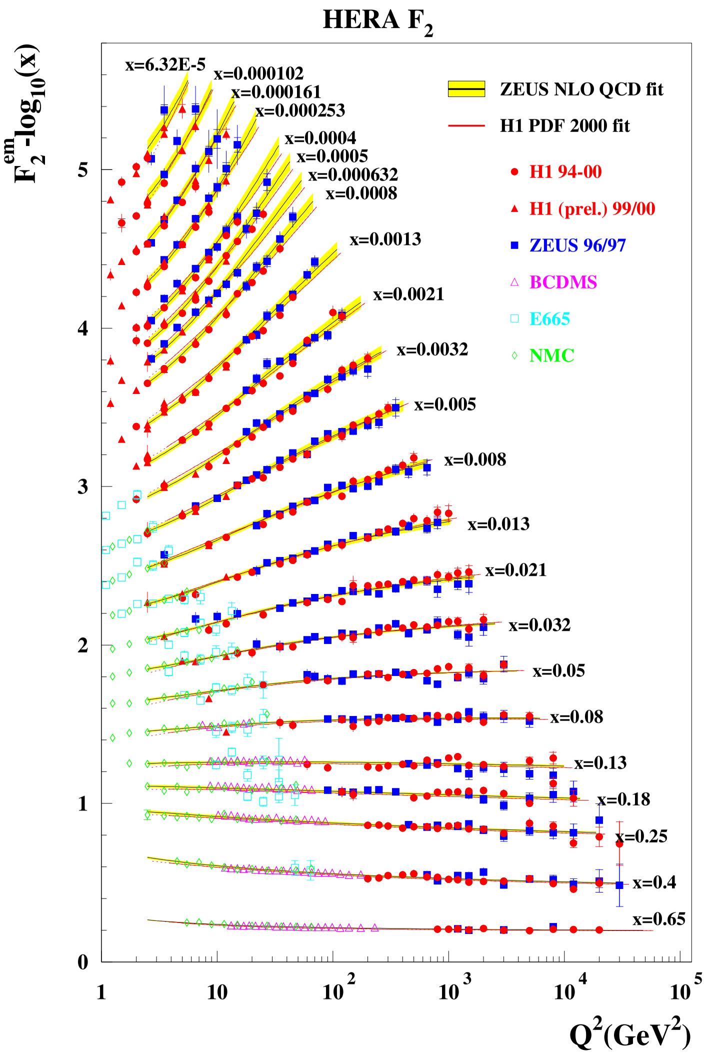
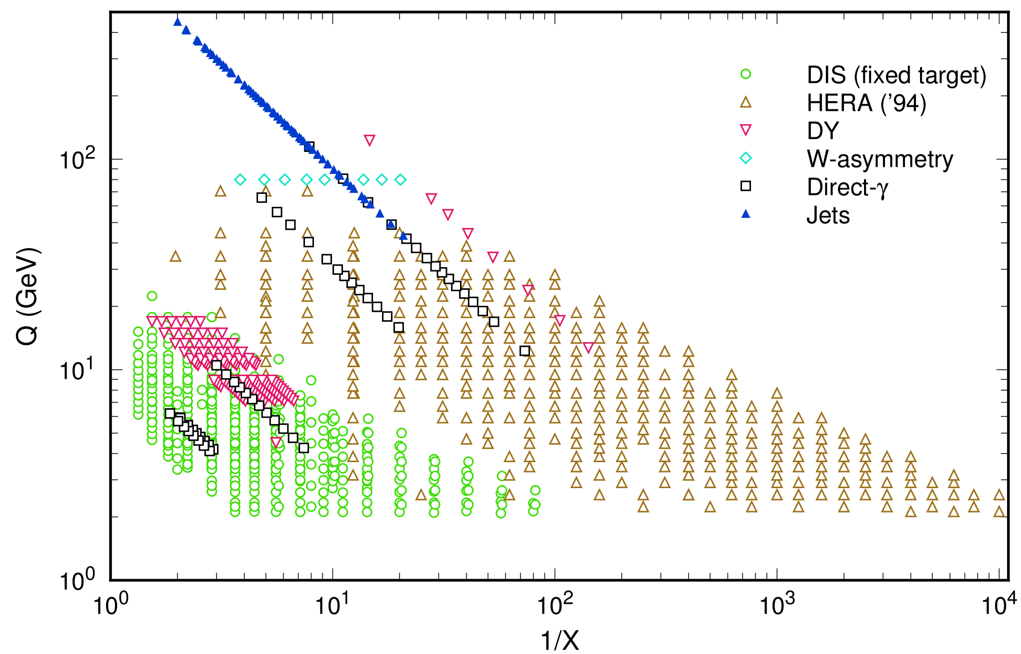
II: FROM PHENOMENOLOGY TO THEORY

THE GLOBAL PICTURE



- PARTONS WITH ERRORS NOW AVAILABLE
- GLOBAL FITS BASED ON LARGE SETS OF DATA:
  MRST, CTEQ ⇒ LARGE DATASETS,
  ALEKHIN: DIS+DY

data included in CTEQ5 parton fit



IS IT CONSISTENT?

PDF ERRORS COMPARABLE TO OR EVEN LARGER THAN THEORY ERRORS

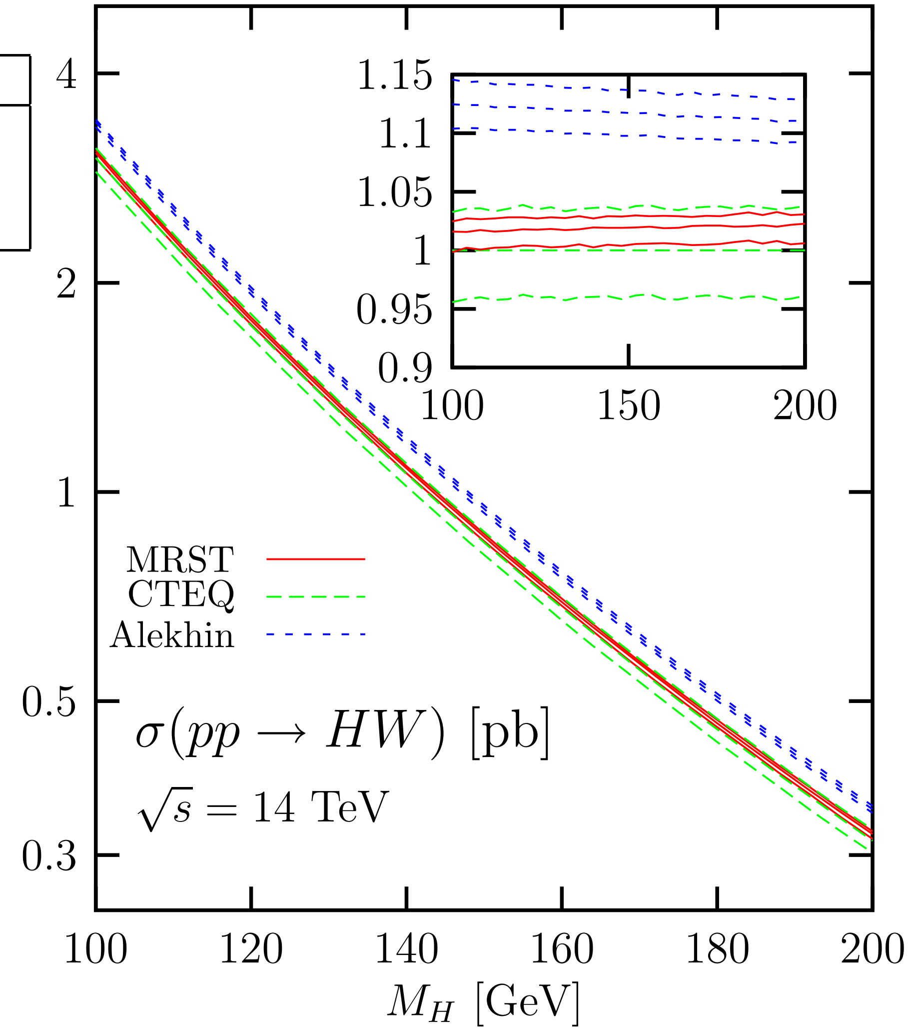
W PRODUCTION CROSS-SECTION TEVATRON

PDF SET	XSEC [NB]	PDF UNCERTAINTY
ALEKHIN	2.73	± 0.05 (TOT)
MRST2002	2.59	± 0.03 (EXPT)
CTEQ6	2.54	± 0.10 (EXPT)

