Lecture II:
Statistical Hadron Production from AGS to Collider Energies

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- Experimental Technique
- Hadron Production
- Statistical Concepts and Choice of Ensemble
- Description of Data
    1. Fixed Target Data
    2. RHIC Data
- Chemical Freeze-Out and the Phase Boundary
- Chemical Freeze-Out vis-a-vis Inital Condition and Thermal Freeze-Out
- Scenario of Chemical Equilibration
- Outlook and Open Questions

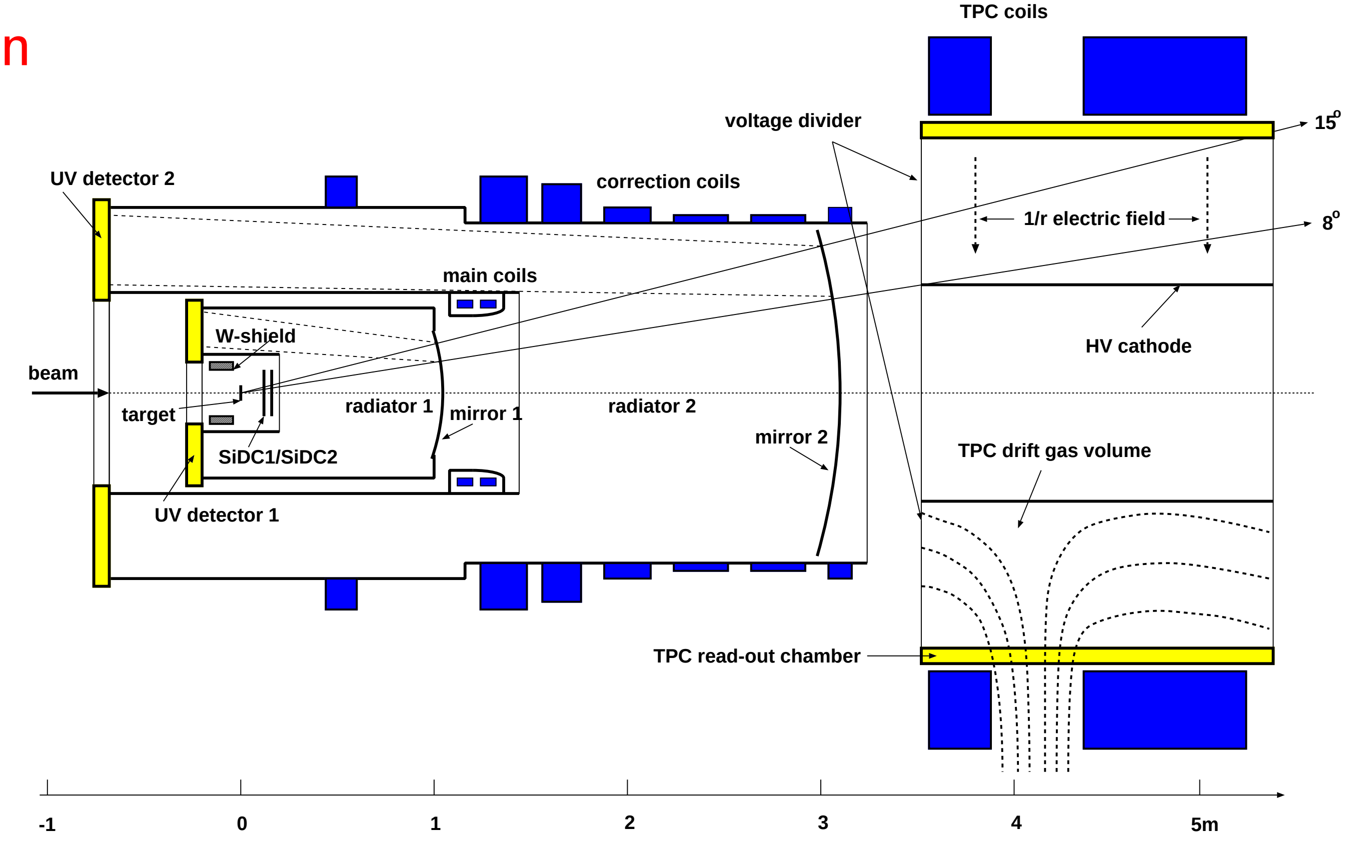
Experimental Challenge of High Energy Heavy Ion Experiments

- Very high multiplicities demanded new developments

1. Time Projection Chambers (TPC)
   developed to unprecedented performance
2. Silicon Pixel (and Drift) Detectors
   with large area and very fine granularity
3. Electron Identification
   in high hadron density environment (RICH and TRD development)

NA45/CERES Experiment at CERN SPS

Study of e^+e^- Production

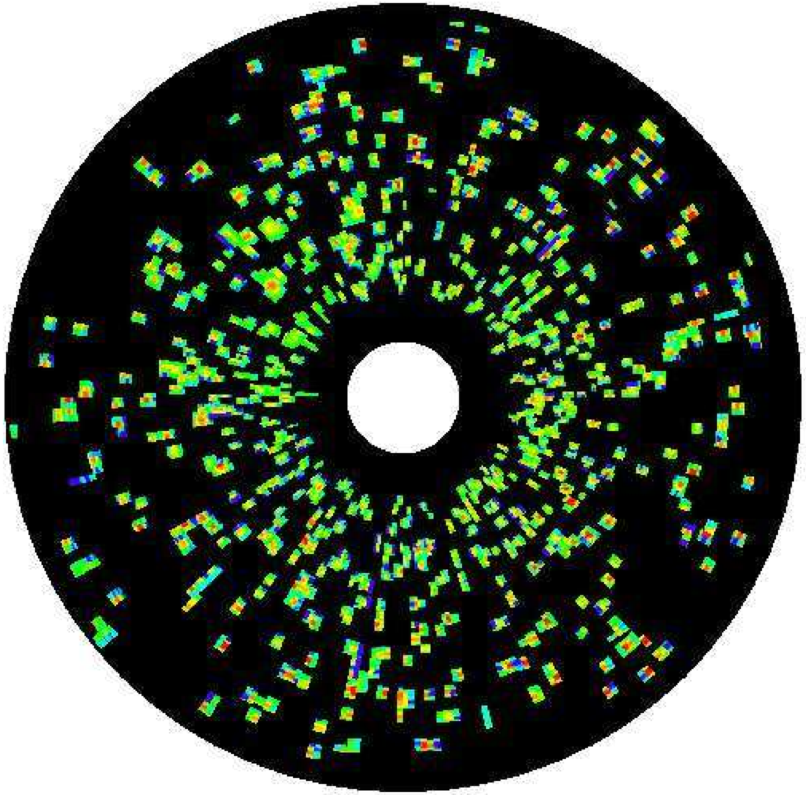


Since 1999:

also hadronic observables

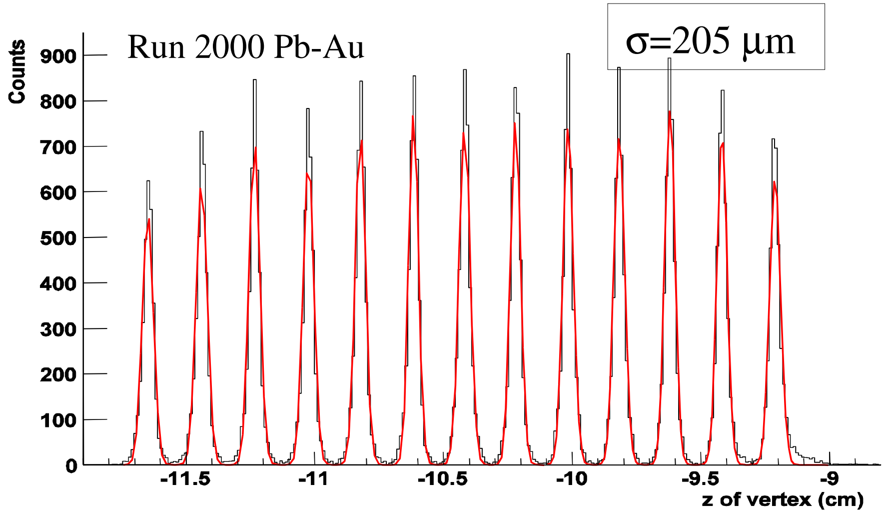
all novel detectors: 2 Si Drift detectors, 2 RICHes, large radial TPC

CERES Silicon Drift Detectors



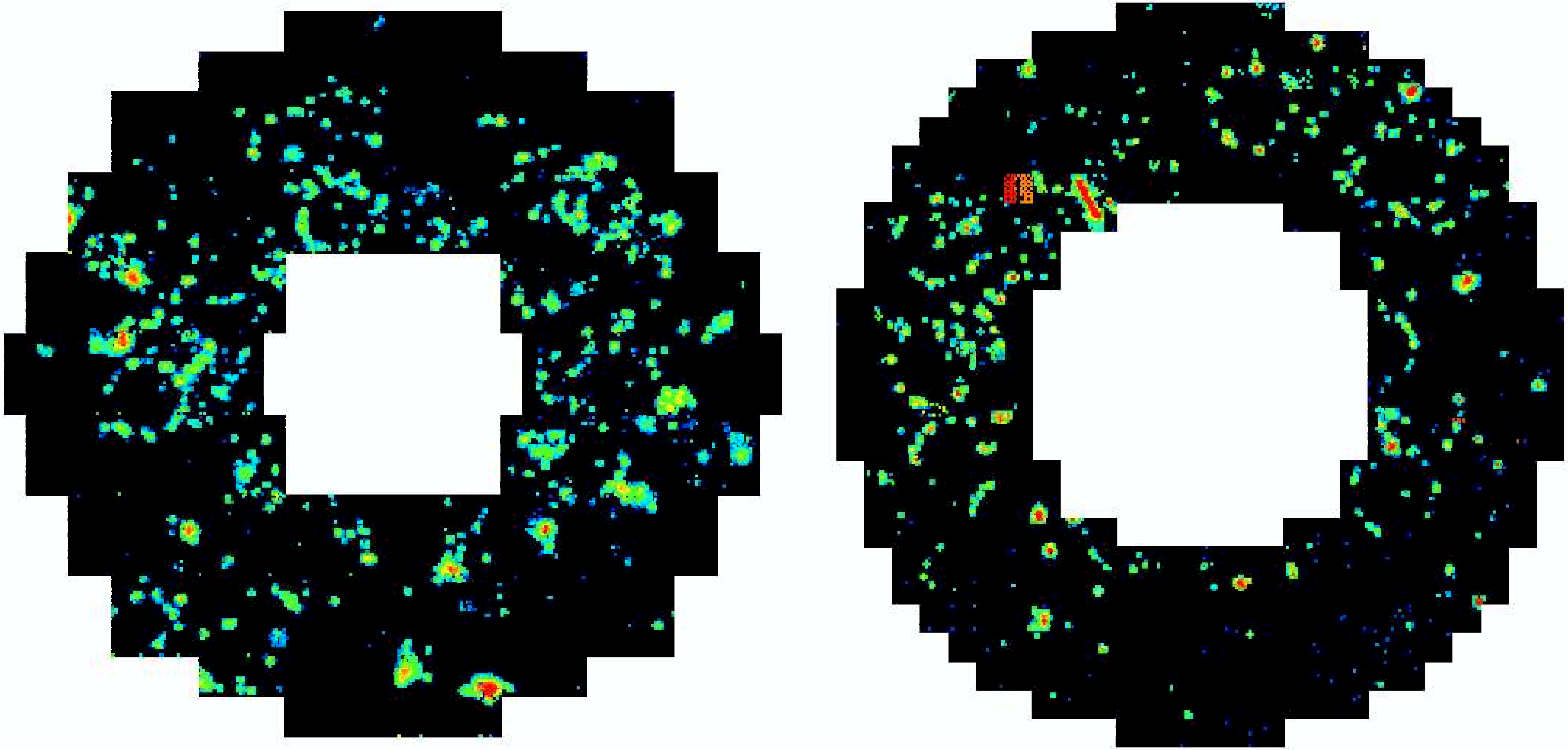
two 4” Silicon wafers

- charged particle tracking
- vertex reconstruction



combination of 2 or more:
form telescopes
in ALICE 6.7 m^2, same resolution
factor 400 in scale

CERES Ring Imaging Cherenkov Counters RICH1/2



electron identification via ring signature in focal plane
about 10 photons per electron ring – limitations in occupancy

