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# Front-end Electronics and Signal Processing - I

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# Introduction – I

Detector signal processing is the set of operations to be performed on the current pulse delivered by the detector in order to extract the information about:

✓ The ***energy released by the radiation*** in the sensitive volume of the detector ⇒ *spectroscopy measurements*

✓ The ***time of occurrence*** of the interaction

⇒ *timing measurements*

✓ The ***position where, in a segmented detector***, the radiation hits its sensitive volume ⇒ *imaging*

*In the following we will limit our attention to capacitive detectors (the large majority, however…)*

# Introduction – II

Parameters to be known in the conception of a system for detector signal processing:

✓ The **amount of charge** made available by the release of the unit energy in the detector volume (sensitivity)

sets the impact of the signal deterioration due to the presence of front-end noise and external disturbances

✓ The **shape of the detector signal** and its duration

defines the time necessary to accumulate a suitable fraction of the total charge in the integration of the signal

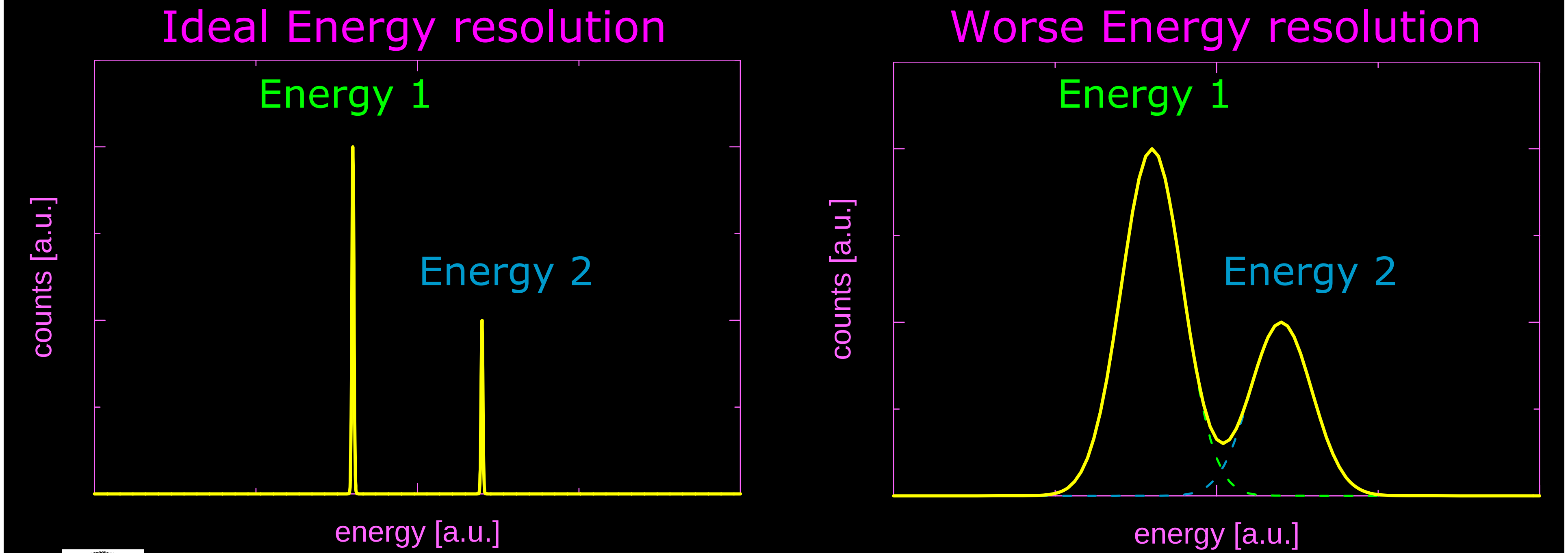
✓ The **rate of interactions**, which defines the number of signals the system has to process per unit time

sets a limit to the time available to process the detector signal

# Introduction: Energy resolution I

**Energy resolution**: ability of an energy dispersive system to distinguish spectral lines that are closely spaced in energy.

If the system broadens the lines, the two lines may merge into a single one, so spectral details are lost.



# Introduction: Energy resolution II

Spectral line broadening due to:

✓ statistical fluctuation of the amount of charge generated inside the detector for a given energy E of the incident radiation.

Average number of electron-hole (ion) pairs created: E/w

⇓

For the Poisson statistics: $\sigma^2=E/w$

**BUT** due to multiple excitation the fluctuation is reduced by the so called **Fano factor, F (F<1):**

$$\sigma^2=FE/w$$

$F\cong0.08$ for Ge and $F\cong0.12$ for Si

✓ trapping effects in the detector bulk

✓ Incomplete induction on the sensitive electrode, due to the low mobility of either type of carrier (as in CdTe and CZT detectors)

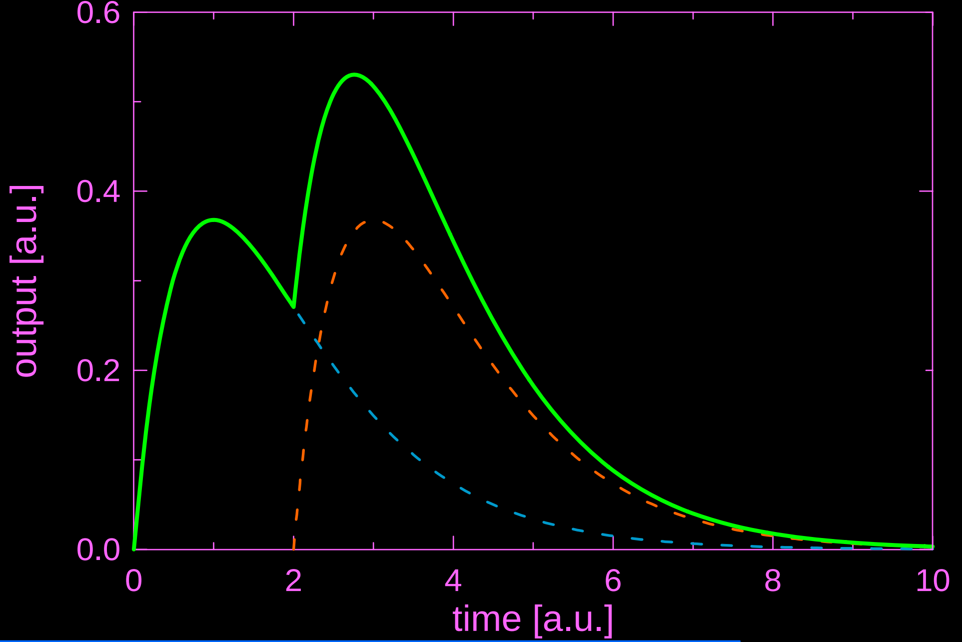
**We will neglect such fluctuations in the following.**

# Introduction: Energy resolution II

Spectral line broadening due to:

- ✓ noise in the front-end system

  **We will mainly deal with it in the following**
- ✓ ballistic deficit in the signal processing system
- ✓ baseline shift at high counting rates
- ✓ pulse-on-pulse pileup



# Introduction – Energy resolution III

Dominant sources of line broadening limiting the achievable energy resolution, for example:

✓ Si(Li) detector or Si detector operating on x-rays of a few tens of keV:
front-end noise

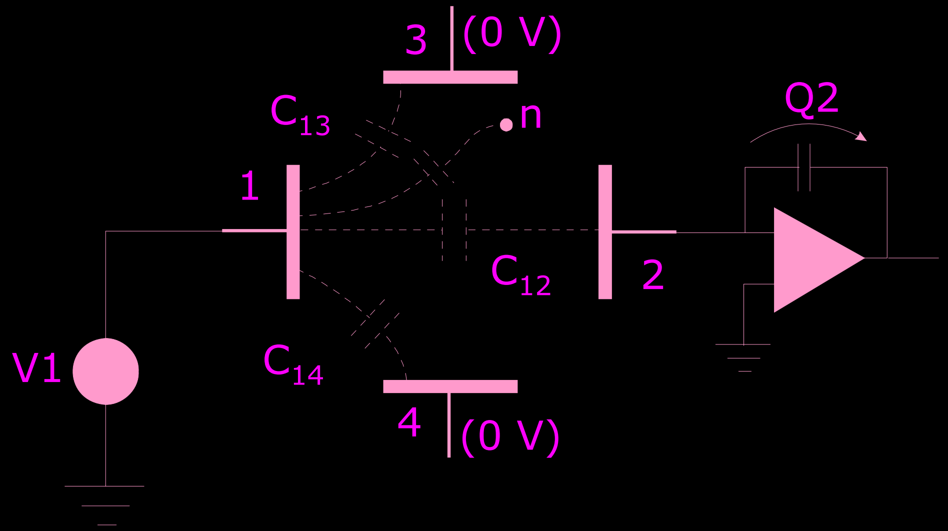
✓ Planar Ge detector operating on $\gamma$-rays of some hundreds of keV at moderately high counting rates:
statistics of pair creation and baseline shifts

✓ Large coaxial Ge detector:
ballistic deficit

✓ CdTe detector:
low mobility of holes and electron trapping

# Signal Formation and Ramo's Theorem - I

- ✓ ***Reciprocity of induced charge***



**Partial capacitance**

$$C_{12} = C_{21} = \left(\frac{Q_2}{V_1}\right)_{V_{2,3,4}=0} = \left(\frac{Q_1}{V_2}\right)_{V_{1,3,4}=0}$$

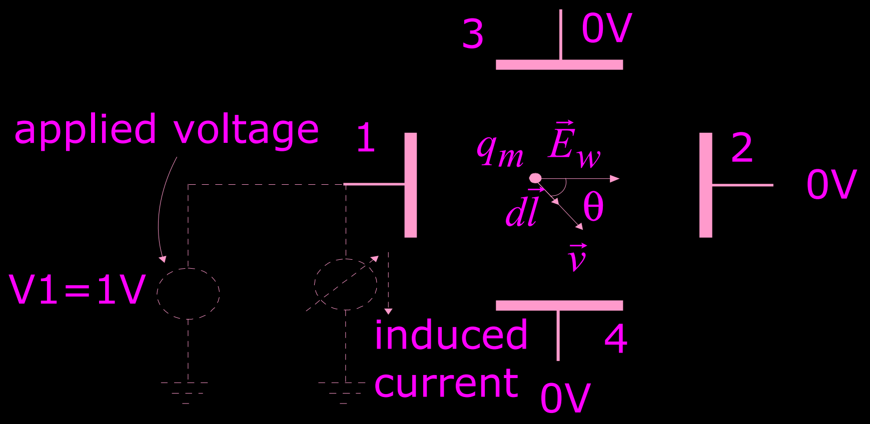
⬇

$$Q_1 V_1 = Q_2 V_2$$

**in general:** $Q_n V_n = Q_1 V_1$

# Signal Formation and Ramo's Theorem - II

✓ ***Current induced by the motion of charge***



**By reciprocity:**

$$q_m V_m = Q_1 V_1 \xrightarrow{V_1=1V} Q_1 = q_m V_m \quad \Longrightarrow \quad i_1(t) = \frac{dQ_1}{dt} = \frac{d(q_m V_m)}{dt} = q_m \frac{dV_m}{dt} \cdot \frac{d\vec{l}}{d\vec{l}}$$

**Weighting field $\vec{E}_w$**

(obtained by applying 1V on electrode 1 and grounding all the others)

$$\boxed{\frac{dV_m}{dl} = -\vec{E}_w \cos\theta}$$

**Induced current - RAMO Theorem**

(S.RAMO, Proc. IRE, 27 (1939) 584)

$$\boxed{i_1(t) = -q_m \vec{E}_w \cdot \vec{v}}$$

**In general:**

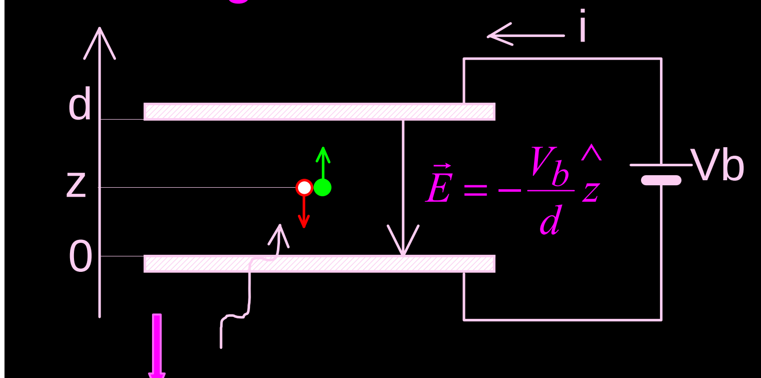
- find weighting field
- find charge velocity
- find $x(t)$, $y(t)$, $z(t)$ $\longrightarrow$ $i(t)$

$$\left.\begin{array}{l}\vec{E}_w(x,y,z)\\ \vec{v}(x,y,z)\end{array}\right\} i(x,y,z) \;\Big\downarrow\; i(t)$$

# Signal Formation and Ramo's Theorem - III

## ✓ *Induced current (charge) in planar electrode geometry*

### Single carrier

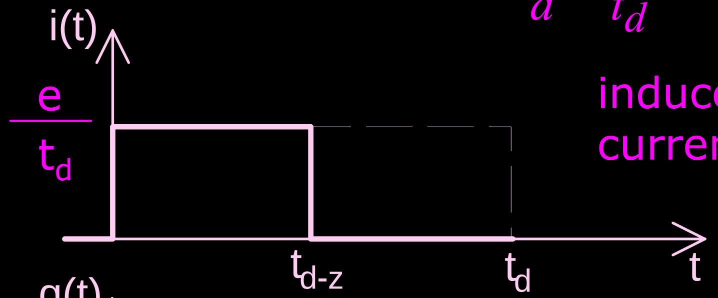


$$\vec{E}=-\frac{V_b}{d}\hat{z}$$

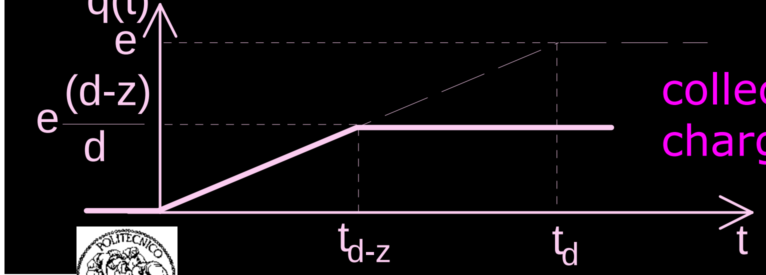
$$\vec{E}_w=-\frac{1}{d}\hat{z}$$

$\vec{E}$ true field

$$i(t)=q\vec{E}_w\cdot\vec{v}=-e\left(-E_w v\right)=e\frac{v}{d}=\frac{e}{t_d}\qquad 0\le t\le t_d$$

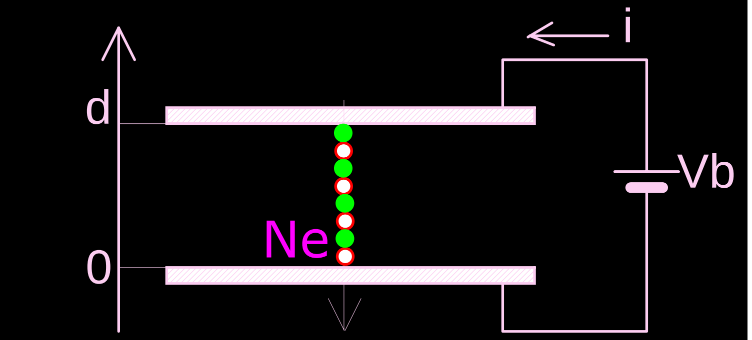


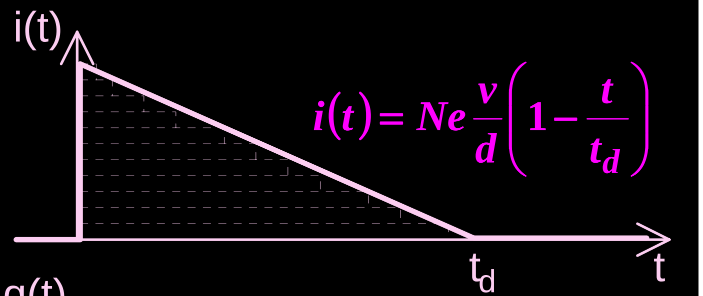
induced current



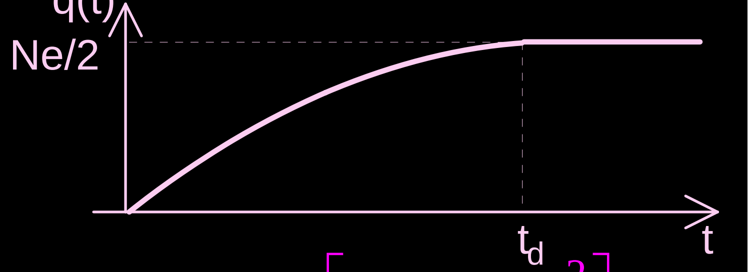
collected charge

### Continuous ionization





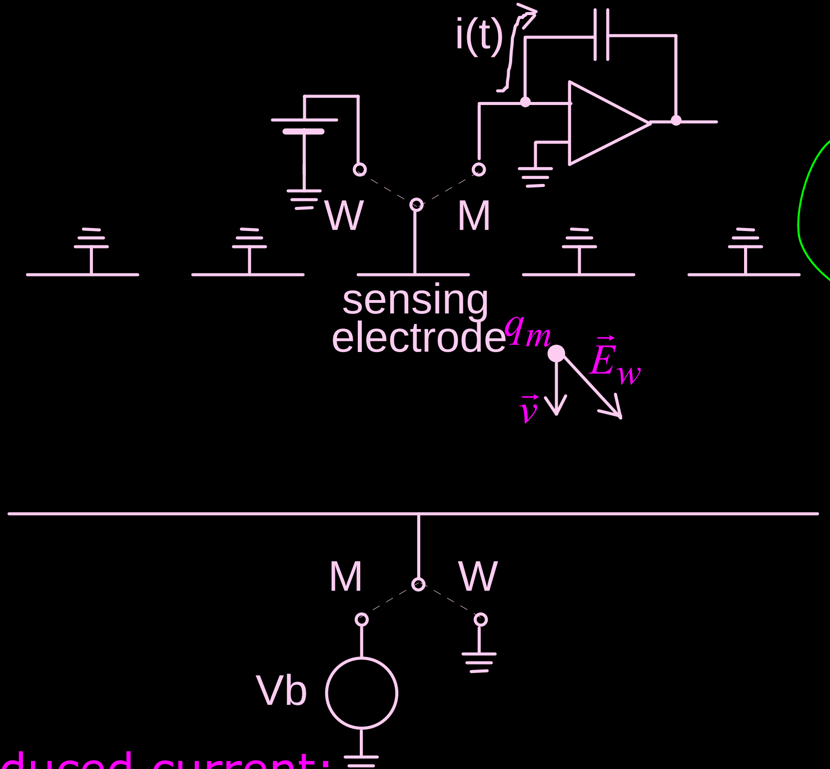
$$i(t)=Ne\frac{v}{d}\left(1-\frac{t}{t_d}\right)$$

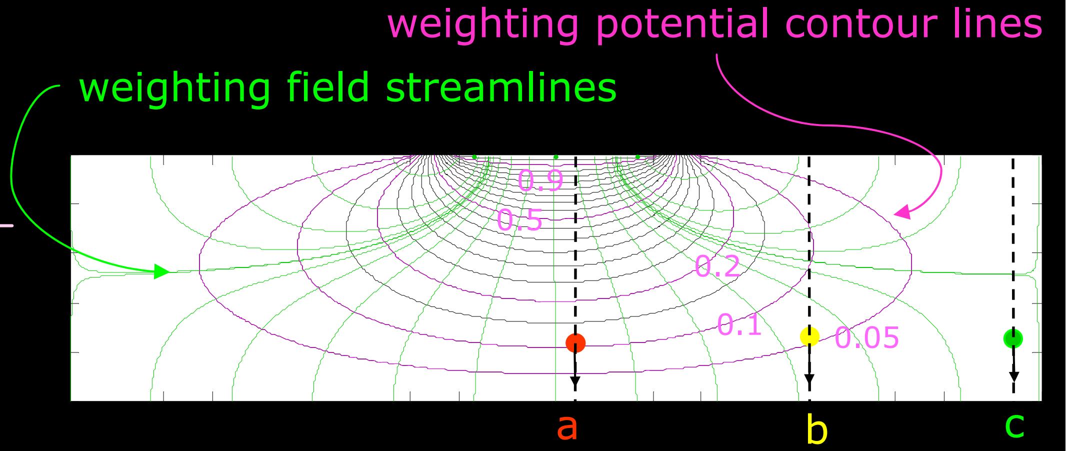


$$Q_s(t)=\int_0^t i(\tau)d\tau=Ne\left[\frac{t}{t_d}-\frac{1}{2}\left(\frac{t}{t_d}\right)^2\right]$$

# Signal Formation and Ramo's Theorem - IV

## ✓ *Induced current in strip electrodes*



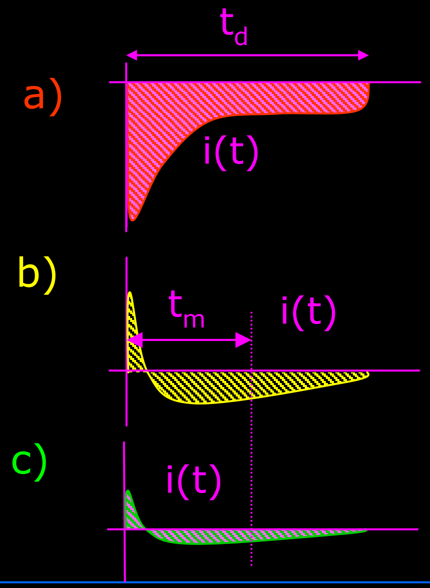


Induced current:

$$i(t) = -q_m \vec{E}_w \cdot \vec{v} = -q_m \vec{E}_w \cdot \frac{d\vec{x}}{dt}$$

Induced charge:

$$Q(t) = \int_0^t i(\tau) d\tau = -q_m \int_0^x \vec{E}_w \cdot d\vec{x}$$



$$\int_0^d \vec{E}_w \cdot d\vec{x} = 1 \Rightarrow Q = -q_m$$

$$\int_0^d \vec{E}_w \cdot d\vec{x} = 0$$

↓

$Q = 0 \qquad t_m > t_d$

$Q \neq 0 \qquad t_m < t_d$

# Noise analysis – definitions – I

## *inherent noise*

- it refers to random noise signals due to fundamental properties of the detector and/or circuit elements;
- therefore it can be never eliminated;
- it can be reduced through proper choice of the preamplifier/shaper design.

## *interference noise*

- it results from unwanted interaction between the detection system and the outside world or between different parts of the system itself;
- it may or may not appear as random signals (power supply noise on ground wires – 50 or 60 Hz, electromagnetic interference between wires, …).

We will deal with *inherent noise* only and we will assume **all noise signals have a mean value of zero.**

For those more rigorously inclined, we assume also that random signals are ergodic therefore their ensemble averages can be approximated by their time averages.

# Noise analysis – Time-domain analysis

- rms (root mean square) value

$$v_{n,rms} \equiv \sqrt{\left[\frac{1}{T}\int_0^T v_n^2(t)dt\right]}$$

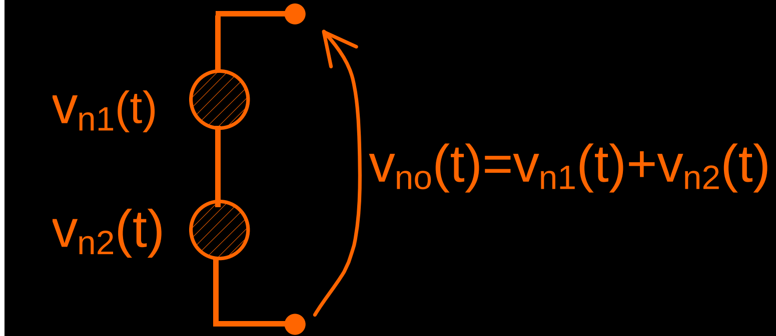
where T is a suitable averaging time interval. A longer T usually gives a more accurate rms measurement.

It indicates the normalized noise power of the signal.

- signal-to-noise ratio (SNR) (in dB)

$$SNR \equiv 10\log\left[\frac{signal\ \ power}{noise\ \ power}\right] = 10\log\left[\frac{v_{x,rms}^2}{v_{n,rms}^2}\right] = 20\log\left[\frac{v_{x,rms}}{v_{n,rms}}\right]$$

- noise summation



$$v_{no,rms}^2 = \frac{1}{T}\int_0^T [v_{n1}(t)+v_{n2}(t)]^2\,dt =$$

$$= \underbrace{v_{n1,rms}^2 + v_{n2,rms}^2}_{\substack{individual\ \ mean \\ squared\ \ values}} + \underbrace{\frac{2}{T}\int_0^T v_{n1}(t)v_{n2}(t)dt}_{\substack{correlation\ between \\ the\ two\ signal\ sources}}$$

# Noise analysis – Frequency-domain analysis I

- **noise spectral density**: average normalized noise power over 1-Hz bandwidth, measured in V$^2$/Hz or A$^2$/Hz.

The rms value of a noise signal can be obtained also in the frequency domain:

$$V_{n,rms}^{2} = \int_0^\infty V_n^2(f)\,df$$

$V_n^2(f)$ is the Fourier transform of the autocorrelation function of the time-domain signal $v_n(t)$ (**Wiener-Khinchin theorem**).

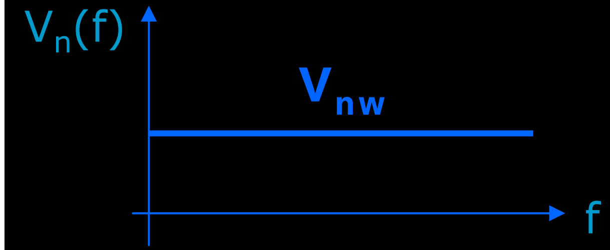
**One-side spectral density**: noise is integrated only over positive frequencies.

**Bilateral spectral density**: noise is integrated over both positive and negative frequencies.

The bilateral definition results in the spectral density being divided by two since, for real-valued signals, the spectral density is the same for positive and negative frequencies.

# Noise analysis – Frequency-domain analysis II

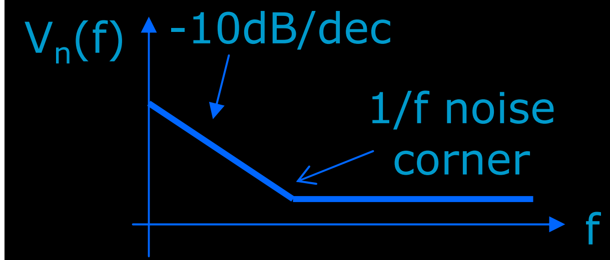
- white noise



$$V_n^2(f) = V_{nw}^2$$ where $V_{nw}$ is a constant.

A noise signal is said to be white if its spectral density is constant over a given frequency, i.e. if it has a flat spectral density.

- 1/f (or flicker) noise



$$V_n^2(f) = \frac{A_f}{f}$$ where $A_f$ is a constant.

The noise power of the 1/f noise is constant in every decade of frequency:

$$\int_{f^*}^{10f^*} V_n^2(f)df = \int_{f^*}^{10f^*} \frac{A_f}{f} df = A_f \ln\frac{10f^*}{f^*} = A_f \ln(10)$$

# Noise analysis – useful theorems

- **Carson's theorem**

noise source with bilateral power spectrum $N(\omega)$ ⟺ superposition (in the time domain) of randomly distributed events with Fourier transform $\Phi(\omega)$ occurring at an average rate $\lambda$

$$N(\omega) = \lambda \left|\Phi(\omega)\right|^2$$

- **Campbell's theorem**

the r.m.s. value of a noise process resulting from the superposition of pulses of a fixed shape $\phi(\tau)$, randomly occurring in time with an average rate $\lambda$ is:

$$\left[\overline{v_n^2}\right]^{1/2} = \left[\lambda \int_{-\infty}^{+\infty} \phi^2(t)\,dt\right]^{1/2}$$

- **Parseval's theorem**

$$\int_{-\infty}^{+\infty} h^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left|H(\omega)\right|^2 d\omega = 2\int_0^{+\infty} \left|H(\omega)\right|^2 df$$

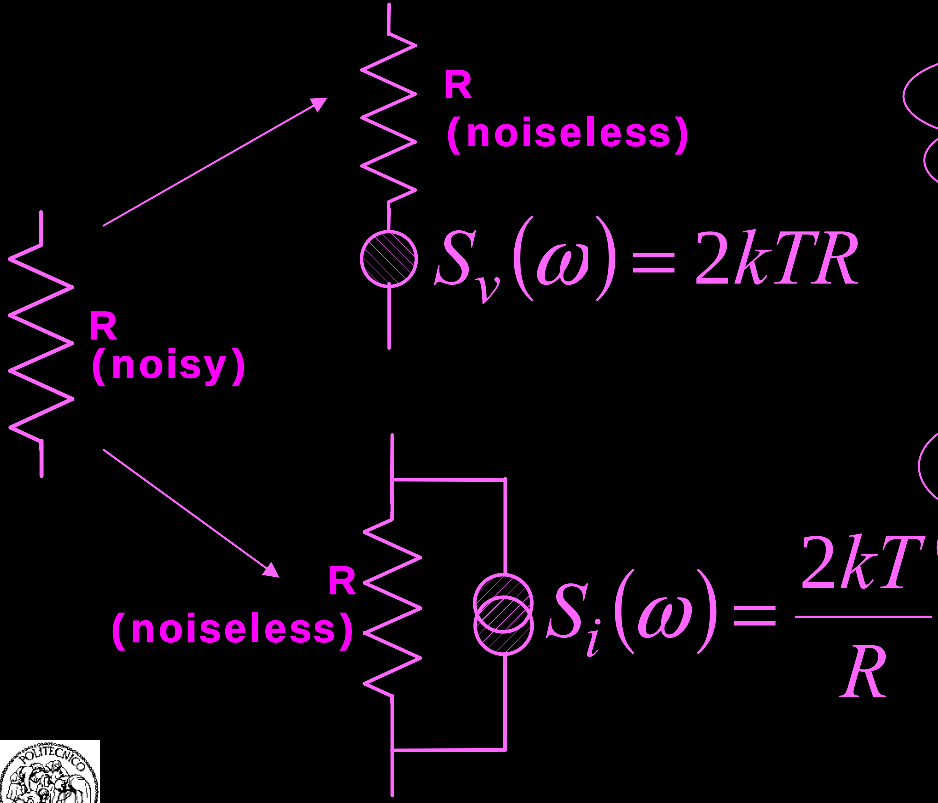
# Main noise mechanisms

- **Thermal noise** (also known as **Johnson** or **Nyquist** noise - 1928):
  - ✓ due to thermal excitation of charge carriers in a conductor;
  - ✓ white spectral density and proportional to absolute temperature;
- **Shot noise** (first studied by Schottky in 1918 in vacuum tubes):
  - ✓ due to the granularity of charge carriers forming the current flow;
  - ✓ white spectral density and dependent on the DC bias current;
- **Flicker noise** (commonly referred to as 1/f noise):
  - ✓ usually arises due to traps in the semiconductor, where carriers constituting the DC current flow are held for some time period and then released;
  - ✓ well modeled as having a $1/f^{\alpha}$ spectral density with $0.8<\alpha<1.3$;
  - ✓ least understood of the noise phenomena.

# Noise in Electronic Devices: Resistors – I

❍ Resistors exhibit **thermal noise**.