THORNE

- ALEKHIN VS. MRST/CTEQ $\rightarrow$ W PRODUCTION XSECT AT TEVATRON DO NOT AGREE WITHIN RESPECTIVE ERRORS
- ALEKHIN VS. MRST/CTEQ $\rightarrow$ PREDICTIONS FOR ASSOCIATE HIGGS W PRODUCTION LHC DO NOT AGREE WITHIN RESPECTIVE ERRORS

HIGGS PRODUCTION AT LHC

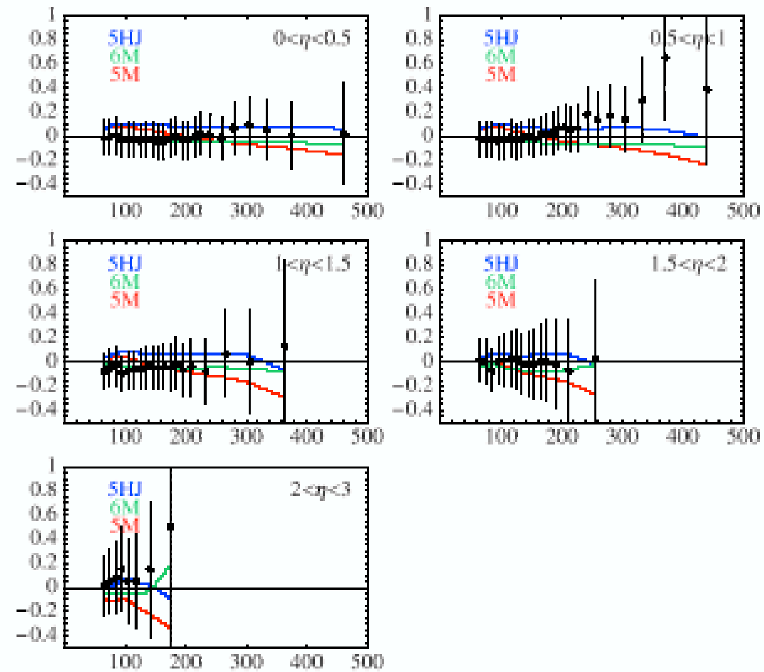


DJOUADI AND FERRAG

PARTON SETS DO NOT AGREE WITHIN RESPECTIVE ERRORS (STIRLING)

CTEQ global fits

- The Tevatron jet data from Run 1 has had increasingly greater importance in the global fits
- D0 jet data over full rapidity range in particular has lead to an a larger gluon at high x
  - but the integrated gluon momentum at very high x is still fairly small



1: The DØ inclusive jet cross section versus E_T for five rapidity bins compared to NLO
ctions using the CTEQ6.1M PDF's. The abscissa is E_T in GeV. The ordinate is (data −
y)/theory. The curves show the CTEQ5M, CTEQ5HJ, and CTEQ6M predictions, as frac-
differences compared to CTEQ6.1M. The error bars are the statistical and systematic errors
ined in quadrature.

J. HUSTON

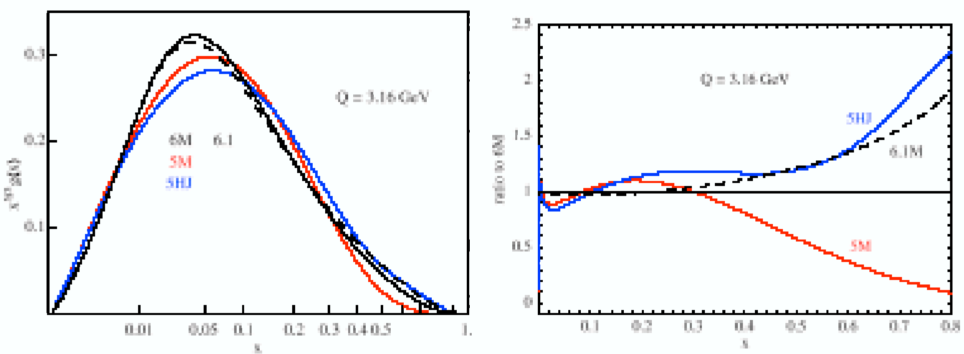


FIG. 2: Left: The CTEQ5M, CTEQ5HJ, and CTEQ6M gluon distributions at $Q^2 = 10\ \mathrm{GeV}^2$. Right: The ratios of CTEQ5M and CTEQ5HJ gluon distribution to that of CTEQ6M. The dashed curves show the CTEQ6.1M gluon distribution.

Theoretical Errors and Fine Details   MRST

It is vital to consider theoretical corrections, and to look at data which determines the small differences in parton distributions. These include ....

- Data determining quark decomposition, e.g. W-asymmetry, dimuon data and Drell-Yan asymmetry.

- possibility of isospin violation, $s(x) \neq \bar{s}(x)$, etc.

- higher orders (NNLO)

- QED (comparable to NNLO ? ($\alpha_s^3 \sim \alpha$)

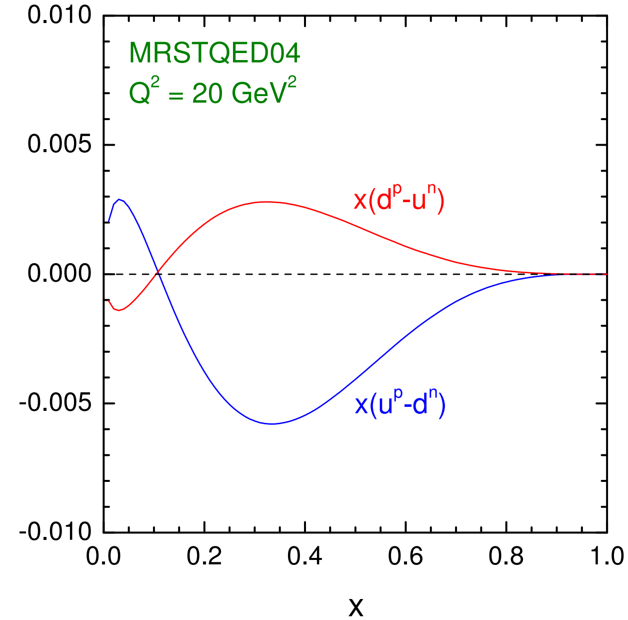
- large x ($\alpha_s^n \ln^{2n-1}(1-x)$)

- low Q^2 (higher twist)

- small x ($\alpha_s^n \ln^{n-1}(1/x)$)

R. Thorne

ISOSPIN VIOLATION



However, QED effects to lead to small isospin violation.

$u_V^p(x)$ quarks radiate more photon than $d_V^n(x)$ quarks.

To rough approximation

$$\gamma(x,Q^2) = \sum_j e_j^2 \frac{\alpha}{2\pi} \ln(Q^2/m_q^2) \int_x^1 \frac{dy}{y} P_{\gamma q}(y) q_j(\frac{x}{y},Q^2).$$

So more photon momentum in proton than neutron due to high-x up quarks radiating more than high-x down quarks.