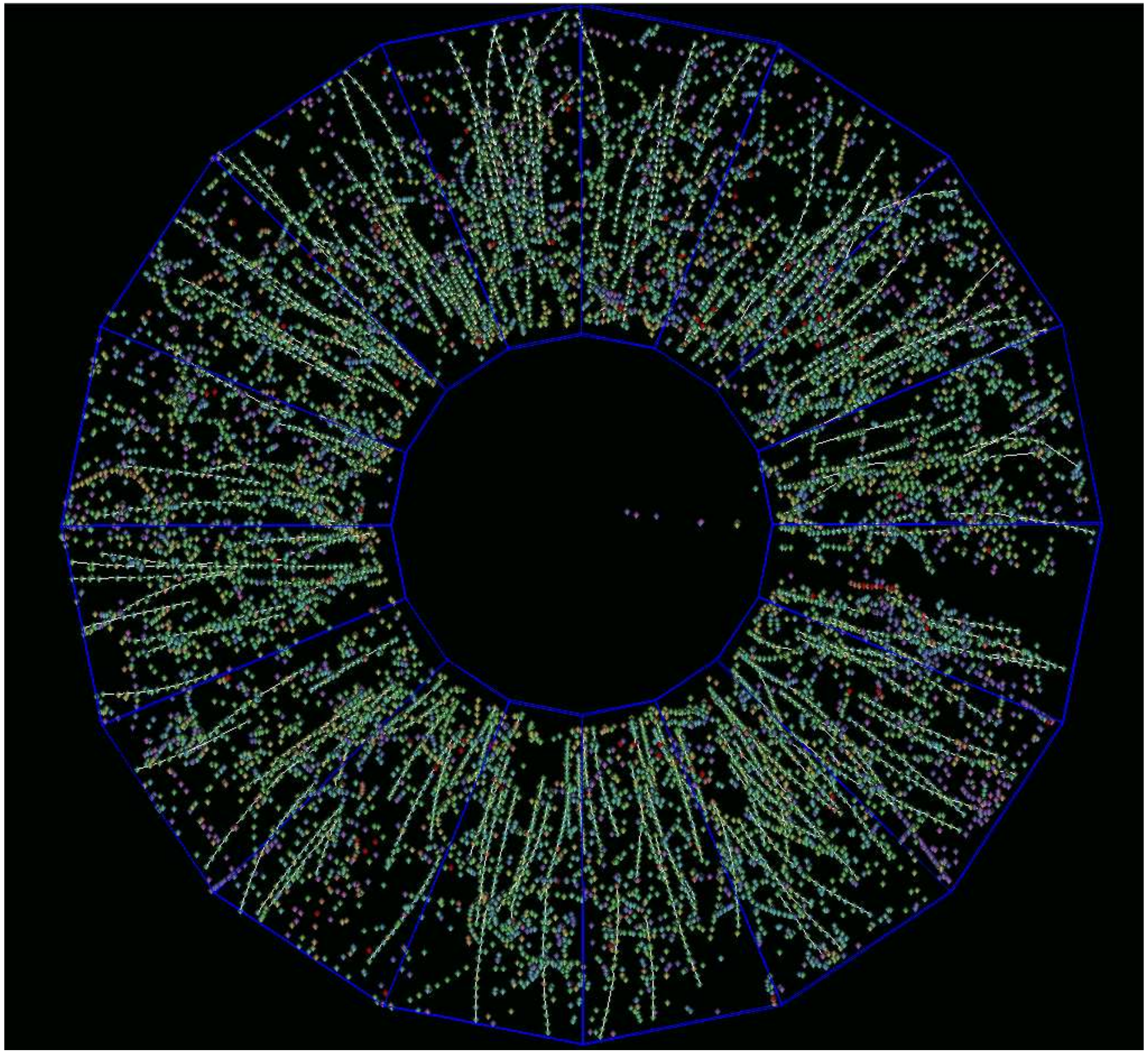
CERES Event Display

charged particle tracking

with TPC

10 m^3, 4·10^6 pixels

up to 400 charged particles



Time Projection Chamber (TPC) Principle

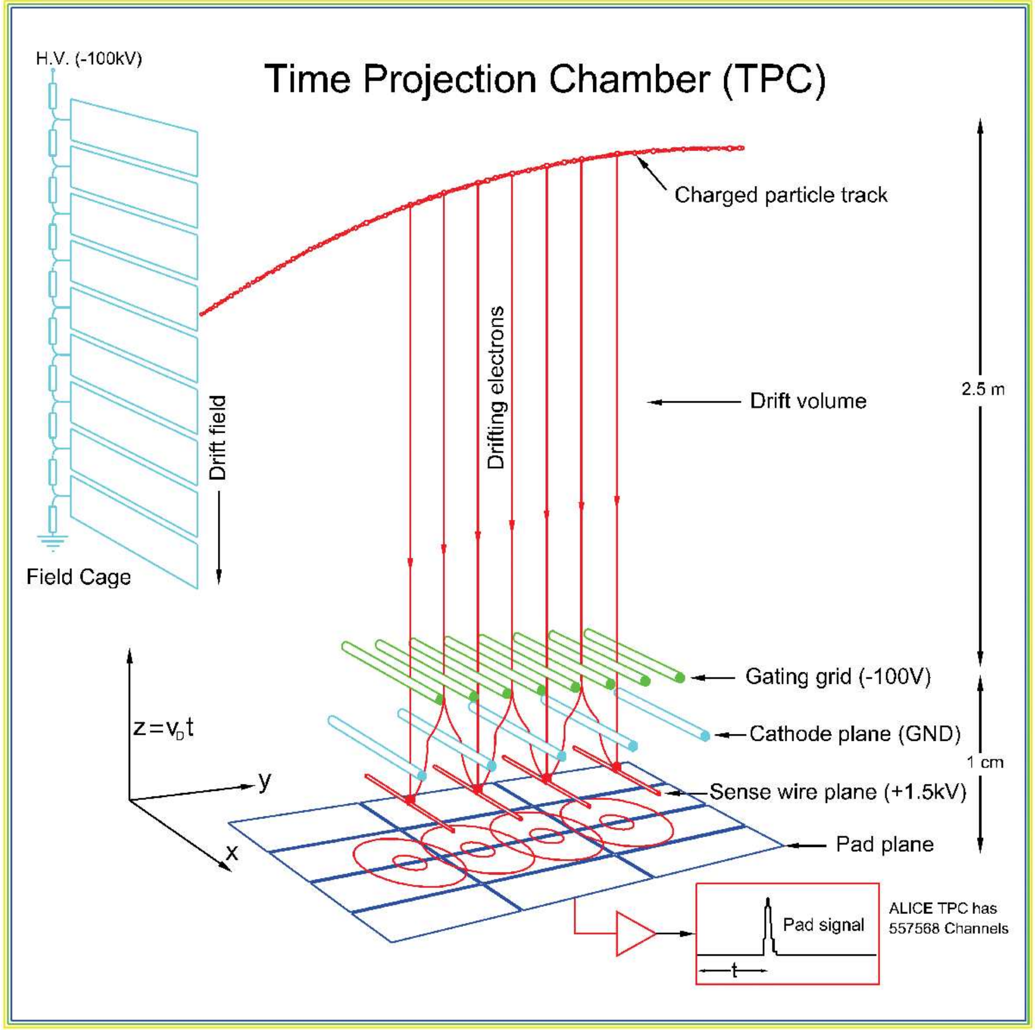
3-dimensional tracking

large volumes possible

90 m^3 and

$3\cdot10^8$ read out pixels

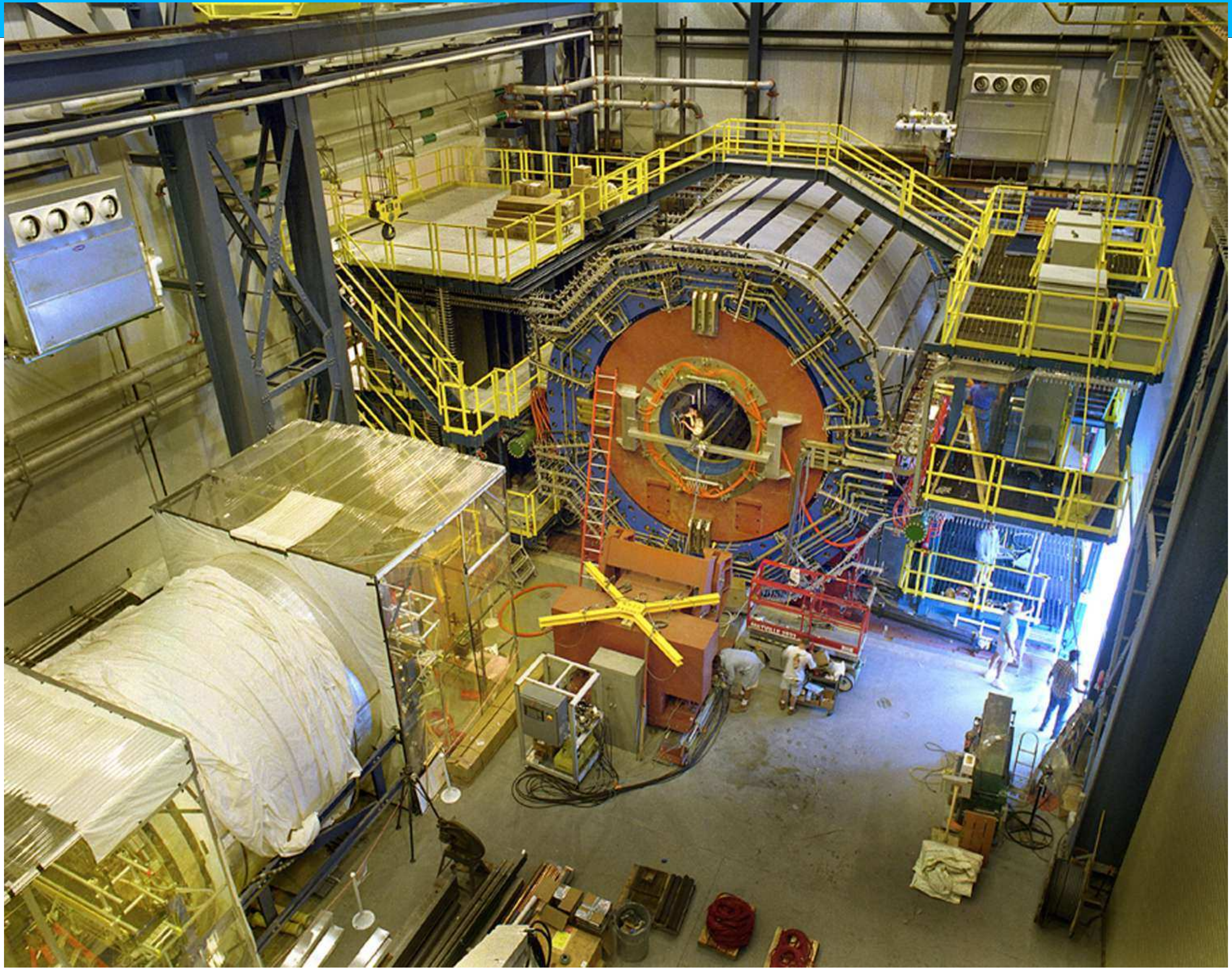
for ALICE TPC



information from drift time and

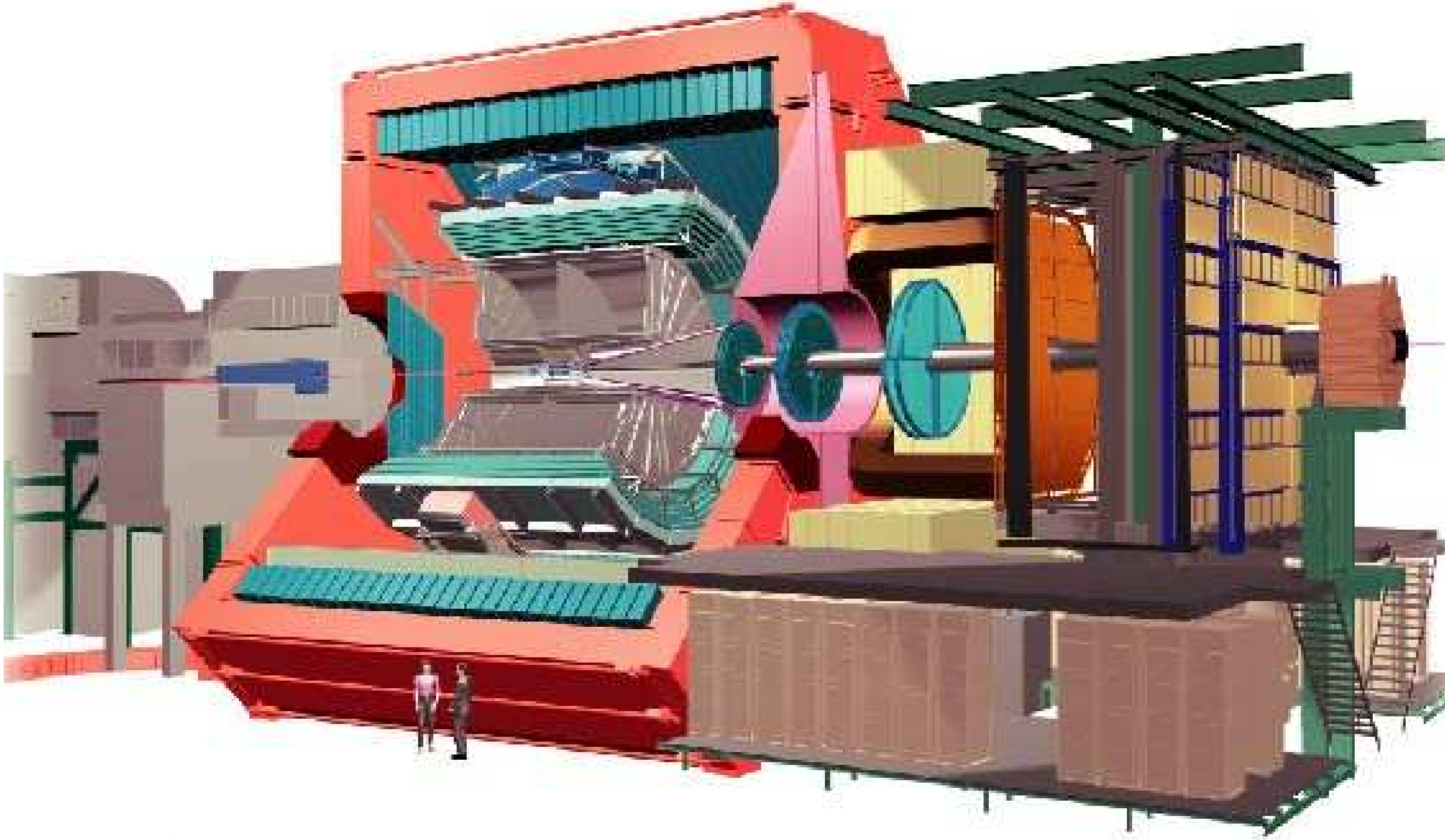
2-dimensional position measurement

STAR Experiment at RHIC



The ALICE Experiment at LHC

running from
2007

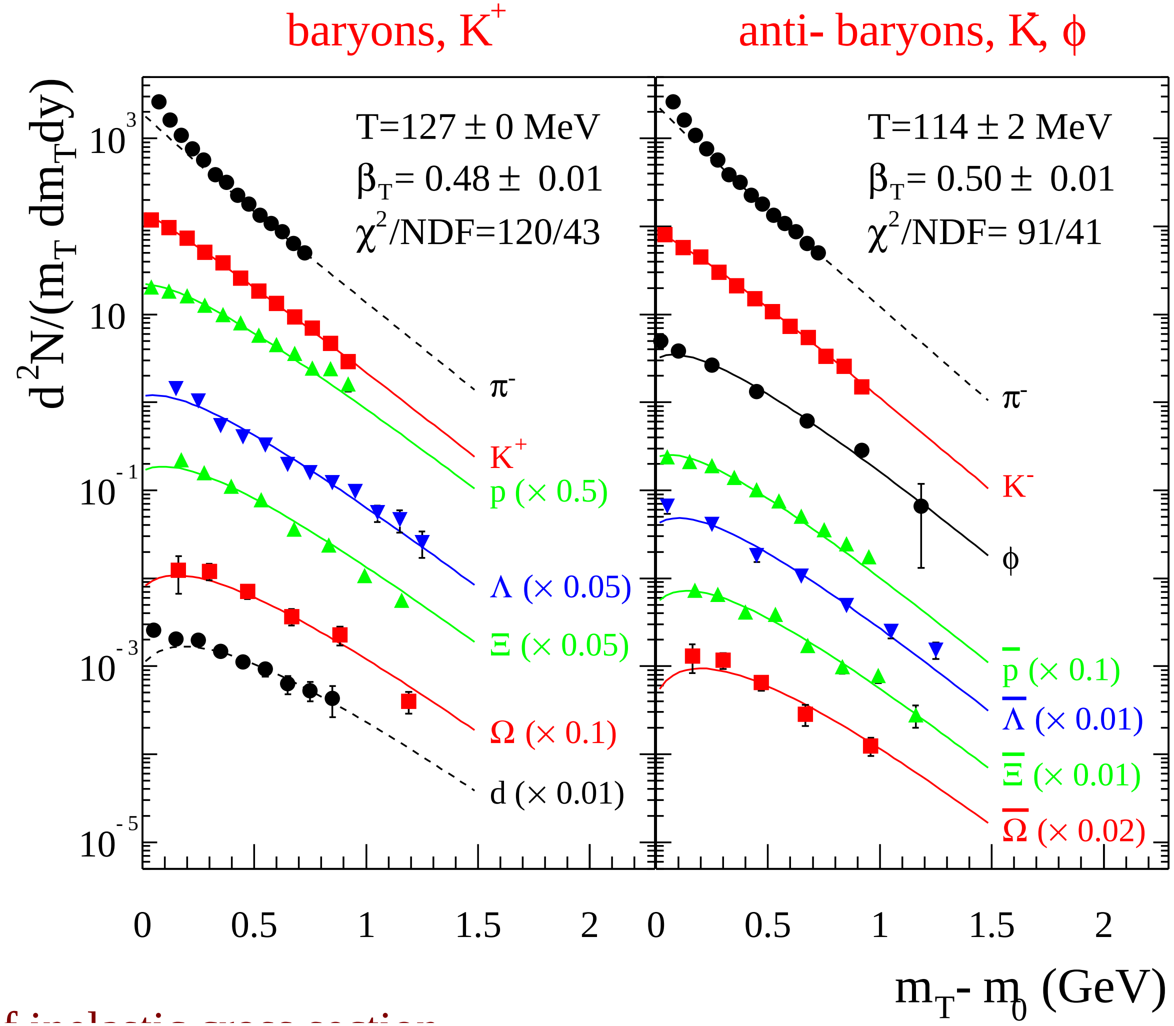


central barrel: ITS, TPC, TRD, TOF +
