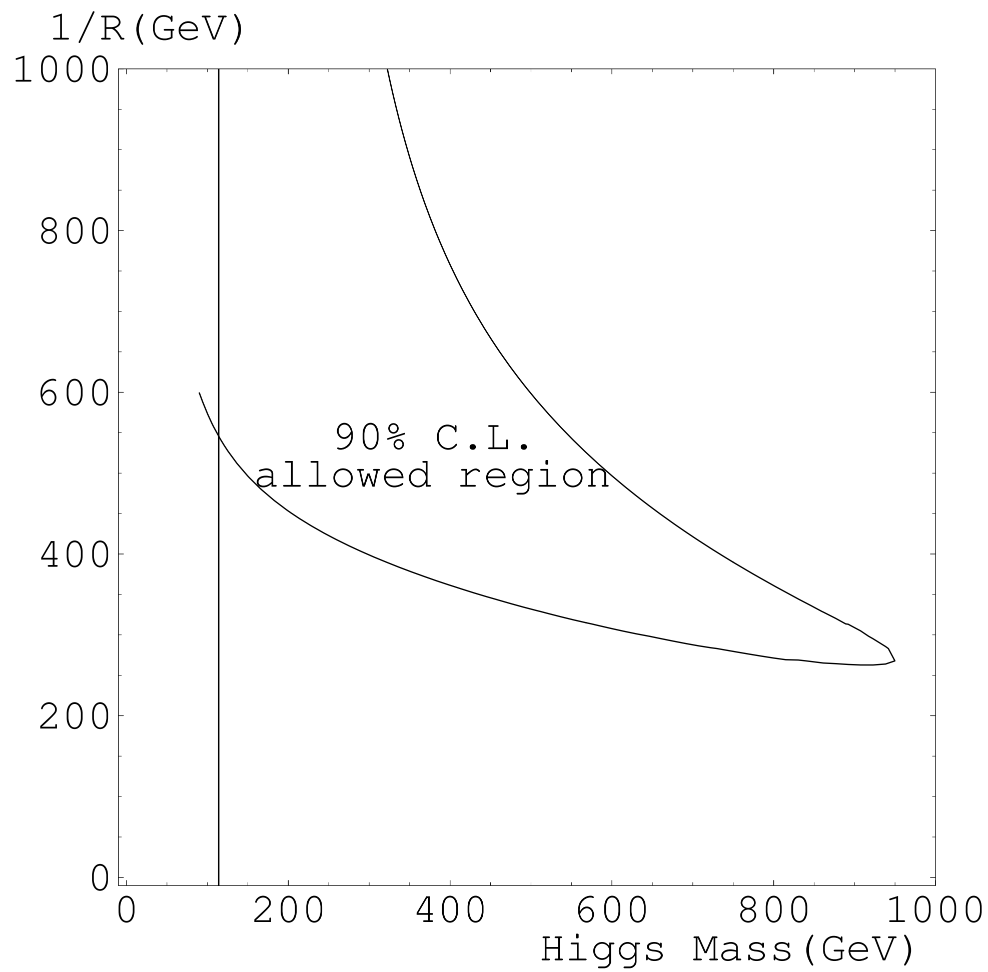
Second-level masses: $\sim 2/R$.

Cascade decay of the 2-mode is followed by $\gamma^{(2)}$ decay into hard leptons:



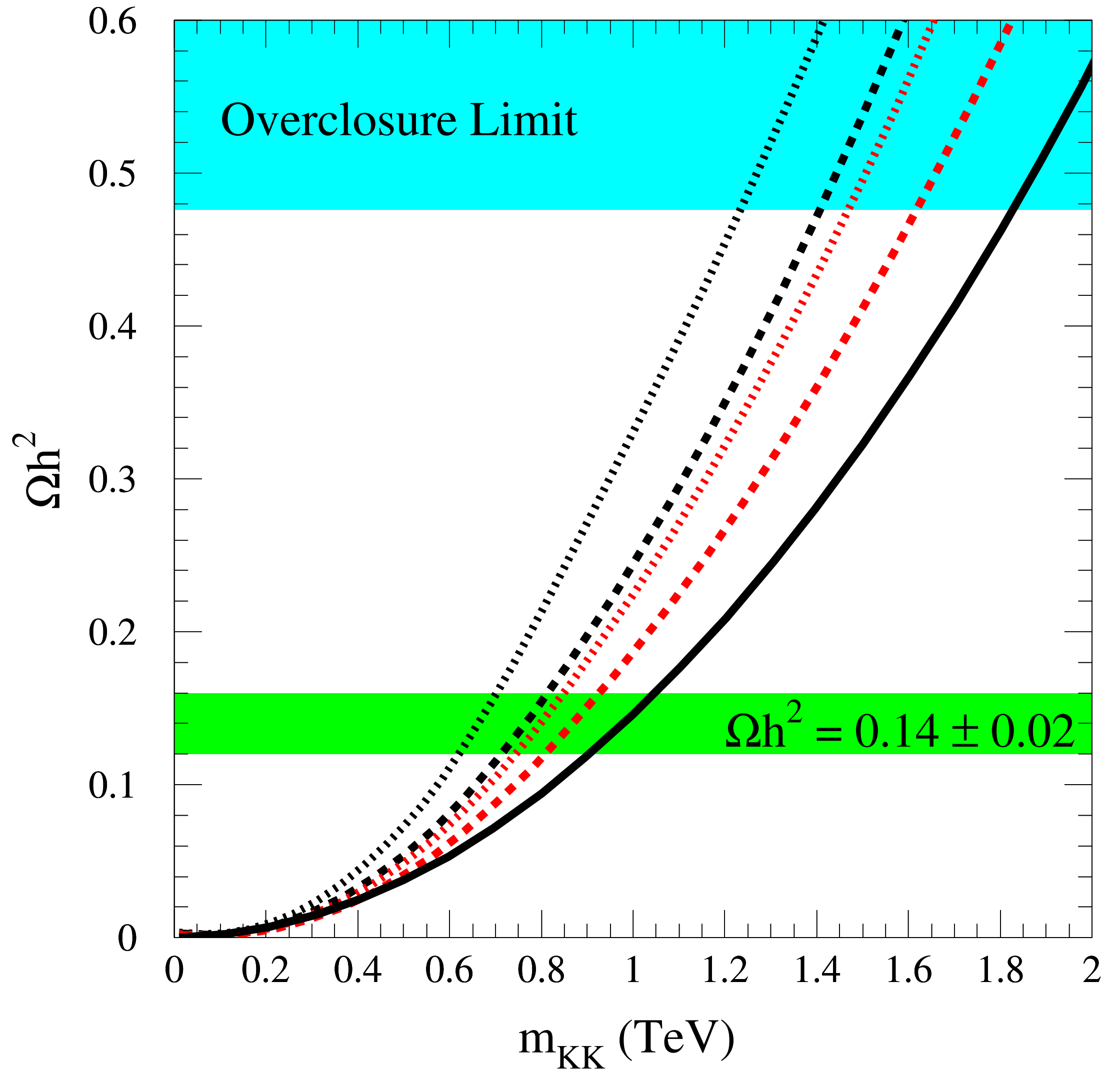
Contributions to the T parameter from top KK modes may compensate for the effect of a heavy Higgs boson on the electroweak fits.

Appelquist, Yee, hep-ph/0211023



Lightest KK particle is stable in UED: $\gamma^{(1)}$ is a viable dark matter candidate

(from Servant, Tait, hep-ph/0206071)



Six-Dimensional Standard Model

work with T. Appelquist, E. Ponton, E. Poppitz, H.-U. Yee

6D is special...

Fermions in $D = 4 + n$ dimensions: chiral representations of the Lorentz group $SO(3 + n, 1)$ exist only for even n.

Properties of chirality in $D = 6$ mod 4 are different than in $D = 4$ mod 4.

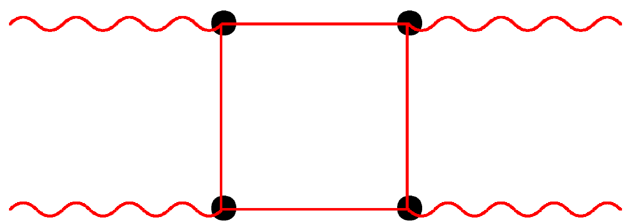
A chiral fermion in $D = 6$ has 4 degrees of freedom.

6D chirality projection operators:

$$P_{\pm} = \frac{1}{2}\left(1 \pm \Gamma^0 ... \Gamma^5\right)$$

Local gauge anomalies in 6 dimensions

"Square anomaly":



• **Reducible anomalies:**

$[SU(3)_C]^2[SU(2)_W]^2$, $[SU(3)_C]^2[U(1)_Y]^2$,
$[SU(2)_W]^2[U(1)_Y]^2$, $[U(1)_Y]^4$, $[SU(2)_W]^4$.