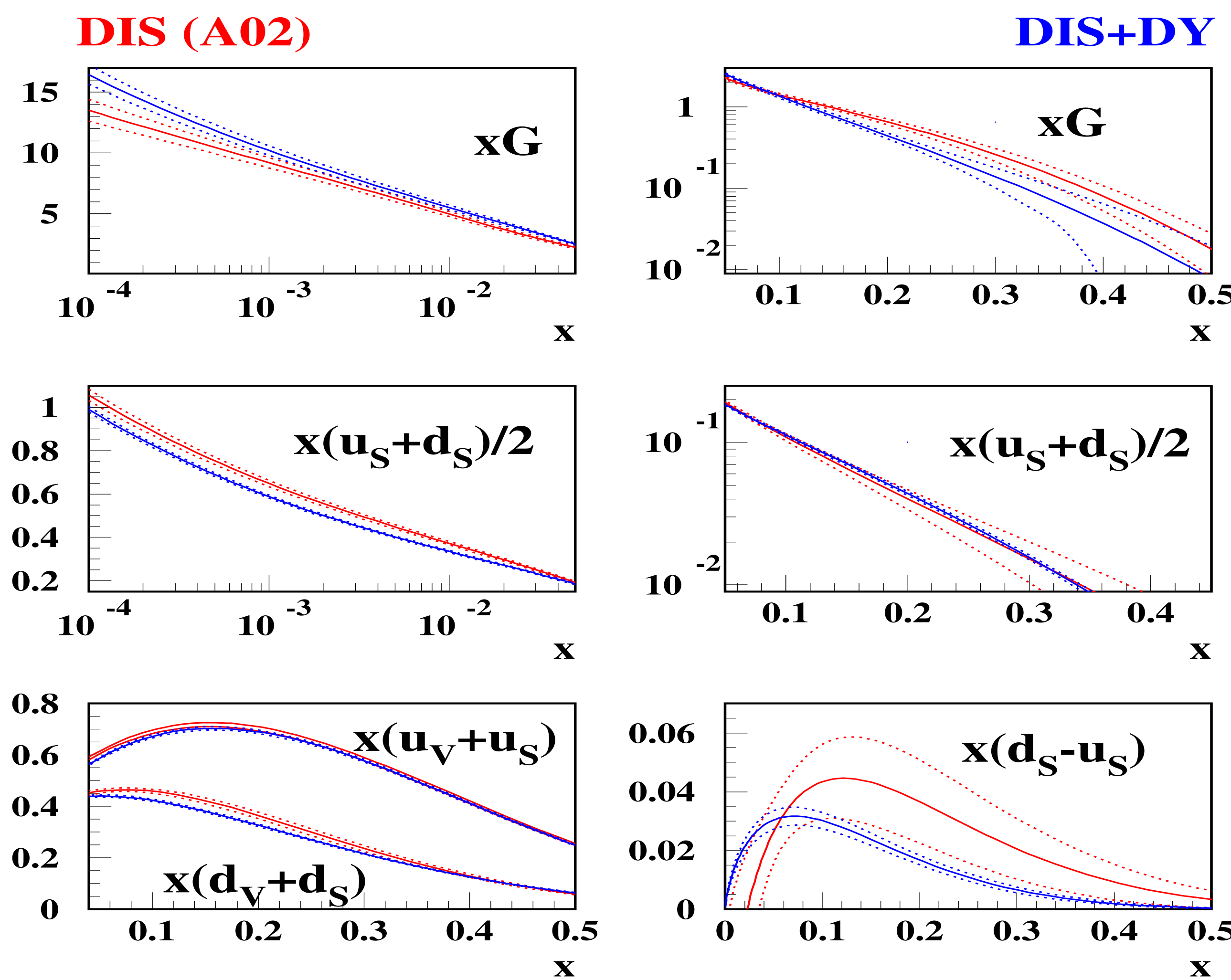
Momentum conservation $\rightarrow u_V^p(x) < d_V^n(x)$ at high .

Hence, $[\delta U_v] < 0$ as required by NuTeV anomaly.

Estimates for $m_u = 6$ MeV and $m_d = 10$ MeV imply similar to isospin violation observed by best fit! Reduces NuTeV anomaly to about $1/2$.

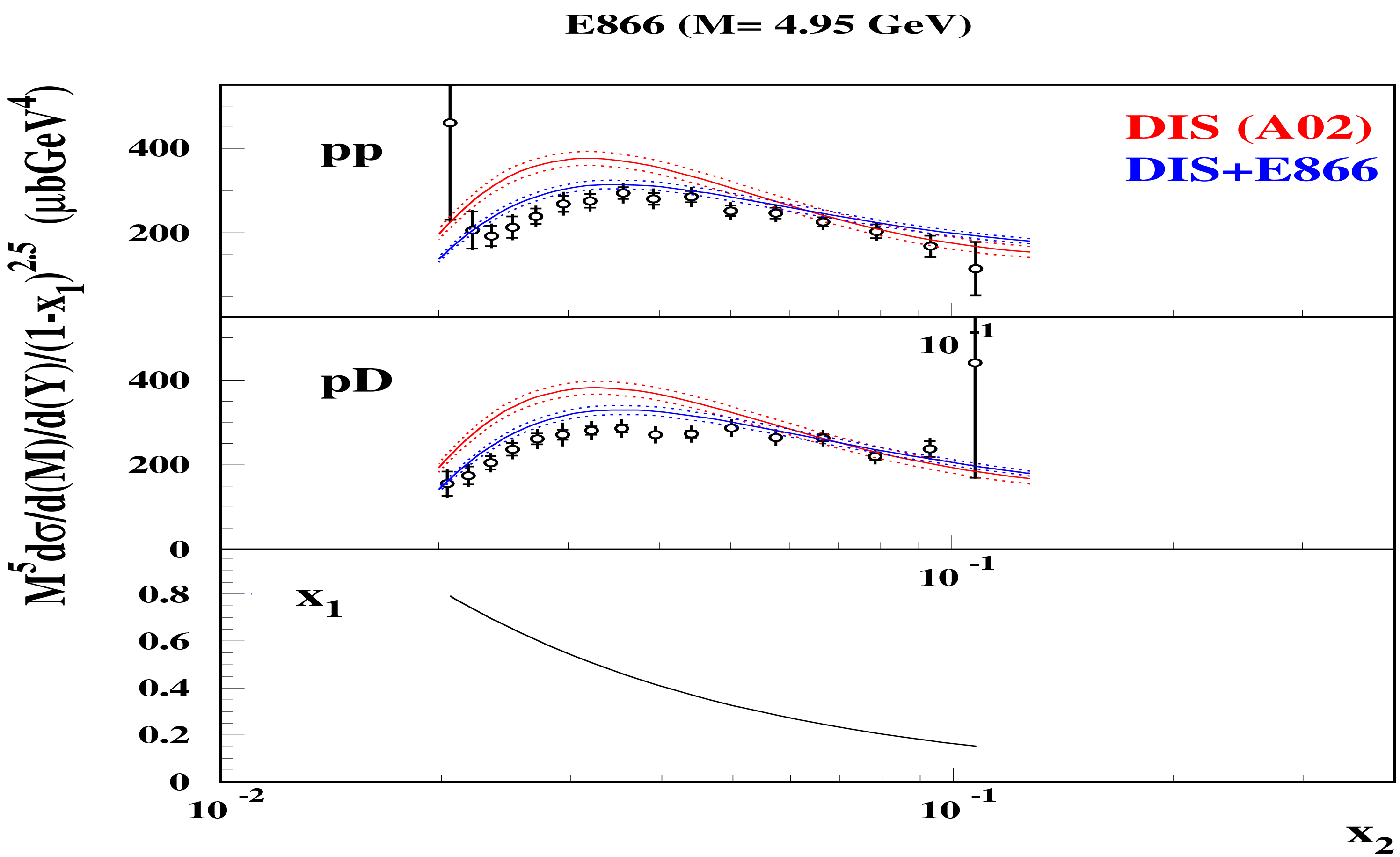
R. THORNE

Drell-Yan necessary for flavour decomposition



S. Alekhin

TROUBLE...