forward muon detector

The ALICE TPC field cage

Hadron Production - Example CERN SPS

NA49



of inelastic cross section

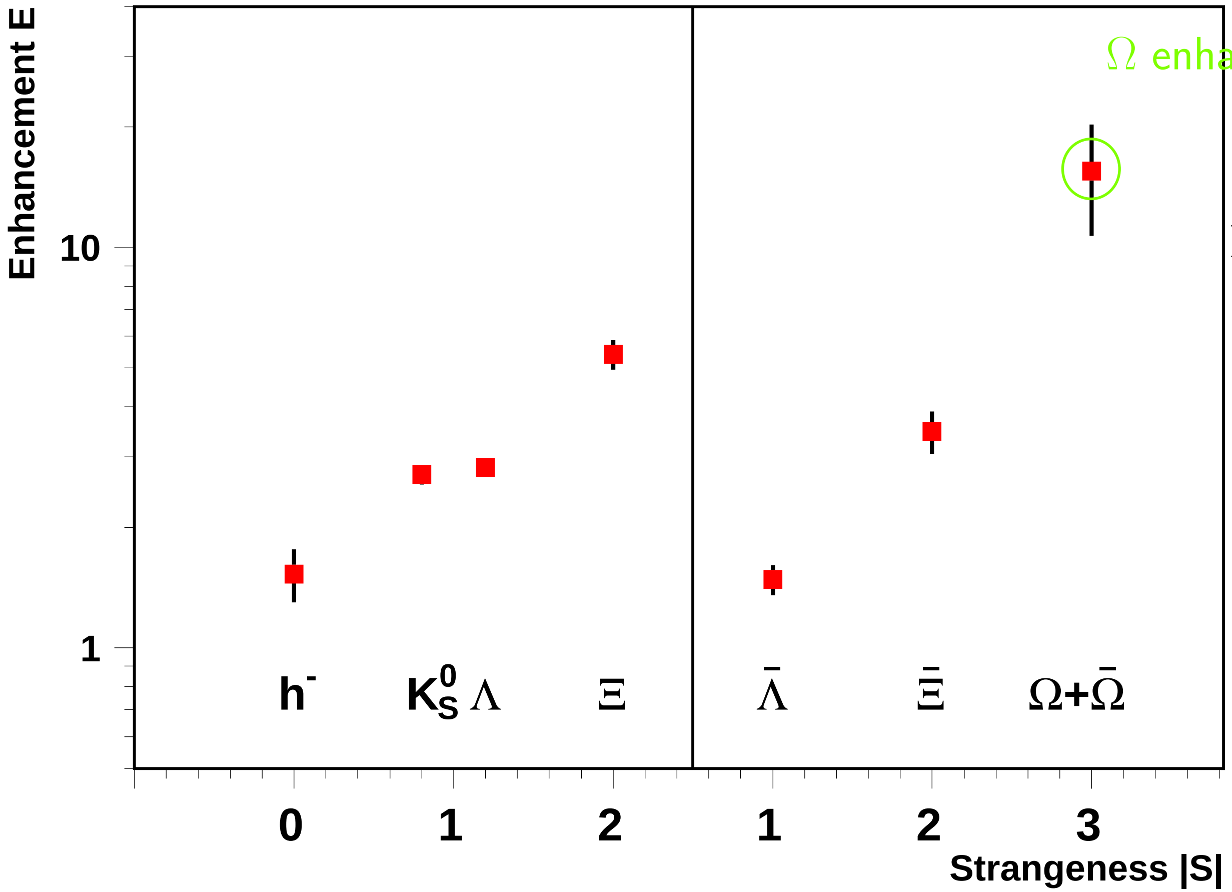
between 5 different experiments

a comprehensive data set for 158 A GeV/c Pb + Pb

Strangeness Enhancement

general feature: hadrons with s quarks enhanced in heavy ion collisions relative to pp

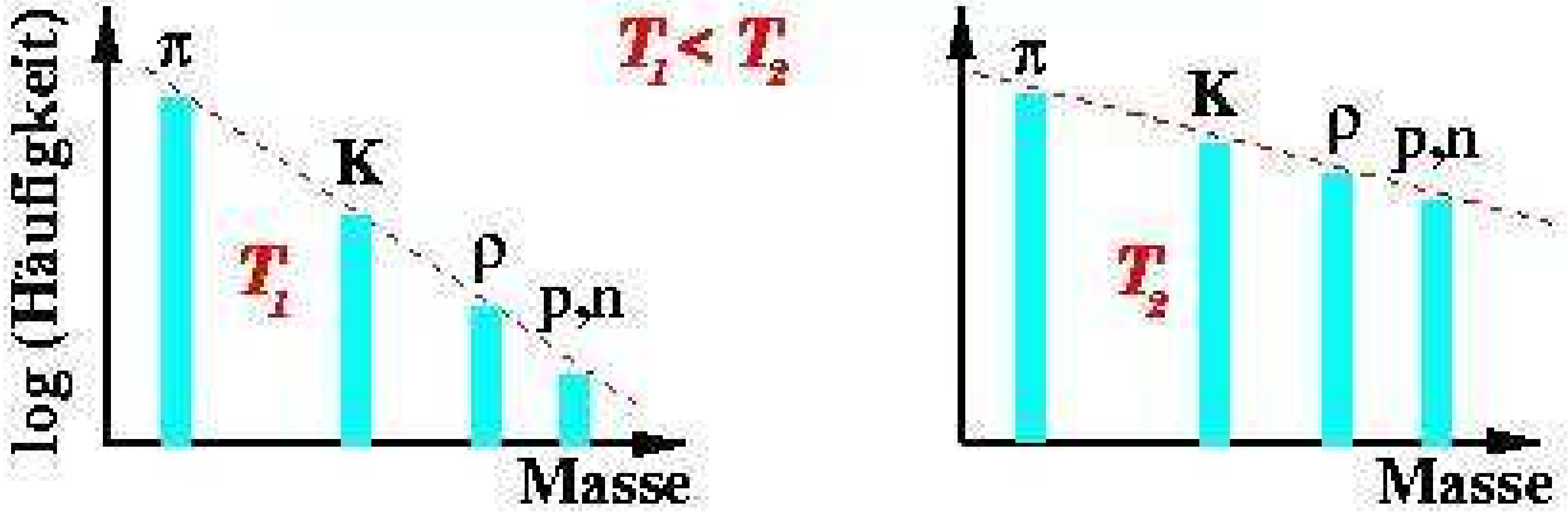
WA97, 158 A GeV/c Pb + Pb Collisions, Phys. Lett. **B449** (1999) 401



Ω enhanced factor 17 in central PbPb!

$$\text{Enhancement E} = \frac{\text{yield(PbPb)}/N_{\text{part}}(\text{PbPb})}{\text{yield(pBe)}/N_{\text{part}}(\text{pBe})}$$