Cancelled by antisymmetric tensor fields, $B_{\mu\nu}$
(Green-Schwarz mechanism)

• **Irreducible anomaly:** $[SU(3)_C]^3U(1)_Y$

$\propto [2(+1/3) \pm (+4/3) \pm (-2/3)]$

Cancels only if: $\mathcal{Q}_+$, $\mathcal{U}_-$, $\mathcal{D}_-$ $\Rightarrow$ QCD is vector-like in 6D.

$n_\pm \equiv$ # of spin–1/2 fields with chirality $\pm$

Local gravitational anomaly in 6D $\propto n_+ - n_-$

$\Rightarrow$ one right-handed neutrino per generation!

Two possible chirality assignments for the 6D quarks and leptons:

$$\mathcal{Q}_+ \, , \, \mathcal{U}_- \, , \, \mathcal{D}_- \, , \begin{cases} \mathcal{L}_+ \, , \; \mathcal{E}_- \, , \; \mathcal{N}_- \\ \qquad \text{or} \\ \mathcal{L}_- \, , \; \mathcal{E}_+ \, , \; \mathcal{N}_+ \end{cases}$$

Global $SU(2)_W$ anomaly

B. Dobrescu, E. Poppitz: PRL 87, 031801 (2001)

Homotopy group in 6D: $\pi_6\,(SU(2)) = Z_{12}$

Anomaly cancellation condition:

(Bershadsky, Vafa, 1997)

$$2\,[n(2_+) - n(2_-)] = 0 \text{ mod } 12$$

One generation: $Q_+ \Rightarrow n_Q(2_+) = 3$

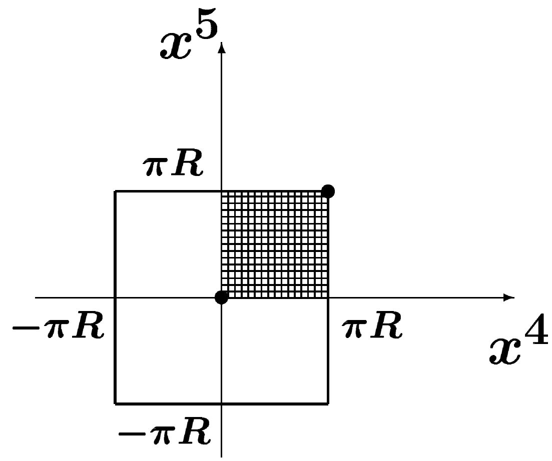
$$L_\pm \Rightarrow n_L(2_\pm) = 1$$

Need more than one generation:

$$(3 \pm 1)\, n_{\rm gen} = 0 \text{ mod } 6 \quad \Rightarrow \quad \underline{n_{\rm gen} = 3 \text{ mod } 3}$$

Compactification of two extra dimensions

T^2/Z_4 orbifold (square torus) of radius R:



Fermions have charge $\pm 1/2$ under rotations
$\rightarrow$ the Lagrangian has an exact Z_8 symmetry!

6D Lorentz symmetry broken by compactification:

$$SO(5,1) \rightarrow SO(3,1) \times Z_8$$

Fermion	Z_8 charge (conserved mod 4)	zero-mode
$\mathcal{Q}_{+L}$	$-1/2$	$q_L = (u_L, d_L)$
$\mathcal{U}_{-R}$, $\mathcal{D}_{-R}$	$-1/2$	u_R, d_R
$\mathcal{Q}_{+R}$, $\mathcal{U}_{-L}$, $\mathcal{D}_{-L}$	$+1/2$	—
$\mathcal{L}_{\pm L}$	$\mp 1/2$	$l_L = (\nu_L, e_L)$
$\mathcal{E}_{\mp R}$, $\mathcal{N}_{\mp R}$	$\mp 1/2$	ν_R, e_R
$\mathcal{L}_{\pm R}$, $\mathcal{E}_{\mp L}$, $\mathcal{N}_{\mp L}$	$\pm 1/2$	—

Z_8 imposes an exact selection rule on the low-energy 4D Lagrangian:

$$3\Delta B + \Delta L = 0 \bmod 8$$

Consequences:

- no Majorana ν masses
- neutrino-less double-beta decays are forbidden

  $(\Delta B = 0,\ \Delta L = 2)$
- no neutron–anti-neutron oscillations

  $(\Delta B = 2,\ \Delta L = 0)$
- long proton lifetime ($\Delta B = 1$ transitions induced by very high-dimension operators)

Perturbativity breaks down at a scale $M_s \approx 5/R$
$\Rightarrow$ 6D standard model is an effective theory valid up to M_s.

Most general operators invariant under Z_8 and $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge transformations are expected to be present at $M_s \geq 2.5$ TeV.