At low M/y the E866 data are in disagreement to the DIS ones

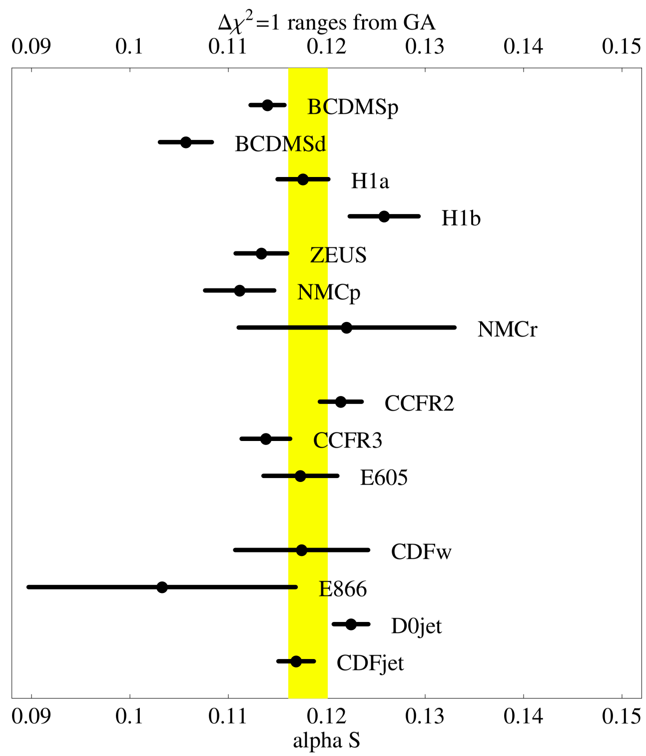
$\sigma_{\rm DY} \sim q(x_1)\bar{q}(x_2)$ and $\Delta q(x_1), \Delta\bar{q}(x_2) \sim O(1\%)$ from DIS