Äquivalenz Energie <=> Masse



Häufigkeit ~ $m^{3/2}\, e^{-m/T}$

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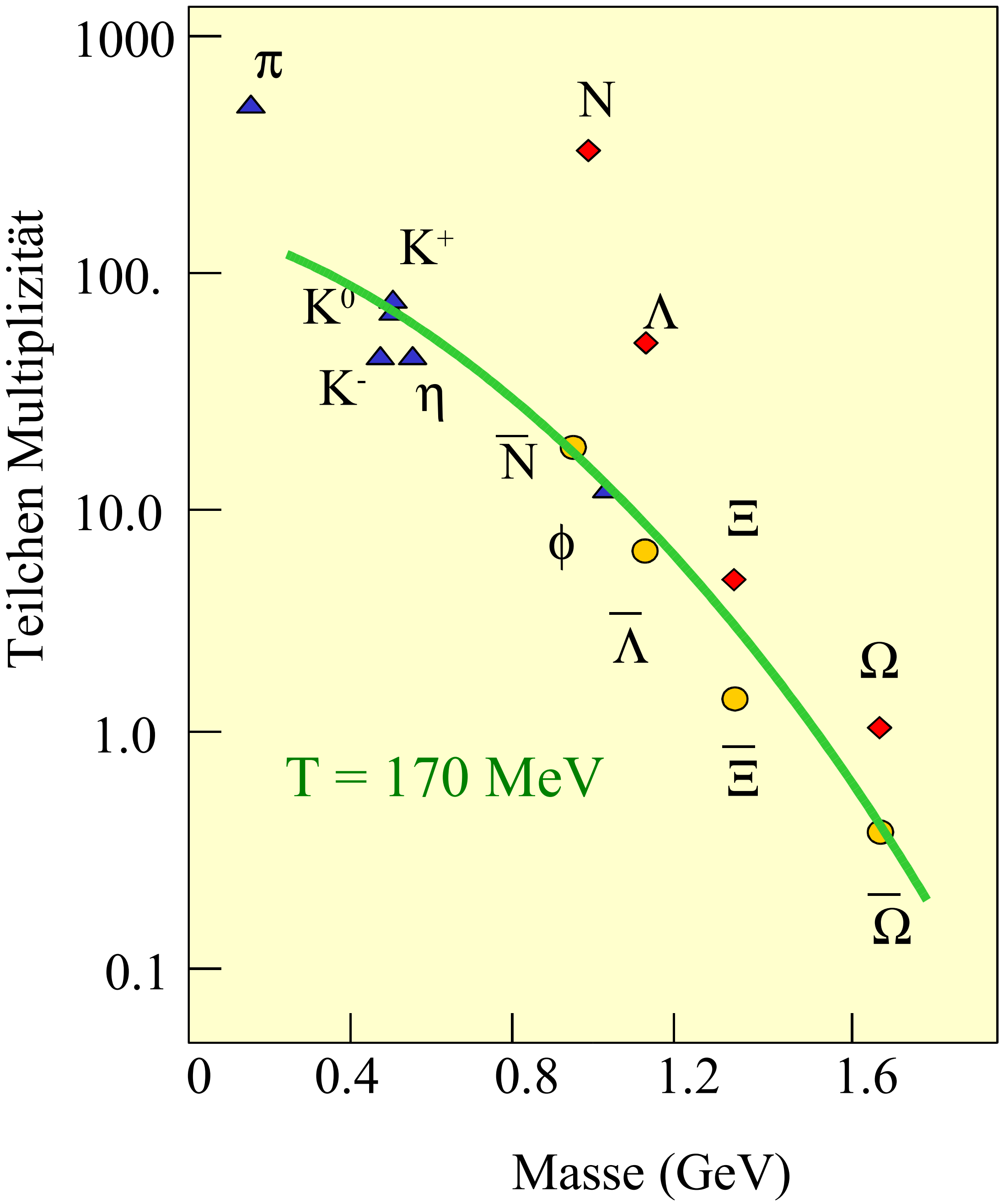


Die gemessene Teilchen-Multiplizität kann man verstehen, wenn alle Teilchen gemeinsam bei einer Temperatur von **170 MeV** produziert werden.

Choice of Statistical Ensemble

- Grand Canonical Emsemble (GC): in large system, with large number of produced particles, conservation
  of additive quantum numbers (B, S, I_3) can be implemented on average by use of chemical potential μ;
  asymptotic realization of exact canonical approach
- Canonical Ensemble (C): in small system, with small particle multiplicity, conservation laws must be
  implemented locally on event-by-event basis (Hagedorn 1971, Shuryak 1972, Rafelski/Danos 1980,
  Hagedorn/Redlich 1985)
  → severe phase space reduction for particle production "canonical suppression"
- C relevant in
  - low energy HI collisions (Cleymans/Redlich/Oeschler 1998/1999)
  - high energy hh or e^+e^- collisions (Becattini/Heinz 1996/1997)

Grand Canonical Ensemble

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T))$$

$$n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\, \mathrm{d}p}{\exp((E_i - \mu_i)/T) \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

for every conserved quantum number there is a chemical potential μ

but can use conservation laws to constrain:

- Baryon number: $V \sum_i n_i B_i = Z + N \quad \rightarrow V$
- Strangeness: $V \sum_i n_i S_i = 0 \quad \rightarrow \mu_S$
- Charge: $V \sum_i n_i I_i^3 = \frac{Z - N}{2} \quad \rightarrow \mu_{I_3}$

This leaves only μ_b and T as free parameter when 4π considered

for rapidity slice fix volume e.g. by dN$_{ch}$/dy

CERN SPS Data and Thermal Model Predictions

P. Braun-Munzinger, I. Heppe, J.Stachel, Phys.Lett.**B465** (1999) 15 + reanalysis in 2003 with more data

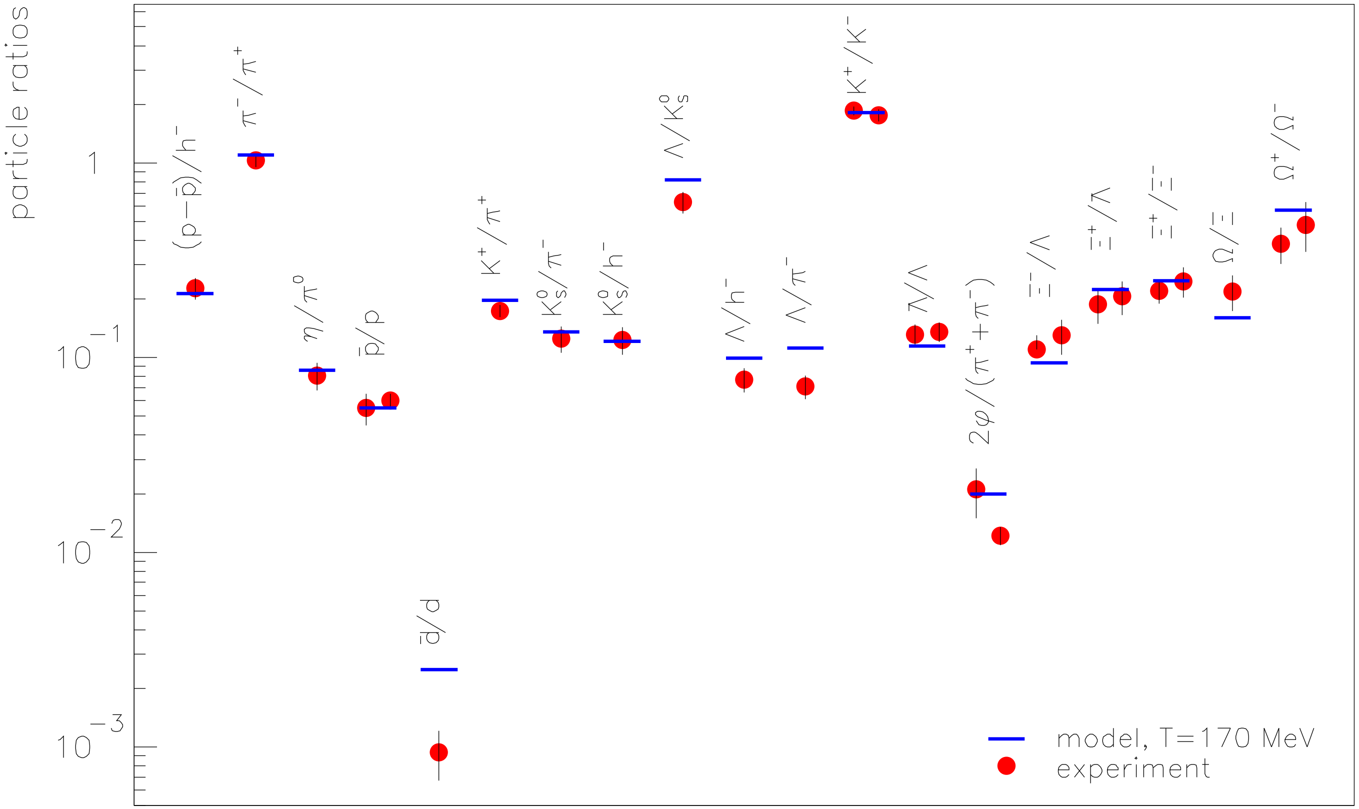
free parameters:
$\mathbf{T} = 0.170 \pm 0.005$ GeV
$\mu_b = 0.255 \pm 0.010$ GeV

fixed by conservation laws:
$\mu_s = 0.074$ GeV from $\Delta S=0$
$\mu_{I_3}=0.005$ GeV from $\Delta Q=0$

reduced χ^2 (excluding ϕ and d)
2.0

largest contribution:
Λ/π, Λ/h^-, Λ/K^0_s
weak feeding in
numerator/denominator
same?

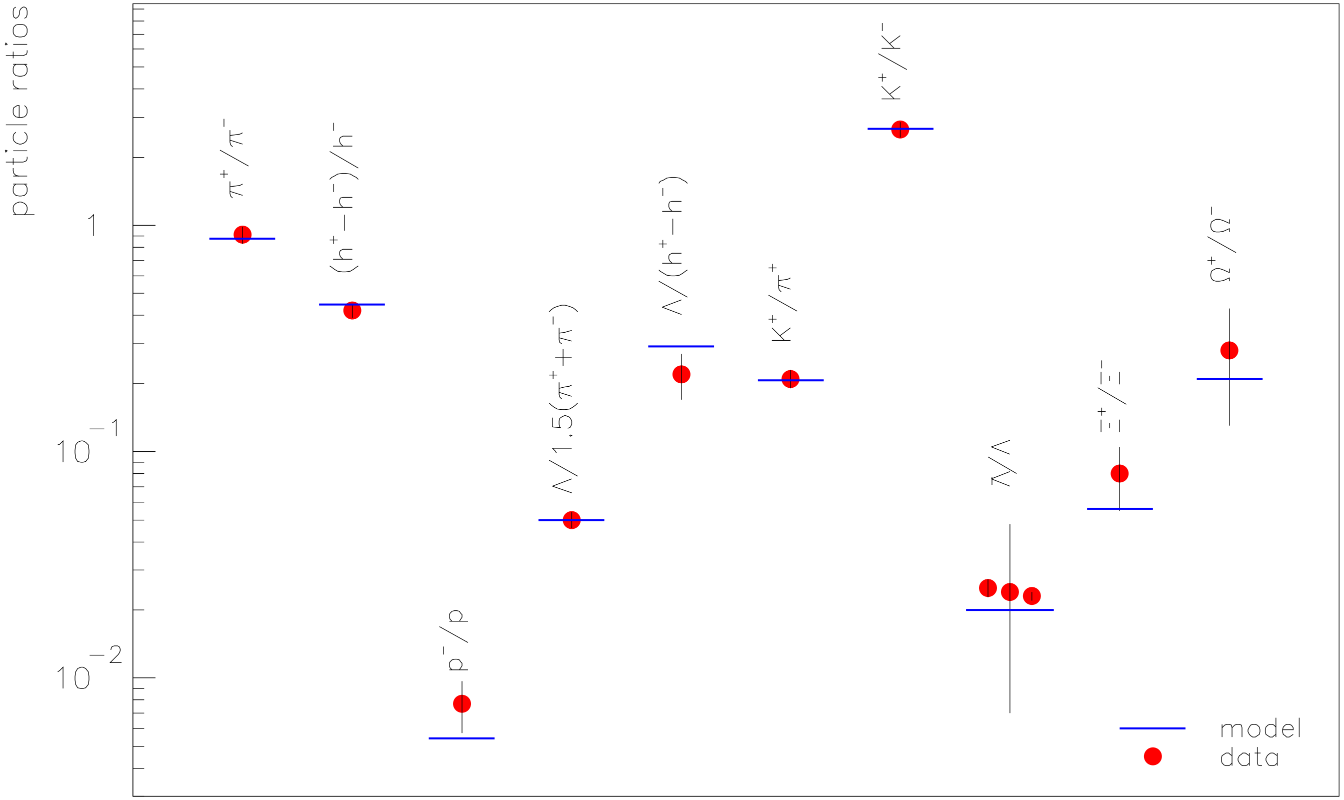
central 158 A GeV/c Pb + Pb collisions



Hadron Yields at SPS and Thermal Model

P. Braun-Munzinger, D. Magestro, J. Stachel, Dec. 02

central 40 A GeV/c Pb + Pb collisions - thermal model parameters: T = 148 MeV, μ_b = 400 MeV