❍ The **power spectral density** of such voltage fluctuations was originally derived by Nyquist in 1928, assuming the law of equipartition of energy states that the energy on average associated with each degree of freedom is the thermal energy.



$$S_v(\omega) = 2kTR$$

$$S_i(\omega) = \frac{2kT}{R}$$

*Only physical resistors (and not resistors used for modeling) contribute thermal noise.*

*1 kΩ resistor exhibits a root spectral density of 4nV/√Hz (4pA/√Hz) of thermal noise at room temperature (300 K).*

k: Boltzmann's constant ($1.38\times10^{-23}$ J/K)

T: absolute temperature in Kelvins

# Noise in Electronic Devices: Resistors – II

❍ At frequencies and temperatures where quantum mechanical effects are significant (hν~kT) each degree of freedom should on average be assigned the energy:

$$\frac{h\nu}{\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]}\xrightarrow{h\nu<<kT}kT \Rightarrow S_v(\omega)=2kTR\frac{h\nu/kT}{\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]}$$

$$h = 6.63\times10^{-34}\,J\cdot s$$

$$k = 1.38\times10^{-23}\,J/K$$

$$\frac{h\nu}{kT}=1\Rightarrow\begin{cases}T=300K & \rightarrow\nu=6\cdot10^{12}\,Hz=6THz\\ T=30K & \rightarrow\nu=6\cdot10^{11}\,Hz=600GHz\\ T=0.3K & \rightarrow\nu=6\cdot10^{9}\,Hz=6GHz\\ T=3\cdot10^{-3}K & \rightarrow\nu=6\cdot10^{7}\,Hz=60MHz\end{cases}$$

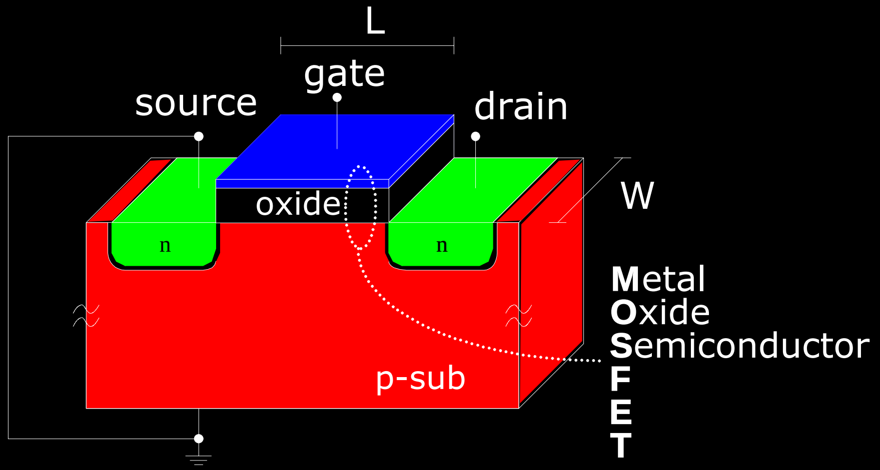
⇓

**at "practical" frequencies and temperatures resistors thermal noise is independent of frequency**

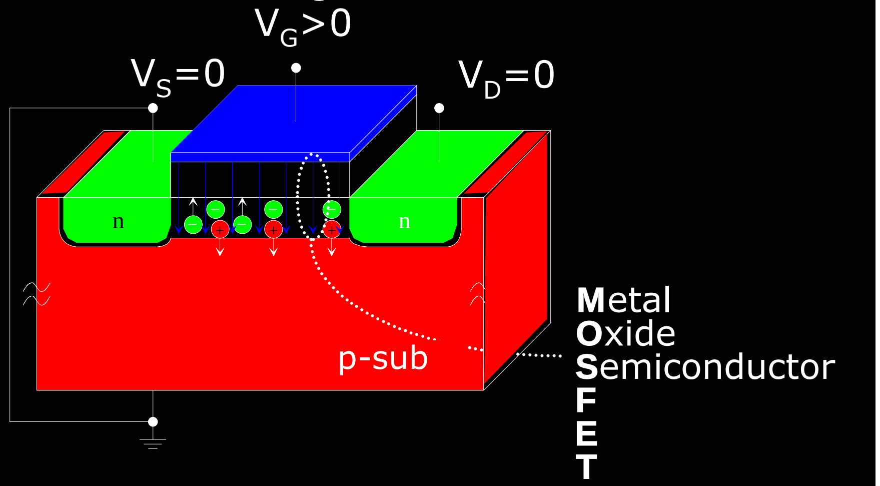
⇒ **white noise**

# MOSFET operating principle – I

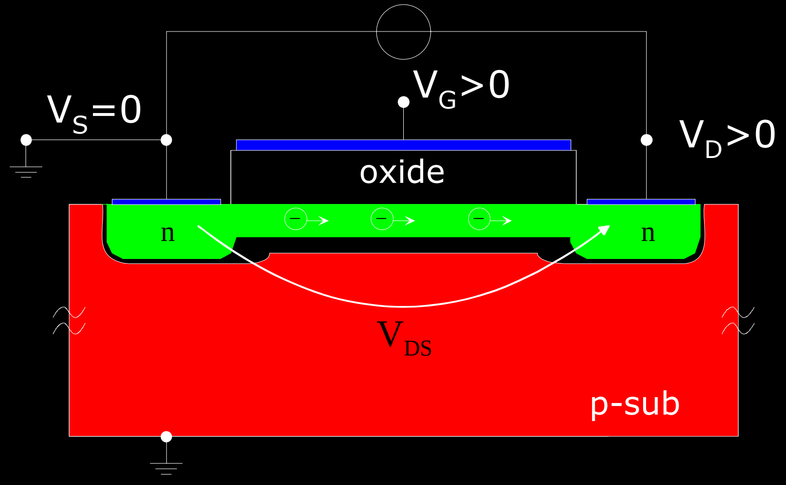
## Basic structure



## The gate contact

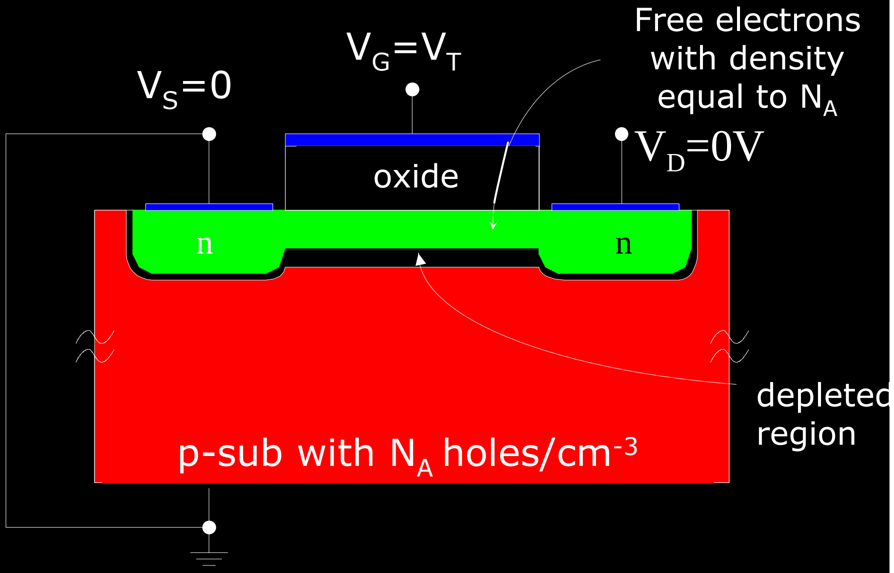


**The conducting channel is formed …**



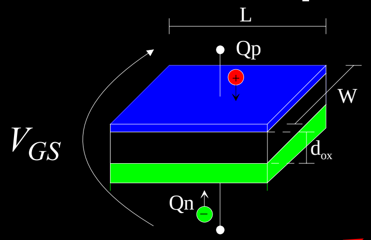
**… current can flow between D and S!**

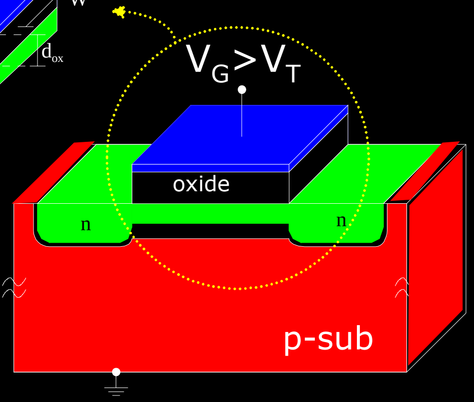
**The threshold voltage: INVERSION**



# MOSFET operating principle – II

## MOS capacitor

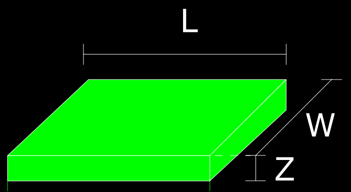


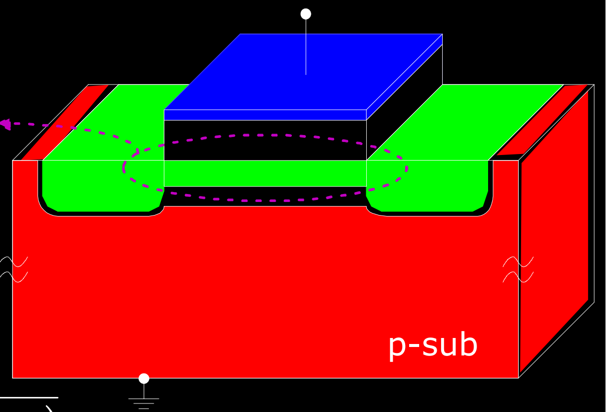


$$C_{gate} = C_{ox} = \frac{\varepsilon_{ox}}{d_{ox}} WL$$

$$Q_n \;=\; C_{ox}\,(V_{GS} - V_T)$$

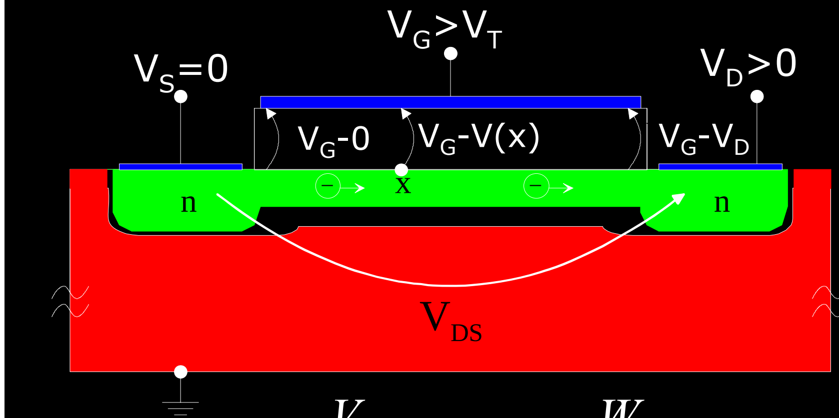
## Channel resistance

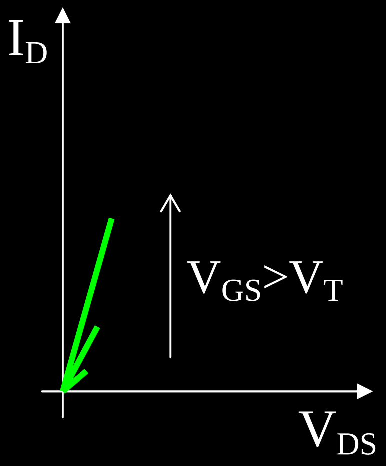




$$R_{ch} = \rho \frac{L}{W \cdot Z} = \frac{1}{\mu_n C_{ox}} \frac{L}{W} \frac{1}{(V_{GS} - V_T)}$$

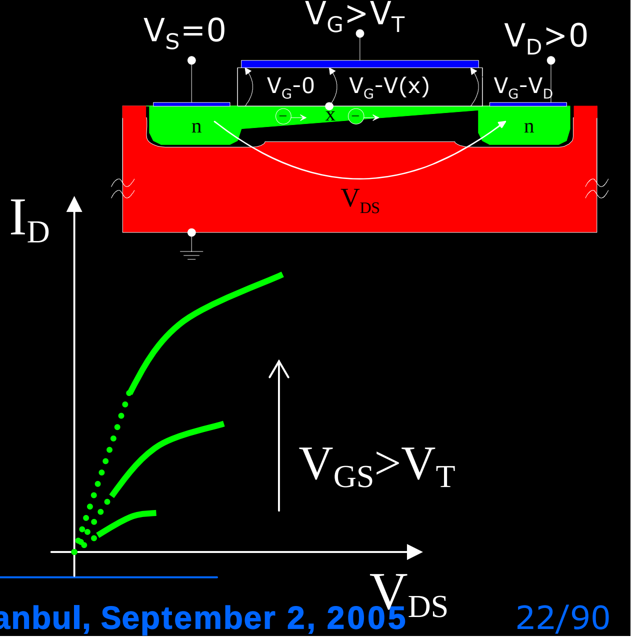
## MOS as variable resistor: OHMIC region





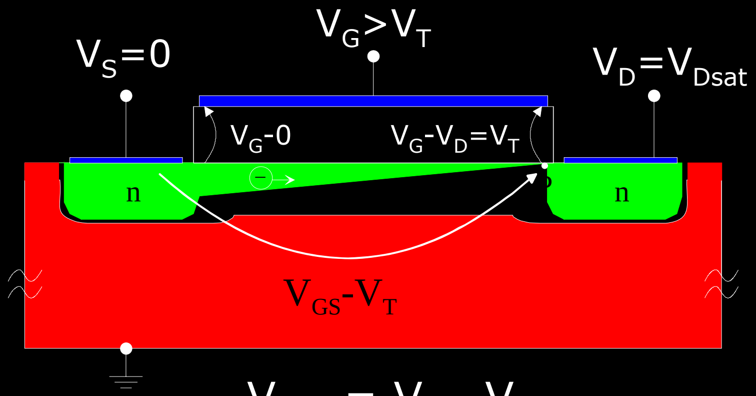
$$I_D = \frac{V_{DS}}{R_{ch}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$$

## as $V_{DS}$ increases ...



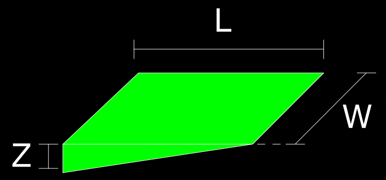
# MOSFET operating principle – III

## Channel pinch-off: saturation

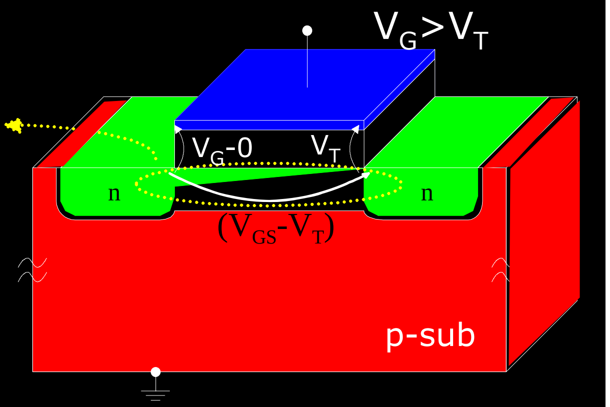


$V_{Dsat}$= $V_{GS}$-$V_T$

## Current at pinch-off voltage

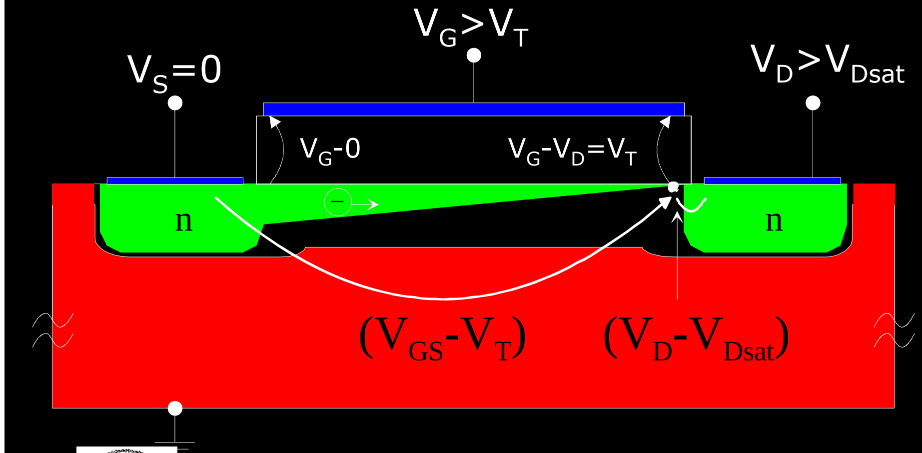


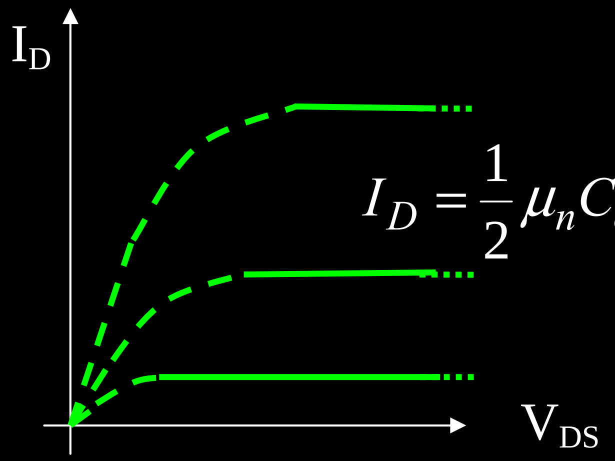
$$R_{sat} = 2 \cdot R_{ch}$$



$$I_D = \frac{(V_{GS} - V_T)}{R_{sat}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2$$

## MOS as transistor: SATURATION region



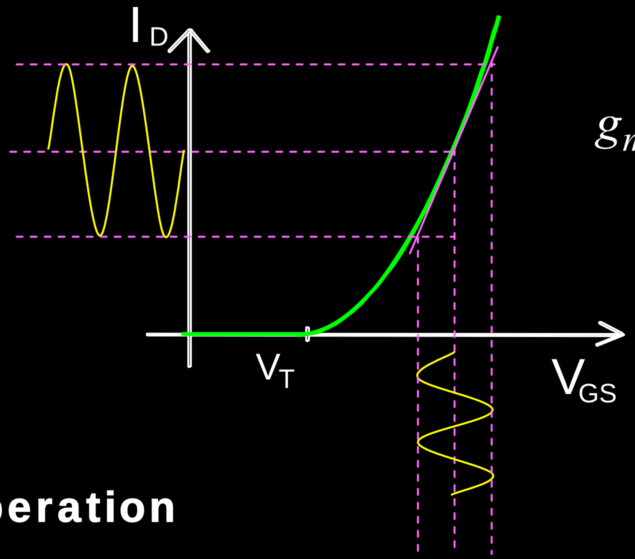


$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2$$

# MOSFET operating principle – IV

## Transcharacteristic curve

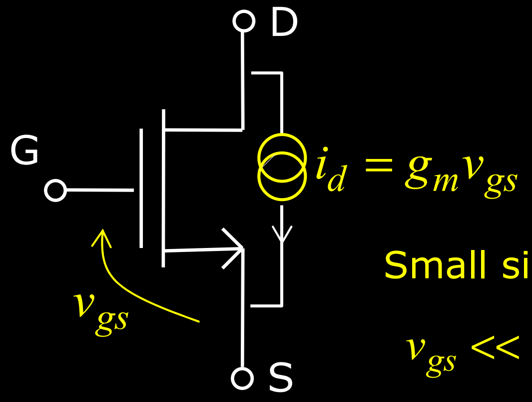
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)^2$$



## Transconductance

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)$$

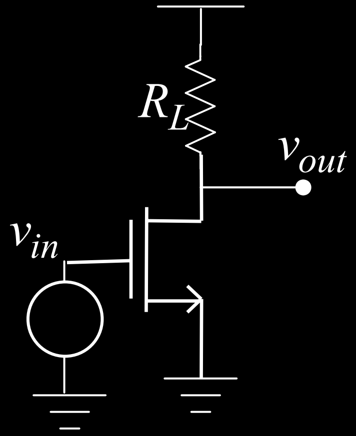
## Small signal operation



Small signal condition:

$$v_{gs} << 2\cdot(V_{GS}-V_T)$$

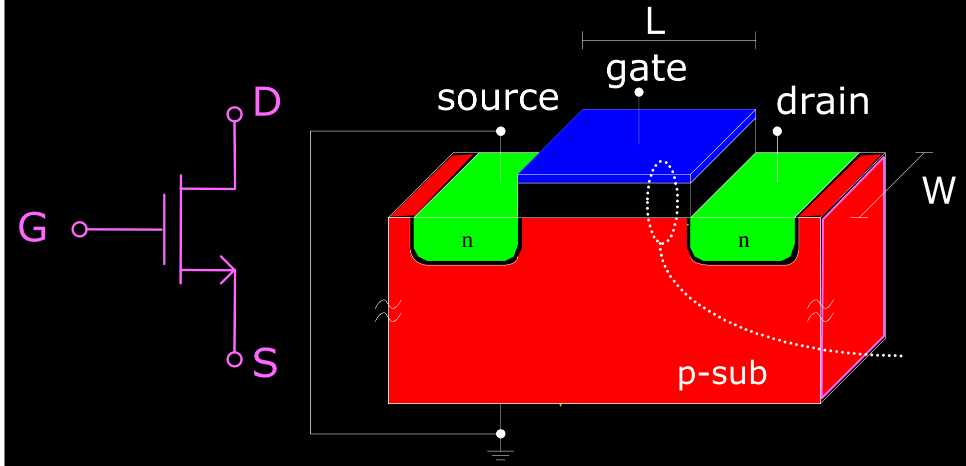
## Basic amplifier configuration (Common source)



**Voltage gain:**

$$A_v = \frac{v_{out}}{v_{in}} = -g_m R_L$$

# Noise in Electronic Devices: MOSFET



(Van der Ziel - 1986)

**1/f noise** (due to random capture and release of carriers by a large number of traps with different time constants):

$$S_{I_D}(\omega)=\frac{B}{|\omega|} \Rightarrow S_{V_g}(\omega)=\frac{K\pi}{WLC_{ox}|\omega|}$$

P-channel MOSFETs feature lower 1/f noise than N-channel MOSFETs.

**Thermal noise** (the channel can be treated as a resistor whose increment resistance is a function of the position coordinate):

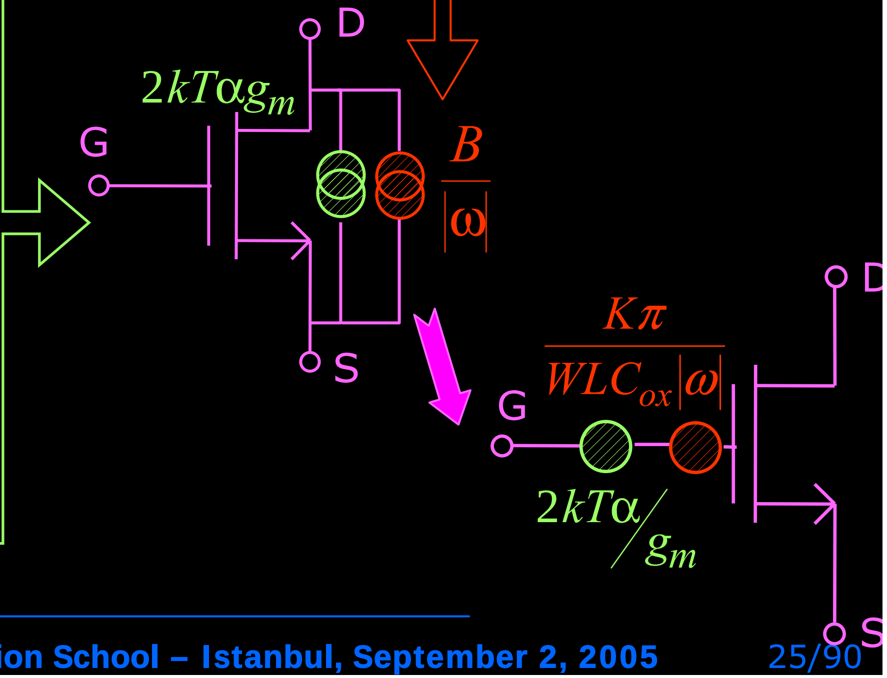
**ohmic region**

$$S_i=\frac{2kT}{R_{ch}}$$

**saturation region**

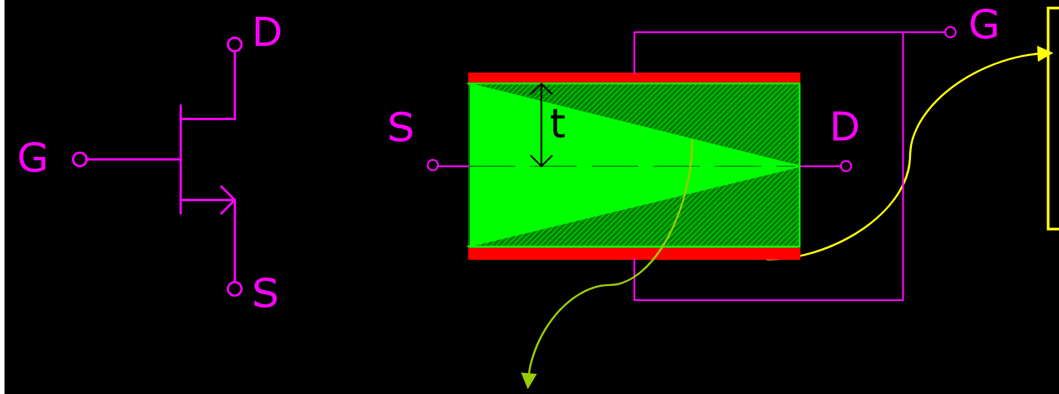
$$S_i=2kT\alpha\, g_m$$

The **thermal noise current** in the channel **is equal to Johnson noise in a conductance equal to $\alpha$ $g_m$** where $\alpha$ =2/3 for long channel and $\alpha$ = $\alpha(V_{GS}-V_T)$ for short channel MOSFETs.



# Noise in Electronic Devices: JFET

(Van der Ziel - 1962)



**Thermal noise** (the channel may be treated as a resistor whose increment resistance is a function of the position coordinate):

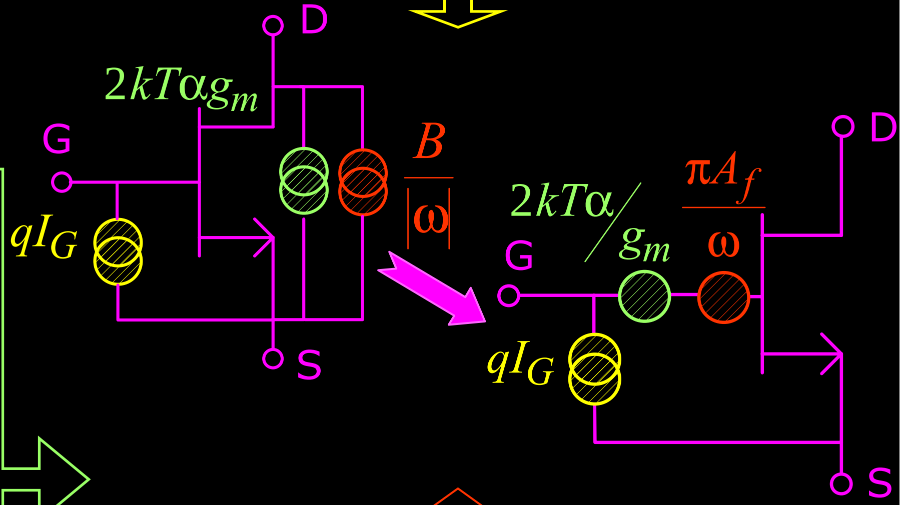
**saturation region**

$$S_i = 2kT\alpha\, g_m$$

The **thermal noise current** in the channel **is equal to Johnson noise in a conductance equal to $\alpha$ $g_m$** where $\alpha$ =2/3.

**Shot noise** (due to leakage current $I_G$ across the gate-channel junction):

$$S_{I_G}(\omega) = qI_G$$



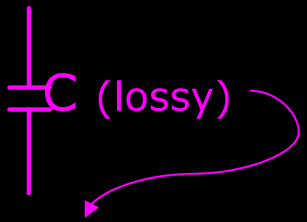
**1/f noise** (due to random capture and release of carriers by traps in the device):

$$S_{I_D}(\omega) = \frac{B}{|\omega|}$$

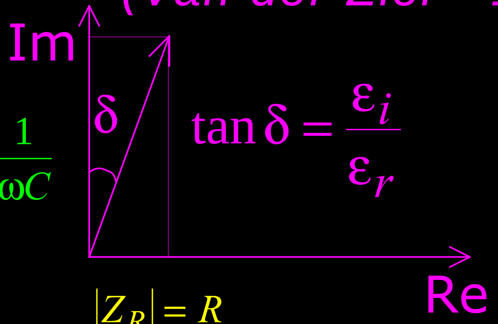
However, much lower than in MOSFET

# Noise in Electronic Devices – lossy capacitor

*(Van der Ziel - 1975)*



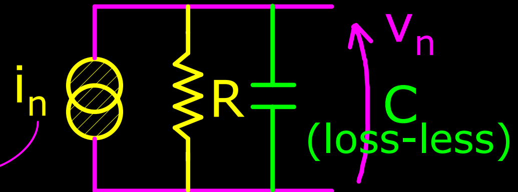
$\varepsilon = \varepsilon_r - j\varepsilon_i$



$|Z_C| = \frac{1}{\omega C}$

$\tan\delta = \frac{\varepsilon_i}{\varepsilon_r}$

$|Z_R| = R$



$$S_i(\omega) = \frac{2kT}{R} = 2kTC\,\omega\tan\delta$$

Power spectral density of the thermal noise current generator

$$\boxed{Y = j\omega(\varepsilon_r - j\varepsilon_i)\frac{\varepsilon_o A}{d} = \frac{1}{R} + j\omega C}$$

$$C = \frac{\varepsilon_o \varepsilon_r A}{d}$$

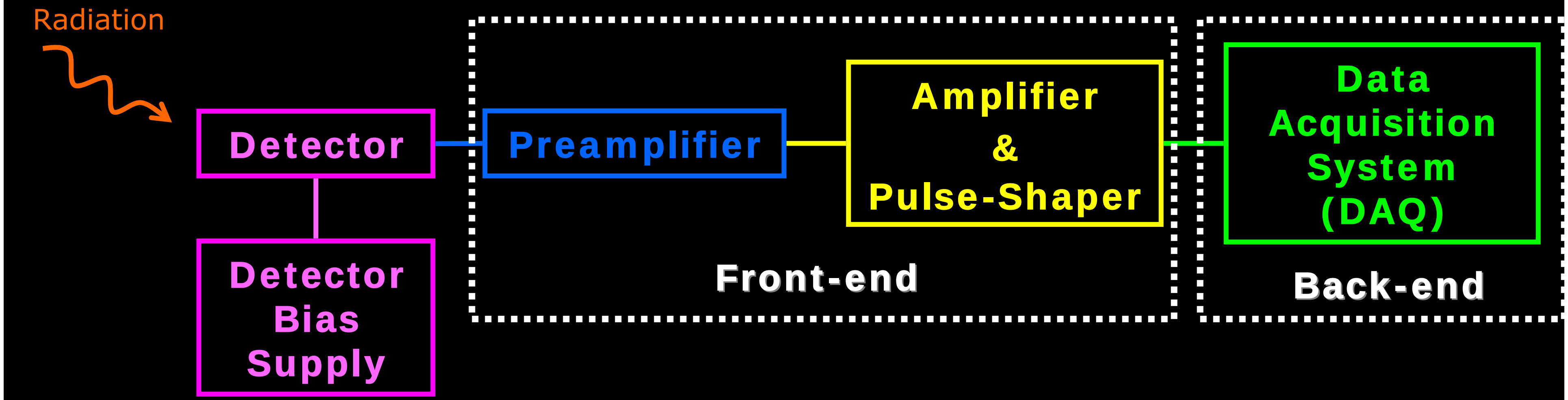
$$R = \frac{d}{\varepsilon_i \varepsilon_o \omega A} = \frac{1}{\omega C \tan\delta}$$

$$S_v(\omega) = S_i(\omega)\frac{R^2}{1+\omega^2 R^2 C^2} = \frac{2kT}{\omega C}\sin\delta\cos\delta$$

As far as the loss angle (δ) is independent of frequency, the output voltage noise shows a 1/f spectrum.

At low frequency the loss resistance is merely a measure of the conductivity (σ) of the dielectric ⟶ $S_v(\omega)$ shows a frequency dependence of the form $\left(1+\omega^2 R_o^2 C^2\right)^{-1}$.