S. ALEKHIN

...MORE TROUBLE

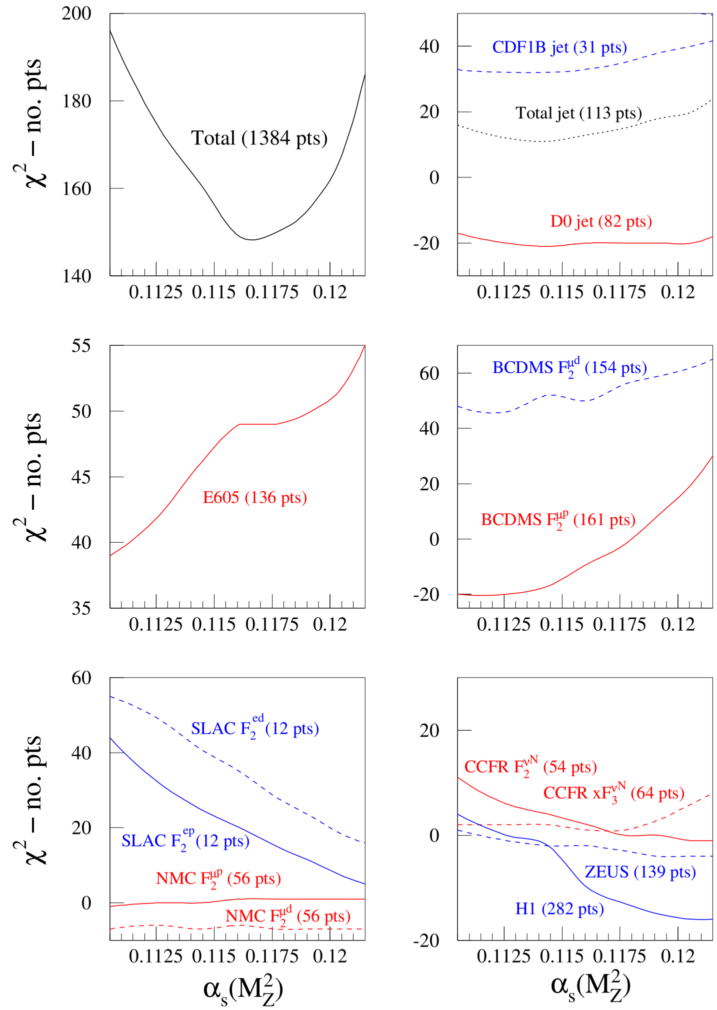
BEST-FIT OF α_s

$\Delta\chi^2 = 1$ INCONSISTENT BETWEEN EXPERIMENTS



CTEQ

GLOBAL χ^2 MINIMUM DOES NOT CORRESPOND TO LOCAL MINIMA

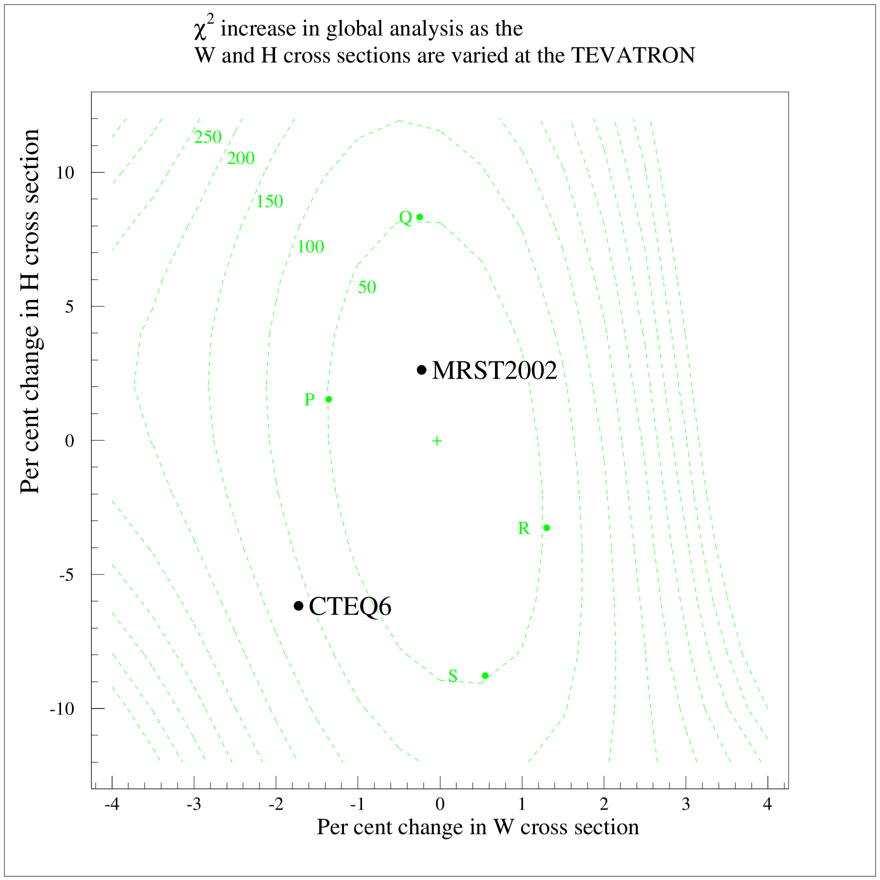


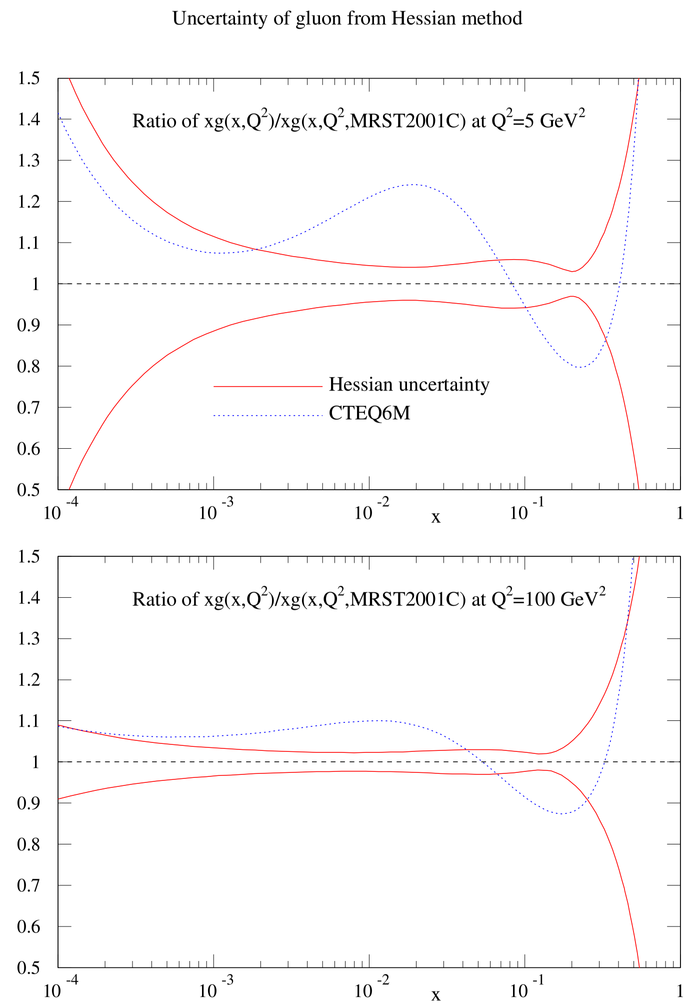
MRST

PROBLEM: INCONSISTENT EXPERIMENTS

THEORETICAL AND EXPERIMENTAL AMBIGUITIES

Different approaches lead to similar accuracy of measured quantities, but can lead to different central values. Must consider effect of assumptions made during fit.





Cuts made on data, data sets fit, correctness of NLO QCD, parameterization for input sets, heavy flavour prescription, no isospin violation, strong coupling ......

Many can be as important as experimental errors on data used (or more so).

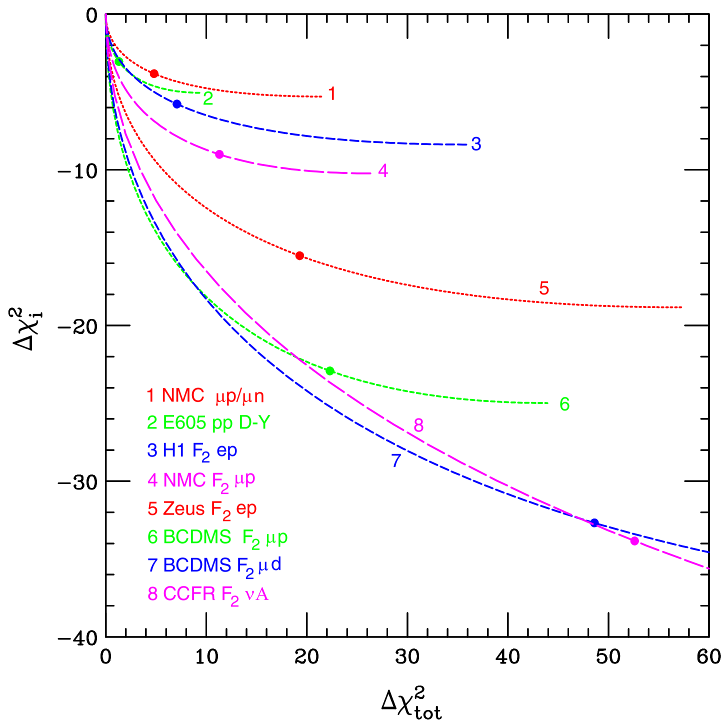
R. THORNE

WAYS OUT: CTEQ TOLERANCE CRITERION

SINGLE OUT INCONSISTENT DATA

- how many parameters are significantly determined by each dataset?
- how consistent are the data from one set with the rest?

STUDY MINIMUM ALLOWED χ_i^2
FOR i-TH EXP. AS
GLOBAL χ^2 ALLOWED TO INCREASE

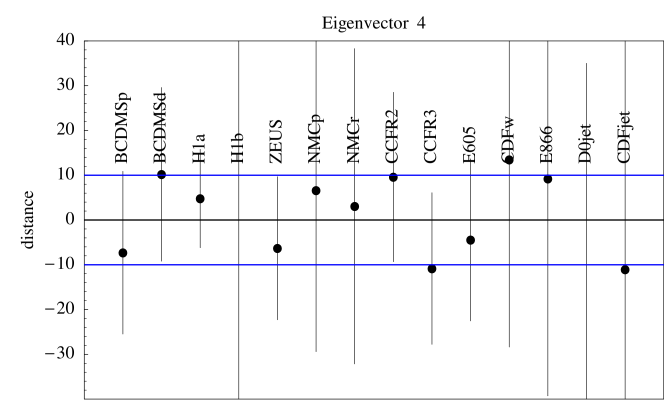


Collins, Pumplin 2001

CCFR, BCDMS
INCOMPATIBLE WITH THE REST

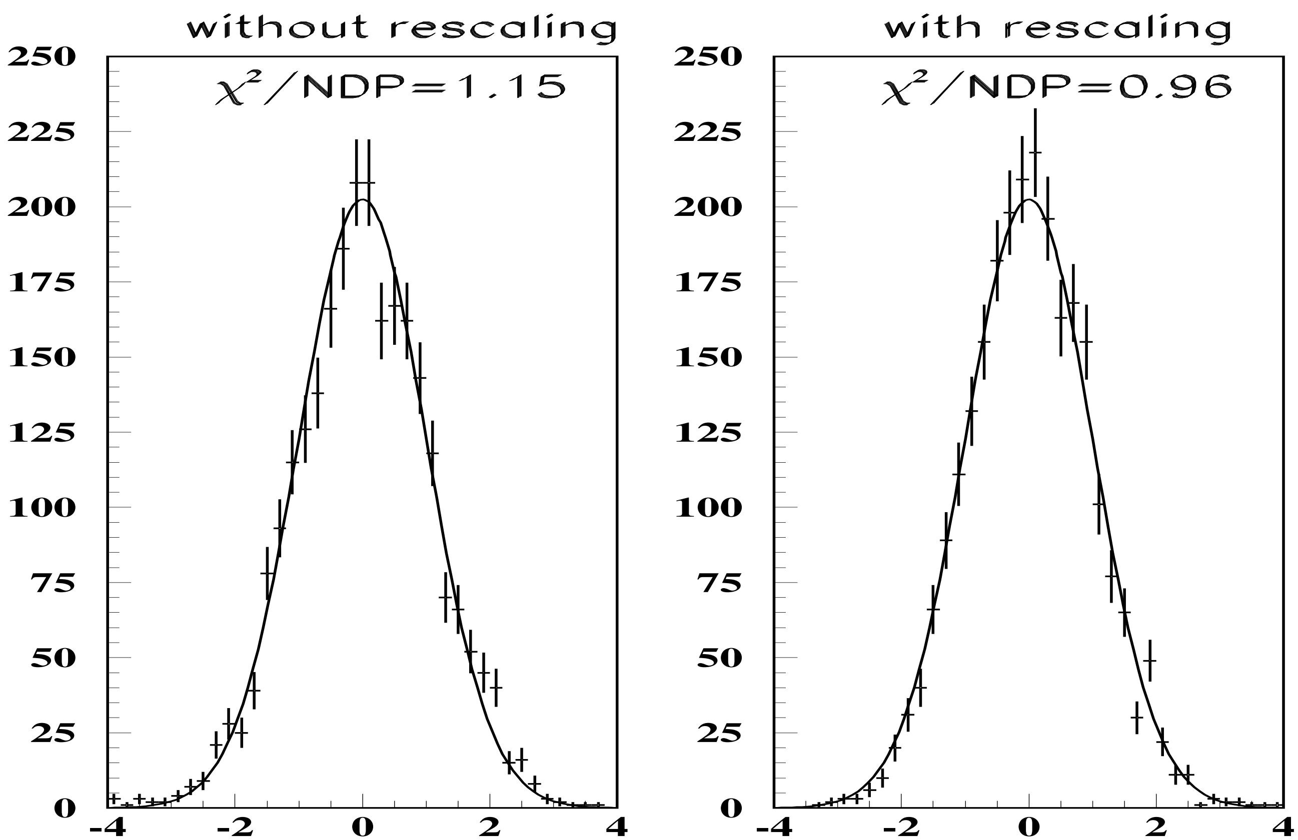
OPTIONS

- discard incompatible experiments
- reweight individual contributions
- INCORPORATE IN ERROR, TOLERATING FIXED MAX DEVIATION FOR EACH EXPERIMENT & EACH FIT PARAMETER