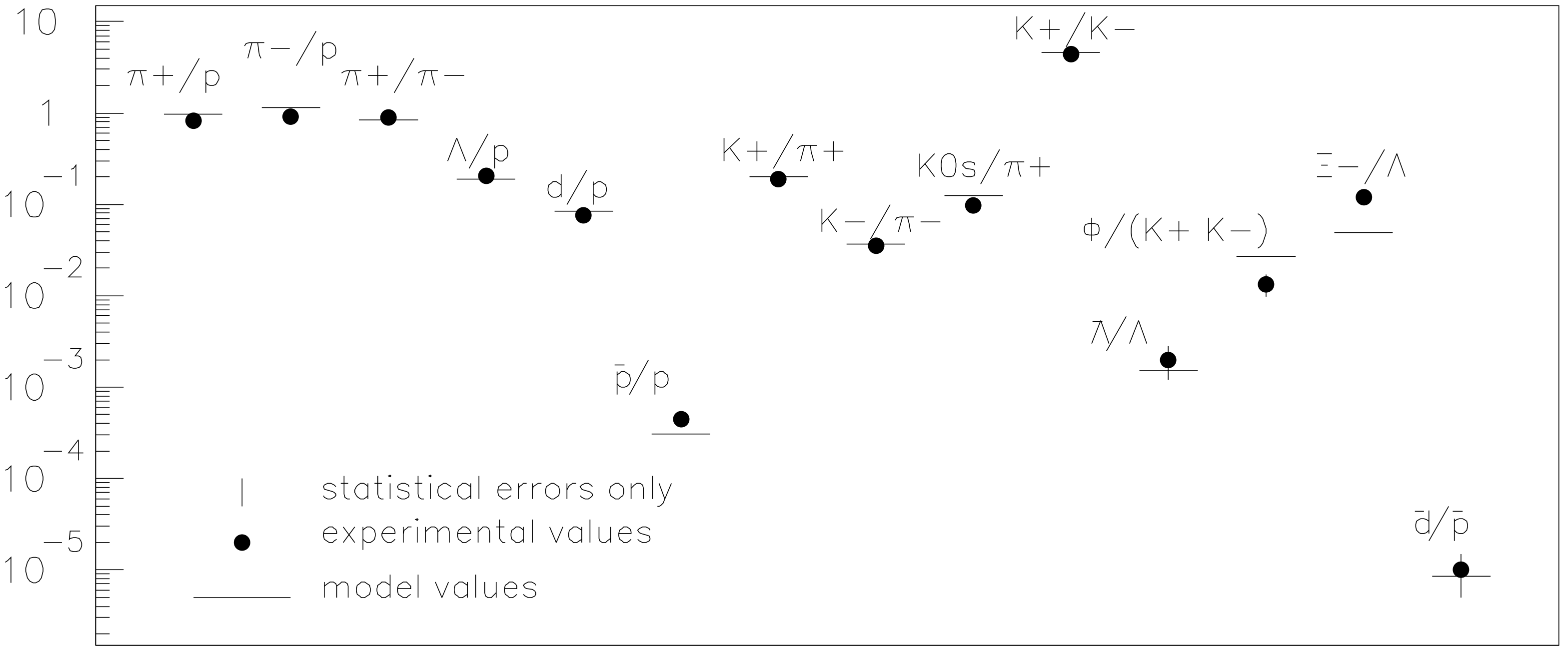


reduced $\chi^2 = 1.1$

Hadron Yields at AGS and Thermal Model

P. Braun-Munzinger, I. Heppe, J. Stachel, Phys. Lett. **B465** (1999) 5
and I. Heppe, Diploma thesis, U. Heidelberg 1998

central 14.6 A GeV/c Si + Au collisions
thermal model parameters: T = 125 MeV, μ_b = 540 MeV



yields for 11.5 A GeV/c Au + Au are very similar

Yields of Light Nuclei at AGS and Thermal Model

ddition of every nucleon → penalty factor R_p=48
but data are at very low p_t
$_t$ int. with A-dependent slope → R_p =26

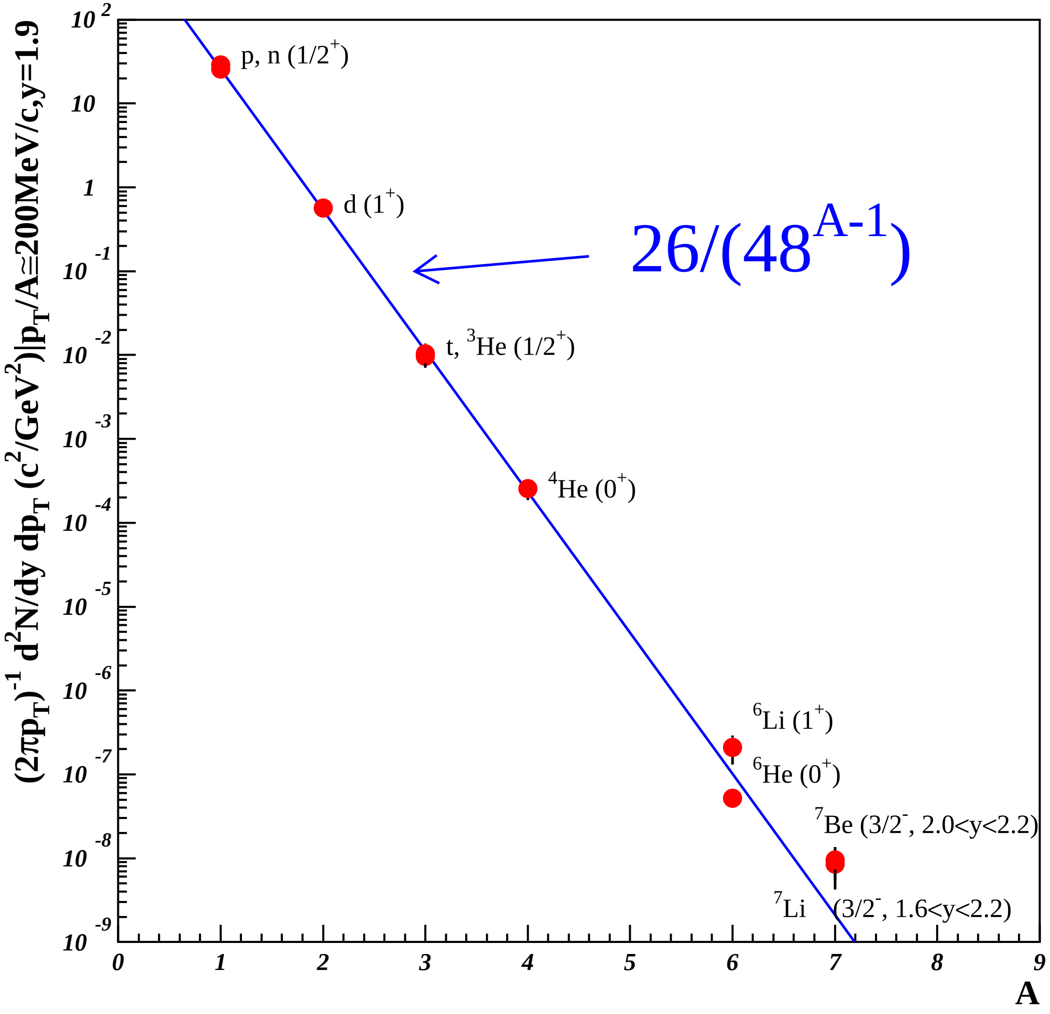
Grand Canonical Ensemble:
$R_p \approx \exp[(m_n \pm \mu_b)/T]$
for T=125 MeV and μ_b = 540 MeV
→ R_p= 23 good agreement!

also good for antideuterons
data: R_p=2±1 10^5 GC: R_p=1.3 10^5

P.Braun-Munzinger, J.Stachel
J.Phys.**G28**(2002)1971

Note: AGS may be special here since
chemical and thermal freeze-out coincide

E864 Collaboration, Phys. Rev. **C61** (2000) 064908

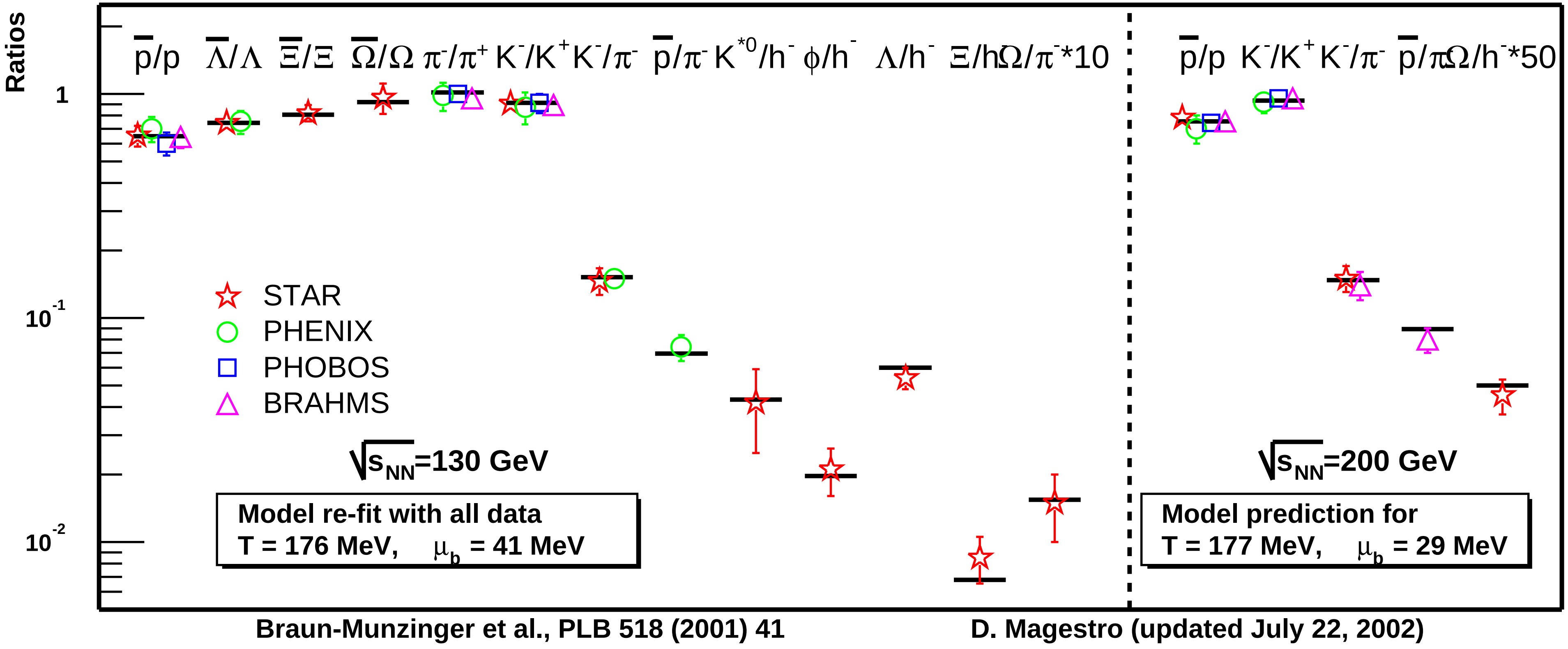


RHIC Data and Thermal Model

P.Braun-Munzinger, D. Magestro, K. Redlich, J.Stachel, Phys. Lett. B518 (2001) 41

central Au + Au collisions, data from all experiments combined

$\chi^2_r = 0.8$ $\chi^2_r = 1.1$



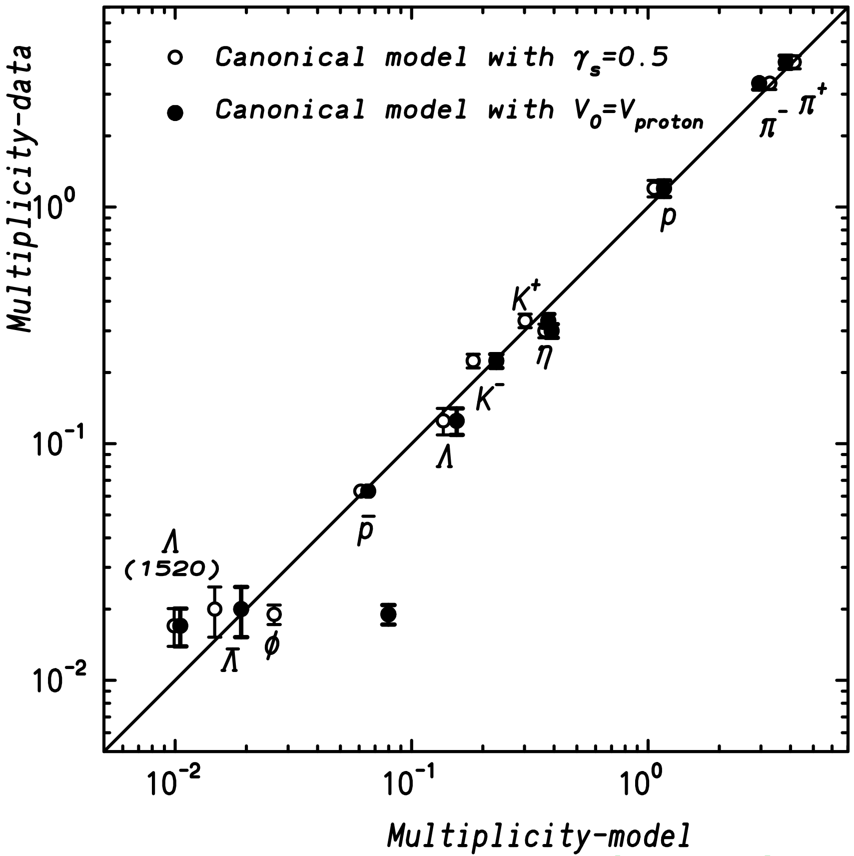
fit result confirmed by Becattini and Kaneta/Xu