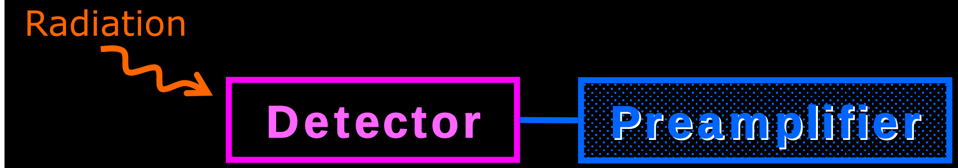
# Basic blocks in a detection system



- ✓ **Detector**: responsible of "converting" radiation in electrical signal
- ✓ **Preamplifier:** first signal amplification and "cable driving"
- ✓ **Amplifier**: signal amplification and filtering
- ✓ **DAQ**: A/D conversion, data acquisition and storage

We will deal with the *front-end section* only.

# Preamplifier

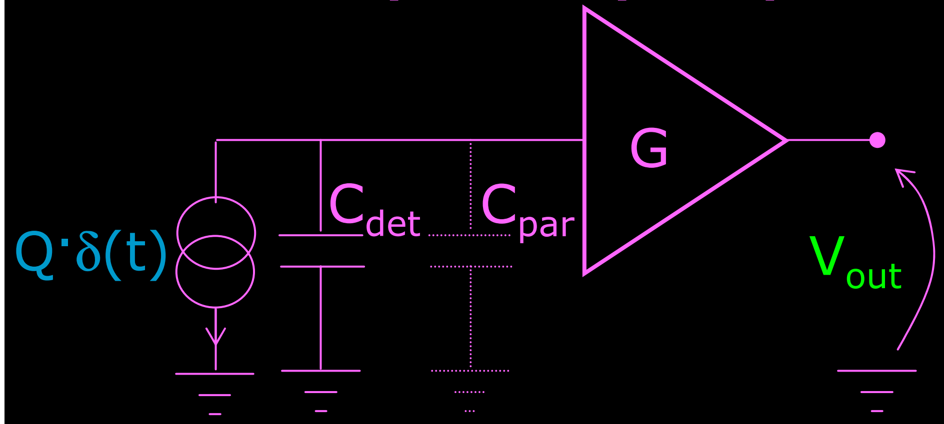


Located as close as possible to the detector to minimize the added electronic noise, the preamplifier must:

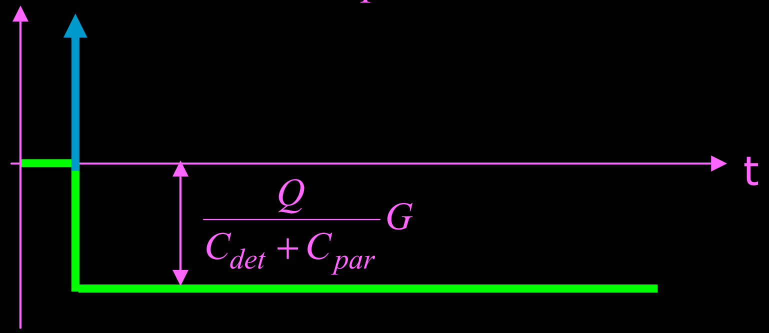
- minimize the internal noise contribution that it adds to the detector signal;
- amplify the detector signal to a level high enough to make negligible the contribution of the noise sources in the following circuits;
- transmit the preamplified signal over a coaxial cable (or other cable) that may have a considerable length (some meters).

# Charge-Sensitive Preamplifier – I

## "Normal" open-loop amplifier

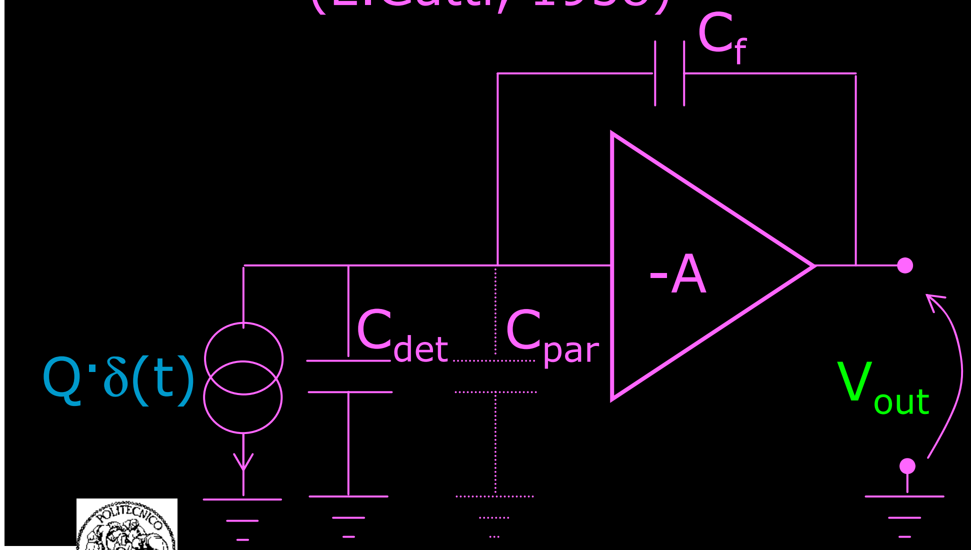


$$V_{out} = \frac{Q}{C_{det} + C_{par}} \cdot G \cdot 1(t)$$

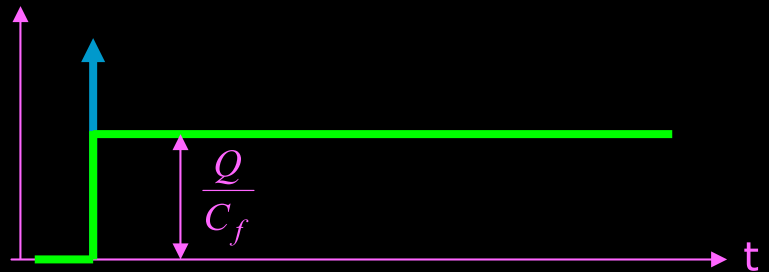


## Charge preamplifier

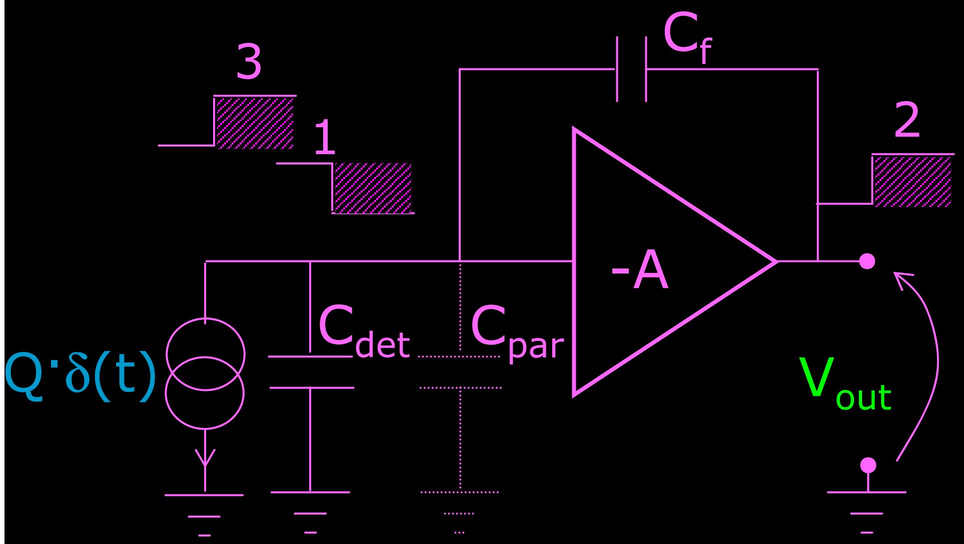
(E.Gatti, 1958)

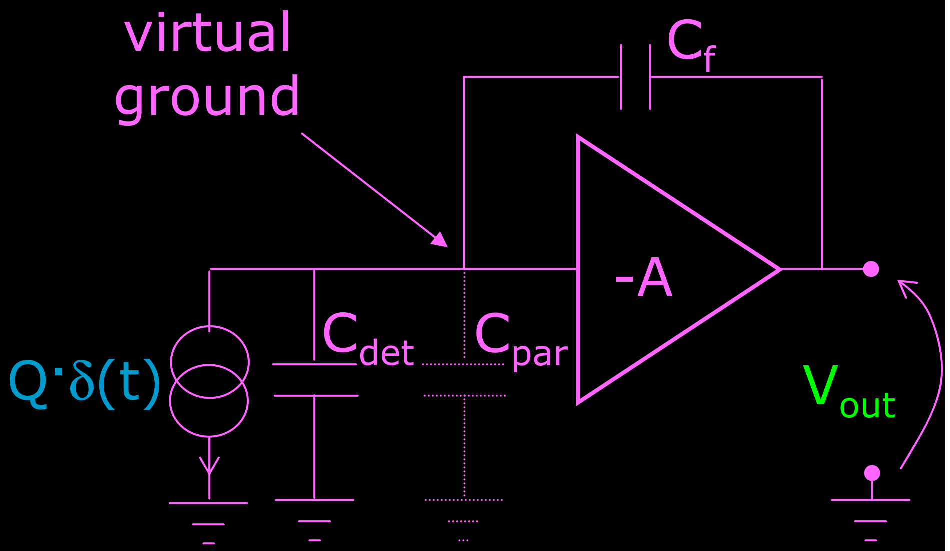


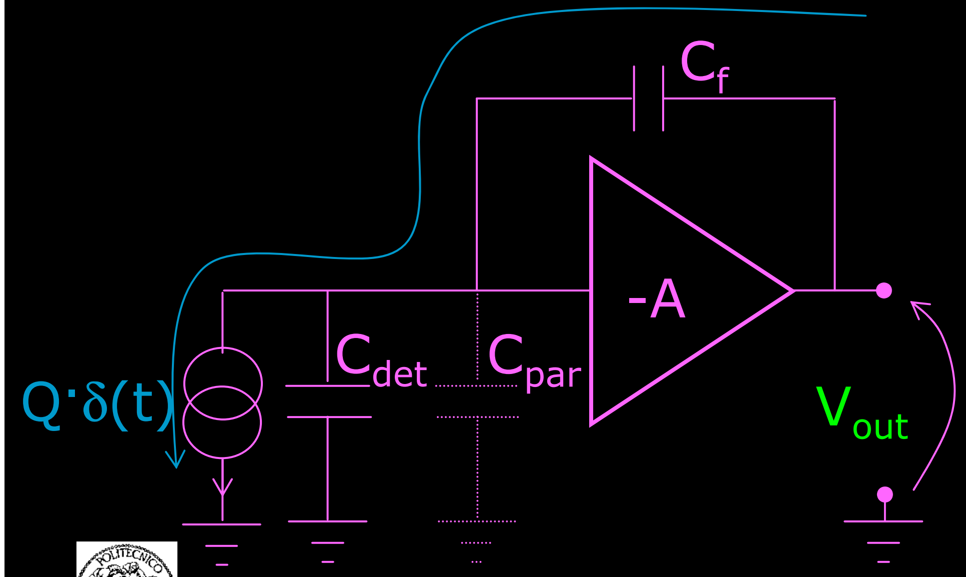
$$V_{out}\Big|_{ideal} = \frac{Q}{C_f} \cdot 1(t)$$



# Charge-Sensitive Preamplifier – II







$$V_{out} = \frac{Q}{C_f}\cdot\frac{1}{1+\frac{1}{A}\left(1+\frac{C_{par}+C_{det}}{C_f}\right)}\cdot 1(t)\xrightarrow{A\to\infty}$$

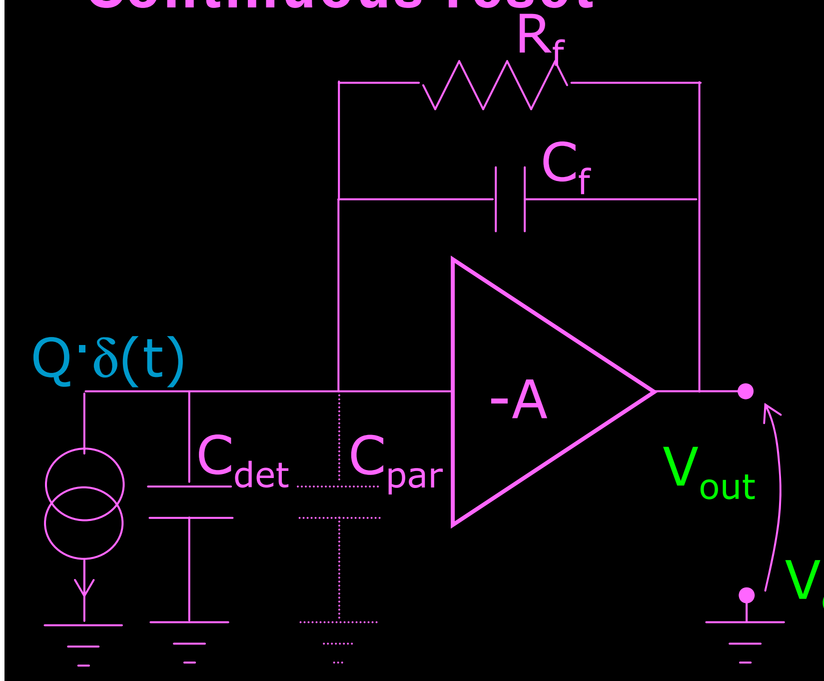
$$\xrightarrow{A\to\infty}\frac{Q}{C_f}\cdot 1(t)$$

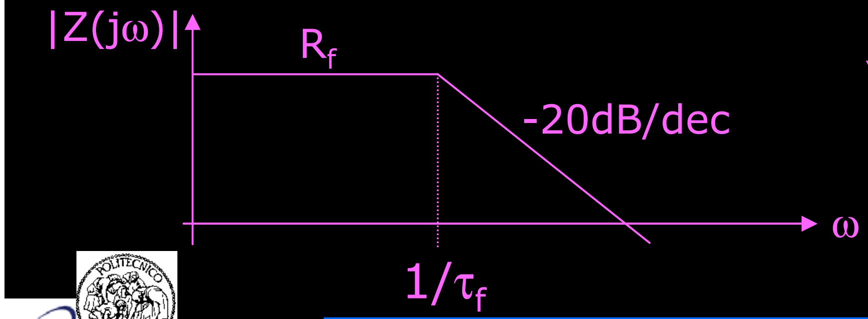
The charge "gain" is determined by a well-controlled component, the feedback capacitor

# Charge-Sensitive Preamplifier – III

**BUT** the charge brought by the radiation pulses on the feedback capacitor must eventually be removed:

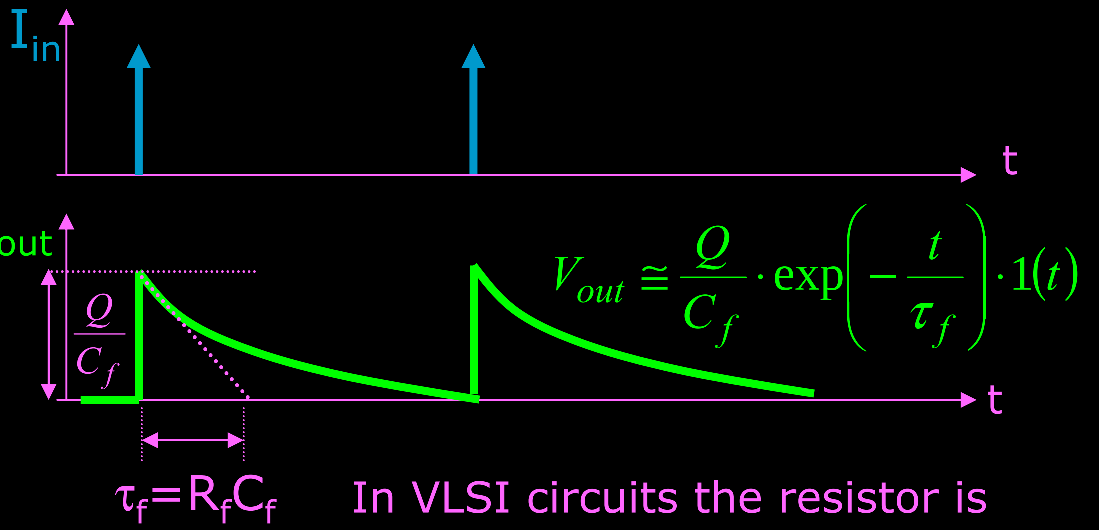
- **Continuous reset**





The signal charge slowly and *continuously* leaks out of the capacitor and the output voltage signal decays exponentially to the baseline level with a characteristic time constant

$\tau_f = R_f C_f$ (typically greater than 40μs)



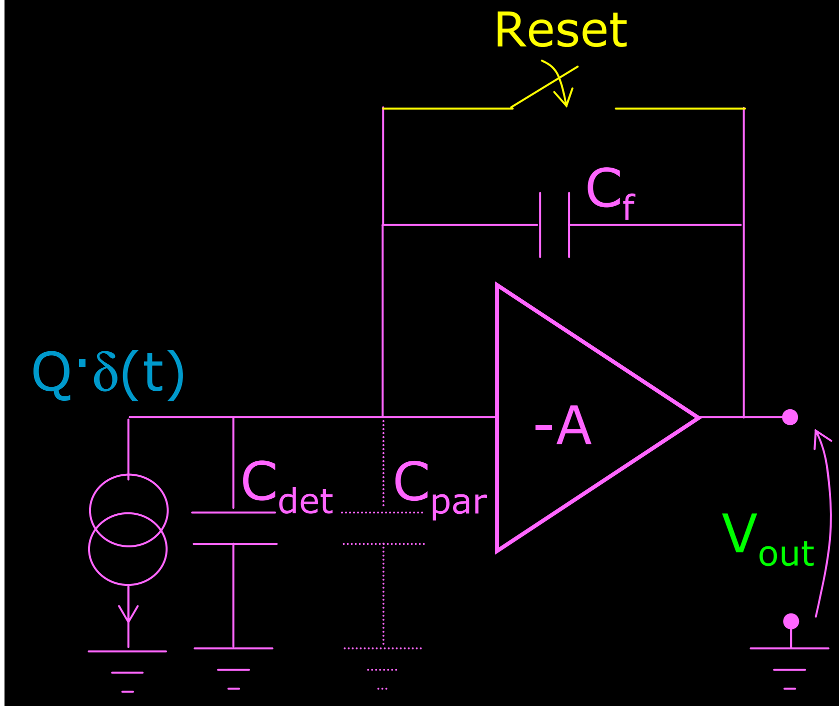
$$V_{out} \cong \frac{Q}{C_f} \cdot \exp\left(-\frac{t}{\tau_f}\right) \cdot 1(t)$$

In VLSI circuits the resistor is substituted by a MOSFET (triode region) or by a transconductor.

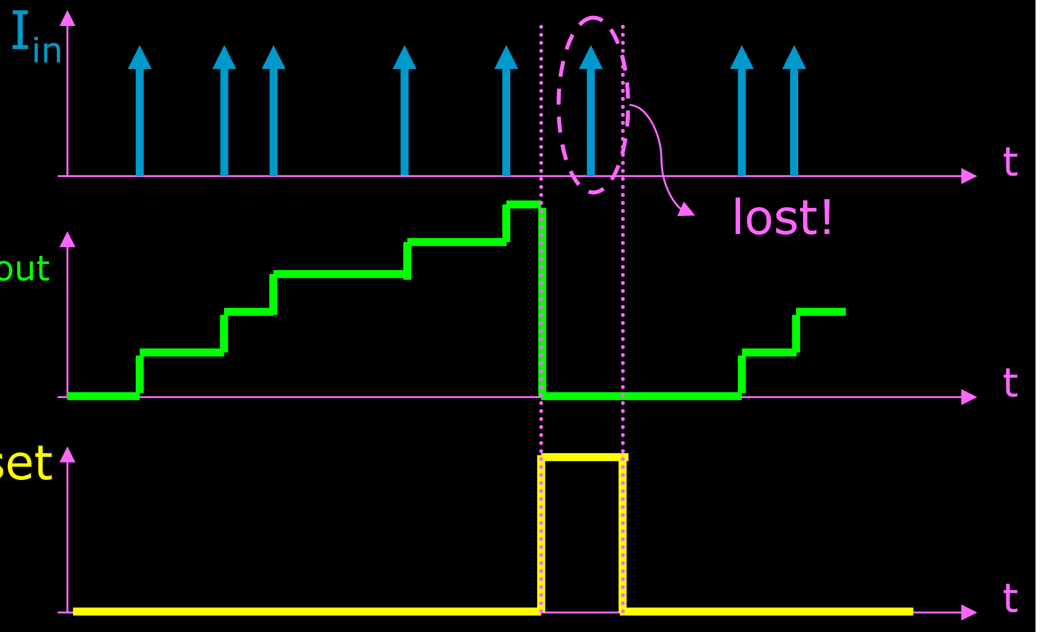
# Charge-Sensitive Preamplifier – IV

**BUT** the charge brought by the radiation pulses on the feedback capacitor must eventually be removed:

- **Pulsed reset**



The charge accumulated is removed periodically or when the output voltage exceeds a preset limit. During this discharge a huge pulse having polarity opposite to the radiation pulse is produced in the amplification chain.

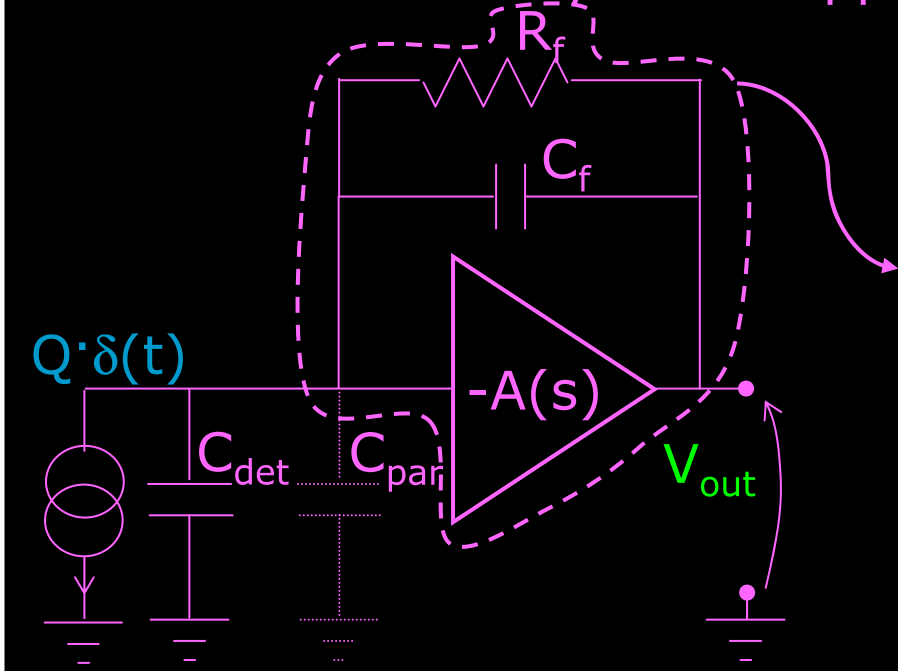


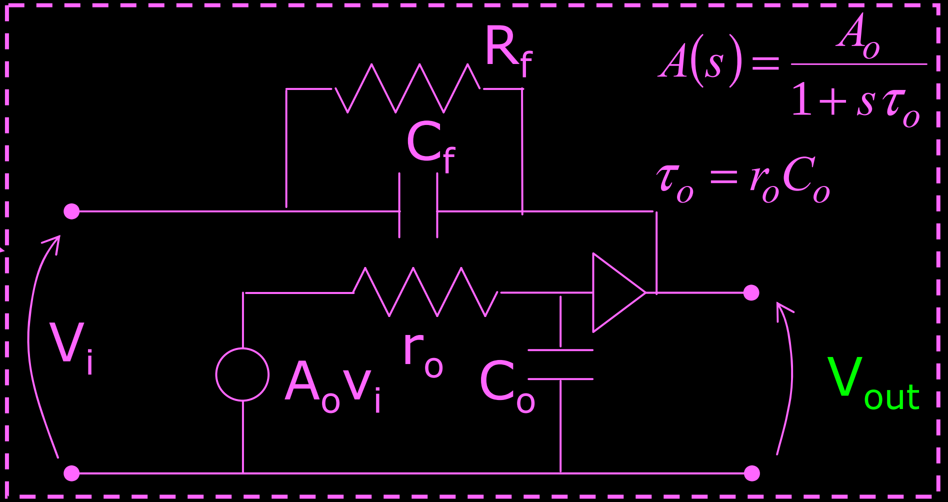
drawbacks:

- ✓ dead time
- ✓ switch noise

# Charge-Sensitive Preamplifier –V

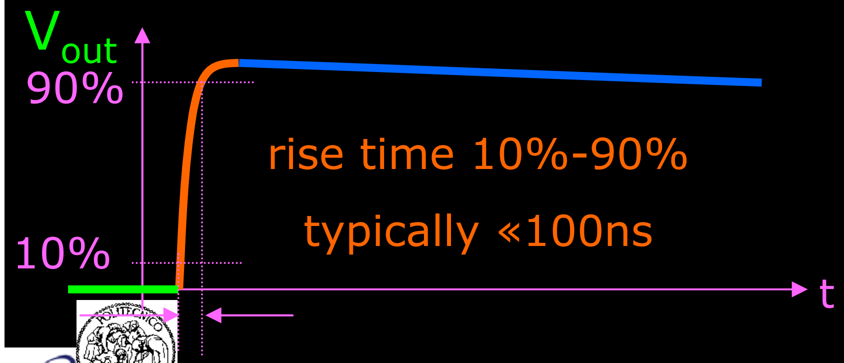
We have assumed that the amplifier is infinitely fast, i.e. it responds instantaneously to the applied signal. This is not the case…





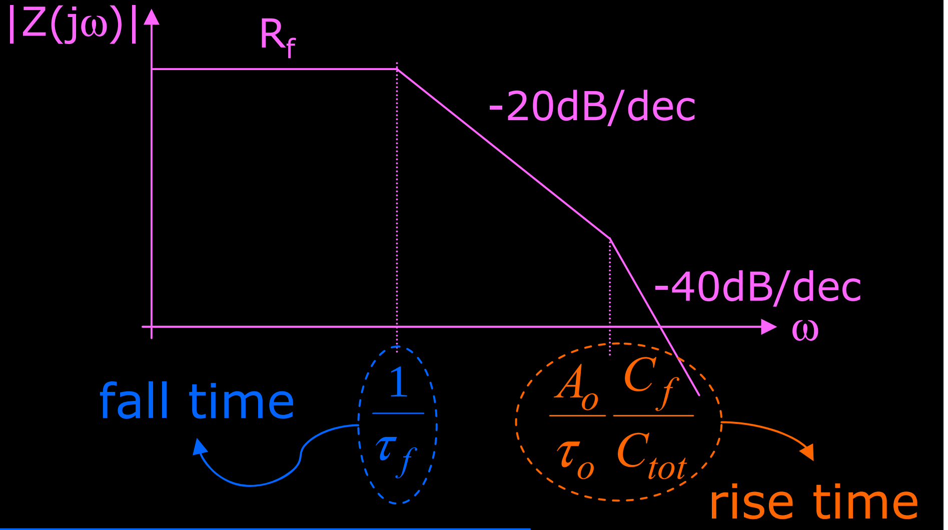
$$A(s)=\frac{A_o}{1+s\tau_o}$$

$$\tau_o = r_o C_o$$



rise time 10%-90%

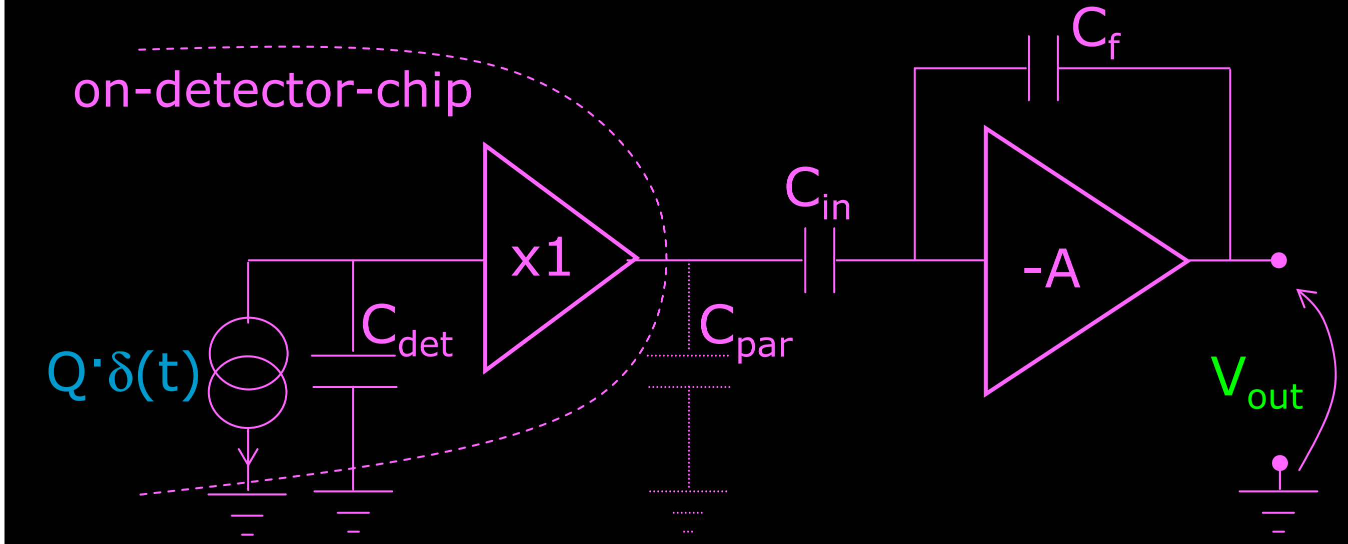
typically «100ns



fall time $\frac{1}{\tau_f}$

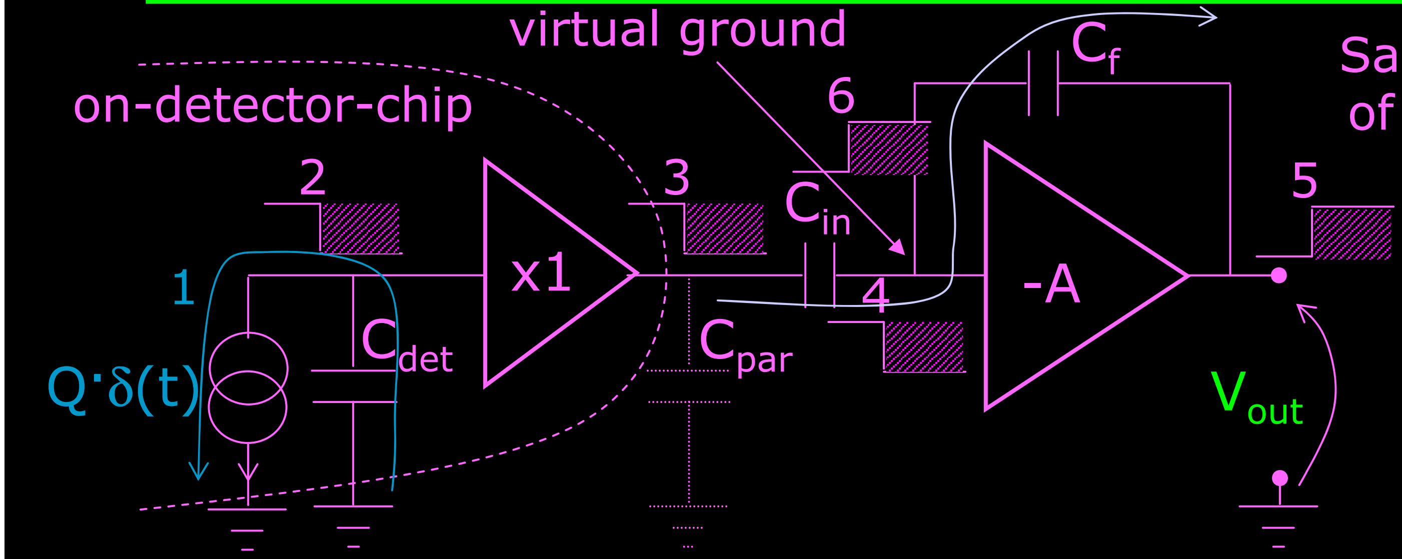
$\frac{A_o}{\tau_o}\frac{C_f}{C_{tot}}$ rise time

# Voltage Preamplifier



Used in pnCCDs and SDDs with *on-chip* source follower (buffer).