Proton Decays

$\mathcal{L}_+$ chirality assignment:

$$\mathcal{O}_{17} = \frac{C_{17}}{M_s^{11}} \left(\overline{\mathcal{L}}_+ \mathcal{D}_-\right)^3 \tilde{\mathcal{H}}$$

Dominant B-violating processes:
$p \to e^- \pi^+ \pi^+ \nu\nu$ and $n \to e^- \pi^+ \nu\nu$

$$\tau_p \approx \frac{10^{35}\mathrm{yr}}{C_{17}^2} \left[\frac{(4\pi)^{-7} 10^{-4}}{\Phi_5 F(\pi\pi)}\right] \left[\frac{1/R}{0.5 \ \mathrm{TeV}}\right]^{12} \left[\frac{R M_s}{5}\right]^{22}$$

$\tau_n/\tau_p \approx 10^{-3}$ (larger phase-space)

Look for nucleon decays into 3 leptons + pions.

Long-live the proton!

Implications for neutrinos:

(Appelquist, Dobrescu, Ponton, Yee: hep-ph/0201131)

- **3 ν_R's required for gravitational anomaly cancellation**
- **Neutrinos are of the Dirac type due to 6D Lorentz invariance**

- **Smallness of neutrino masses due to small wave-function overlap with the ν_R's**

  Typical prediction: **large θ_{13} & large-mixing-angle solution for solar ν's.**

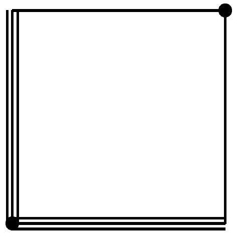
Chiral boundary conditions on a square

(Dobrescu, Ponton, hep-ph/0401032; work with G. Burdman and E. Ponton)

Identify pairs of adjacent sides:

$\Phi(y, 0) = e^{i\theta}\Phi(0, y)$, ...

$\Rightarrow \ \theta = n\pi/2$



Symmetry: $Z_8 \times Z_2$

Complete sets of functions satisfying the boundary conditions:

$$f_{0,2}^{(j,k)}(x^4,x^5) = \frac{1}{1+\delta_{j,0}}\left[\cos\left(\frac{jx^4+kx^5}{R}\right) \pm \cos\left(\frac{kx^4-jx^5}{R}\right)\right]$$

$$f_{1,3}^{(j,k)}(x^4,x^5) = i\sin\left(\frac{jx^4+kx^5}{R}\right) \mp \sin\left(\frac{kx^4-jx^5}{R}\right)$$

Spectrum of KK modes:

(j,k)	(1,0)	(1,1)	(2,0)	$(2,1)$ $(1,2)$	(2,2)	(3,0)	$(3,1)$ $(1,3)$
$M_{j,k}R$	1	$\sqrt{2}$	2	$\sqrt{5}$	$2\sqrt{2}$	3	$\sqrt{10}$

The KK expansions of a 6D fermion of $+$ chirality with a left-handed zero mode:

$$\Psi_{+L} = \frac{1}{L}\left[\Psi_{+L}^{(0,0)}(x^\nu) + \sum_{j\geq 1}\sum_{k\geq 0} f_0^{(j,k)}(x^4,x^5)\Psi_{+L}^{(j,k)}(x^\nu)\right] \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Psi_{+R} = -\frac{i}{L}\sum_{j\geq 1}\sum_{k\geq 0} \frac{j+ik}{\sqrt{j^2+k^2}} f_3^{(j,k)}(x^4,x^5)\Psi_{+R}^{(j,k)}(x^\nu) \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Similar decompositions for fermions having a zero mode of the type Ψ_{+R}, Ψ_{-L} or Ψ_{+L}.

KK decomposition of the gauge fields:

$$A_\mu(x^\nu, x^4, x^5) = \frac{1}{L}\left[A_\mu^{(0,0)}(x^\nu) + \sum_{j\geq 1}\sum_{k\geq 0} f_0^{(j,k)}(x^4, x^5) A_\mu^{(j,k)}(x^\nu)\right]$$

$$A_+(x^\nu, x^4, x^5) = -\frac{1}{L}\sum_{j\geq 1}\sum_{k\geq 0} f_3^{(j,k)}(x^4, x^5) A_+^{(j,k)}(x^\nu)$$

$$A_-(x^\nu, x^4, x^5) = \frac{1}{L}\sum_{j\geq 1}\sum_{k\geq 0} f_1^{(j,k)}(x^4, x^5) A_-^{(j,k)}(x^\nu)$$