Statistical model and pp data

F. Becattini, Z. Phys. C69 (1996) 485; F. Becattini and U. Heinz, Z. Phys. C76 (1997) 269

pp data, $\sqrt{s} = 27.6$ GeV

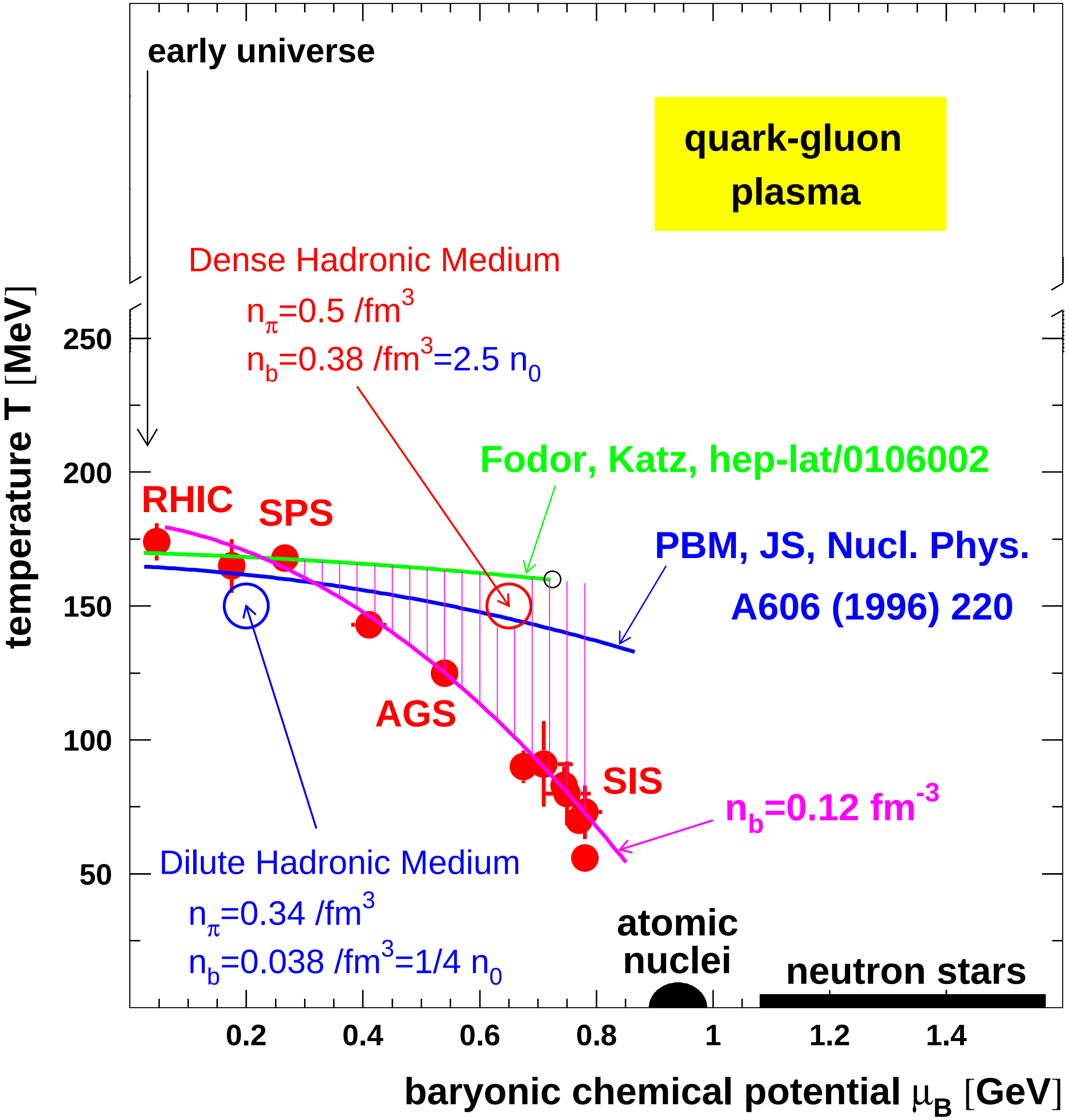
strangeness suppression factor γ_s introduced (non-equilibrium), T $=$ 165 MeV



ϕ discrepancy: $\longrightarrow$ non-equilibrium!

Phase Diagram of Nuclear Matter

1. hadron yields equilibrated
2. for full SPS energy and above: hadron yields frozen at phase boundary
3. how is equilibrium achieved? at SPS and RHIC not with hadronic cross sections → QGP much more efficient equilibrator



P. Braun-Munzinger, J. Stachel, J. Phys. G: Nucl. Part. Phys. 28 (2002) 1971

Estimates of Maximum Density from Data

1. from transverse energy distributions $\mathrm{d}E_T/\mathrm{d}\eta$

   $\longrightarrow$ max. energy density ϵ_{max}

   using Bjorken formula

   $$\epsilon = \frac{1}{A_\perp} dE_T/d\eta \; d\eta/dz$$

2. from net baryon rapidity distribution $\mathrm{d}N_{b-\bar{b}}/\mathrm{d}\eta$,

   $\longrightarrow$ max. baryon density $\mathrm{n}_{baryon}^{max}$

   again using Bjorken formula

   $$\mathrm{n}_{baryon} = \frac{1}{A_\perp} dN_{b-\bar{b}}/d\eta \; d\eta/dz$$

3. Jacobian $\mathrm{d}\eta/\mathrm{dz} = 1/\tau$

   maximal values at initial time given by $\tau \approx 1\mathrm{fm}$.

Estimate of Initial Conditions

energy and baryon density a la Bjorken:

$$\epsilon = \frac{1}{A_\perp}\frac{dE_t}{d\eta}\frac{d\eta}{dz} \qquad \text{and} \qquad \rho_b = \frac{1}{A_\perp}\frac{dN_b}{d\eta}\frac{d\eta}{dz}$$

$d\eta/dz$ typically 1 fm for AGS and SPS

- **AGS 11 A GeV/c Au+Au**

$$dE_t/d\eta = 200\text{GeV} \quad dN_b/d\eta = 150$$

$\rightarrow \epsilon = 1.4\ \text{GeV/fm}^3$ **and** $\rho_b = 1.0/\text{fm}^3 \approx 6\rho_0$

T_i = 170 MeV

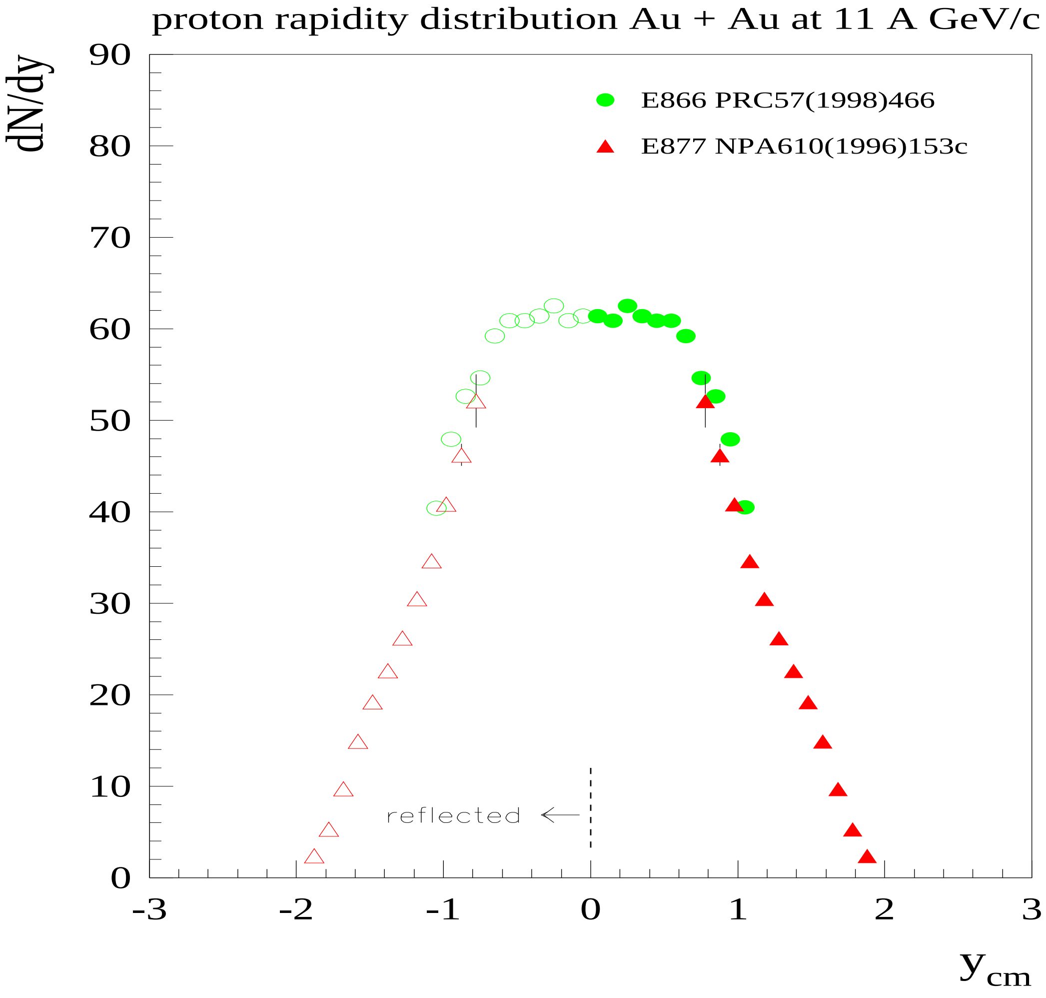
- **SPS 158 A GeV/c Pb+Pb**

$$dE_t/d\eta = 450\text{GeV} \quad dN_b/d\eta = 80$$

$\rightarrow \epsilon = 3\ \text{GeV/fm}^3$ **and** $\rho_b = 0.5/\text{fm}^3 \approx 3\rho_0$

T_i = 210 MeV

$p = 0.7\ \text{GeV/fm}^3 = 10^{35}$ Pa



initial energy density from transverse energy

from transverse energy rapidity density using Bjorken formula:

$\varepsilon_0 = dE_t/d\eta/(\tau_0 \pi R^2)$

PHENIX & STAR central Au-Au collisions: $dE_t/d\eta \approx 600$ GeV

(nucl-ex/0407003 and nucl-ex/0409015)

conservatively: $\tau_0 =$ 1 fm/c → $\varepsilon_0 = 5.5\ \mathrm{GeV/fm^3}$

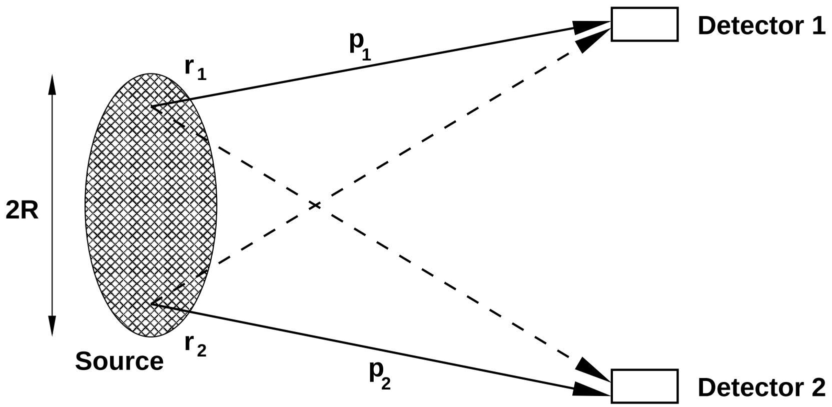
(factor 2 higher than at SPS top energy)

$\tau_0 = 1/Q_s = 0.14$ fm/c → $\varepsilon_0 = 40\ \mathrm{GeV/fm^3}$

in any case this appears significantly above critical energy density from lattice QCD of $0.7\ \mathrm{GeV/fm^3}$

Pion HBT interferometry

When phase space volume smaller than $\Delta p_x \Delta x \approx \hbar$ is considered, chaotic system of identical non-interacting particles exhibits quantum fluctuations following Bose-Einstein or Fermi statistics