# Voltage Preamplifier



Same reset mechanism of charge preamplifiers can be used.

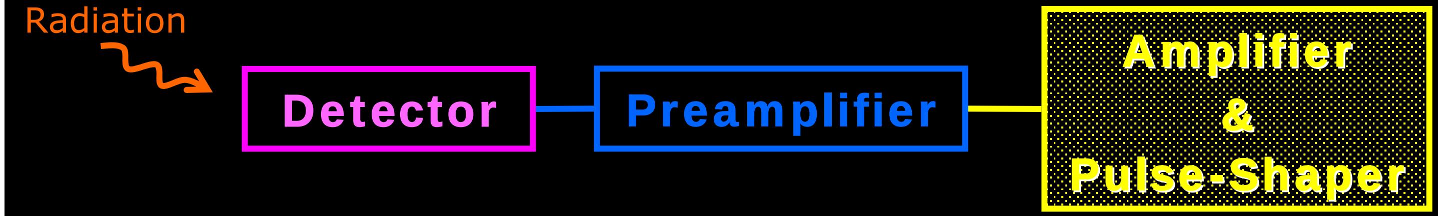
Same considerations on rise time can be obtained.

$$V_{out} = \frac{Q}{C_{det}} \cdot \frac{1}{1+\frac{1}{A}\left(1+\frac{C_{in}}{C_f}\right)} \cdot 1(t) \xrightarrow{A\to\infty} \frac{Q}{C_{det}} \frac{C_{in}}{C_f} \cdot 1(t)$$ (assuming ideal buffer)

☹ Charge-to-voltage conversion on the detector capacitance.

☺ On-chip source follower minimizes the impact of parasitics.

# Shaping amplifier – I



The amplifier and pulse-shaper (also called shaping amplifier):

- amplify the signals to a level sufficient for further analysis;
- shape the signals to optimize the system performances:
  - ✓ obtain the best practically possible S/N
  - ✓ (permit the operation at high counting rates)
  - ✓ (make the output pulse amplitude almost insensitive to fluctuations in the signal rise-time)

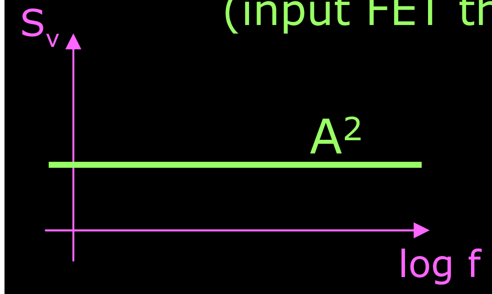
# Shaping amplifier - II

The noise can be viewed either in the frequency ($f$) domain – *power spectra S* – or in the time ($t$) domain– *random sequences of small pulses with given shape and rate of occurrence.*

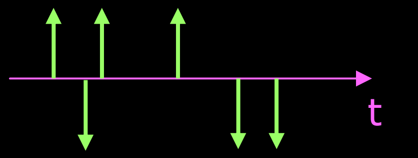
The noise at the preamp output (i.e. the shaper input noise) has two main components:

series noise

(input FET thermal noise)

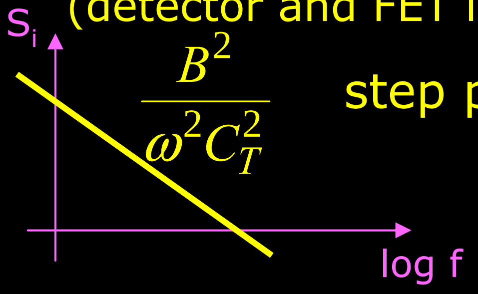


δ pulses – rate $A^2$

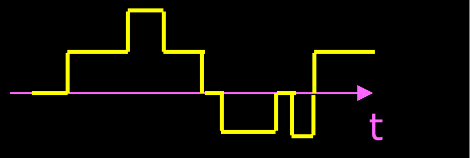


parallel noise

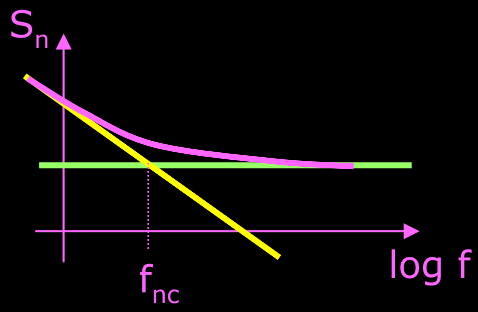
(detector and FET leakage current)



step pulses – rate $B^2/C_T^2$



total noise spectrum



noise corner frequency (time-constant)

$$f_{nc} = \frac{1}{2\pi C_T}\frac{B}{A} = \frac{1}{2\pi C_T}\sqrt{\frac{b}{a}} \qquad \left(\tau_{nc} = C_T\sqrt{\frac{a}{b}}\right)$$

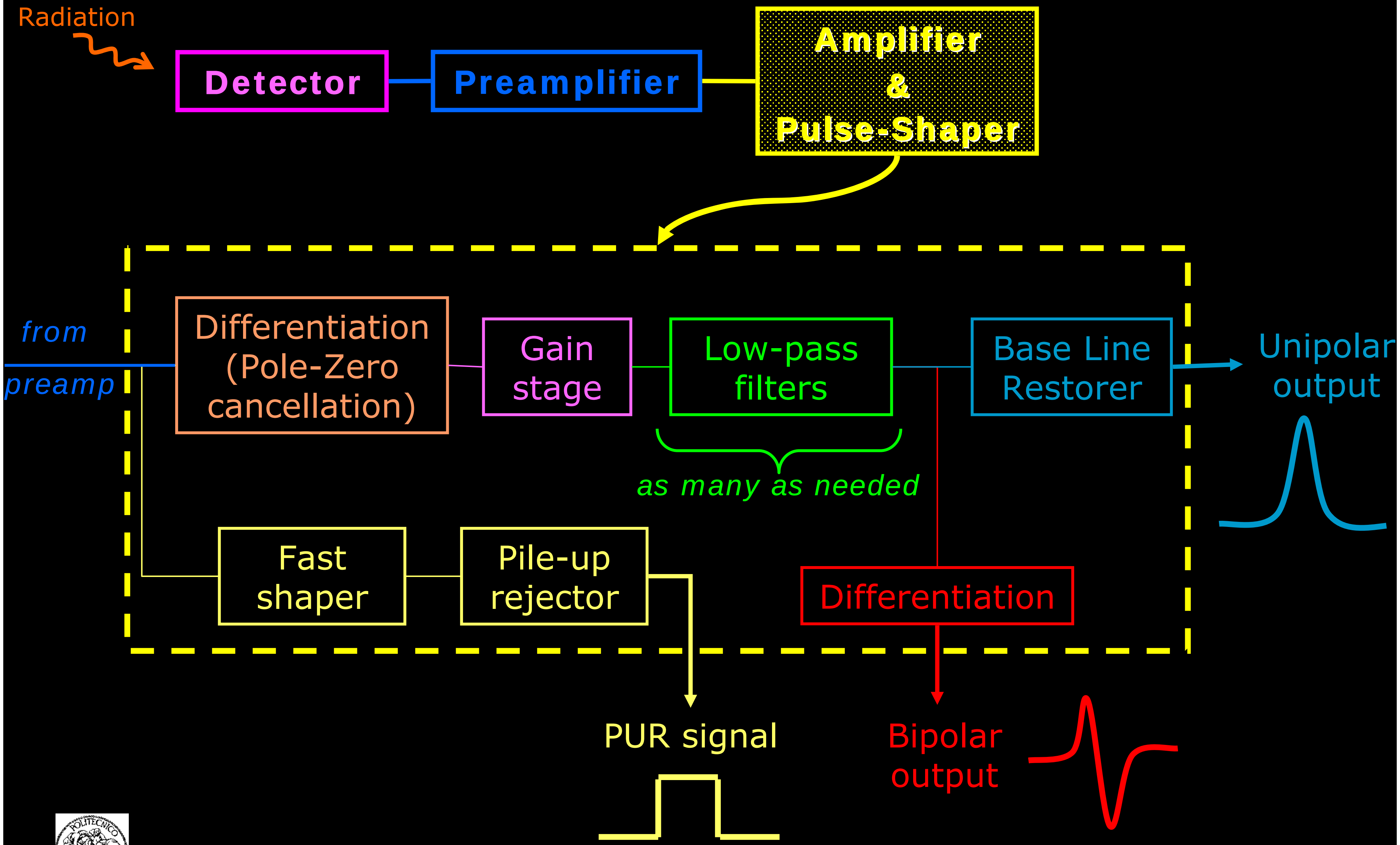
**suitable filtering includes:**

✓ **high-pass filter to cut $S_i$**

✓ **low-pass filter to cut $S_v$**

**filters' bandwidth depends on $f_{nc}$**

# Shaping amplifier – III



Radiation

**Detector**

**Preamplifier**

**Amplifier**
**&**
**Pulse-Shaper**

*from preamp*

Differentiation (Pole-Zero cancellation)

Gain stage

Low-pass filters

*as many as needed*

Base Line Restorer

Unipolar output

Fast shaper

Pile-up rejector

Differentiation

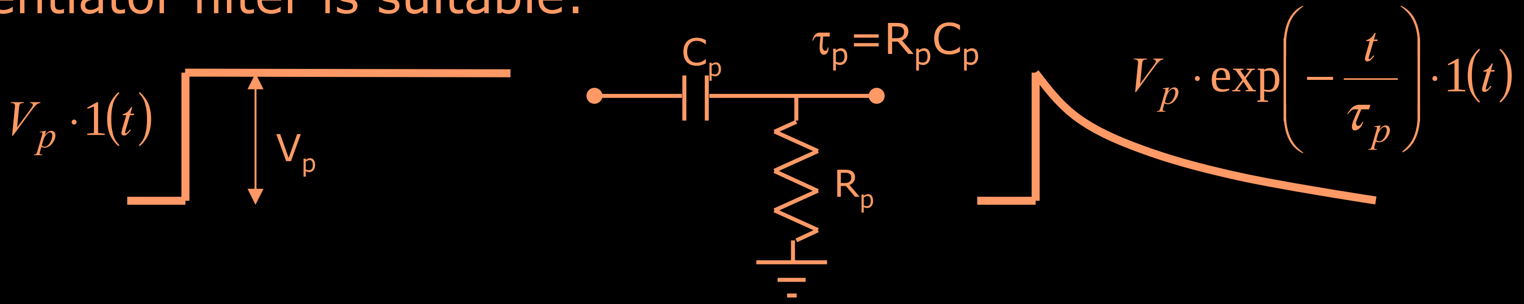
PUR signal

Bipolar output

# Shaping amplifier – IV

## ✓ *Pole-zero cancellation - I*

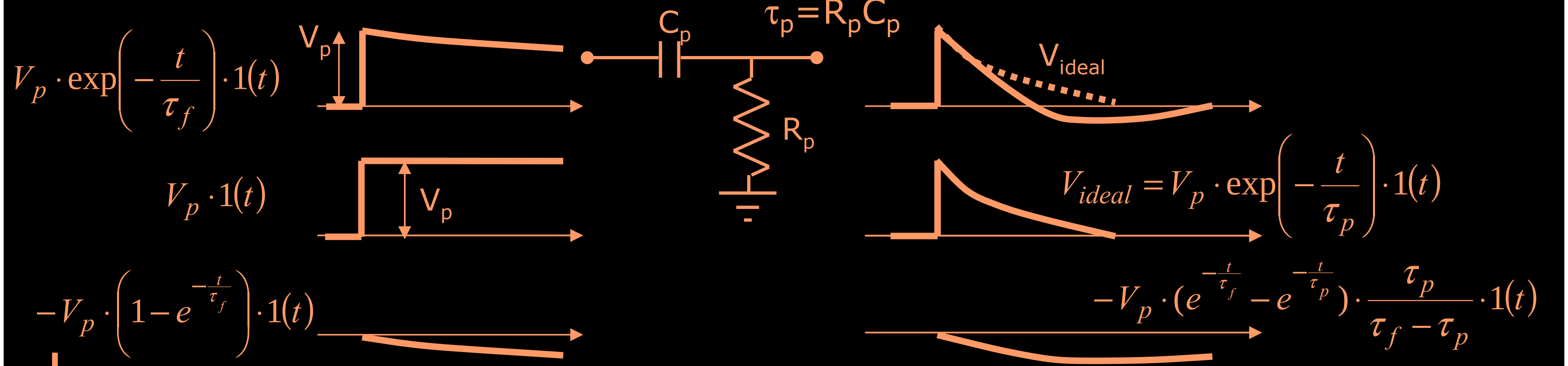
When the signal coming from the preamp has a “flat-top” a simple CR differentiator filter is suitable:



$$V_p \cdot 1(t)$$

$$V_p \cdot \exp\left(-\frac{t}{\tau_p}\right) \cdot 1(t)$$

**BUT** if the preamp features a resistive discharge, the CR differentiator is not suitable. In fact:



$$V_p \cdot \exp\left(-\frac{t}{\tau_f}\right) \cdot 1(t)$$

$$V_p \cdot 1(t)$$

$$-V_p \cdot \left(1-e^{-\frac{t}{\tau_f}}\right) \cdot 1(t)$$

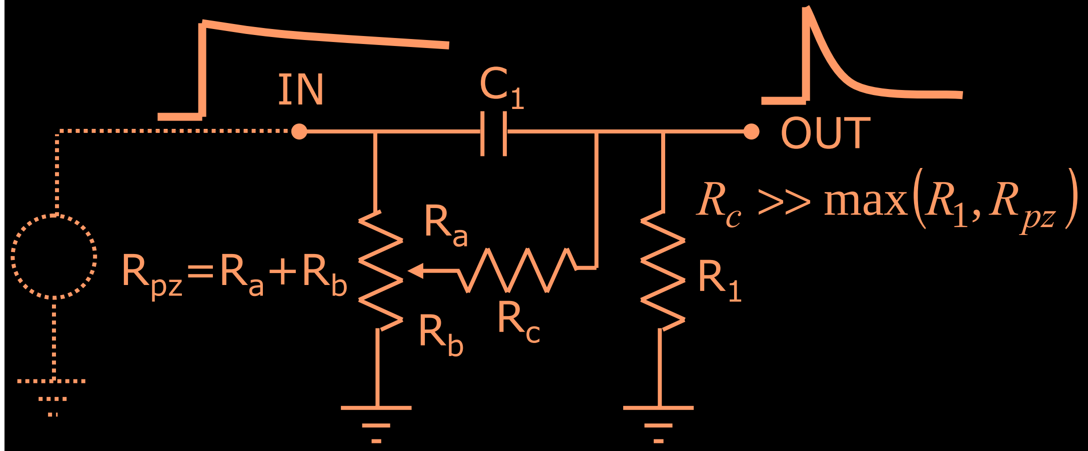
$$V_{ideal} = V_p \cdot \exp\left(-\frac{t}{\tau_p}\right) \cdot 1(t)$$

$$-V_p \cdot (e^{-\frac{t}{\tau_f}} - e^{-\frac{t}{\tau_p}}) \cdot \frac{\tau_p}{\tau_f - \tau_p} \cdot 1(t)$$

↳ spurious “tail” added to the correct pulse (although of lowered amplitude)

# Shaping amplifier – V

## ✓ *Pole-zero cancellation - II*



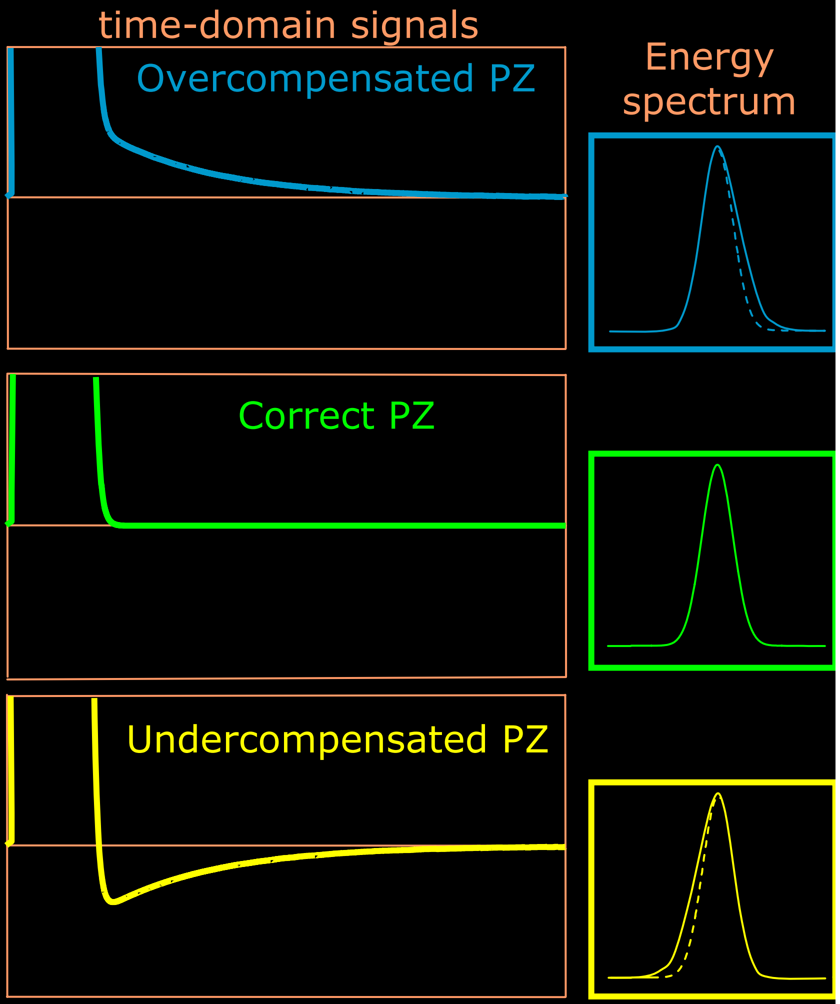
$$R_c >> \max\left(R_1, R_{pz}\right)$$

Fixed pole $\tau_p \cong C_1\left(R_1 // R_c\right)$

Adjustable zero $\tau_z \cong C_1\left(R_a + R_c\right)$

Attenuation factor $\left.\frac{OUT}{IN}\right|_{DC} \cong \frac{R_1}{R_1 + R_c}\frac{R_b}{R_{pz}}$

A proper cancellation implies to transmit through the input stage a fraction $\tau_p/\tau_f$ of the DC level at the input.

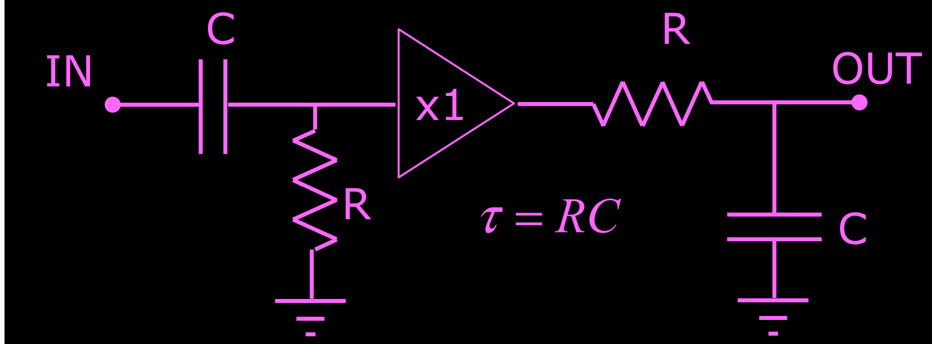


*Note:* the name “Pole-Zero cancellation” originates from the Laplace transform domain analysis. The exponential signal at the preamp output is represented by a real pole at $1/\tau_f$ and the network provides an adjustable zero to neutralize such pole, besides the desired pole at $1/\tau_p$.

# Shaping amplifier – VI

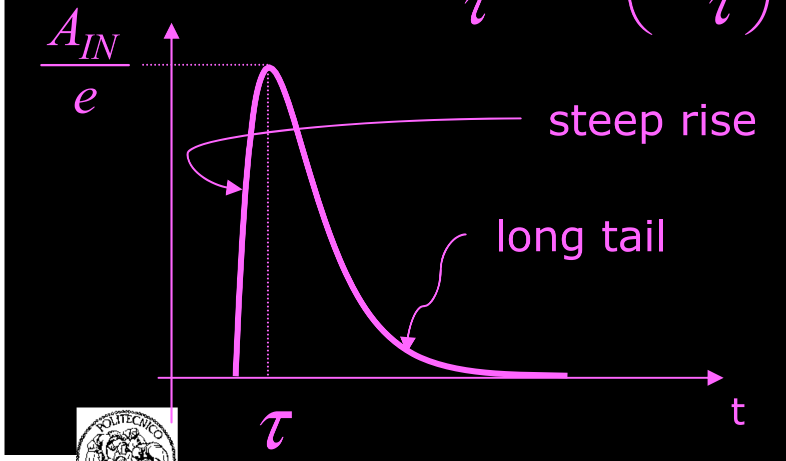
## ✓ *Practical shaper implementation – I*

- ***RC-CR***



✓ time response:

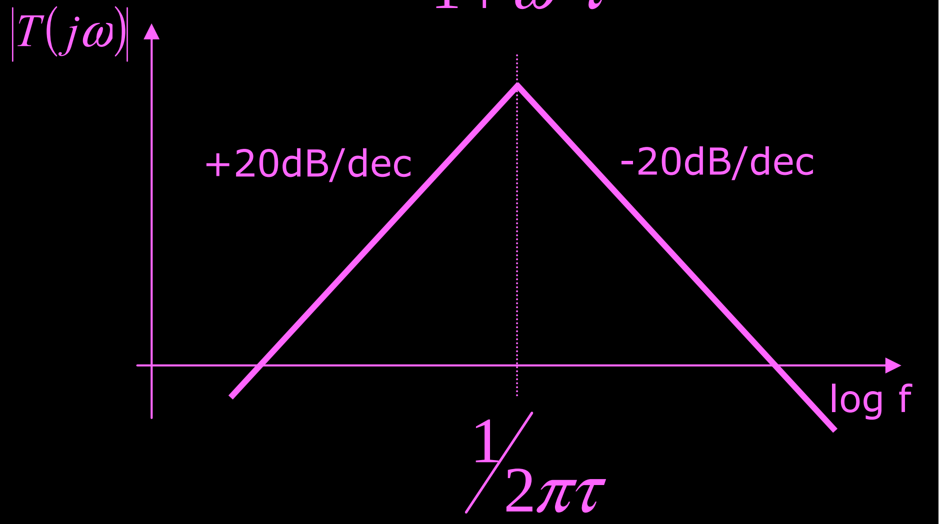
$$OUT(t) = A_{IN} \cdot \frac{t}{\tau} \cdot \exp\left(-\frac{t}{\tau}\right)$$



✓ transfer function:

$$T(s) = \frac{sCR}{1+sCR}\frac{1}{1+sRC}$$

$$|T(j\omega)| = \frac{\omega\tau}{1+\omega^2\tau^2}$$



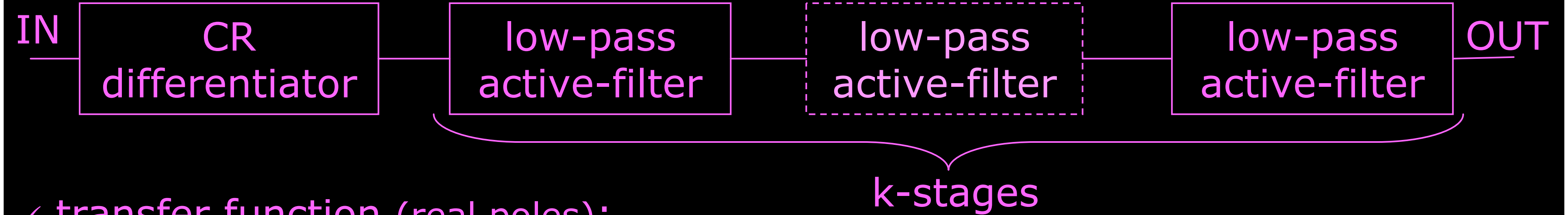
$\tau = RC$ shaping time constant

$\tau = RC$ peaking time

# Shaping amplifier –VII

✓ ***Practical shaper implementation – II***

- ***(pseudo) Gaussian*** (polinomial approximation of a Gaussian shape)



IN

CR differentiator

low-pass active-filter

low-pass active-filter

low-pass active-filter

OUT

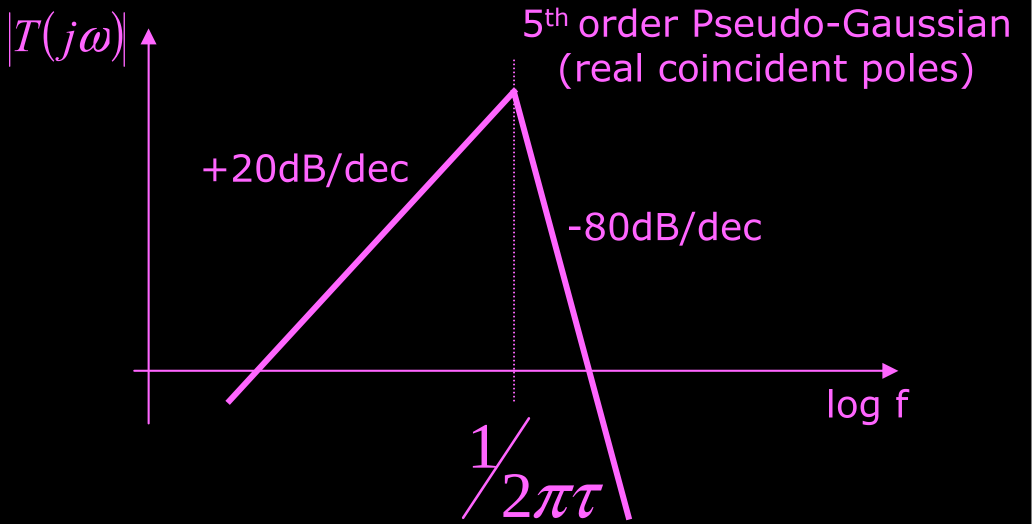
k-stages

✓ transfer function (real poles):

$$T(s) = \frac{s\tau}{\prod_{i=1}^{n}(1+s\tau_i)}$$

✓ time-domain response:

$$OUT(t) = A_{IN} \cdot \frac{t^{n-1}}{\tau^{n-1}} \cdot \exp\left(-\frac{t}{\tau}\right)$$

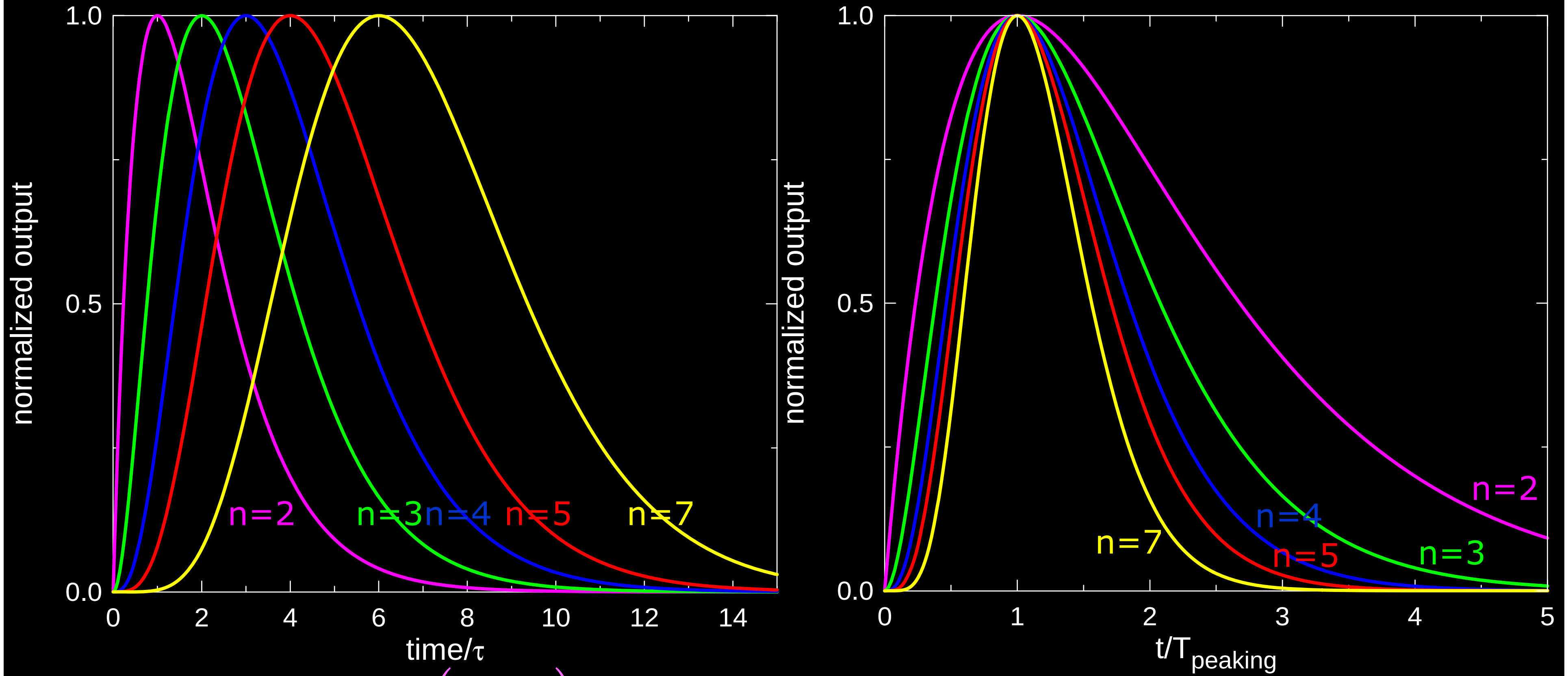


The number of poles gives the order of the pseudo-Gaussian shaping.

# Shaping amplifier –VIII

✓ *Practical shaper implementation – II*

- *(pseudo) Gaussian: pulse shape vs shaping order*



$$T_{peak} = (n-1) \cdot \tau$$

(for real poles)

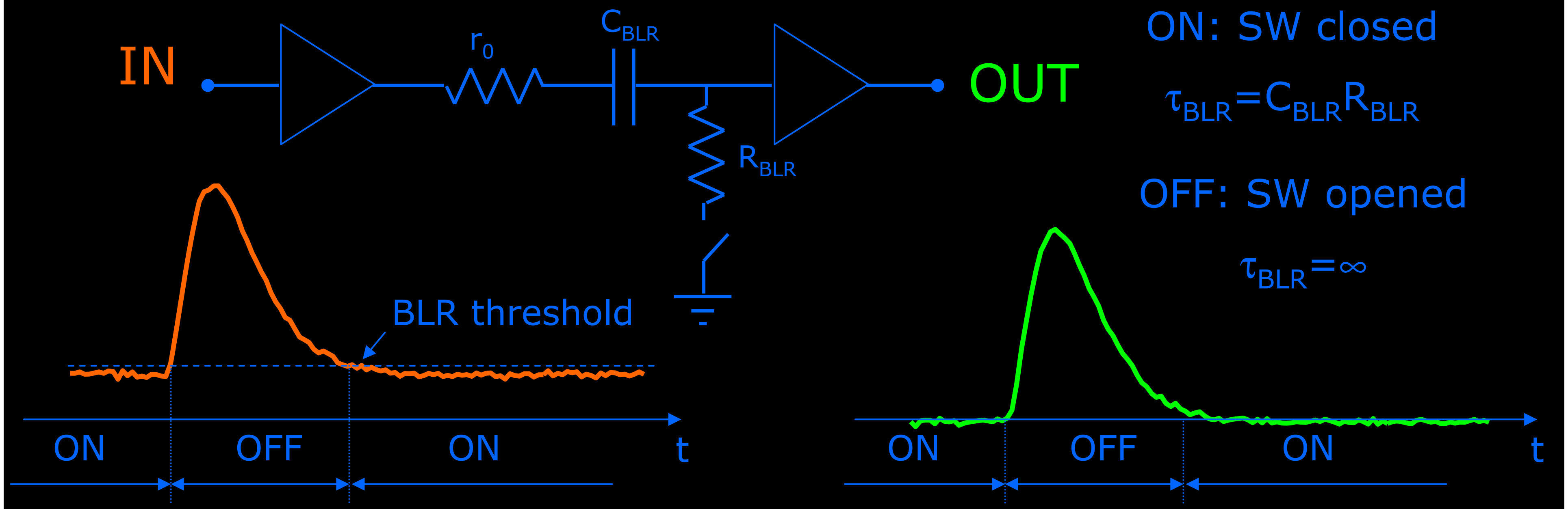
Better approximation of the Gaussian shape with increasing number of poles.