$\Delta\chi^2 = 100$ (CTEQ6)

WAYS OUT: ERROR RESCALING



For the experiments with $\chi^2 > 1$ the statistical errors in data were rescaled in order to get $\chi^2 = 1$

S. ALEKHIN

ERROR RESCALING

Experiment	NDP	χ^2/NDP	scale factor
SLAC-E-49A	118	0.57	–
SLAC-E-49B	299	1.22	1.10
SLAC-E-87	218	0.95	–
SLAC-E-89A	148	1.38	1.18
SLAC-E-89B	162	0.83	–
SLAC-E-139	26	1.28	1.14
SLAC-E-140	17	0.57	–
BCDMS	605	1.13	1.07
NMC	490	1.24	1.12
H1(96-97)	135	1.18	1.09
ZEUS(96-97)	161	1.28	1.14
E605	119	1.5	1.22
E866	39	1.4	1.19

S. ALEKHIN

WAYS OUT: MRST CONSERVATIVE PARTONS

IMPOSE RESTRICTIVE KINEMATIC CUTS

cut in Q^2 raised from 2 to 10 GeV^2; cut off $x < 0.005$; cut in W^2 raised from 12.5 to 15 GeV^2

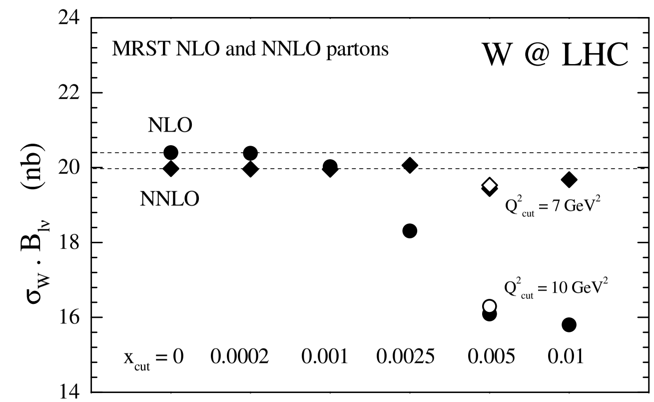
- ABOUT 800 DATAPOINTS REMOVED OUT OF ABOUT 2000
- $\Delta\chi^2 = 5$ SUFFICIENT FOR REASONABLE 1-σ

VARIATION OF THE χ^2

$x_{\rm cut}$:	0	0.0002	0.001	0.0025	0.005	0.01
# DATA POINTS	2097	2050	1961	1898	1826	1762
$\chi^2(x > 0)$	2267					
$\chi^2(x > 0.0002)$	2212	2203				
$\chi^2(x > 0.001)$	2134	2128	2119			
$\chi^2(x > 0.0025)$	2069	2064	2055	2040		
$\chi^2(x > 0.005)$	2024	2019	2012	1993	1973	
$\chi^2(x > 0.01)$	1965	1961	1953	1934	1917	1916
Δ_i^{i+1}	0.19	0.10	0.24	0.28	0.02	

DATA-THEORY AGREEMENT FOR EVOLUTION OF F_2 IMPROVES IF SMALL x DATA REMOVED χ^2 improves with fixed # of pts (same row)

VARIATION OF THE W XSECT.



SIZABLE EFFECT BECAUSE SMALL x REGION IMPORTANT

PARTLY STABILIZED BY NNLO

THEORETICAL ISSUES: NNLO

Splitting functions in x-space

with

$$f_0(x) \;=\; \frac{1}{x}\,, \qquad f_{\pm 1}(x) \;=\; \frac{1}{1\mp x}\,. \tag{4.3}$$

A useful short-hand notation is

$$H_{\underbrace{0,\ldots,0}_{m},\pm 1,\underbrace{0,\ldots,0}_{n},\pm 1,\ldots}(x) \;=\; H_{\pm(m+1),\pm(n+1),\ldots}(x)\,. \tag{4.4}$$

For $w \le 3$ the harmonic polylogarithms can be expressed in terms of standard polylogarithms; a complete list can be found in appendix A of Ref. [45]. All harmonic polylogarithms of weight $w=4$ in this article can be expressed in terms of standard polylogarithms, Nielsen functions [74] or, by means of the defining relation (4.2), as one-dimensional integrals over these functions. A FORTRAN program for the functions up to weight $w=4$ has been provided in Ref. [75].

For completeness we recall the one- and two-loop non-singlet splitting functions [3, 8]

$$P^{(0)}_{\rm ns}(x) \;=\; C_F\,(2p_{\rm qq}(x)+3\delta(1-x)) \tag{4.5}$$

and

$$\begin{aligned}
P^{(1)+}_{\rm ns}(x) \;=&\; 4C_AC_F\Big(p_{\rm qq}(x)\Big[\frac{67}{18}-\zeta_2+\frac{11}{6}H_0+H_{0,0}\Big]+p_{\rm qq}(-x)\Big[\zeta_2+2H_{-1,0}-H_{0,0}\Big]\\
&+\frac{14}{3}(1-x)+\delta(1-x)\Big[\frac{17}{24}+\frac{11}{3}\zeta_2-3\zeta_3\Big]\Big)-4C_Fn_f\Big(p_{\rm qq}(x)\Big[\frac{5}{9}+\frac{1}{3}H_0\Big]+\frac{2}{3}(1-x)\\
&+\delta(1-x)\Big[\frac{1}{12}+\frac{2}{3}\zeta_2\Big]\Big)+4C_F{}^2\Big(2p_{\rm qq}(x)\Big[H_{1,0}-\frac{3}{4}H_0+H_2\Big]-2p_{\rm qq}(-x)\Big[\zeta_2+2H_{-1,0}\\
&-H_{0,0}\Big]-(1-x)\Big[1-\frac{3}{2}H_0\Big]-H_0-(1+x)H_{0,0}+\delta(1-x)\Big[\frac{3}{8}-3\zeta_2+6\zeta_3\Big]\Big)\,,
\end{aligned} \tag{4.6}$$

$$\begin{aligned}
P^{(1)-}_{\rm ns}(x) \;=&\; P^{(1)+}_{\rm ns}(x)+16C_F\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[\zeta_2+2H_{-1,0}-H_{0,0}\Big]-2(1-x)\\
&-(1+x)H_0\Big)\,.
\end{aligned} \tag{4.7}$$

Here and in Eqs. (4.9) – (4.11) we suppress the argument x of the polylogarithms and use

$$p_{\rm qq}(x) \;=\; 2\,(1-x)^{-1}-1-x\,. \tag{4.8}$$

All divergences for $x\to 1$ are understood in the sense of $+$-distributions.