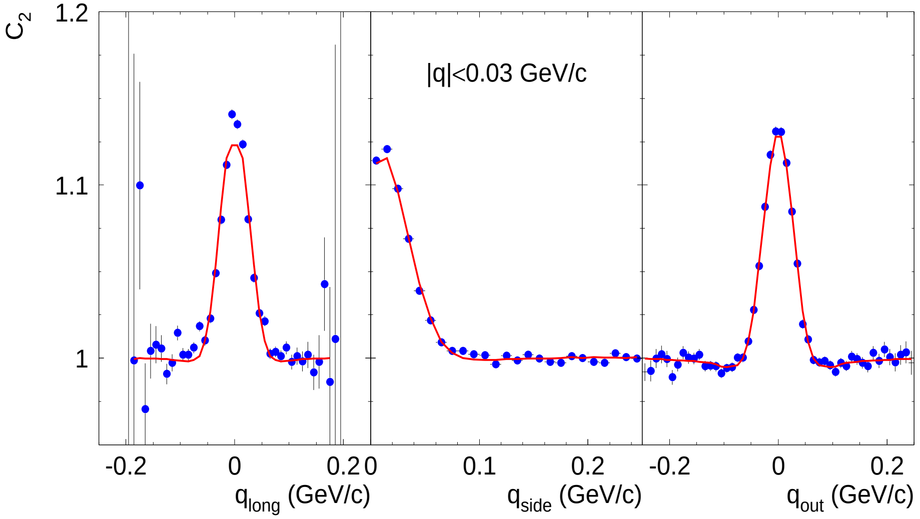


First application in astrophysics (Hanbury Brown and Twiss) → size of stars

$$C_2 \propto \frac{P_2(\vec{p_1},\vec{p_2})}{P_1(\vec{p_1})P_2(\vec{p_2})} = 1 \pm \chi(\vec{p_1}-\vec{p_2})$$

$$\Delta r = \frac{\hbar c}{q} = \frac{197\mathrm{MeV}}{q}\mathrm{fm}$$

in heavy ion physics typical dimensions 1-10 fm → momentum differences of 20-200 MeV/c



more complications, but also more information for non-static source:
duration of emission, space-momentum correlations due to expansion, strong & EM interaction, decays of resonances ...
measure C_2 as function of
Δp_x, Δp_y, Δp_z for all y, p_t, m

Longitudinal expansion

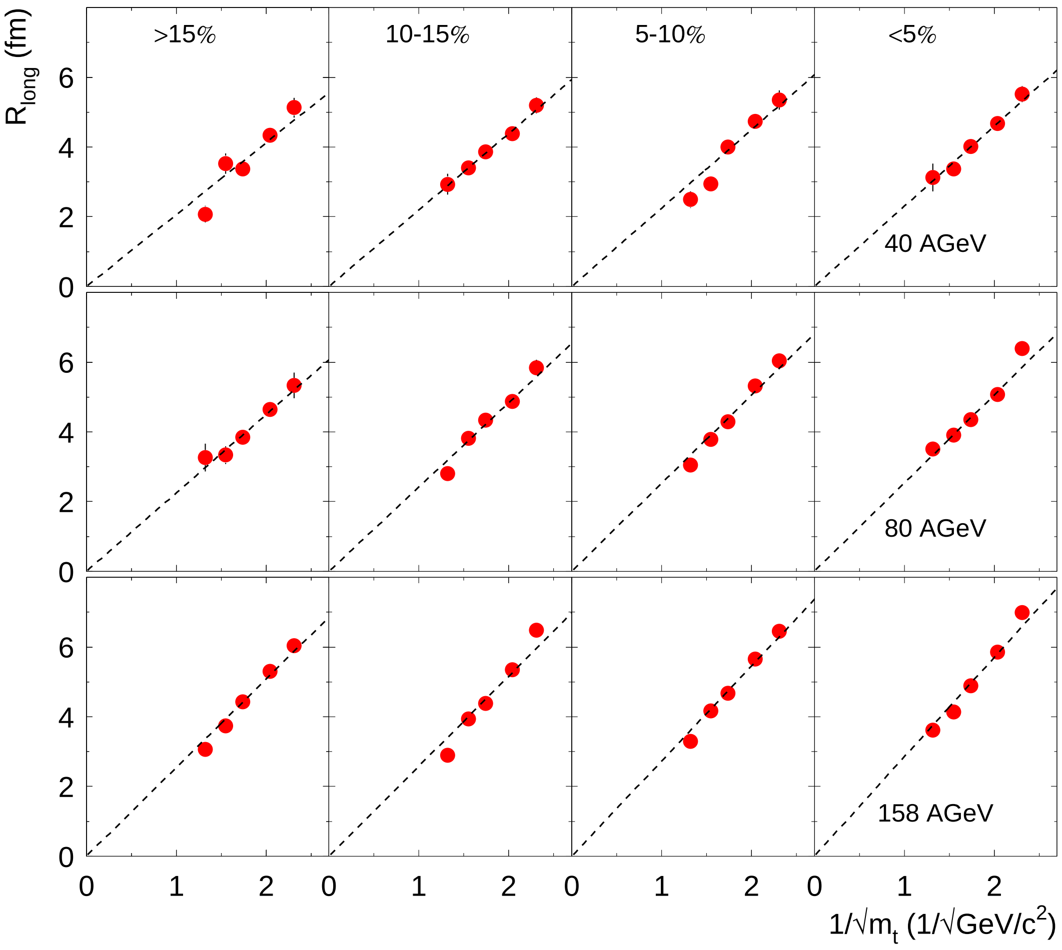
Duration of expansion (lifetime) τ **of the system can be estimated from the transverse momentum dependence of R_{long}:**

$$R_{\text{long}} \approx \tau \cdot \sqrt{\frac{T_f}{m_t}} \quad \textbf{Y. Sinyukov}$$

$\Rightarrow$

$$\tau = 6.5 - 8 \ \textbf{fm/c} \quad \textbf{for } T_f = 120 \ \textbf{MeV}$$

● CERES collaboration, Nucl.Phys.**A714**(2003)124



Transverse expansion

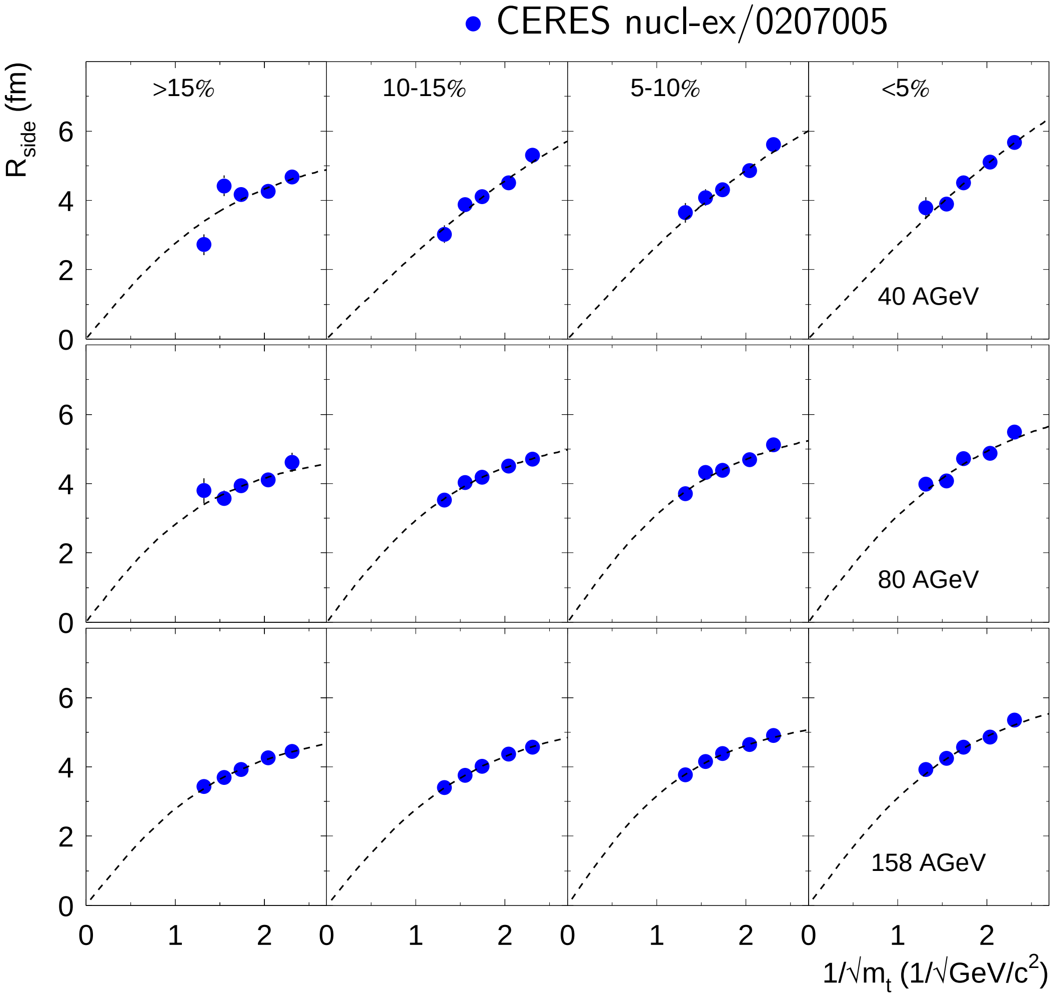
Transverse momentum dependence of R_{side} allows determination of geometric source size R_{geo} and
average transverse flow velocity β_t:

$$R_{\text{side}} \approx R_{\text{geo}}/(1 + m_t \cdot F(T_f, \beta_t))^{\frac{1}{2}}$$

U. Heinz *et al.*

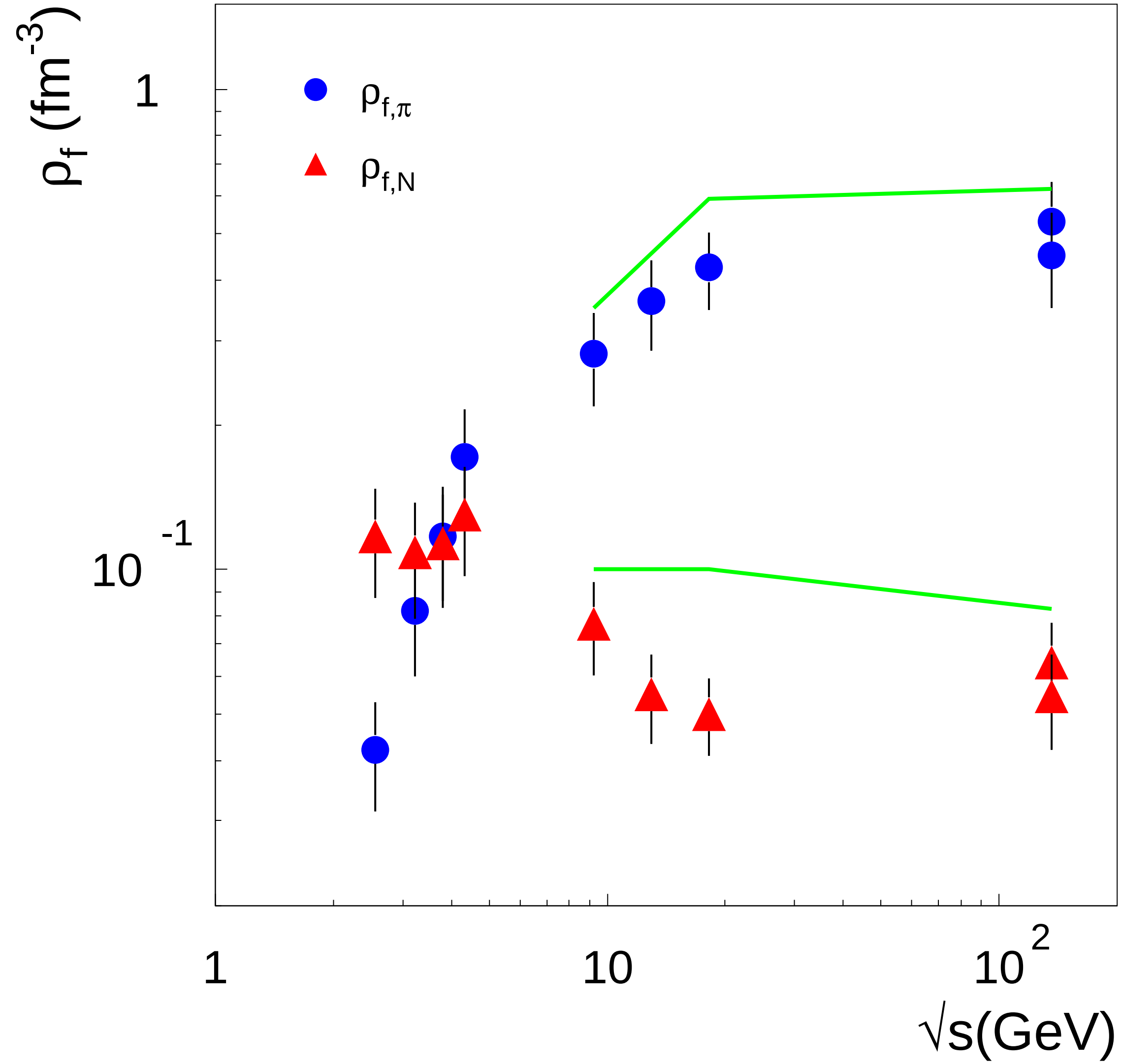
$\Rightarrow$

$$\beta_t \approx 0.55 \text{ for } T_f = 120 \text{ MeV}$$



Freeze-Out Density from Pion HBT

HBT gives density at thermal freeze-out



← pion density at chemical freeze-out

← pion density at thermal freeze-out

← nucleon density at chemical freeze-out

← nucleon density at thermal freeze-out

1/2 - 1/3 ρ_o

Volume appears to only grow 30 % between chemical and thermal freeze-out!