# Shaping amplifier – IX

✓ ***Base-line restorer (BLR)***

It has to cut-off the low-frequency noise and disturbances and the drift of the DC level.

This function cannot be performed by a simple high-pass filter since it would alter also the signal pulse shape ➡ **Time-variant differentiator filter**



ON: SW closed

$\tau_{BLR}=C_{BLR}R_{BLR}$

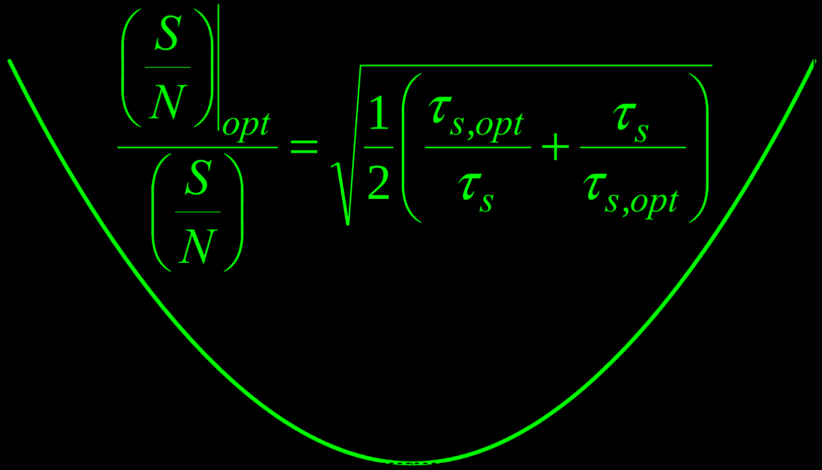
OFF: SW opened

$\tau_{BLR}=\infty$

The optimum BLR threshold level corresponds to the edge of the noise amplitude distribution , i.e. to the border of the "noise band" visible on the oscilloscope.

# Shaping amplifier –X

## ✓ *Filter parameter setting and signal to noise ratio*



$$\frac{\left(\frac{S}{N}\right)\Big|_{opt}}{\left(\frac{S}{N}\right)} = \sqrt{\frac{1}{2}\left(\frac{\tau_{s,opt}}{\tau_s}+\frac{\tau_s}{\tau_{s,opt}}\right)}$$

In a linear filter the output amplitude receives contribution from the input values over a preceding interval (similar to weighted average).

For a uniform weighting (integration for time $\tau_s$):

- signal amplitude $\propto \tau_s$
- series contribution $\propto$ (mean n. of $\delta$ pulses in $\tau_s$)

  $\propto A^2\,\tau_s$
- parallel contribution $\propto$ (n. of step pulses in $\tau_s$* step amplitude),

  $\propto (B/C_T)^2\,\tau_s{*}\,\tau_s^2 = (B/C_T)^2\,\tau_s^3$

$$\tau_s\big|_{opt} \propto \frac{AC_T}{B} = \sqrt{\frac{a}{b}}\,C_T = \tau_c$$

(see slide 39)

# Pile-Up Rejection – I

✓***Pile-up probability***

Pile-up of shaper output pulses is an unavoidable effect caused by:

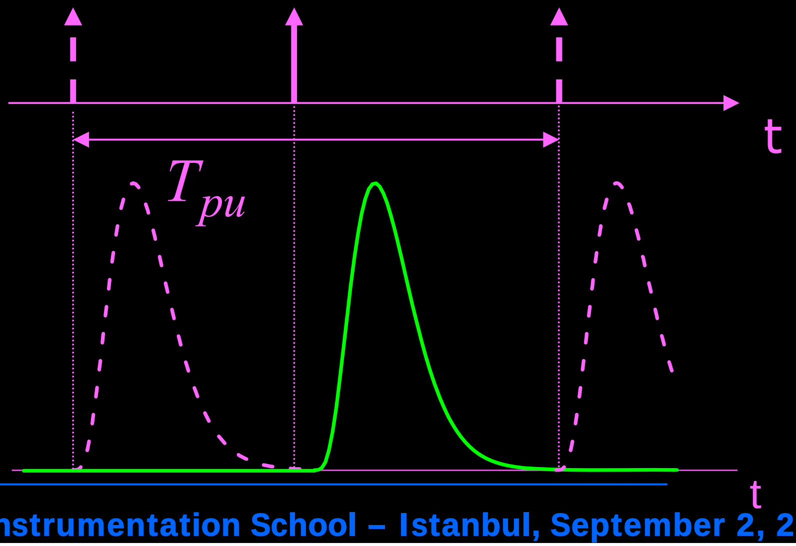
- random distribution in time of radiation pulses
- finite width of output pulses

From Poisson statistics:

- probability of being free from pile-up: $P_{np} = \exp\left(-\lambda_{in} T_{pu}\right)$
- probability of being piled-up: $P_{pu} = 1 - P_{np} = 1 - \exp\left(-\lambda_{in} T_{pu}\right)$

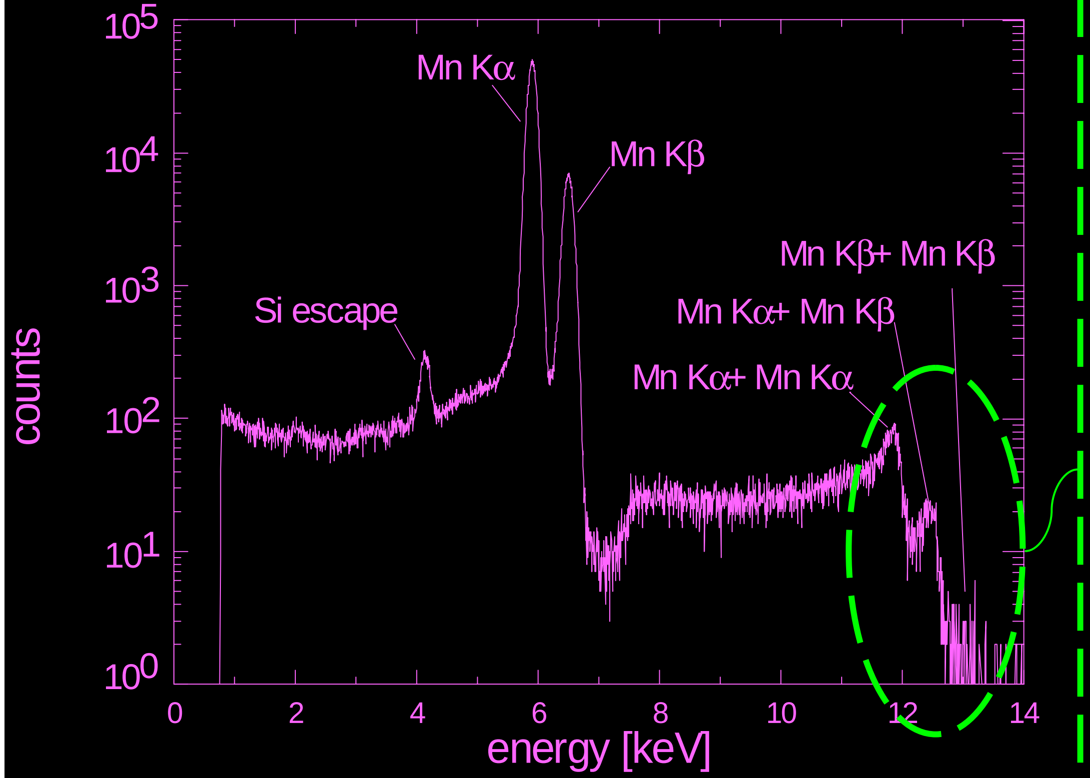
$\lambda_{in}$ mean rate of detected radiation

$T_{pu}$ pile-up guard interval



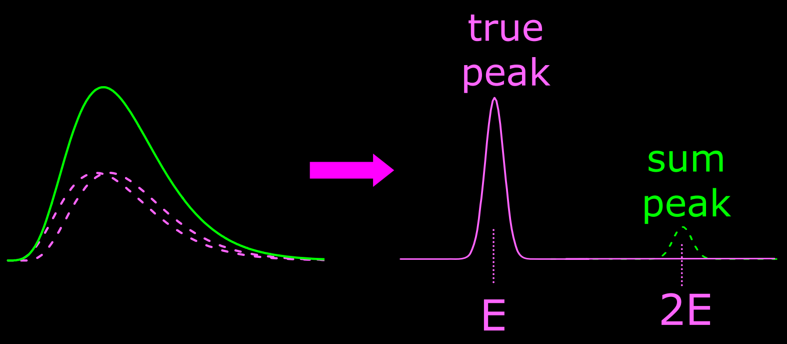
# Pile-Up Rejection – II

✓ ***Effects on the collected spectra***



$^{55}$Fe energy spectrum detected by a 5 mm$^2$ Peltier-cooled Silicon Drift Detector (7th order pseudo-Gaussian shaper t=1μs) at 8 kcps with no pile-up rejection.

**Full pile-up:**

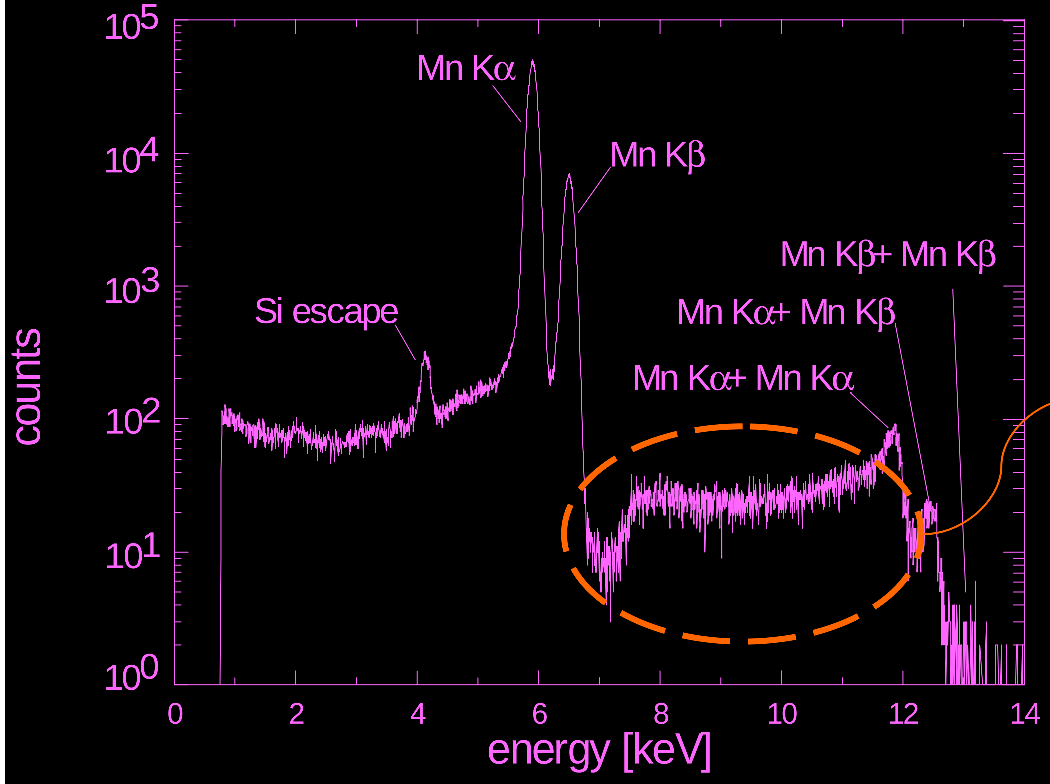


Sum-peaks are distinguishable because:

- ✓ apparent energy is the double of the energy of a true peak or the sum of energies of couples of true peaks;
- ✓ intensity of sum peaks increases as counting rate increase, while that of the true peaks decreases.

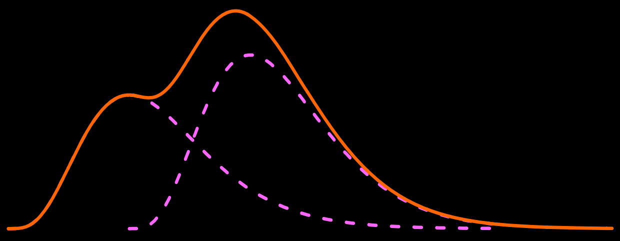
# Pile-Up Rejection – III

✓ *Effects on the collected spectra*



$^{55}$Fe energy spectrum detected by a 5 mm$^2$ Peltier-cooled Silicon Drift Detector (7th order pseudo-Gaussian shaper $t=1\mu s$) at 8 kcps with no pile-up rejection.

**"Intermediate" pile-up:**

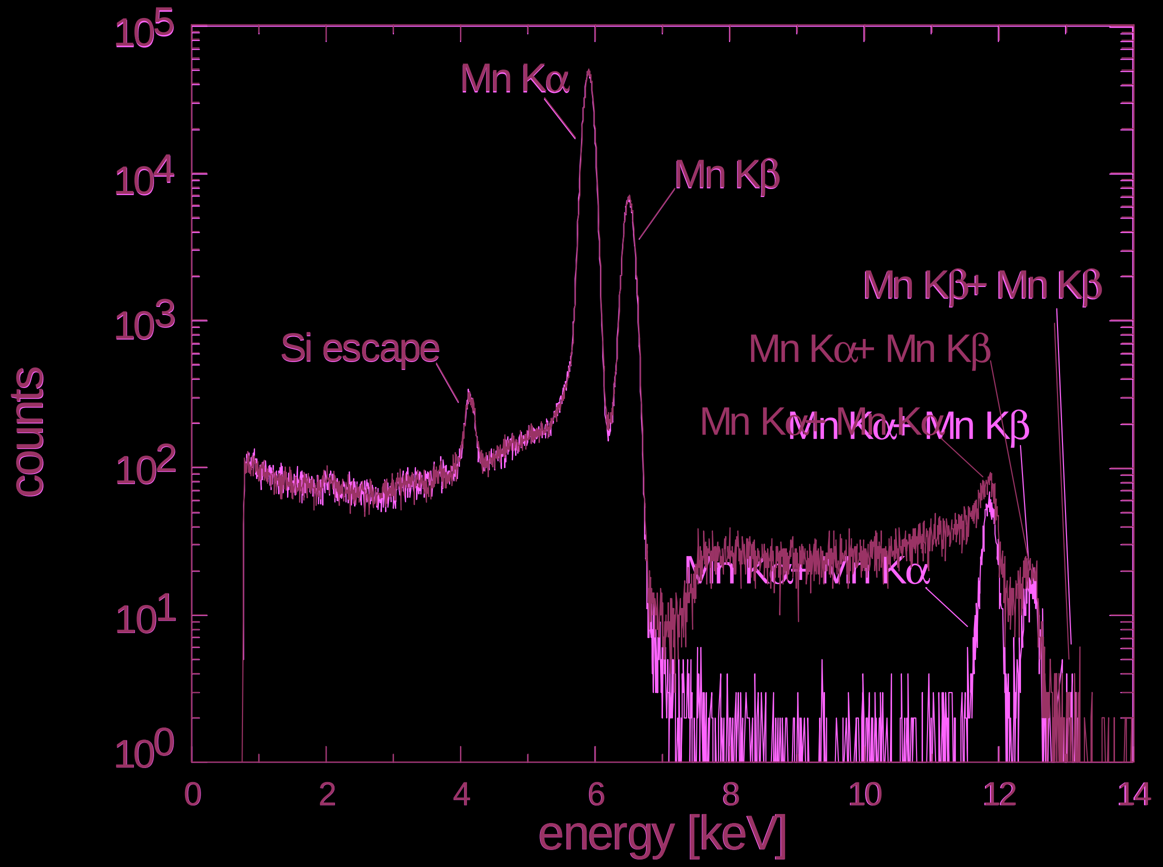


Any amplitude in between the true peak (or peaks) and its double (or their sum) can be obtained.

This is particularly undesirable when trace elements spectroscopy has to be performed.

# Pile-Up Rejection – IV

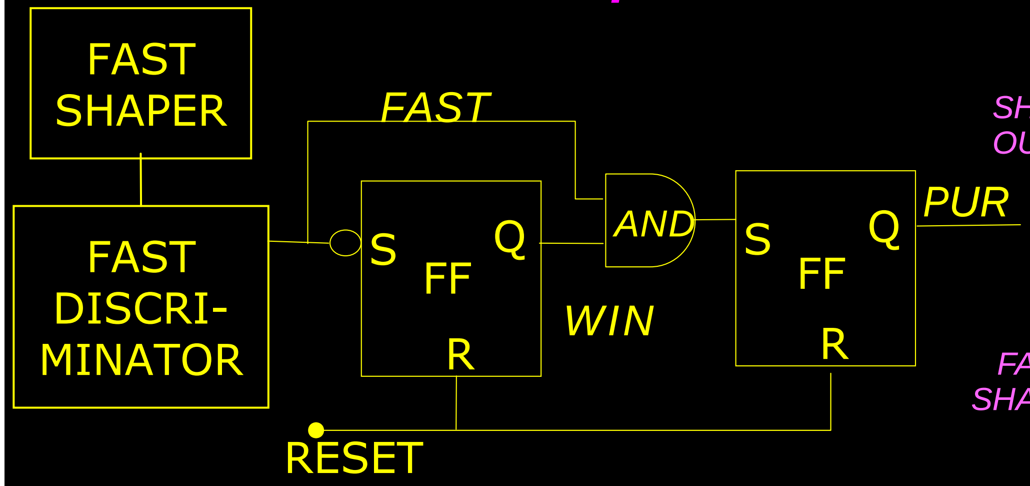
- ✓ ***Effects on the collected spectra***



$^{55}$Fe energy spectrum detected by a 5 mm$^2$ Peltier-cooled Silicon Drift Detector (7th order pseudo-Gaussian shaper $t$=1μs) at 8 kcps with pile up rejection.

# Pile-Up Rejection –V

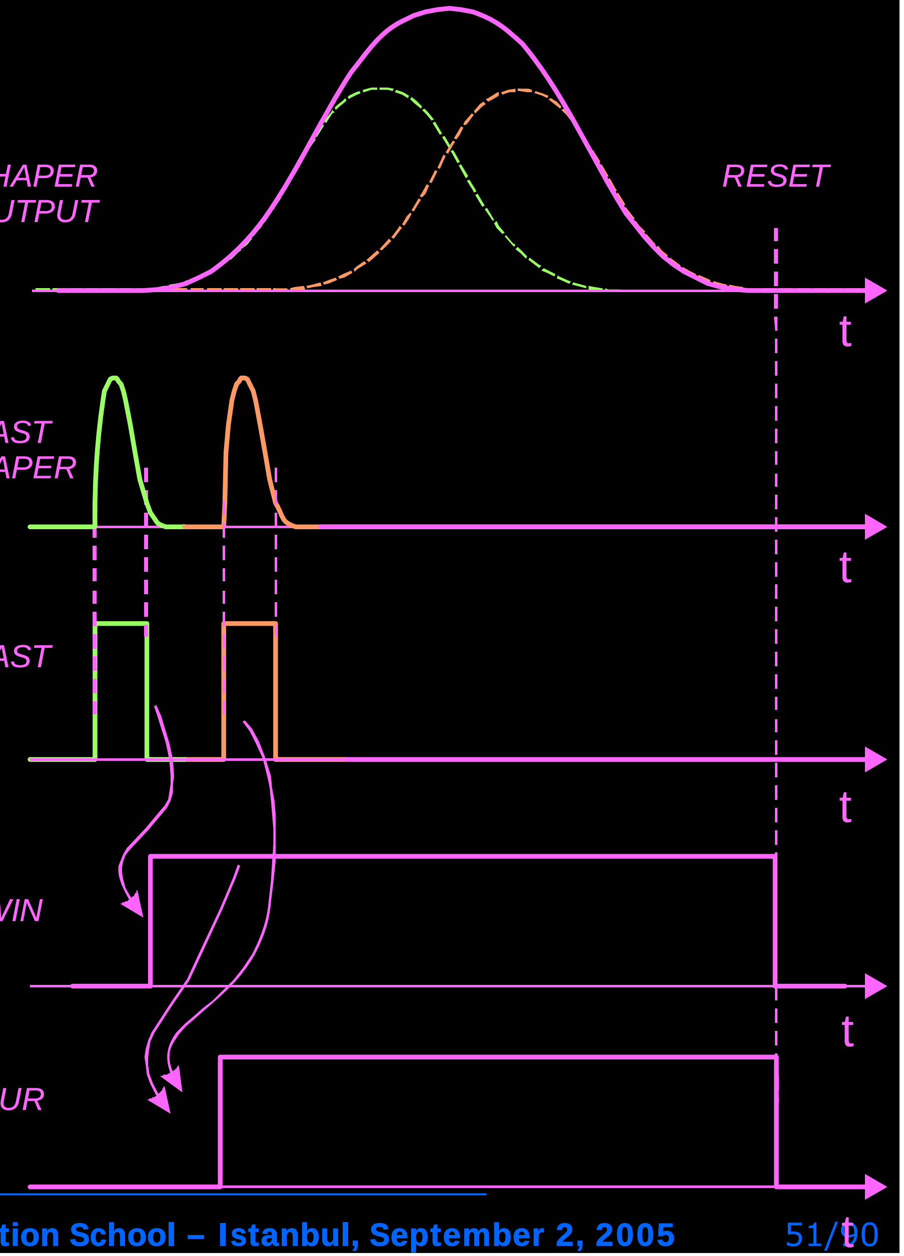
## ✓*PUR circuit example*



Setting of the fast discriminator threshold is very important:

- if too high, pile-up by small signals not recognized
- if too low, discriminator frequently triggered by noise and large fraction of signals unnecessarily rejected

The correct setting of the threshold is at the edge of the noise band.



# The end

# Front-end Electronics and Signal Processing - II

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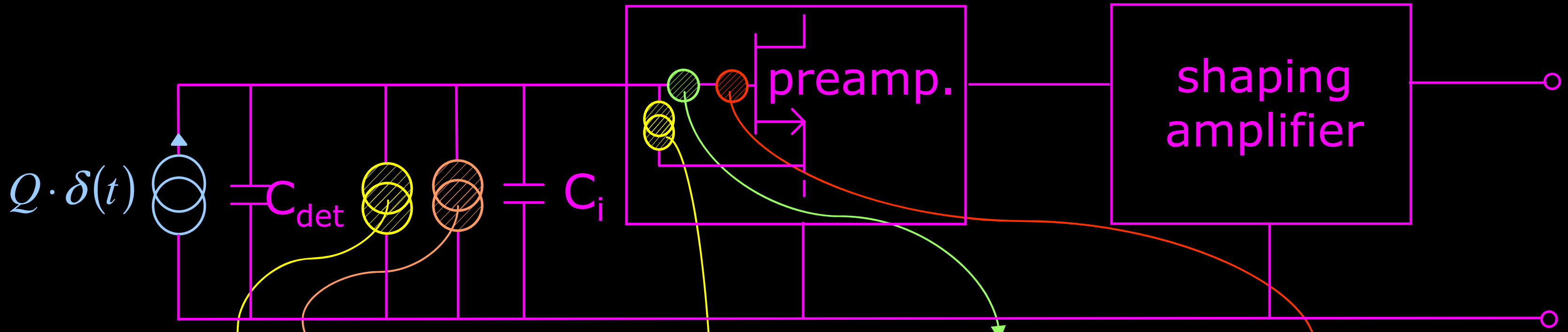
*http://www.elet.polimi.it/upload/guazzoni*

# Table of Contents – Part II

- Introduction
- Signal formation and Ramo's Theorem
- Noise analysis and noise modeling in electronic devices
- Basic blocks in a detection system
- Equivalent noise charge concept and calculation
- Capacitive matching
- Time measurements
- Time-variant filters

# Equivalent Noise Charge - I

✓ *Identification of the detector and preamplifier noise sources*



$qI_L + \frac{2kT}{R_f}$

(leakage current feedback resistor)

$2kTC\tan(\delta)\omega$

(capacitor dielectric losses)

$qI_G$

(gate current shot noise)

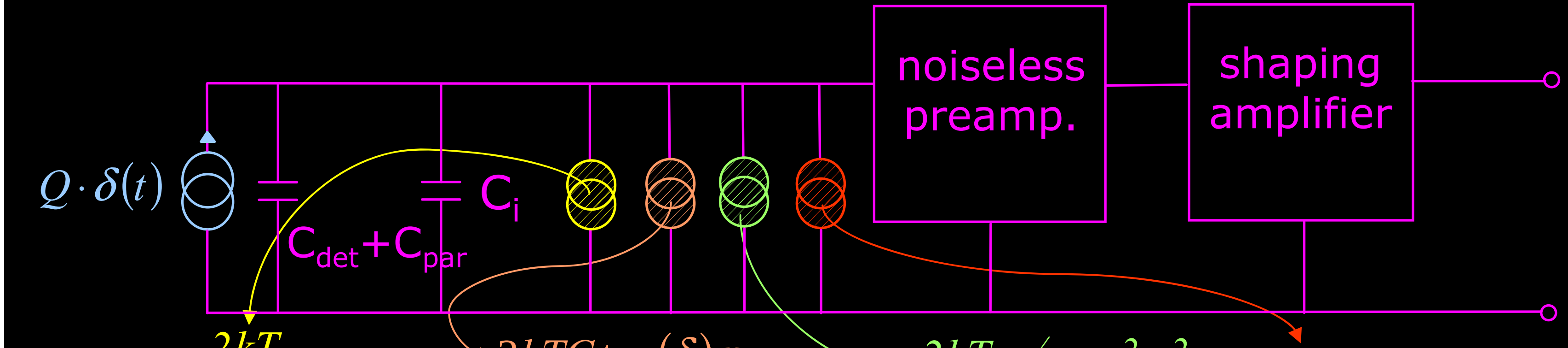
$2kT\alpha/g_m$

(white voltage noise)

$\frac{\pi A_f}{\omega}$ (1/f voltage noise)

# Equivalent Noise Charge - II

✓ *Equivalent circuit for ENC calculation*



$qI_L + \frac{2kT}{R_f} + qI_G$ (leakage current feedback resistor gate current)

$2kTC\tan(\delta)\omega$ (capacitor dielectric losses)

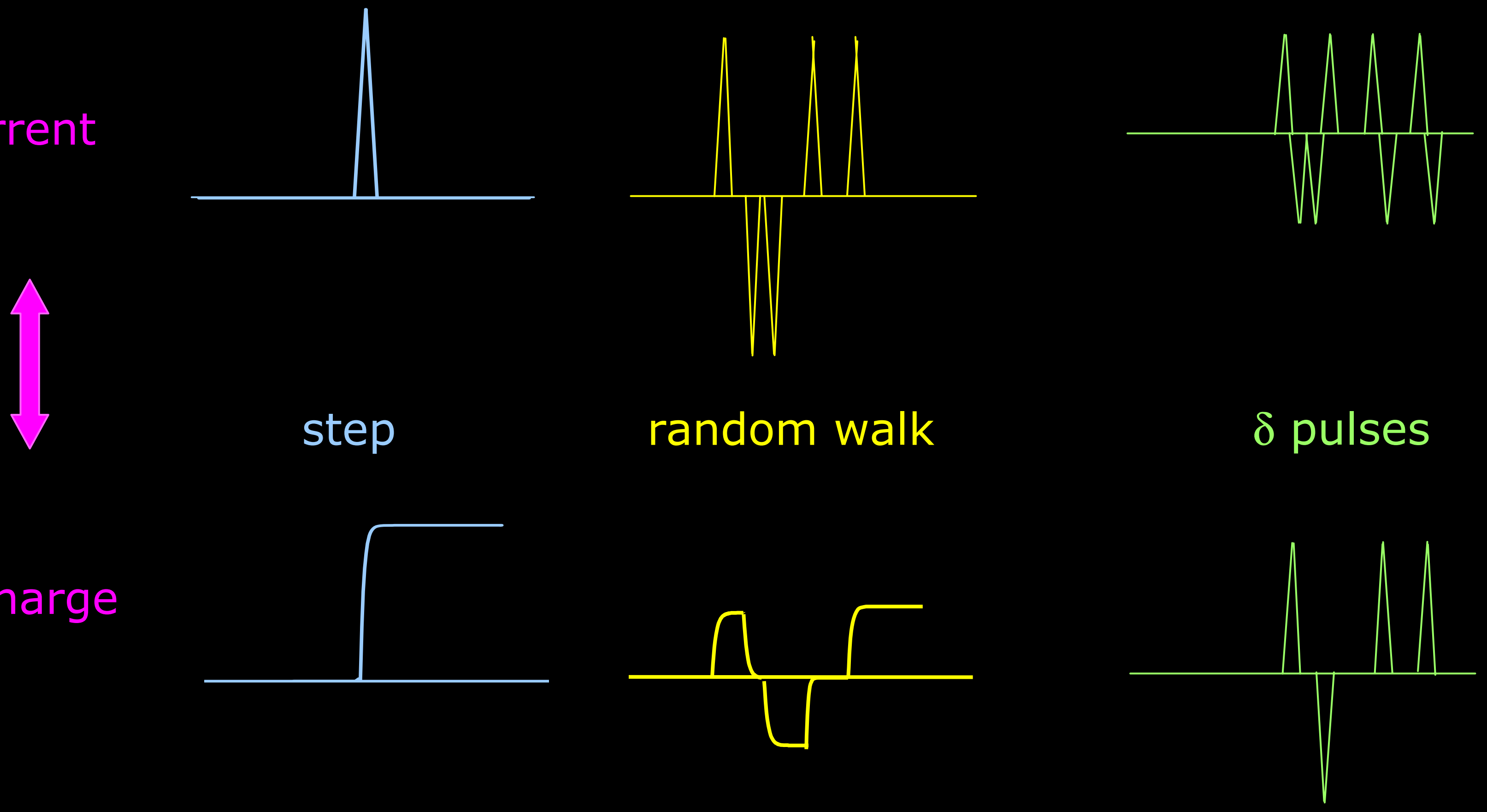
$2kT\,\alpha/g_m \; \omega^2 C_T^2$ (white voltage noise)

$\pi A_f \omega C_T^2$ (1/f voltage noise)

**Equivalent Noise Charge** is the value of charge that injected across the detector capacitance by a δ-like pulse produces at the output of the shaping amplifier a signal whose amplitude equals the output r.m.s. noise, i.e. is the amount of charge that makes the S/N ratio equal to 1.