The three-loop splitting function for the evolution of the 'plus' combinations of quark densities in Eq. (2.2), corresponding to the anomalous dimension (3.8) reads

$$\begin{aligned}
P^{(2)+}_{\rm ns}(x) \;=&\; 16C_AC_Fn_f\Big(\frac{1}{6}p_{\rm qq}(x)\Big[\frac{10}{3}\zeta_2-\frac{209}{36}-9\zeta_3-\frac{167}{18}H_0+2H_0\zeta_2-7H_{0,0}-2H_{0,0,0}\\
&+3H_{1,0,0}-H_3\Big]+\frac{1}{3}p_{\rm qq}(-x)\Big[\frac{3}{2}\zeta_3-\frac{5}{3}\zeta_2-H_{-2,0}-2H_{-1}\zeta_2-\frac{10}{3}H_{-1,0}-H_{-1,0,0}\\
&+2H_{-1,2}+\frac{1}{2}H_0\zeta_2+\frac{5}{3}H_{0,0}+H_{0,0,0}-H_3\Big]+(1-x)\Big[\frac{1}{6}\zeta_2-\frac{257}{54}-\frac{43}{18}H_0-\frac{1}{6}H_{0,0}-H_1\Big]
\end{aligned}$$

$$\begin{aligned}
&-(1+x)\Big[\frac{2}{3}H_{-1,0}+\frac{1}{2}H_2\Big]+\frac{1}{3}\zeta_2+H_0+\frac{1}{6}H_{0,0}+\delta(1-x)\Big[\frac{5}{4}-\frac{167}{54}\zeta_2+\frac{1}{20}\zeta_2{}^2+\frac{25}{18}\zeta_3\Big]\Big)\\
&+16C_AC_F{}^2\Big(p_{\rm qq}(x)\Big[\frac{5}{6}\zeta_3-\frac{69}{20}\zeta_2{}^2-H_{-3,0}-3H_{-2}\zeta_2-14H_{-2,-1,0}+3H_{-2,0}+5H_{-2,0,0}\\
&-4H_{-2,2}-\frac{151}{48}H_0+\frac{41}{12}H_0\zeta_2-\frac{17}{2}H_0\zeta_3-\frac{13}{4}H_{0,0}-4H_{0,0}\zeta_2-\frac{23}{12}H_{0,0,0}+5H_{0,0,0,0}+\frac{2}{3}H_3\\
&-24H_1\zeta_3-16H_{1,-2,0}+\frac{67}{9}H_{1,0}-2H_{1,0}\zeta_2+\frac{31}{3}H_{1,0,0}+11H_{1,0,0,0}+8H_{1,1,0,0}-8H_{1,3}+H_4\\
&+\frac{67}{9}H_2-2H_2\zeta_2+\frac{11}{3}H_{2,0}+5H_{2,0,0}+H_{3,0}\Big]+p_{\rm qq}(-x)\Big[\frac{1}{4}\zeta_2{}^2-\frac{67}{9}\zeta_2+\frac{31}{4}\zeta_3+5H_{-3,0}\\
&-32H_{-2}\zeta_2-4H_{-2,-1,0}-\frac{31}{6}H_{-2,0}+21H_{-2,0,0}+30H_{-2,2}-\frac{31}{3}H_{-1}\zeta_2-42H_{-1}\zeta_3+\frac{9}{4}H_0\\
&-4H_{-1,-2,0}+56H_{-1,-1}\zeta_2-36H_{-1,-1,0,0}-56H_{-1,-1,2}-\frac{134}{9}H_{-1,0}-42H_{-1,0}\zeta_2-H_{3,0}\\
&+32H_{-1,3}-\frac{31}{6}H_{-1,0,0}+17H_{-1,0,0,0}+\frac{31}{3}H_{-1,2}+2H_{-1,2,0}+\frac{13}{12}H_0\zeta_2+\frac{29}{2}H_0\zeta_3+\frac{67}{9}H_{0,0}\\
&+13H_{0,0}\zeta_2+\frac{89}{12}H_{0,0,0}-5H_{0,0,0,0}-7H_2\zeta_2-\frac{31}{6}H_3-10H_4\Big]+(1-x)\Big[\frac{133}{36}+4H_{0,0,0,0}\\
&-\frac{167}{4}\zeta_3-2H_0\zeta_3-2H_{-3,0}+H_{-2}\zeta_2+2H_{-2,-1,0}-3H_{-2,0,0}+\frac{77}{4}H_{0,0,0}-\frac{209}{6}H_1-7H_1\zeta_2\\
&+4H_{1,0,0}+\frac{14}{3}H_{1,0}\Big]+(1+x)\Big[\frac{43}{2}\zeta_2-3\zeta_2{}^2+\frac{25}{2}H_{-2,0}-31H_{-1}\zeta_2-14H_{-1,-1,0}-\frac{13}{3}H_{-1,0}\\
&+24H_{-1,2}+23H_{-1,0,0}+\frac{55}{2}H_0\zeta_2+5H_{0,0}\zeta_2+\frac{1457}{48}H_0-\frac{1025}{36}H_{0,0}-\frac{155}{6}H_2+H_2\zeta_2-15H_3\\
&+2H_{2,0,0}-3H_4\Big]-5\zeta_2-\frac{1}{2}\zeta_2{}^2+50\zeta_3-2H_{-3,0}-7H_{-2,0}-H_0\zeta_3-\frac{37}{2}H_0\zeta_2-\frac{242}{9}H_0\\
&-2H_{0,0}\zeta_2+\frac{185}{6}H_{0,0}-22H_{0,0,0}-4H_{0,0,0,0}+\frac{28}{3}H_2+6H_3+\delta(1-x)\Big[\frac{151}{64}+\zeta_2\zeta_3-\frac{205}{24}\zeta_2\\
&-\frac{247}{60}\zeta_2{}^2+\frac{211}{12}\zeta_3+\frac{15}{2}\zeta_5\Big]\Big)+16C_A{}^2C_F\Big(p_{\rm qq}(x)\Big[\frac{245}{48}-\frac{67}{18}\zeta_2+\frac{12}{5}\zeta_2{}^2+\frac{1}{2}\zeta_3+\frac{1043}{216}H_0\\
&+H_{-3,0}+4H_{-2,-1,0}-\frac{3}{2}H_{-2,0}-H_{-2,0,0}+2H_{-2,2}-\frac{31}{12}H_0\zeta_2+4H_0\zeta_3+\frac{389}{72}H_{0,0}-2H_{2,0,0}\\
&-H_{0,0,0,0}+9H_1\zeta_3+6H_{1,-2,0}-H_{1,0}\zeta_2-\frac{11}{4}H_{1,0,0}-3H_{1,0,0,0}-4H_{1,1,0,0}+4H_{1,3}+\frac{31}{12}H_{0,0,0}\\
&+\frac{11}{12}H_3+H_4\Big]+p_{\rm qq}(-x)\Big[\frac{67}{18}\zeta_2-\zeta_2{}^2-\frac{11}{4}\zeta_3-H_{-3,0}+8H_{-2}\zeta_2+\frac{11}{6}H_{-2,0}-4H_{-2,0,0}\\
&-3H_{-1,0,0,0}+\frac{11}{3}H_{-1}\zeta_2+12H_{-1}\zeta_3-16H_{-1,-1}\zeta_2+8H_{-1,-1,0,0}+16H_{-1,-1,2}+\frac{67}{9}H_{-1,0}\\
&-8H_{-2,2}+11H_{-1,0}\zeta_2+\frac{11}{6}H_{-1,0,0}-\frac{11}{3}H_{-1,2}-8H_{-1,3}-\frac{3}{4}H_0-\frac{1}{6}H_0\zeta_2-4H_0\zeta_3-\frac{67}{18}H_{0,0}\\
&-3H_{0,0}\zeta_2-\frac{31}{12}H_{0,0,0}+H_{0,0,0,0}+2H_2\zeta_2+\frac{11}{6}H_3+2H_4\Big]+(1-x)\Big[\frac{1883}{108}-\frac{1}{2}H_{0,0,0,0}+11H_1\\
&-H_{-2,-1,0}+\frac{1}{2}H_{-3,0}-\frac{1}{2}H_{-2}\zeta_2+\frac{1}{2}H_{-2,0,0}+\frac{523}{36}H_0+H_0\zeta_3-\frac{13}{3}H_{0,0}-\frac{5}{2}H_{0,0,0}+2H_1\zeta_2\\
&-2H_{1,0,0}\Big]+(1+x)\Big[8H_{-1}\zeta_2+4H_{-1,-1,0}+\frac{8}{3}H_{-1,0}-5H_{-1,0,0}-6H_{-1,2}-\frac{13}{3}\zeta_2+\frac{3}{8}\zeta_2{}^2
\end{aligned}$$