Rapid Equilibration at $\mathrm{T}_{ch} \approx \mathrm{T}_c$

P.Braun-Munzinger, J.S., C. Wetterich, nucl-th/0311005

- Ω baryons with 3 strange quarks enhanced by factor 16 as compared to p p collisions
- how to get them into equilibrium??
- successive 2-body collisions much too slow (consensus in literature)
- multi-particle reactions:
- rate of change of density for n_{in} ingoing and n_{out} outgoing particles

$$r(n_{in}, n_{out}) = \bar{n}(\mathrm{T})^{n_{in}} |\mathcal{M}|^2 \phi$$

with

$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^{\mu} \right)$$

- The phase space factor ϕ depends on $\sqrt{s}$
  needs to be weighted by the probability $f(s)$ that multiparticle scattering occurs
  at a given value of $\sqrt{s}$

Multi-particle Reactions to Achieve Equilibrium Densities

reaction $2\pi + 3K \rightarrow \Omega + \bar{N}$

$\Rightarrow$ can achieve final density starting from 0 in 2.2 fm/c at T=176 MeV!

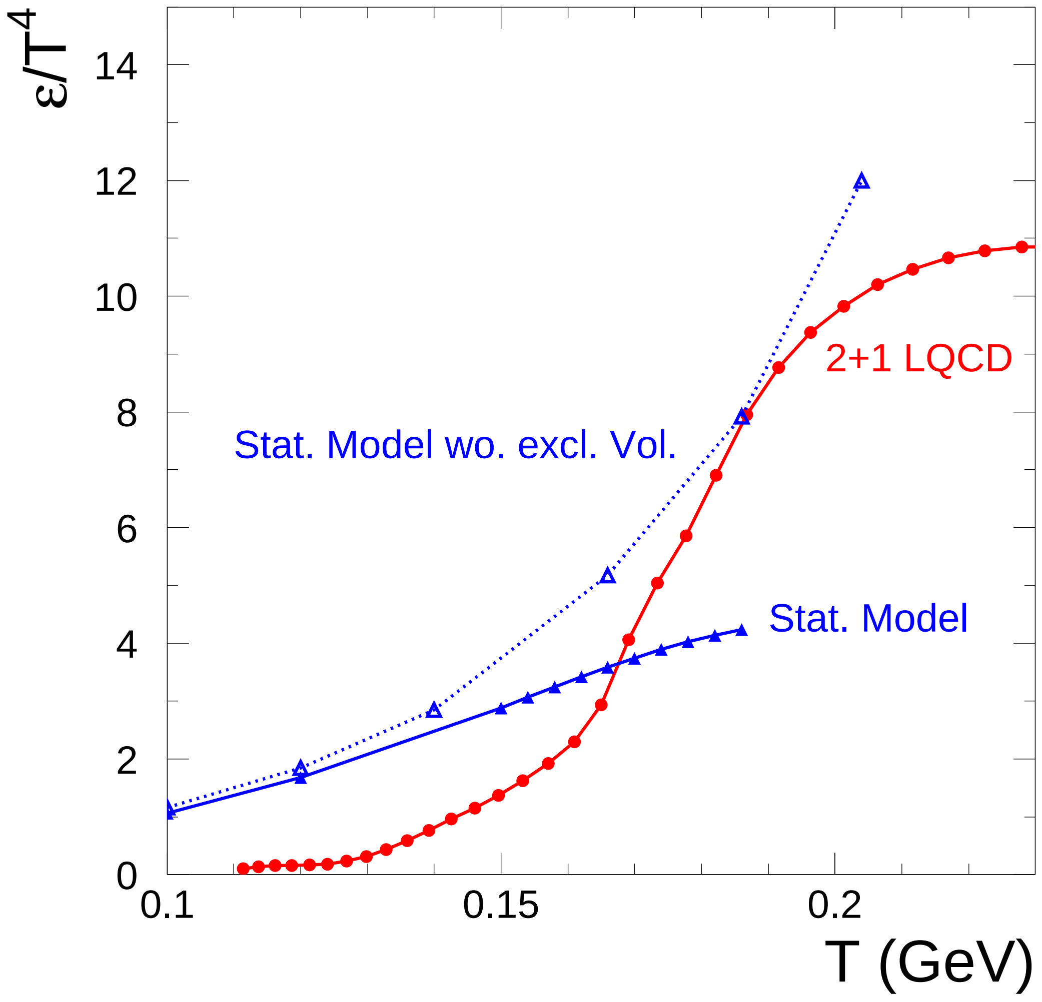
similarly one obtains

for $3\pi + 2K \rightarrow \Xi + \bar{N}$ $\tau_{\Xi} =$ 0.71 fm/c

and

for $4\pi + K \rightarrow \Lambda + \bar{N}$ $\tau_{\Lambda} =$ 0.66 fm/c

Rapid Density Growth near T_c

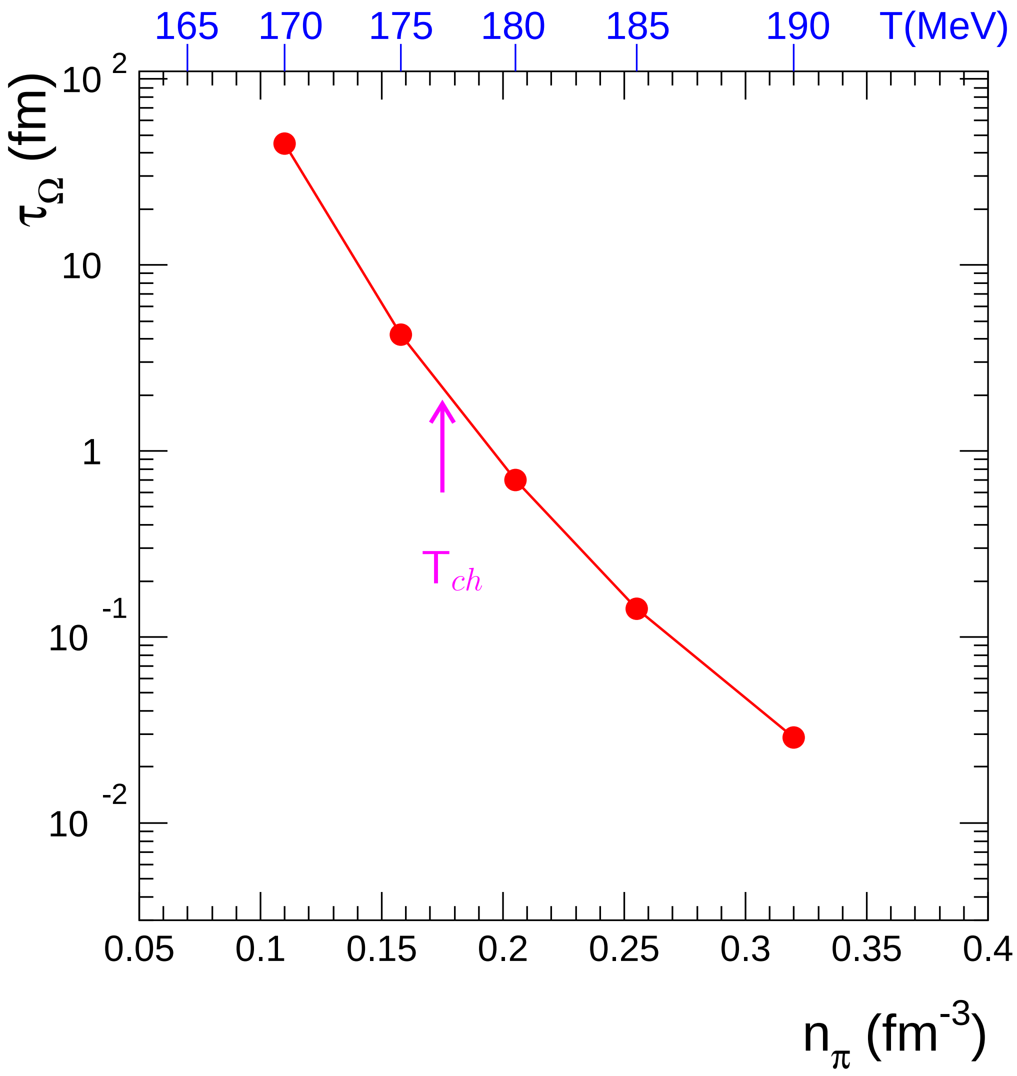


Karsch et al.

at T_c very large increase in energy density and particle density due to increase in degrees of freedom in QGP

Super-rapid Equilibration near T_c

both densities and phasespace very sensitive to T



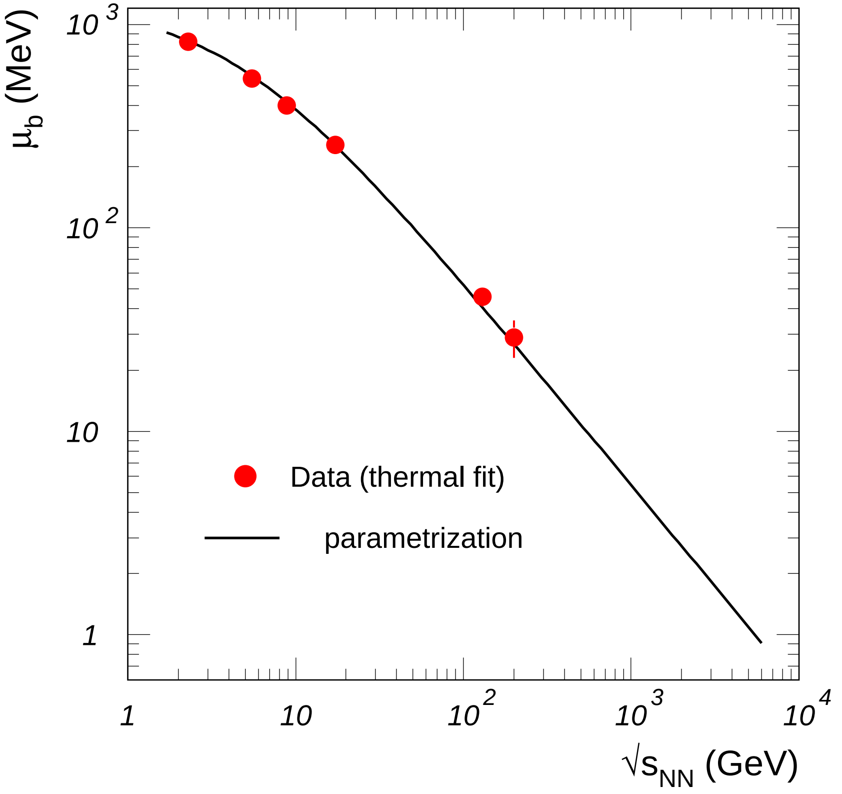
increase ρ_π by 1/3: $\tau_\Omega = 0.2$ fm/c

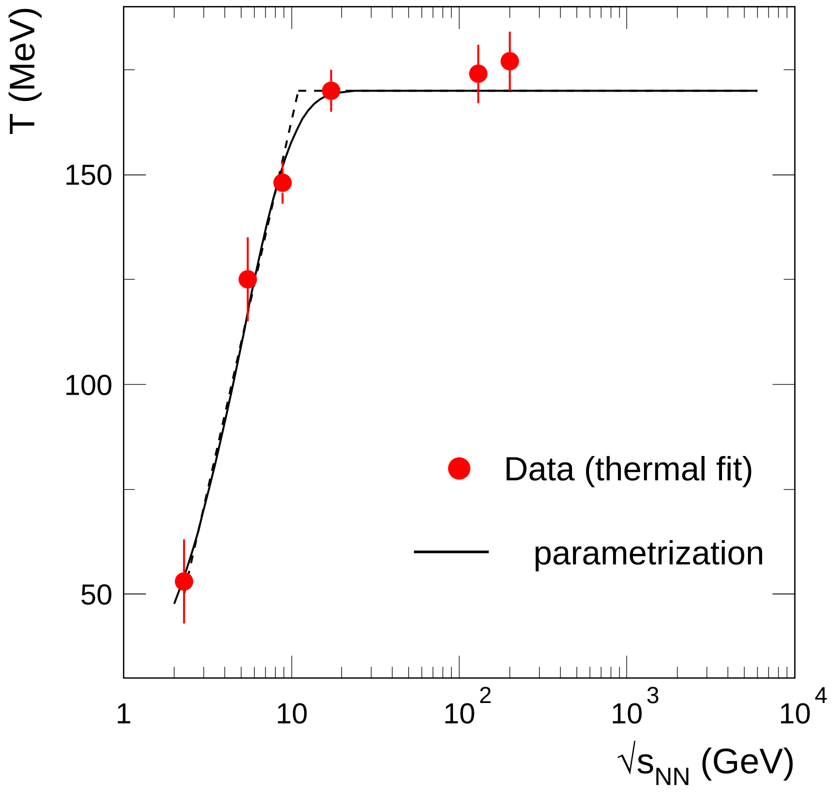
decrease ρ_π by 1/3: $\tau_\Omega = 27$ fm/c

$\tau_\Omega \propto T^{-60}$!

within very narrow interval of T all hadron yields can thermalize

Evolution of Thermal Parameters with $\sqrt{s}$





with increasing beam energy μ_b decreases and T saturates at T_c