Physical degrees of freedom:

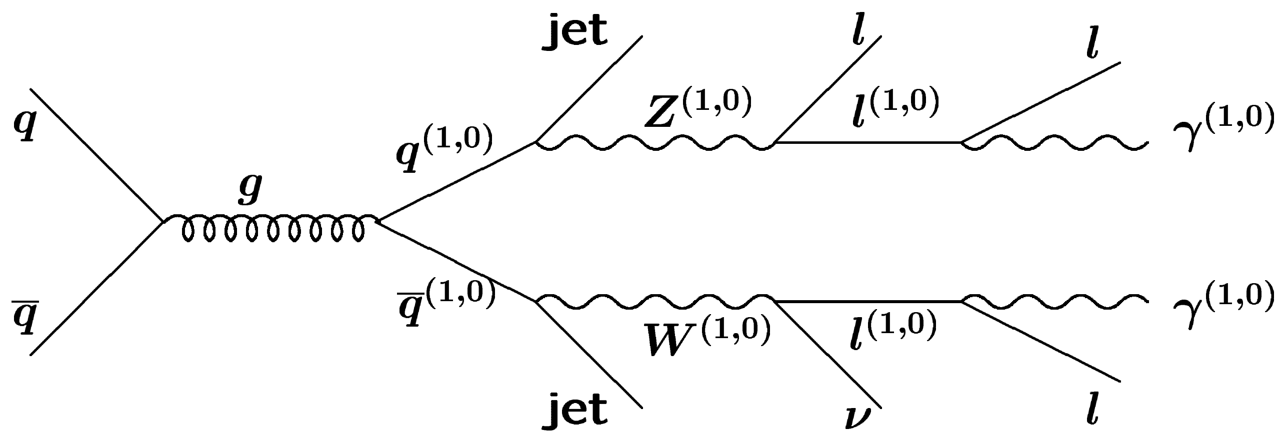
$$A_\pm^{(j,k)} = \frac{j + ik}{\sqrt{j^2 + k^2}}\left(A_H^{(j,k)} \mp i A_G^{(j,k)}\right)$$

$A_G^{(j,k)}$ is the longitudinal polarization of $A_\mu^{(j,k)}$

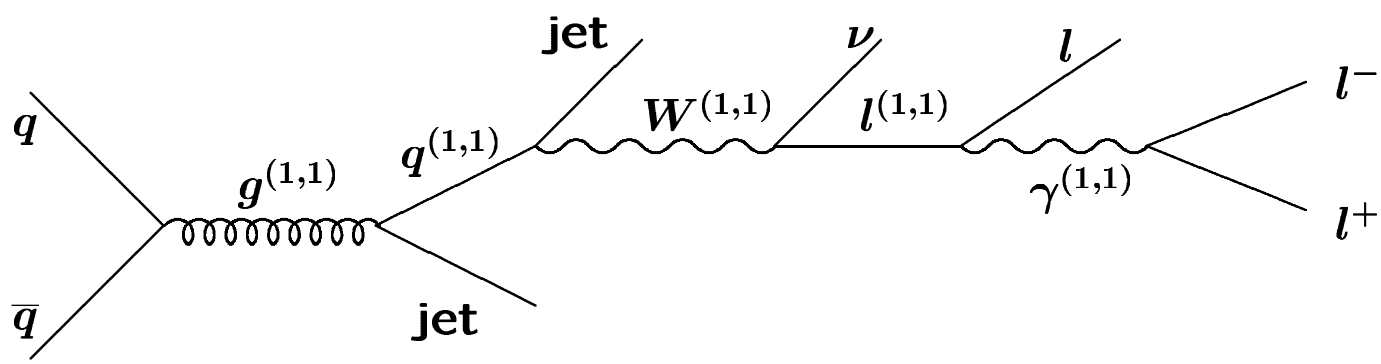
$A_H^{(j,k)}$ is a real scalar field

Signals at colliders:

Pair production of (1,0) modes: $3l + \not{E}_T$ (soft leptons)



s-channel production of a (1,1) mode of mass $\sqrt{2}/R$
$\rightarrow$ l^+l^- resonance + soft jets and leptons:



Conclusions

- Universal Extra Dimensions
  - compactification scale can be as low as 300 GeV.
  - lightest KK mode is a dark matter candidate

- 6-Dimensional Standard Model
  - 3 generations of quarks and leptons are required for global $SU(2)_W$ anomaly cancellation
  - proton is long-lived due to 6D Lorentz invariance
  - neutrinos are special

- Test: look for Kaluza-Klein modes at colliders.