# Equivalent Noise Charge - III

✓*Time domain representation of signal and noise*



$\delta$ pulse

$\delta$ pulses

$\delta$` pulses doublets

Current

step

random walk

$\delta$ pulses

Charge

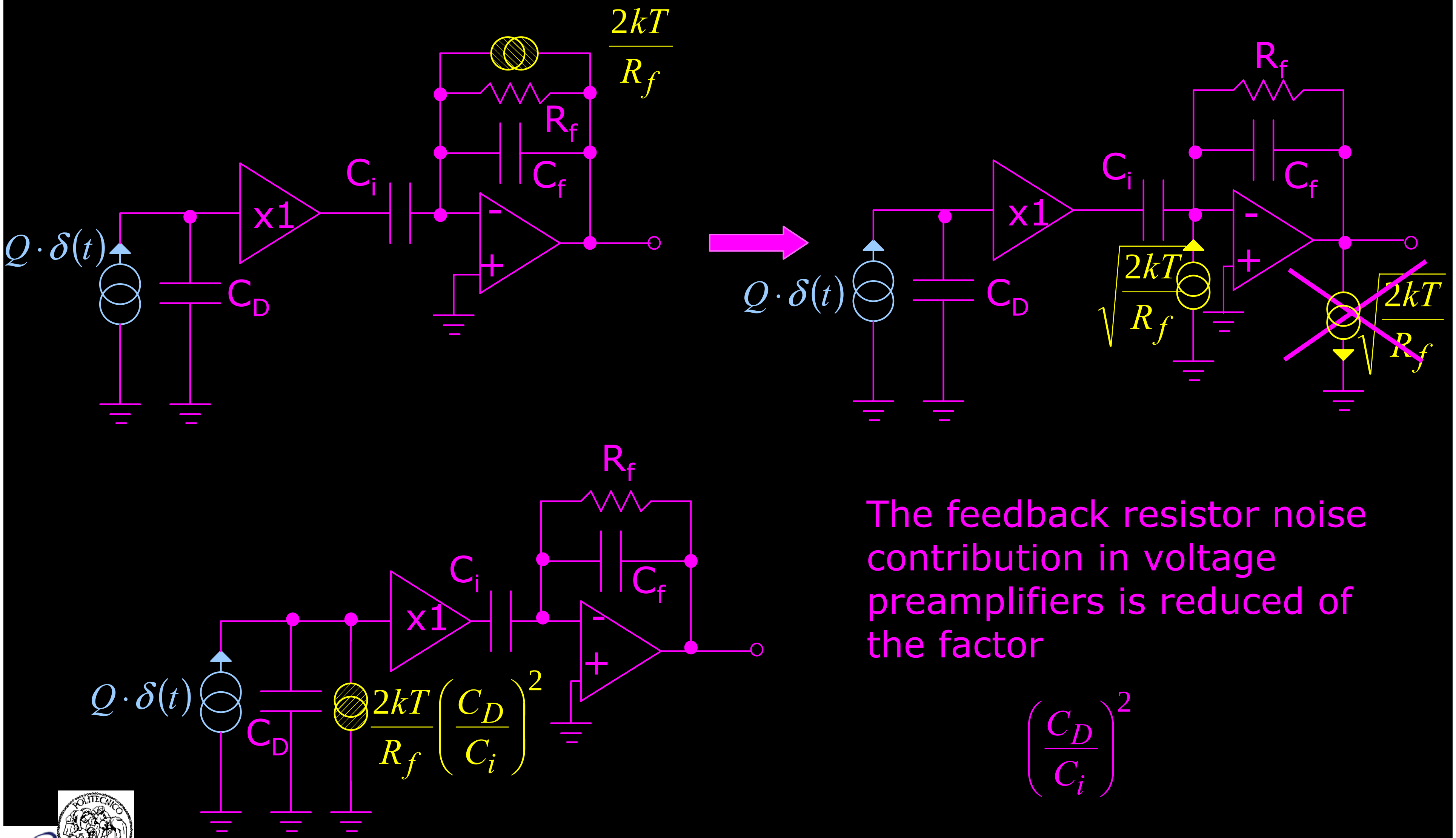
**signal**

**white parallel**

**white series**

# Equivalent Noise Charge - IV

✓ ***Feedback resistor noise contribution in voltage preamplifiers***

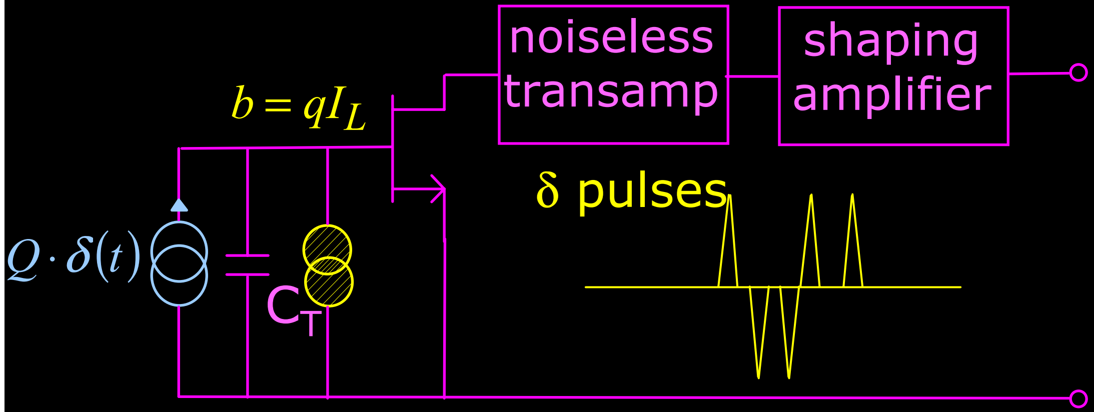


The feedback resistor noise contribution in voltage preamplifiers is reduced of the factor

$$\left(\frac{C_D}{C_i}\right)^2$$

# Equivalent Noise Charge - V

✓ ***ENC calculation in presence of white parallel noise***

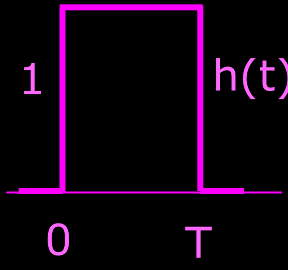


$$ENC_p^2 = \int_{-\infty}^{+\infty} N(\omega)|H(j\omega)|^2 df =$$

$$= qI_L \int_{-\infty}^{+\infty} |H(j\omega)|^2 df = qI_L \int_{-\infty}^{+\infty} |h(t)|^2 dt$$

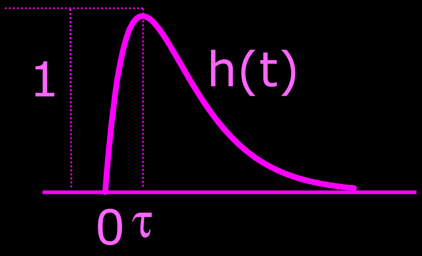
Parseval's theorem

gated integrator



$$\int_{-\infty}^{+\infty} |h(t)|^2 dt = T$$

$$ENC_p^2 = qI_L T = q(N_{el}q)$$



RC-CR shaping

$$\int_{-\infty}^{+\infty} |h(t)|^2 dt = \frac{e^2}{4}\tau = 1.85 \cdot \tau$$

$$\Rightarrow \boxed{ENC_p^2 = bA_3\tau}$$

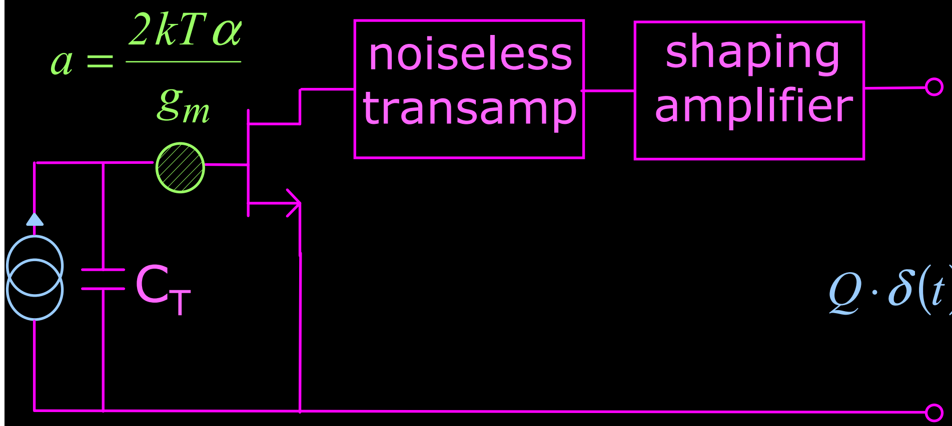
$ENC_p^2 \propto \tau$

$ENC_p^2$ independent of $C_T$

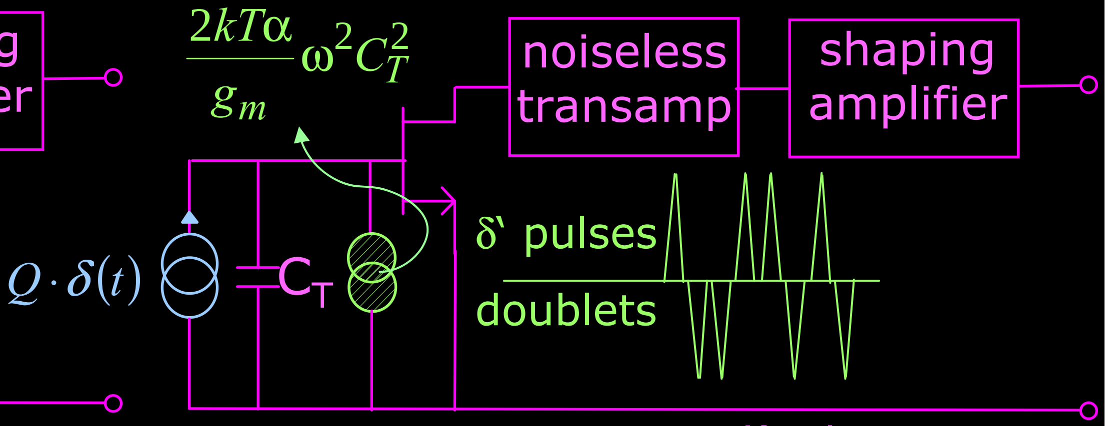
# Equivalent Noise Charge - VI

✓ ***ENC calculation in presence of white series noise***

$$a=\frac{2kT\alpha}{g_m}$$

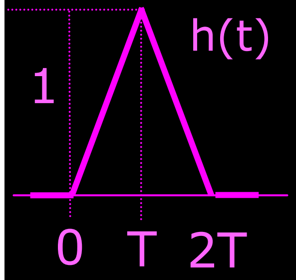


$$\frac{2kT\alpha}{g_m}\omega^2 C_T^2$$



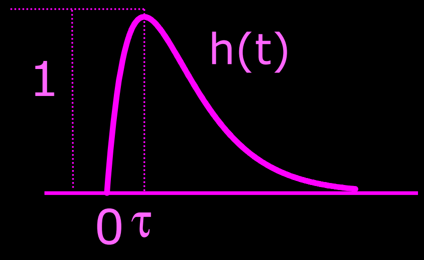
Parseval's theorem

$$ENC_s^2=\int_{-\infty}^{+\infty}N(\omega)|H(j\omega)|^2df=\frac{2kT\alpha}{g_m}C_T^2\int_{-\infty}^{+\infty}\omega^2|H(j\omega)|^2df=\frac{2kT\alpha}{g_m}C_T^2\int_{-\infty}^{+\infty}|h^{'}(t)|^2dt$$



triangular shaping

$$\int_{-\infty}^{+\infty}|h^{'}(t)|^2dt=\frac{2}{T}$$



RC-CR shaping

$$\int_{-\infty}^{+\infty}|h^{'}(t)|^2dt=\frac{e^2}{4}\frac{1}{\tau}=\frac{1.85}{\tau}$$

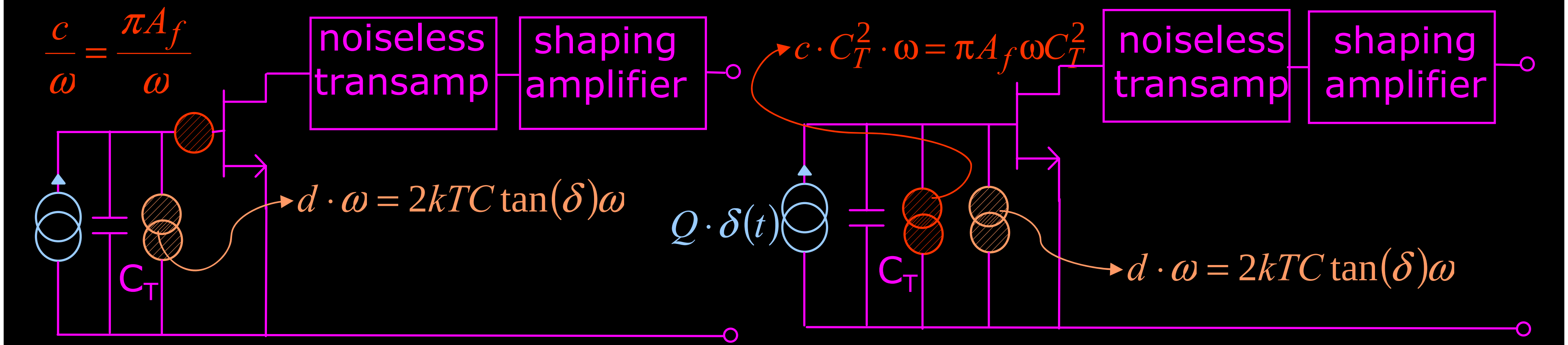
$$\boxed{ENC_s^2=a\,C_T^2A_1\frac{1}{\tau}}$$

$$ENC_s^2\propto C_T^2$$

$$ENC_s^2\propto\frac{1}{\tau}$$

# Equivalent Noise Charge - VII

✓ ***ENC calculation in presence of 1/f noise and/or dielectric losses***



$\frac{c}{\omega} = \frac{\pi A_f}{\omega}$

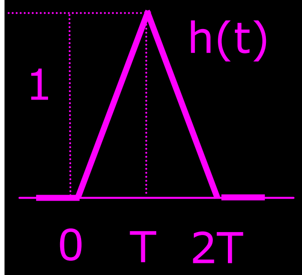
$d \cdot \omega = 2kTC\tan(\delta)\omega$

$Q \cdot \delta(t)$

$c \cdot C_T^2 \cdot \omega = \pi A_f \omega C_T^2$

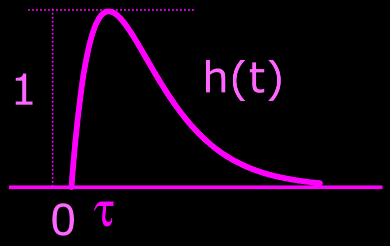
$d \cdot \omega = 2kTC\tan(\delta)\omega$

$$ENC_{1/f}^2 = \int_{-\infty}^{+\infty} N(\omega)|H(j\omega)|^2 df = \left(\pi A_f C_T^2 + 2kTC\tan\delta\right)\int_{-\infty}^{+\infty} |\omega||H(j\omega)|^2 df$$



triangular shaping

$$\int_{-\infty}^{+\infty} |\omega||H(j\omega)|^2 df = 4\frac{\ln 2}{\pi} \cong 0.88$$



RC-CR shaping

$$\int_{-\infty}^{+\infty} |\omega||H(j\omega)|^2 df = 1.18$$

$$\boxed{ENC_{1/f}^2 = \left(cC_T^2 + d\right)A_2}$$

$ENC_{1/f}^2$ independent of shaping time

$ENC_{1/f}^2 \propto C_T^2$

# Equivalent Noise Charge - VIII

✓ ***ENC calculation in presence of white and 1/f + dielectric noises***

$$ENC^2 = \underbrace{\left(aC_T^2\right)\int_{-\infty}^{+\infty}\left|h'(t)\right|^2 dt}_{series} + \underbrace{\left(cC_T^2 + d\right)\int_{-\infty}^{+\infty}\left|\omega\right|\left|H(\omega)\right|^2 df}_{1/f\,+dielectric} + \underbrace{b\int_{-\infty}^{+\infty}\left|h(t)\right|^2 dt}_{parallel}$$

Introducing $x = t/\tau$, where $\tau$ is a typical width of h(t) as the peaking time or the FWHM:

$$ENC^2 = \underbrace{\left(\frac{aC_T^2}{\tau}\right)\int_{-\infty}^{+\infty}\left|h'(x)\right|^2 dx}_{series} + \underbrace{\left(cC_T^2 + d\right)\int_{-\infty}^{+\infty}\left|\omega\right|\left|H(\omega)\right|^2 df}_{1/f\,+dielectric} + \underbrace{b\tau\int_{-\infty}^{+\infty}\left|h(x)\right|^2 dx}_{parallel} =$$

$$= \underbrace{A_1\left(\frac{aC_T^2}{\tau}\right)}_{series} + \underbrace{A_2\left(cC_T^2 + d\right)}_{1/f\,+dielectric} + \underbrace{A_3\, b\, \tau}_{parallel}$$

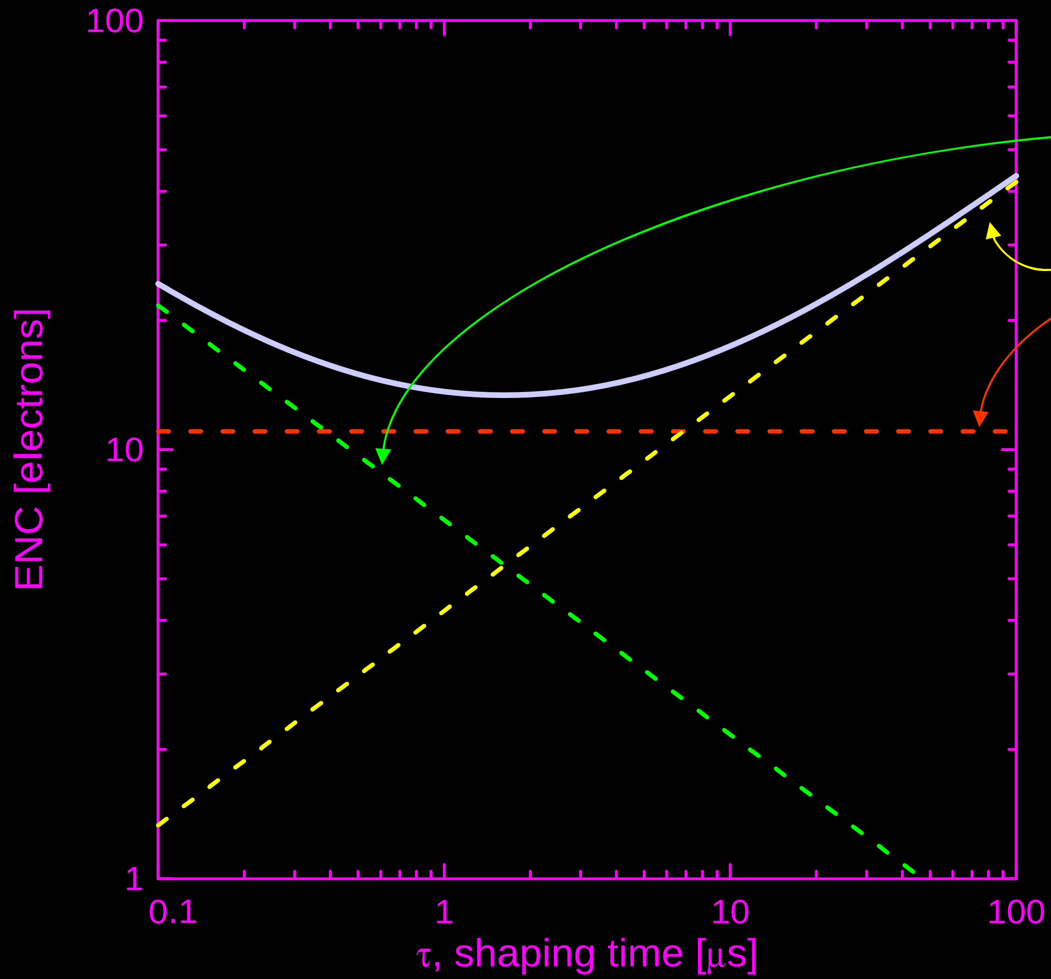
$A_1, A_2, A_3$ are shape factors depending only on the shape of the filter:

$$A_1 = \int_{-\infty}^{+\infty}\omega^2\left|H(\omega)\right|^2 df = \int_{-\infty}^{+\infty}\left|h'(t)\right|^2 dt \quad A_2 = \int_{-\infty}^{+\infty}\left|\omega\right|\left|H(\omega)\right|^2 df \quad A_3 = \int_{-\infty}^{+\infty}\left|H(\omega)\right|^2 df = \int_{-\infty}^{+\infty}\left|h(t)\right|^2 dt$$

# Equivalent Noise Charge - IX

- ✓ ***ENC vs. shaping time ($\tau$)***

**5mm² SDD (on-chip JFET)**



$C_d = 0.15\,pF$

$C_G = 0.15\,pF$

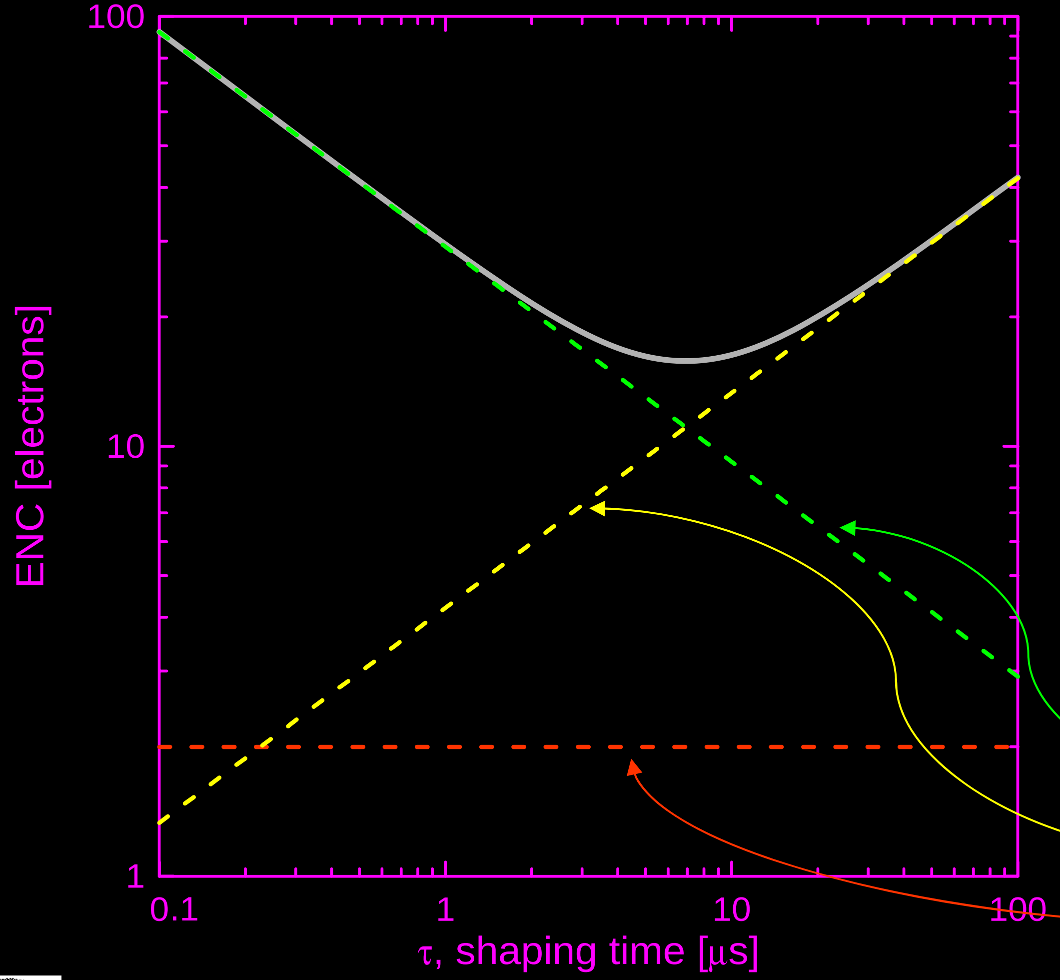
$g_m = 0.3mS$

$I_L = 1.6\,pA$

$A_f = 1.1 \cdot 10^{-11}\,V^2$

# Equivalent Noise Charge - IX

✓ ***ENC vs. shaping time ($\tau$)***



**5mm² pn-diode (NJ14 JFET)**

$C_d = 1.7\,pF$

$C_{par} = 2\,pF$

$C_G = 2\,pF$

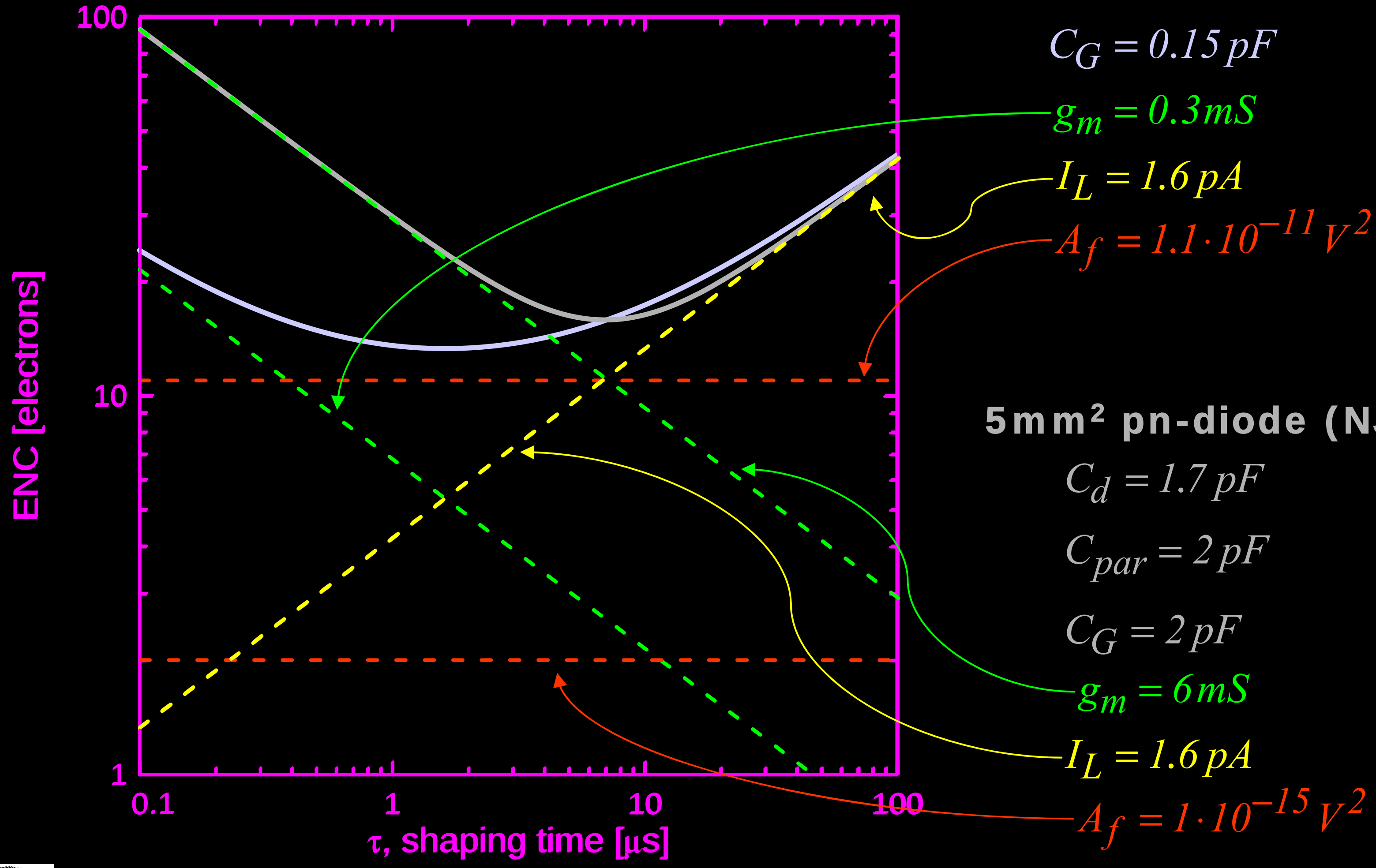
$g_m = 6\,mS$

$I_L = 1.6\,pA$

$A_f = 1 \cdot 10^{-15}\,V^2$

# Equivalent Noise Charge - IX

- ✓ *ENC vs. shaping time ($\tau$)*



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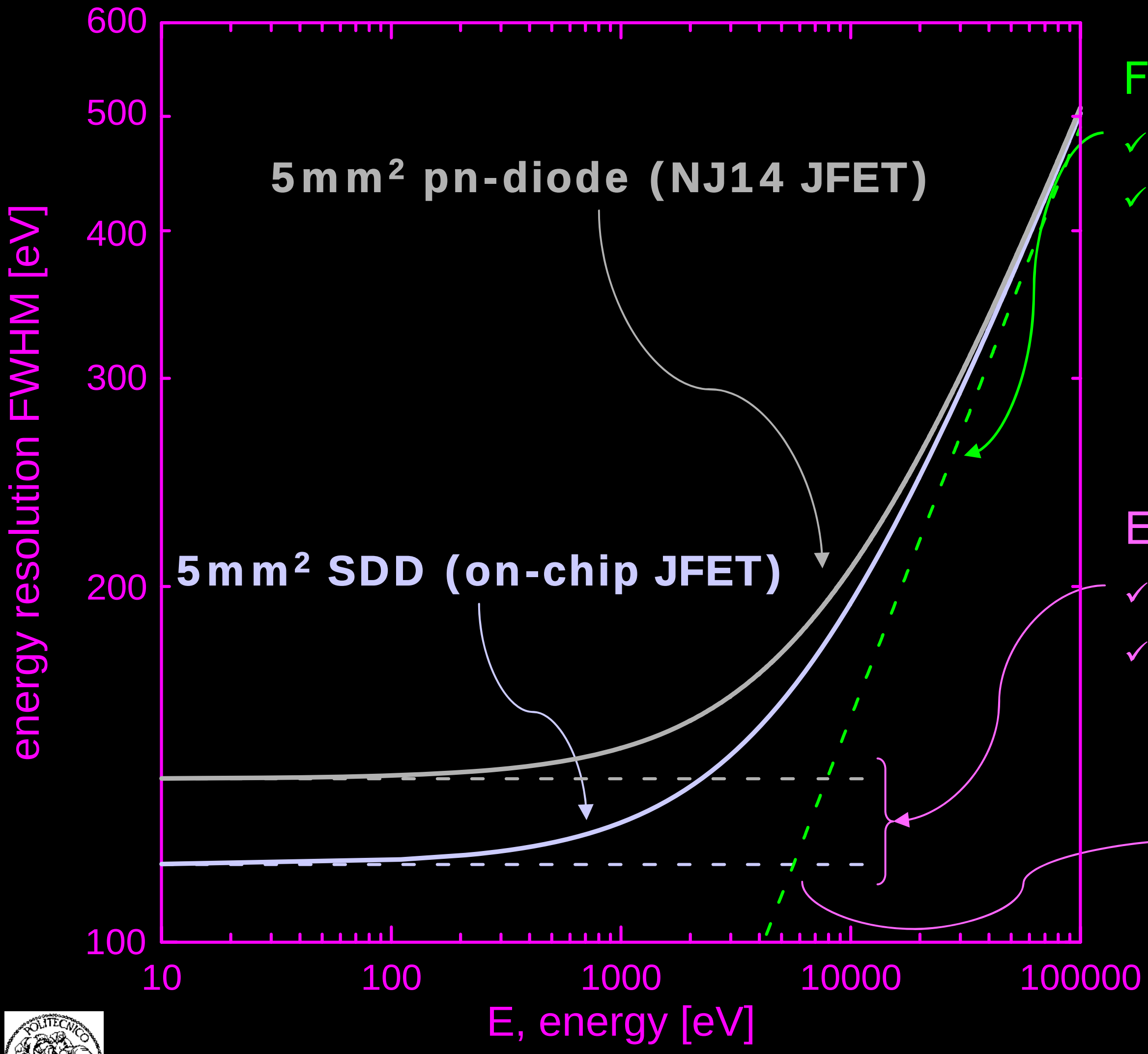
$g_m = 6\,mS$

$I_L = 1.6\,pA$

$A_f = 1 \cdot 10^{-15}\,V^2$

# Equivalent Noise Charge - X

✓ ***Energy resolution vs. incident energy***



Fano contribution depends:

- ✓ on the detector material
- ✓ on the energy

$$\propto \sqrt{\frac{FE}{w}}$$

Electronic noise contribution:

- ✓ independent of E
- ✓ depends on the shaping time

corner energy

$$E_c = \frac{w}{F} ENC^2$$

# Equivalent Noise Charge - XI

✓ ***Ballistic deficit***

- Ballistic deficit occurs when the peak value of the signal at the filter output does not correspond to the complete collection of the charge delivered by the detector, but only to a fraction of it.

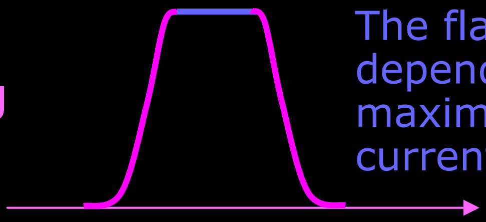
- If the detector current pulses cannot be considered as δ-pulses, though all events yield the same charge, the amplitude of the signal at the filter output depends on the detector current pulse duration.

- If the duration of the detector current pulses varies from event to event, the fraction of the charge lost is a random variable which introduces a dispersion in the amplitude of the signal at the filter output.

⇓

To reduce the ballistic deficit the shape of the filter and its duration (shaping time) must be chosen with the criterion that the δ-response of the entire analog channel (preamplifier and filter) feature a low curvature at the peaking point.

**FLAT-TOP**



The flat-top duration depends on the maximum detector current pulse duration.

# Equivalent Noise Charge - XII

***In conclusion:***

The value of the shaping time $\tau$ to be used:

- must take into account in any case the ballistic deficit, i. e. the maximum collection time of the detector

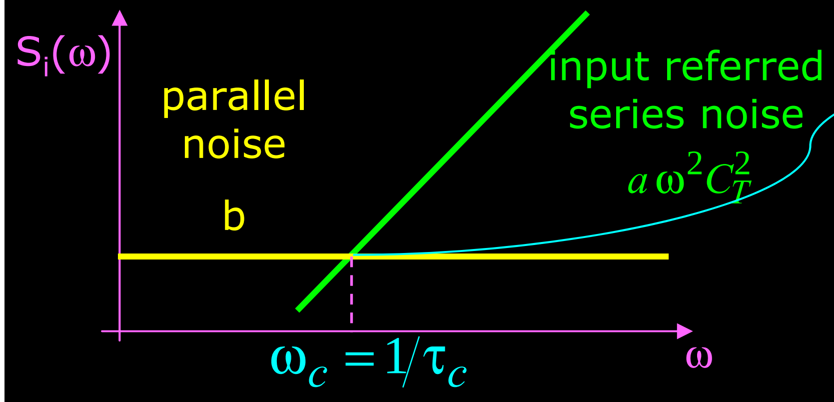
- must take into account S/N optimization (minimum of the ENC)

- must take into account the pile-up effects, unless it is $\lambda_{in}\,\tau << 1$

  If the input rate $\lambda_{in}$ has a prescribed high value and the S/N is important, the shaping time is chosen as a compromise between the conflicting requirements such that to optimize the experimental results.

# Equivalent Noise Charge - XIII

## ✓*Shape of the optimum filter*



$$\tau_c = C_T \sqrt{\frac{a}{b}}$$

noise corner time constant

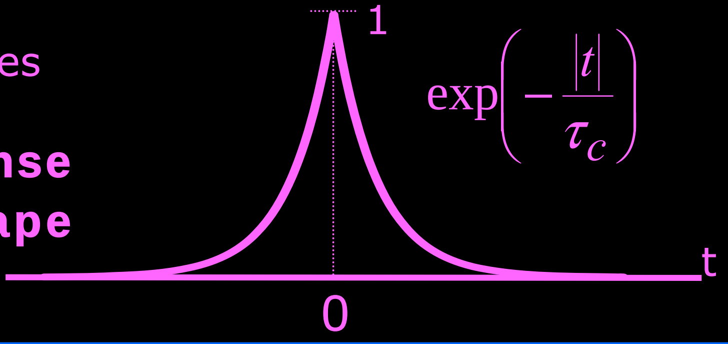
(reciprocal of the angular frequency at which the contributions from white series and parallel noise at the preamplifier input become equal)

$$\left(\frac{S}{N}\right)^2 = \frac{Q^2\{\max[h(t)]\}^2}{ENC^2} = \frac{Q^2}{b\int_{-\infty}^{+\infty}|h(t)|^2\,dt + aC_T^2\int_{-\infty}^{+\infty}|h'(t)|\,dt}$$

**signal-to-noise ratio**

Search for h(t) which minimizes the denominator of S/N (variational method)

(in presence of only white noises and with infinite time)
the **sought impulse response**
has the **indefinite cusp shape**



$$\exp\left(-\frac{|t|}{\tau_c}\right)$$

$$ENC_{opt} = \sqrt[4]{4abC_T^2}$$

# Equivalent Noise Charge - XIV

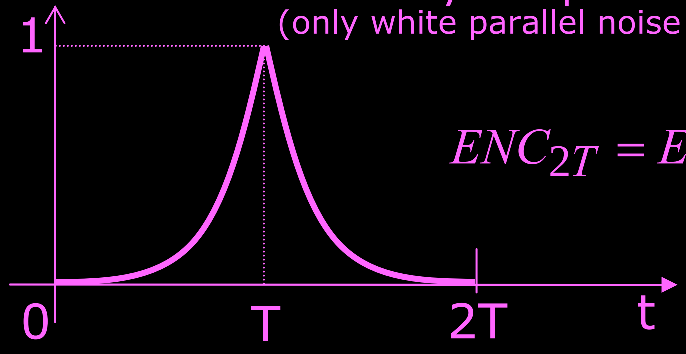
✓ ***Shape of the optimum filter in presence of additional constraints - I***

- **finite width (2T) ⟶ truncated cusp**

Practically a triangle when $T<\tau_c$
(only white series noise is "active")

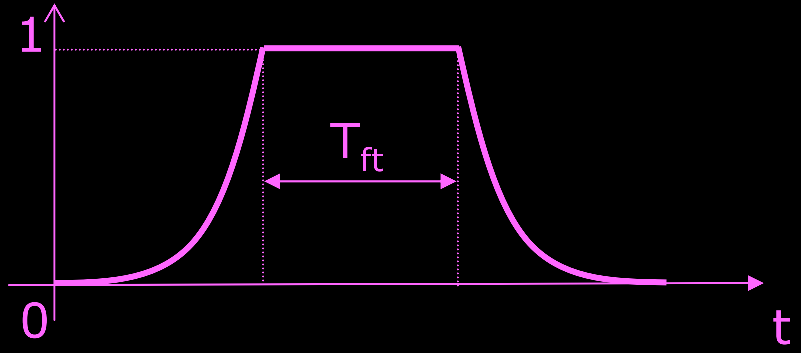
Practically a $\delta$-pulse when $T>\tau_c$
(only white parallel noise is "active")

$$h(t)=\begin{cases}\dfrac{\sinh\left(\dfrac{t}{\tau_c}\right)}{\sinh\left(\dfrac{T}{\tau_c}\right)} & 0\le t\le 2T \\ 0 & t>2T\end{cases}$$



$$ENC_{2T}=ENC_{opt}\sqrt{\coth\left(\frac{T}{\tau_c}\right)}$$

- **ballistic deficit ⟶ flat-top is needed**



The shaper "sees" the finite-width input pulse as a δ function

"flat-top" regions contributes only to parallel (and1/f noise)

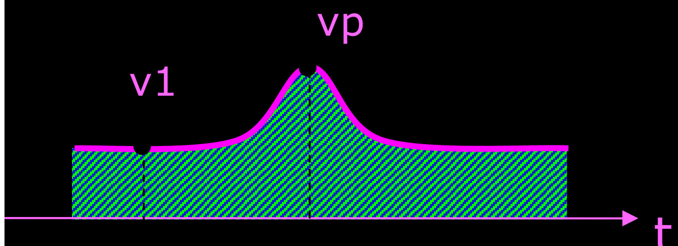
$$ENC_{ft}^2=ENC_\delta^2+bT_{ft}$$

(in presence of only white noises)

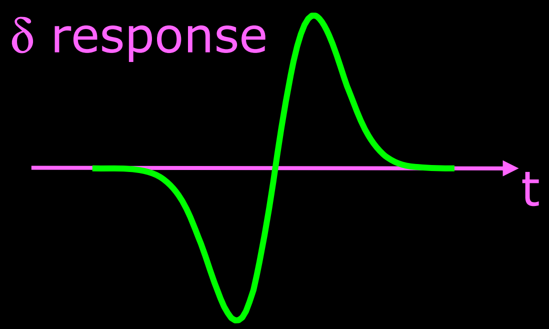
# Equivalent Noise Charge - XV

## ✓ *Shape of the optimum filter in presence of additional constraints - II*

- **constant offset**

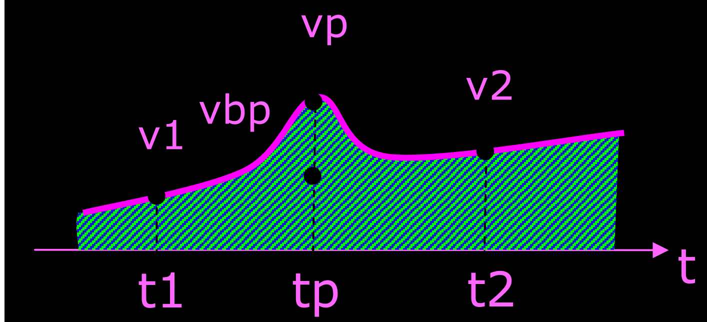


input signal

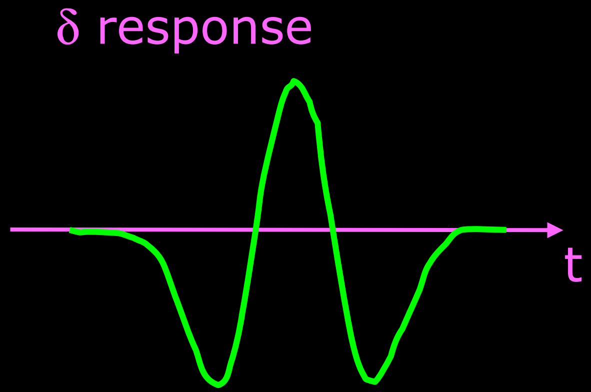


The value of the offset is filtered prior to the signal pulse and this value is “subtracted” from the signal measurement.

- **baseline drift**



input signal



$$v_{bp} = v_1 \frac{t_2 - t_p}{t_2 - t_1} + v_2 \frac{t_p - t_1}{t_2 - t_1}$$

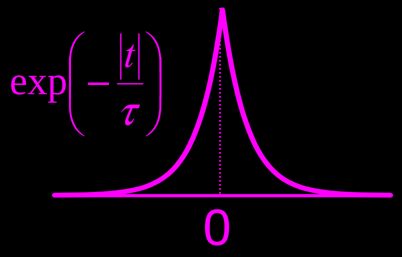
If t1, t2 and tp are equidistant

$$v_{bp} = \frac{v_1 + v_2}{2}$$

# Equivalent Noise Charge - XVI

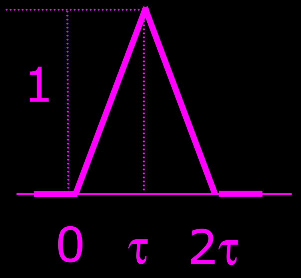
## ✓ *Shape factors for different shapers*

- **Indefinite cusp** (optimum shape for white noises)



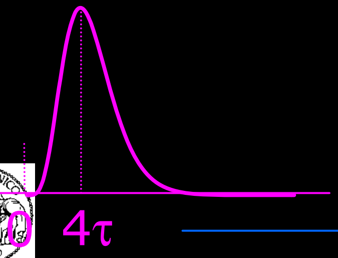
A1=1
A2=2/π
A3=1
F=1 worsening factor

- **Triangular** (optimum shape for white voltage noise and finite measurement time)



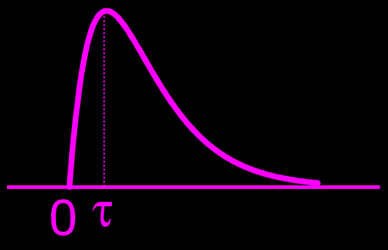
A1=2
A2=(4ln2)/π
A3=2/3
F=1.075

- **Pseudo-Gaussian (4th order)**



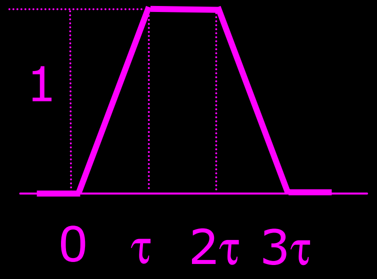
A1=0.51
A2=1.04
A3=3.58
F=1.165

- **RC-CR**



A1=1.85
A2=1.18
A3=1.85
F=1.359

- **Trapezoidal**



A1=2
A2=1.38
A3=5/3

° The “flat-top” regions contributes only to A2 and A3.

° A1 is equal to the triangular case having the same leading and trailing edges

# Equivalent Noise Charge - XVII

✓ ***What you have to do to calculate the shape factors:***

1. Know the shaper impulse response in the time domain [h(t)] and in the frequency domain [H(ω)] (either analytically or via numerical samples).

2. Consider the functions h(t) and H(ω) as a function of a dimensionless variables $x=t/\tau$ and $\xi=\omega\tau$.

3. Normalize the function in such a way that max[h(x)]=1.

4. Compute the following integrals (either analytically or with numerical methods):

$$A_1 = \int_{-\infty}^{+\infty} \xi^2 |H(\xi)|^2 d\xi = \int_{-\infty}^{+\infty} |h'(x)|^2 dx$$

white series noise

$$A_2 = \int_{-\infty}^{+\infty} |\xi| |H(\xi)|^2 d\xi$$

1/f series noise

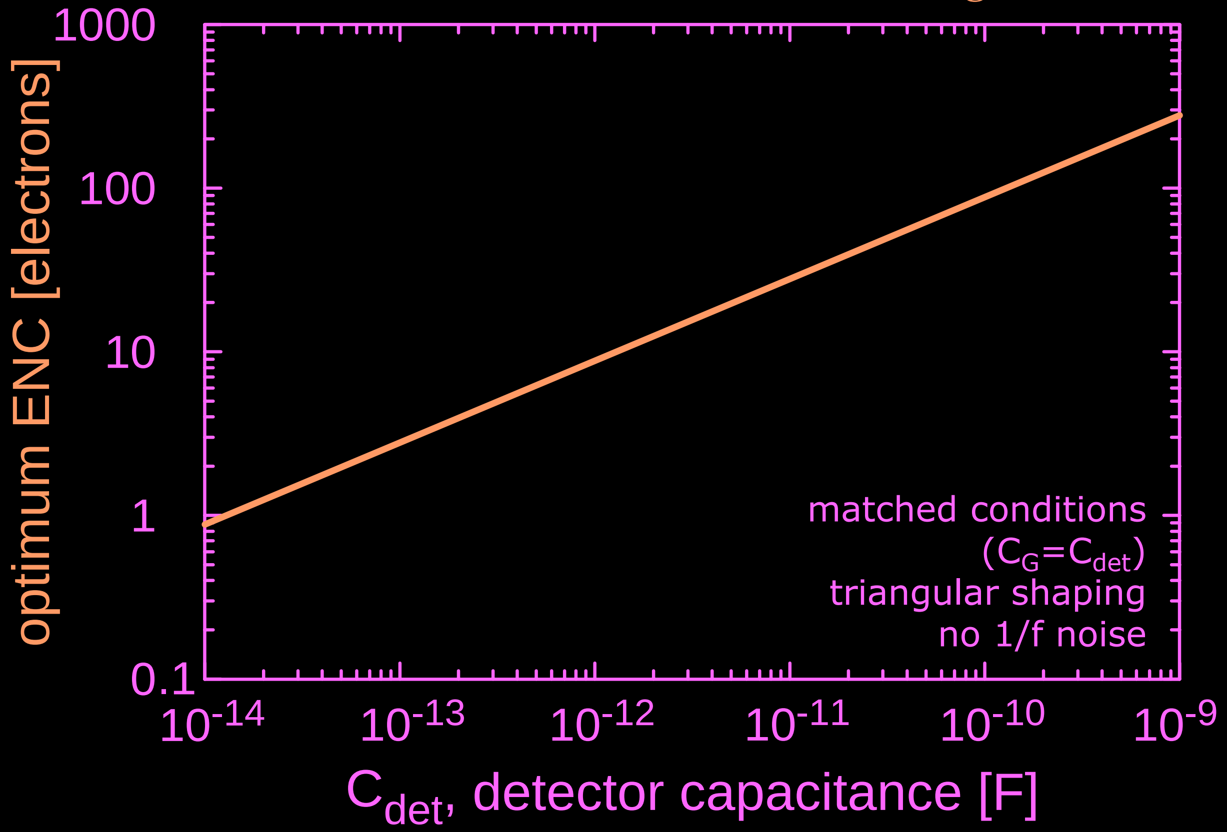
$$A_3 = \int_{-\infty}^{+\infty} |H(\xi)|^2 d\xi = \int_{-\infty}^{+\infty} |h(x)|^2 dx$$

white parallel noise

# Equivalent Noise Charge - XVIII

✓ ***Optimum ENC vs. detector capacitance***

$$ENC_{opt}^2 = \sqrt{A_1 A_3}\sqrt{2kTq}\underbrace{\sqrt{\frac{\alpha}{\omega_T}}}_{FET}\underbrace{\sqrt{C_D I_L}}_{detector}\underbrace{\left(\sqrt{\frac{C_G}{C_{det}}}+\sqrt{\frac{C_{det}}{C_G}}\right)}_{matching}+ENC_{1/f}^2\left(C_T^2\right)$$

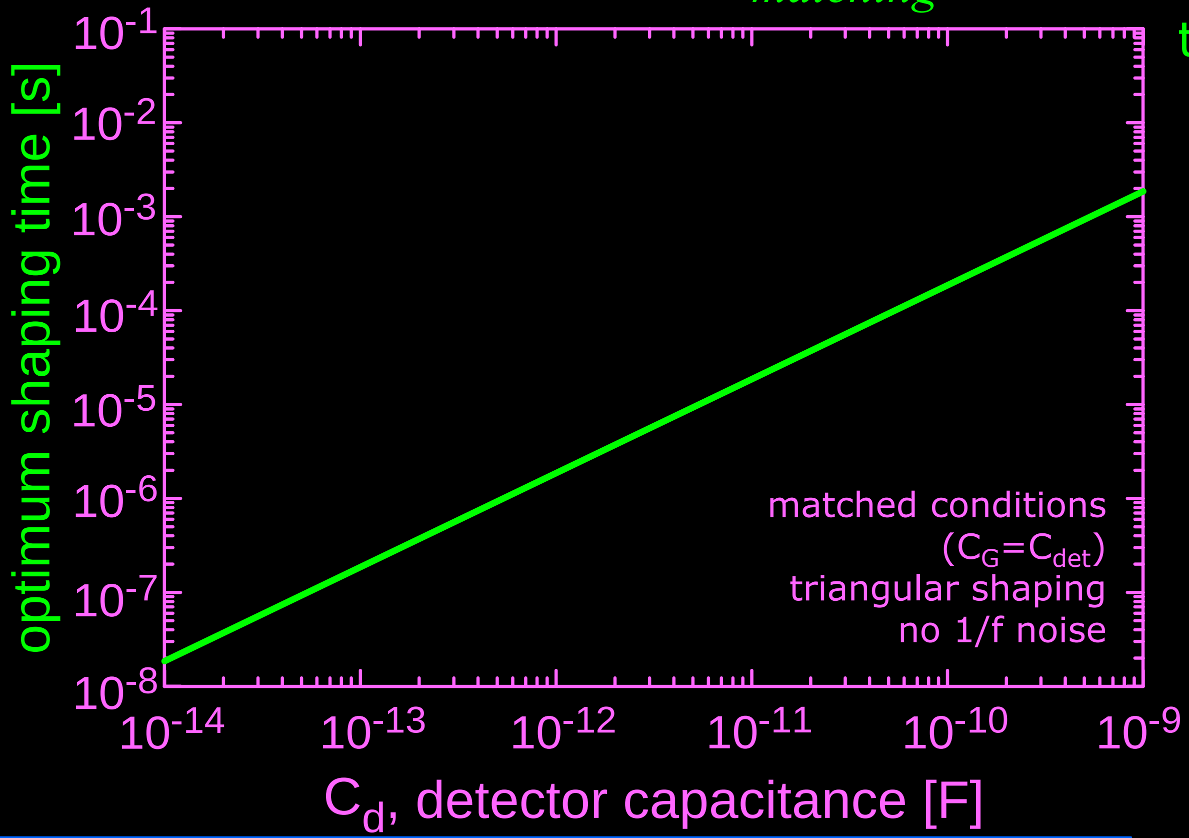


# Equivalent Noise Charge - XIX

- ✓ ***Optimum shaping time vs. detector capacitance***

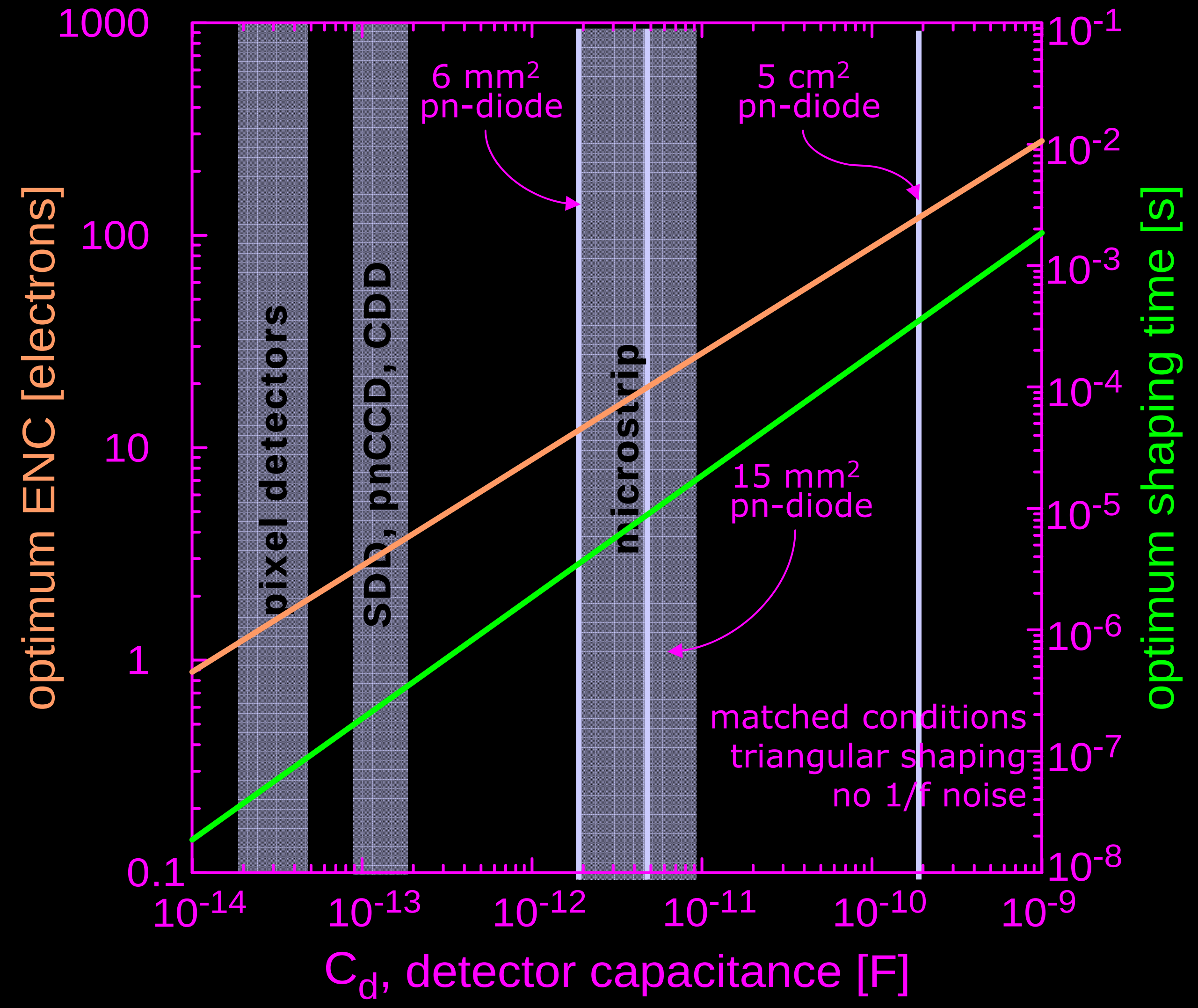
$$\tau_{opt} = \sqrt{\frac{A_1}{A_3}}\sqrt{2V_{th}}\underbrace{\sqrt{\frac{\alpha}{\omega_T}}}_{FET}\underbrace{\sqrt{\frac{C_{det}}{I_L}}}_{detector}\underbrace{\left(\sqrt{\frac{C_G}{C_{det}}}+\sqrt{\frac{C_{det}}{C_G}}\right)}_{matching} = \sqrt{\frac{A_1}{A_3}}\tau_c$$

noise corner time constant



# Equivalent Noise Charge - XX

- ✓ ***Optimum ENC and shaping time vs. detector type***



# Capacitive Matching - I

✓ ***Fixed current density*** → FET cut-off frequency independent of size

**MOSFET frontend**

$$ENC_s^2 = aC_{tot}^2 A_1 \frac{1}{\tau}$$ white series noise

$$a = \frac{2kT\alpha}{g_m} \qquad \alpha = \frac{2}{3}$$ Input voltage spectral noise

$$g_m = 2\sqrt{kI_D} = \sqrt{2\mu C_{ox}\frac{W}{L}J_D W}$$ MOSFET transconductance - strong inversion

$$A_1\frac{1}{\tau} = \int_{-\infty}^{\infty}\left|h'(t)\right|^2 dt$$ Series noise integral with *h(t)* impulse response

→ Optimum size of the input MOSFET (length *L* and impulse response *h(t)* constant)

$$0 = \frac{d\left(ENC_s^2\right)}{dW} = \frac{kT\alpha A_1}{\tau\sqrt{2\mu C_{ox}\frac{J_D}{L}}}\left[-2\frac{\left(C_{\det}+C_{ox}WL\right)^2}{W^2} + 4\frac{\left(C_{\det}+C_{ox}WL\right)C_{ox}L}{W}\right]$$

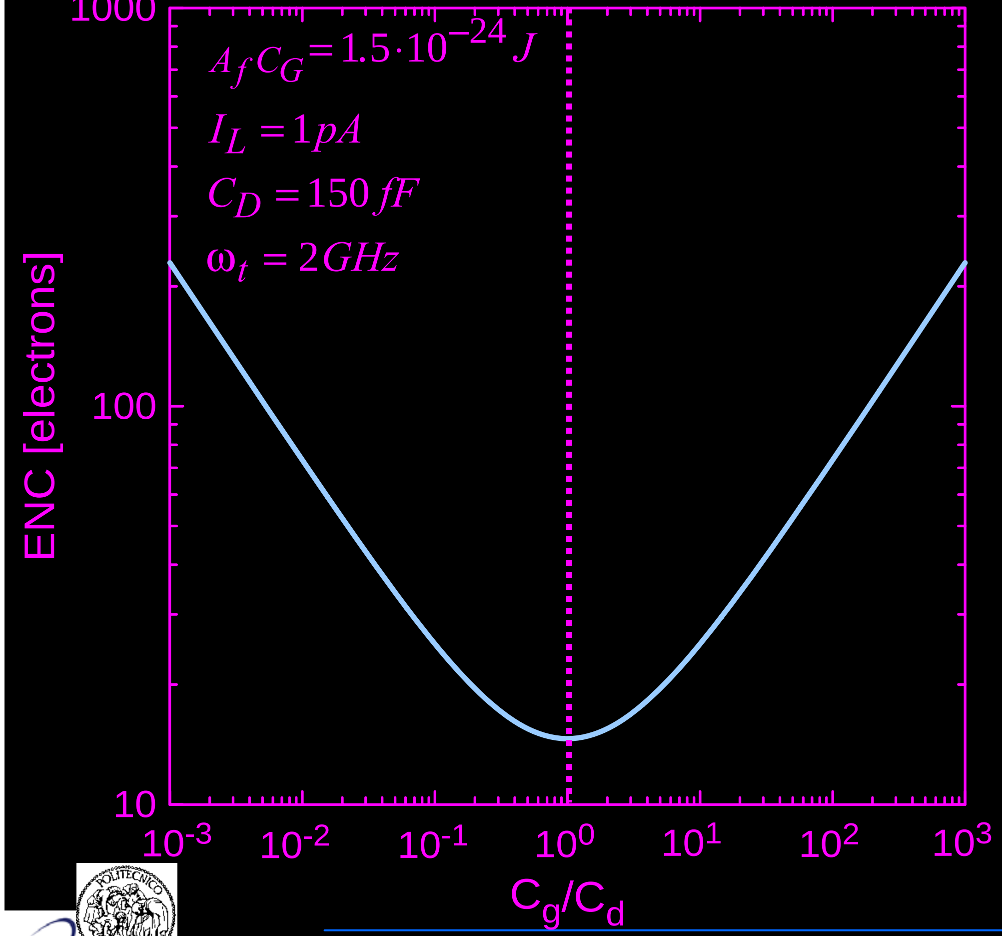
$$\boxed{C_{ox}WL = C_{det} \Rightarrow W_{opt} = \frac{C_{det}}{C_{ox}L}}$$

Valid also in the presence of 1/f noise and applies to JFET frontend

# Capacitive Matching - I

✓ ***Fixed current density*** ⟶ FET cut-off frequency independent of size

$$ENC^2 = A_1\frac{2kT\alpha}{\omega_t}C_D\left(\sqrt{\frac{C_G}{C_D}}+\sqrt{\frac{C_D}{C_G}}\right)^2\frac{1}{\tau}+A_2\left(\pi A_f C_G\right)C_D\left(\sqrt{\frac{C_G}{C_D}}+\sqrt{\frac{C_D}{C_G}}\right)^2+A_3\left(qI_L\right)\tau$$



**Optimum size of the input FET**

$$\boxed{C_g = C_{det}}$$

**In the MOSFET case:**

$$\boxed{C_{ox}WL = C_{det} \Rightarrow W_{opt} = \frac{C_{det}}{C_{ox}L}}$$

# Capacitive Matching - II

✓ ***Fixed power dissipation*** → fixed drain current, white series noise

**MOSFET frontend**

$$ENC_s^2 = aC_{tot}^2 A_1 \frac{1}{\tau}$$ white series noise

$$a = \frac{2kT\alpha}{g_m} \qquad \alpha = \frac{2}{3}$$ Input voltage spectral noise

$$g_m = 2\sqrt{kI_D} = \sqrt{2\mu C_{ox}\frac{W}{L}I_D}$$ MOSFET transconductance - strong inversion

$$g_m = \frac{qI_D}{nkT}$$ MOSFET transconductance - weak inversion

$$A_1\frac{1}{\tau} = \int_{-\infty}^{\infty}\left|h'(t)\right|^2 dt$$ Series noise integral with *h(t)* impulse response

Optimum size of the input MOSFET (length *L*, current $I_D$ and impulse response *h(t)* constant)

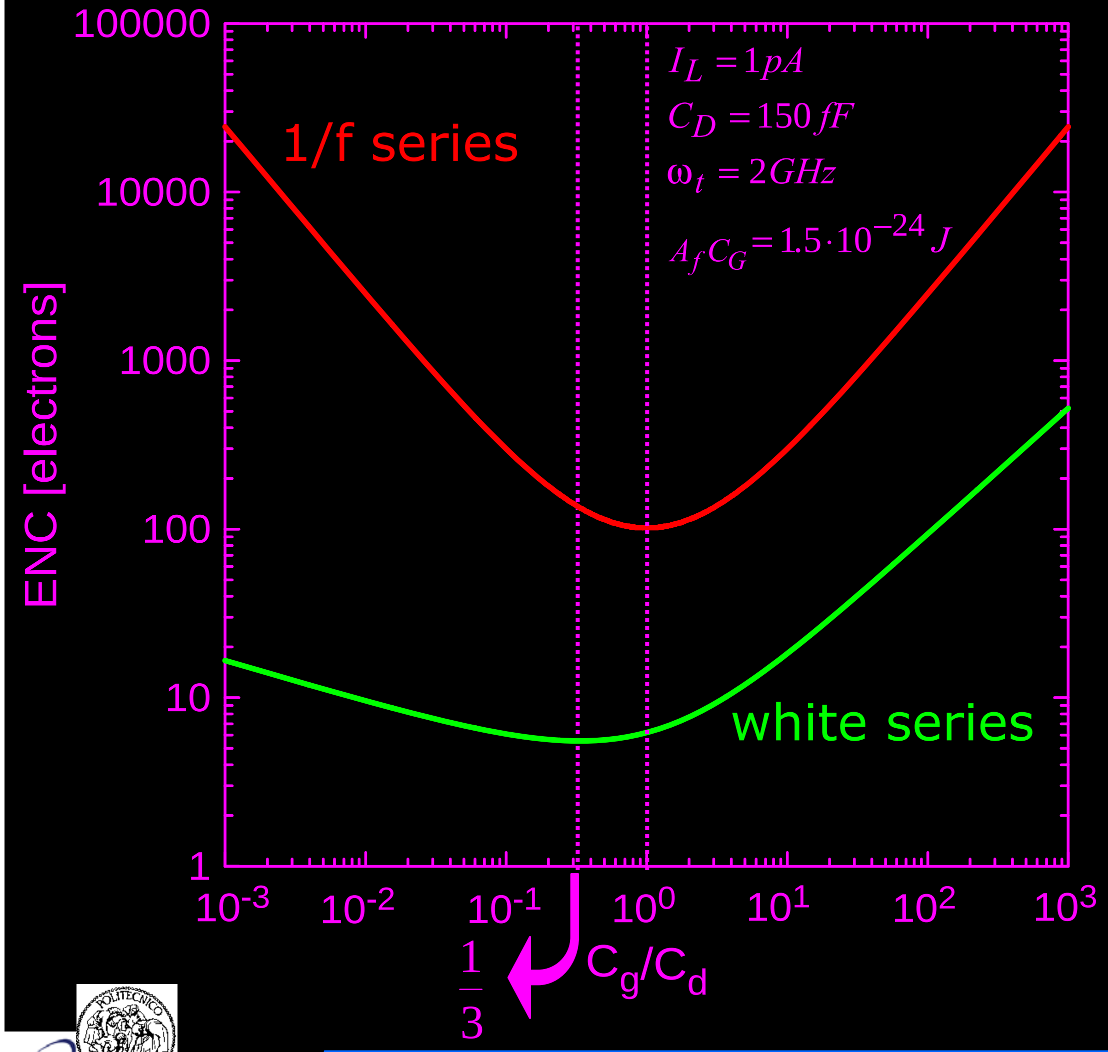
$$0 = \frac{d\left(ENC_s^2\right)}{dW} = \frac{2kT\alpha A_1}{\tau\sqrt{2\mu C_{ox}\frac{I_D}{L}}}\left[-\frac{1}{2}\frac{(C_{\det}+C_{ox}WL)^2}{W^{3/2}} + 2\frac{(C_{\det}+C_{ox}WL)C_{ox}L}{W^{1/2}}\right]$$

$$\boxed{C_{ox}WL = \frac{1}{3}C_{det} \Rightarrow W_{opt} = \frac{C_{det}}{3C_{ox}L}}$$

# Capacitive Matching - II

✓ ***Fixed power dissipation*** → ***fixed drain current***

**MOSFET frontend**



**Optimum size of the input MOSFET**

- ***1/f noise component***

$$\boxed{C_g = C_{det}}$$

$$\boxed{W_{opt} = \frac{C_{det}}{C_{ox}L}}$$

- ***white series noise component***

$$\boxed{C_g = \frac{1}{3}C_{det}}$$

$$\boxed{W_{opt} = \frac{C_{det}}{3C_{ox}L}}$$

# Weighting Function - I

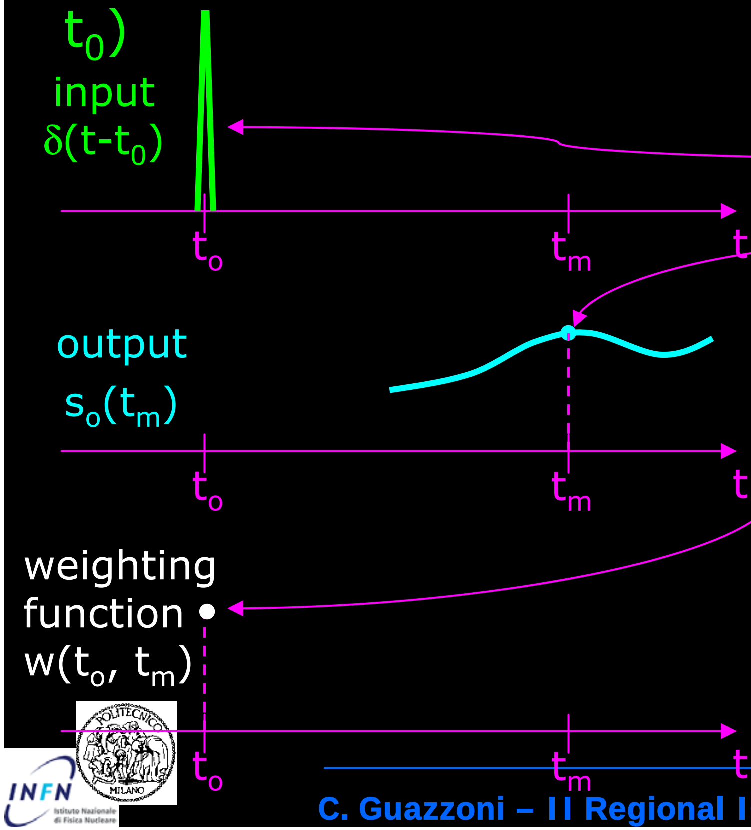
✓ **definition**

**defined by an implicit expression** $s_o(t_m) = \int_{-\infty}^{t_m} f(\tau)\; w(\tau,t_m)\; d\tau$

output at the measurement time $t_m$ due to the signal f(t)

weighting function peculiar to the point $t_m$

$f(t)=\delta(t-t_0) \longrightarrow s_o(t_m) = w(t_o,t_m)$



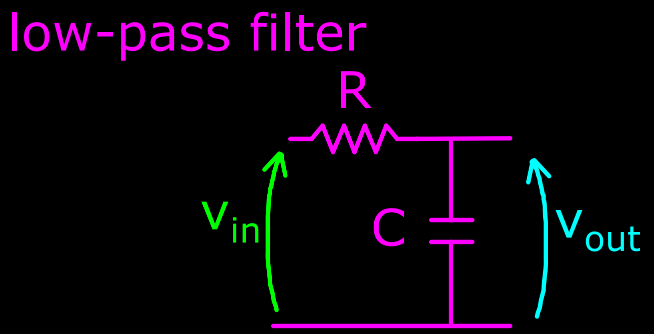
**operative definition for a given point $t_o$:**

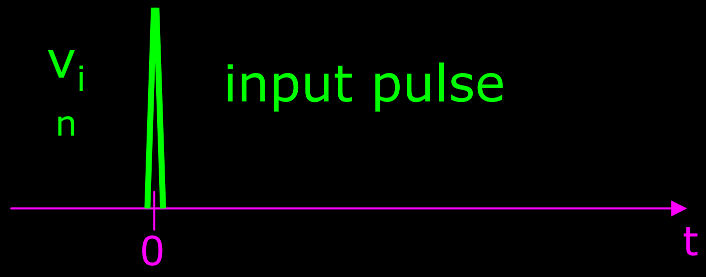
- δ pulse is applied at $t=t_o$.
- response $s_o(t_m)$ is observed at the measurement time $t_m$.
- a point of the weighting function $w(t_o, t_m)$ is obtained.

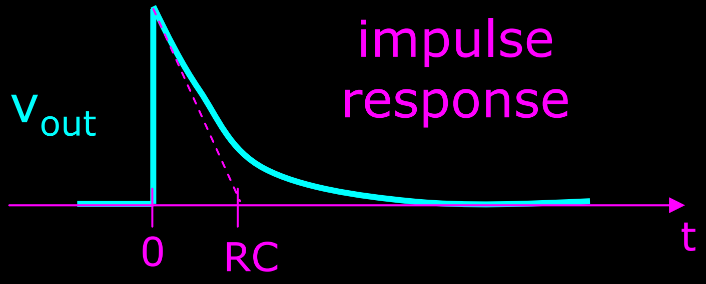
**spanning all $t_o$ values before $t_m$ the complete weighting function can be reconstructed.**

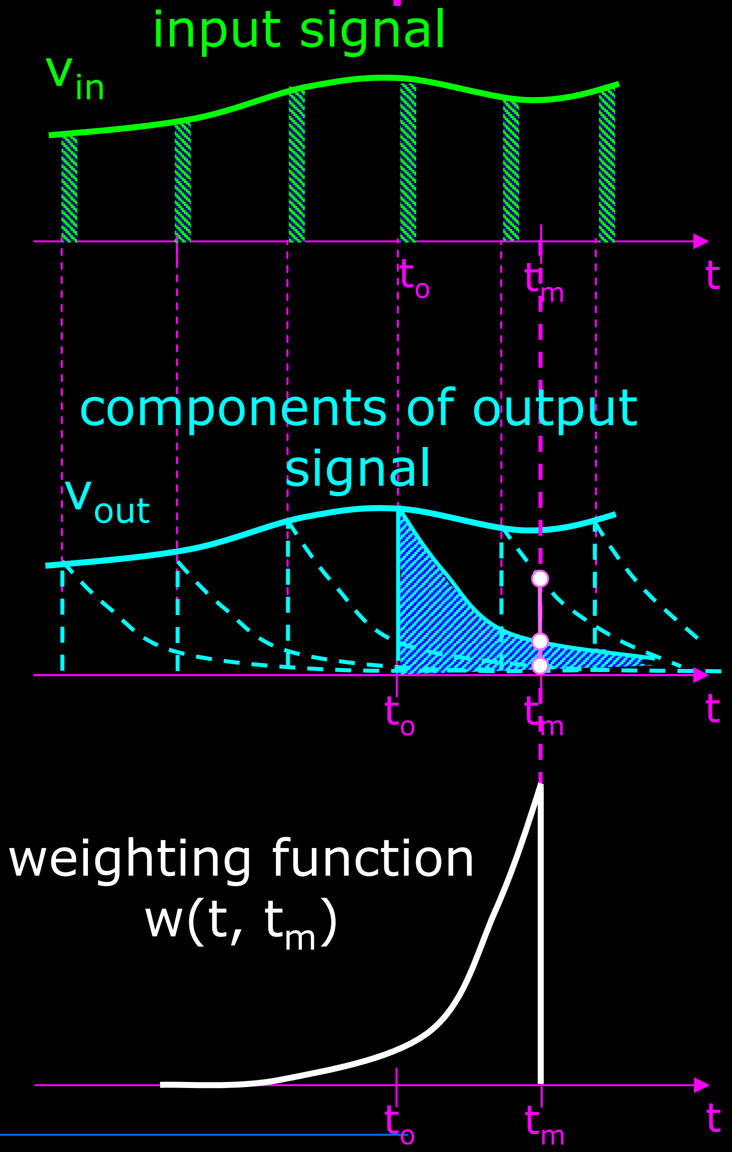
# Weighting Function - II

- ✓ **calculation of the weighting function for the low-pass filter**



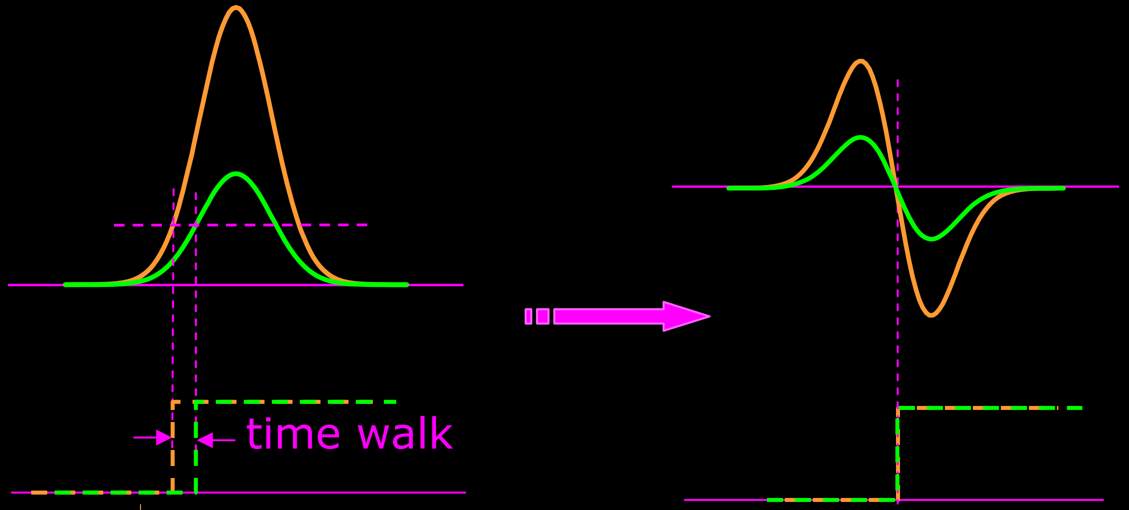




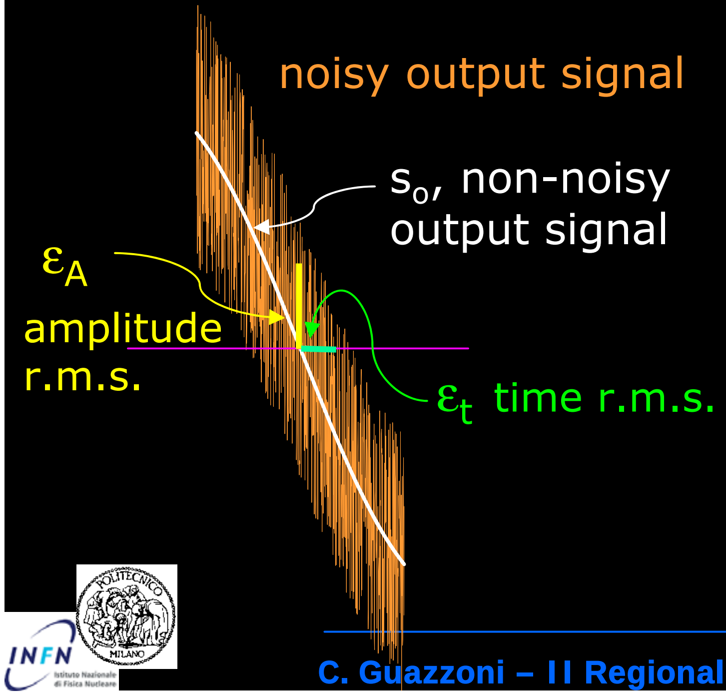


# Time Measurements - I

We want to measure the arrival time of the signal pulse



as time information we choose the 0-crossing time of the output signal

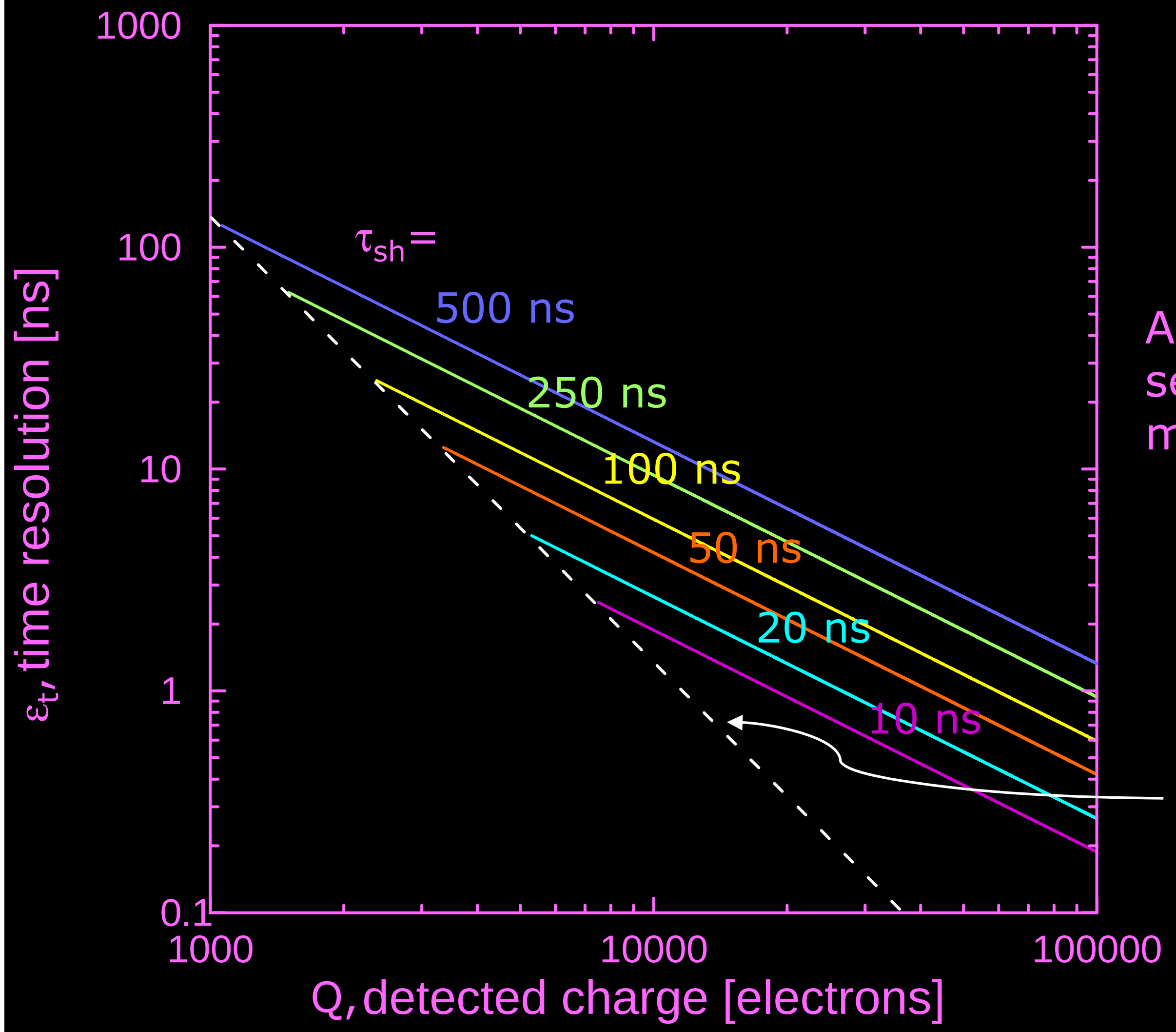


- for simplicity we fix the 0-crossing time in t=0
- due to geometrical considerations:

$$\frac{\varepsilon_A}{\varepsilon_t} = \left(\frac{ds_o}{dt}\right)_{t=0} \Rightarrow \boxed{\varepsilon_t^2 = \frac{\varepsilon_A^2}{\left(\frac{ds_o}{dt}\right)_{t=0}^2}}$$

**time resolution improves as the slope at the 0-crossing increases**

# Time Measurements - II



$$\varepsilon_t^2 = \frac{\varepsilon_A^2}{\left(\frac{ds_o}{dt}\right)_{t=0}^2} = \frac{ENC^2}{\left(\frac{ds_o}{dt}\right)_{t=0}^2}$$

Assuming that only the white series noise is relevant (true in most of the cases…):

$$\varepsilon_t = \sqrt{A_1 a C_T^2}\,\frac{\tau_{sh}}{Q}$$

Minimum detectable charge, arbitrarily assumed equal to (4*ENC)

$$ENC = \sqrt{\frac{A_1 a C_T^2}{\tau_{sh}}}$$

# Time Measurements - III

**output signal** $s_0(t) = Q\int_{-\infty}^{+\infty} f(t-\tau)h(\tau)d\tau$

input current pulse

shaper δ response

$$\left.\begin{aligned}
s_0(t) &= Q\int_{-\infty}^{+\infty} f(t-\tau)w(-\tau)d\tau \\
\frac{ds_0}{dt}(t) &= Q\int_{-\infty}^{+\infty} f'(t-\tau)w(-\tau)d\tau \\
\left(\frac{ds_0}{dt}\right)^2_{t=0} &= Q^2\left[\int_{-\infty}^{+\infty} f'(t-\tau)w(-\tau)d\tau\right]^2
\end{aligned}\right\}$$

- weighting function $w(\tau,t) = h(t-\tau)$
- t=0 nominal crossing time for the noiseless signal

$$h(\tau) = w(-\tau,0) = w(-\tau)$$

$$\varepsilon_t^2 = \frac{\varepsilon_A^2}{\left(\frac{ds_o}{dt}\right)^2_{t=0}} = \frac{aC_T^2\int_{-\infty}^{+\infty}[w'(\tau)]^2 d\tau + b\int_{-\infty}^{+\infty}[w(\tau)]^2 d\tau}{Q^2\left[\int_{-\infty}^{+\infty} f'(\tau)w(\tau)d\tau\right]^2}$$

$$\Big\downarrow$$

by a variational method, minimising $\varepsilon_t$

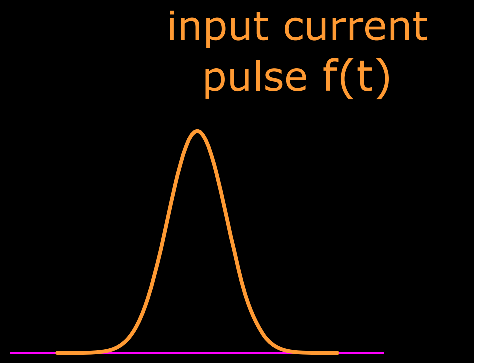
$$w_{opt}(t) = K_o\left[\frac{\tau_c}{2}\exp\left(-\frac{|t|}{\tau_c}\right)\otimes f'(t)\right] \qquad K_o = \frac{\varepsilon^2_{t,\min}Q^2}{aC_T^2}\int_{-\infty}^{+\infty} f'(x)w(x)dx$$

**Optimum WF for time measurements obtained as convolution of the cusp filter with the derivative of the input current pulse.**

# Time Measurements - IV

## Optimum time resolution

$$\boxed{\varepsilon_{t,\min}^2 = \frac{\sqrt{4abC_T}}{Q^2}\frac{1}{\int_{-\infty}^{+\infty}\left\{f'(t)\left[\exp\left(-\frac{|t|}{\tau_c}\right)\otimes f'(t)\right]\right\}dt}}$$

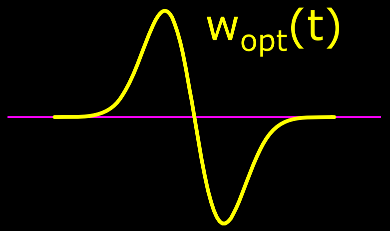


- **only white parallel noise ($\tau_c \to 0$)**

$$\frac{1}{2\tau_c}\exp\left(-\frac{|t|}{\tau_c}\right) \to \delta(t)$$

$$w_{opt}(t) \propto f'(t)$$

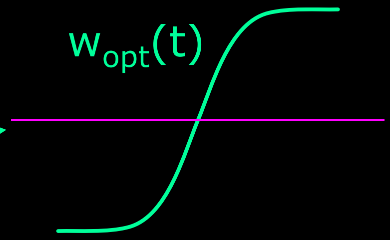
$$\boxed{\varepsilon_{t,\min}^2 = \frac{b}{Q^2}\frac{1}{\int_{-\infty}^{+\infty}[f'(t)]^2 dt}}$$



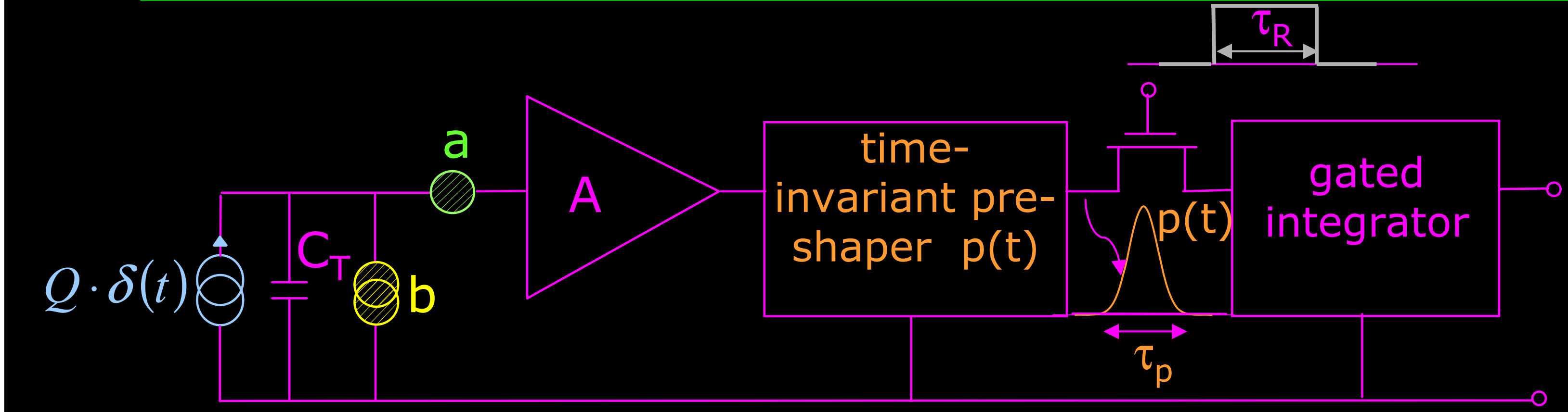
- **only white series noise ($\tau_c \to \infty$)**

$$w_{opt}(t) \propto \left[\int_{-\infty}^{t} f(\tau)d\tau - \frac{1}{2}\right]$$

$$\boxed{\varepsilon_{t,\min}^2 = \frac{aC_T^2}{Q^2}\frac{1}{\int_{-\infty}^{+\infty}[f(t)]^2 dt}}$$



# Time-variant Filters - I



- The switch conduction is synchronised with the detector-signal arrival and the switch remains conductive for $\tau_R \geq \tau_p$.
- The contribution of the δ pulses describing series and parallel noises generator to the r.m.s. noise at the measuring instant depends on their relationship with the signal.

**Knowledge of the processor response to the δ -pulse-like detector current not sufficient to evaluate the noise.**

**Noise evaluation based upon a time domain approach which requires the knowledge of the so-called "noise weighting function"**

A detector signal of charge Q occurring at $t=t_1$ will produce at the pre-shaper output the signal $\frac{QA}{C_T}p(t-t_1)1(t-t_1)$ that is integrated over the time interval $[t_1, t_1 + \tau_R]$

# Time-variant Filters - II

As signals arriving to the gate have a finite width $\tau_p$, all the $\delta$-pulses delivered by the parallel and series noise generator in the time interval $[t_1-\tau_p, t_1+\tau_R]$ contribute to the noise at the measuring instant $t_1+\tau_R$

**Noise weigthing function [$WF_N(t_o)$]:** contribution to the noise at the measuring time instant given by a $\delta$-pulse delivered by the parallel noise generator at a time $t_o$. ($t_o \in [t_1-\tau_p, t_1+\tau_R]$)

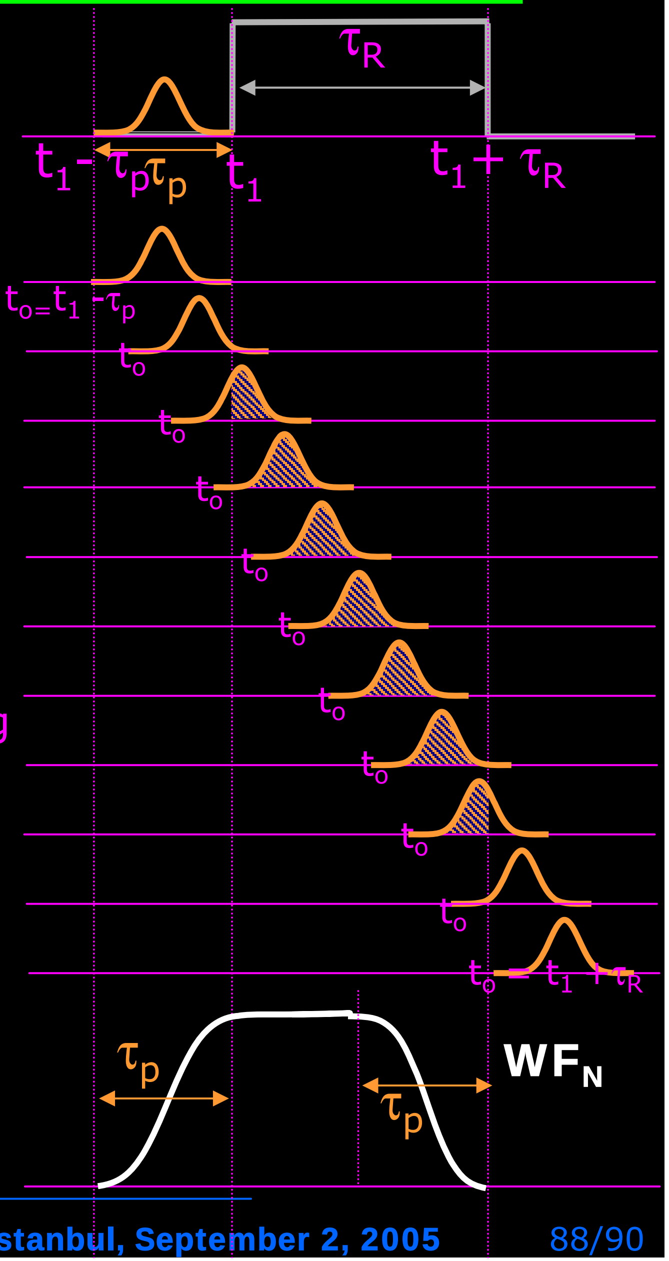
- $WF_N$ ($t_o$) is given by the area of the shaded region of the signal induced at the pre-shaper output by a $\delta$-pulse of the parallel noise generator
- In fact the portion of this signal entering the gate is integrated and stored in the integrator therefore contributing to the noise at $t_m=t_1+\tau_R$

$$WF_N(t_o)=0 \qquad t_o < t_1-\tau_p \; and \; t_o > t_1+\tau_R$$

$$WF_N(t_o)=\frac{A}{C_T}\int_0^{t_o-(t_1-\tau_p)} p(\tau_p-x)dx \qquad t_1-\tau_p < t_o < t_1$$

$$WF_N(t_o)=\frac{A}{C_T}\int_0^{\tau_p} p(\tau_p-x)dx \qquad t_1 < t_o < t_1+\tau_R-\tau_p$$

$$WF_N(t_o)=\frac{A}{C_T}\int_0^{t_1+\tau_R-t_o} p(x)dx \qquad t_1+\tau_R-\tau_p < t_o < t_1+\tau_R$$



# Time-variant Filters - III

- The noise contribution from the parallel source can be evaluated by adding quadratically all the elementary contributions appearing at the integrator output and caused by δ-pulses occurring in time intervals $[t_o, t_o+dt_o]$ as $t_o$ varies.

$$v_n^2\Big|_{parallel} = \frac{A^2}{C_T^2} b\int_{t_1+\tau_p}^{t_1+\tau_R} \left|WF_N(x)\right|^2 dx$$

- By sliding the derivative of $(A/C_T)p(t)$ through the integrator time window, the noise weighting function for doublet of current injected across the $C_T$ capacitor can be determined.
- The resulting function, which is the derivative of $WF_N$, allows the calculation of the output noise arising from the white series generator.

$$v_n^2\Big|_{series} = A^2 a\int_{t_1+\tau_p}^{t_1+\tau_R} \left|WF_N'(x)\right|^2 dx$$

$$\boxed{ENC^2 = \frac{aC_T^2\int_{t_1+\tau_p}^{t_1+\tau_R}\left|WF_N'(x)\right|^2 dx + b\int_{t_1+\tau_p}^{t_1+\tau_R}\left|WF_N(x)\right|^2 dx}{\left[\int_0^{\tau_R} p(x)dx\right]^2}}$$

**The knowledge of two different functions, *p(x)* for the signal and *$WF_N(x)$* for the noise, is required in the analysis of a time-variant shaper.**

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The end