$$\begin{aligned}
&-\frac{43}{4}\zeta_3-\frac{5}{2}H_{-2,0}-\frac{11}{2}H_0\zeta_2-\frac{1}{2}H_2\zeta_2-\frac{5}{4}H_{0,0}\zeta_2+7H_2-\frac{1}{4}H_{2,0,0}+3H_3+\frac{3}{4}H_4\Big]+\frac{1}{2}H_{0,0}\zeta_2\\
&+\frac{1}{4}\zeta_2{}^2-\frac{8}{3}\zeta_2+\frac{17}{2}\zeta_3+H_{-2,0}-\frac{19}{2}H_0+\frac{5}{2}H_0\zeta_2-H_0\zeta_3+\frac{13}{3}H_{0,0}+\frac{5}{2}H_{0,0,0}+\frac{1}{2}H_{0,0,0,0}\\
&-\delta(1-x)\Big[\frac{1657}{576}-\frac{281}{27}\zeta_2+\frac{1}{8}\zeta_2{}^2+\frac{97}{9}\zeta_3-\frac{5}{2}\zeta_5\Big]\Big)+16C_Fn_f^2\Big(\frac{1}{18}p_{\rm qq}(x)\Big[H_{0,0}-\frac{1}{3}+\frac{5}{3}H_0\Big]\\
&+(1-x)\Big[\frac{13}{54}+\frac{1}{9}H_0\Big]-\delta(1-x)\Big[\frac{17}{144}-\frac{5}{27}\zeta_2+\frac{1}{9}\zeta_3\Big]\Big)+16C_F{}^2n_f\Big(\frac{1}{3}p_{\rm qq}(x)\Big[5\zeta_3-4H_{1,0,0}\\
&-\frac{55}{16}+\frac{5}{8}H_0+H_0\zeta_2+\frac{3}{2}H_{0,0}-H_{0,0,0}-\frac{10}{3}H_{1,0}-\frac{10}{3}H_2-2H_{2,0}-2H_3\Big]+\frac{2}{3}p_{\rm qq}(-x)\Big[\frac{5}{3}\zeta_2\\
&-\frac{3}{2}\zeta_3+H_{-2,0}+2H_{-1}\zeta_2+\frac{10}{3}H_{-1,0}+H_{-1,0,0}-2H_{-1,2}-\frac{1}{2}H_0\zeta_2-\frac{5}{3}H_{0,0}-H_{0,0,0}+H_3\Big]\\
&-(1-x)\Big[\frac{10}{9}+\frac{19}{18}H_{0,0}-\frac{4}{3}H_1+\frac{2}{3}H_{1,0}+\frac{4}{3}H_2\Big]+(1+x)\Big[\frac{4}{3}H_{-1,0}-\frac{25}{24}H_0+\frac{1}{2}H_{0,0,0}\Big]+\frac{2}{9}H_0\\
&+\frac{7}{9}H_{0,0}+\frac{4}{3}H_2-\delta(1-x)\Big[\frac{23}{16}-\frac{5}{12}\zeta_2-\frac{29}{30}\zeta_2{}^2+\frac{17}{6}\zeta_3\Big]\Big)+16C_F{}^3\Big(p_{\rm qq}(x)\Big[\frac{9}{10}\zeta_2{}^2-2H_{-3,0}\\
&+6H_{-2}\zeta_2+12H_{-2,-1,0}-6H_{-2,0,0}-\frac{3}{16}H_0-\frac{3}{2}H_0\zeta_2+H_0\zeta_3+\frac{13}{8}H_{0,0}-2H_{0,0,0,0}+8H_{1,3}\\
&+12H_1\zeta_3+8H_{1,-2,0}-6H_{1,0,0}-4H_{1,0,0,0}+4H_{1,2,0}-3H_{2,0}+2H_{2,0,0}+4H_{2,1,0}+4H_{2,2}\\
&+4H_{3,0}+4H_{3,1}+2H_4\Big]+p_{\rm qq}(-x)\Big[\frac{7}{2}\zeta_2{}^2-\frac{9}{2}\zeta_3-6H_{-3,0}+32H_{-2}\zeta_2+8H_{-2,-1,0}+3H_{-2,0}\\
&-26H_{-2,0,0}-28H_{-2,2}+6H_{-1}\zeta_2+36H_{-1}\zeta_3+8H_{-1,-2,0}-48H_{-1,-1}\zeta_2+40H_{-1,-1,0,0}\\
&+48H_{-1,-1,2}+40H_{-1,0}\zeta_2+3H_{-1,0,0}-22H_{-1,0,0,0}-6H_{-1,2}-4H_{-1,2,0}-32H_{-1,3}-\frac{3}{2}H_0\\
&-\frac{3}{2}H_0\zeta_2-13H_0\zeta_3-14H_{0,0}\zeta_2-\frac{9}{2}H_{0,0,0}+6H_{0,0,0,0}+6H_2\zeta_2+3H_3+2H_{3,0}+12H_4\Big]\\
&+(1-x)\Big[2H_{-3,0}-\frac{31}{8}+4H_{-2,0,0}+H_{0,0}\zeta_2-3H_{0,0,0,0}+35H_1+6H_1\zeta_2-H_{1,0}+\frac{5}{2}H_{2,0}\Big]\\
&+(1+x)\Big[\frac{37}{10}\zeta_2{}^2-\frac{93}{4}\zeta_2-\frac{81}{2}\zeta_3-15H_{-2,0}+30H_{-1}\zeta_2+12H_{-1,-1,0}-2H_{-1,0}-26H_{-1,0,0}\\
&-24H_{-1,2}-\frac{539}{16}H_0-28H_0\zeta_2+\frac{191}{8}H_{0,0}+20H_{0,0,0}+\frac{85}{4}H_2-3H_{2,0,0}-2H_{3,0}+13H_3\\
&-H_4\Big]+4\zeta_2+33\zeta_3+4H_{-3,0}+10H_{-2,0}+\frac{67}{2}H_0+6H_0\zeta_3+19H_0\zeta_2-25H_{0,0}-17H_{0,0,0}\\
&-2H_2-H_{2,0}-4H_3+\delta(1-x)\Big[\frac{29}{32}-2\zeta_2\zeta_3+\frac{9}{8}\zeta_2+\frac{18}{5}\zeta_2{}^2+\frac{17}{4}\zeta_3-15\zeta_5\Big]\Big)\,.
\end{aligned} \tag{4.9}$$

The x-space counterpart of Eq. (3.8) for the evolution of the 'minus' combinations (2.2) is given by

$$\begin{aligned}
P^{(2)-}_{\rm ns}(x) \;=&\; P^{(2)+}_{\rm ns}(x)+16C_AC_F\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[\frac{134}{9}\zeta_2-4\zeta_2{}^2-11\zeta_3-4H_{-3,0}\\
&+32H_{-2}\zeta_2+\frac{22}{3}H_{-2,0}-16H_{-2,0,0}-32H_{-2,2}+\frac{44}{3}H_{-1}\zeta_2+48H_{-1}\zeta_3-64H_{-1,-1}\zeta_2\\
&+32H_{-1,-1,0,0}+64H_{-1,-1,2}+\frac{268}{9}H_{-1,0}+44H_{-1,0}\zeta_2+\frac{22}{3}H_{-1,0,0}-12H_{-1,0,0,0}-\frac{44}{3}H_{-1,2}\\
&-32H_{-1,3}-3H_0-\frac{2}{3}H_0\zeta_2-16H_0\zeta_3-\frac{134}{9}H_{0,0}-12H_{0,0}\zeta_2-\frac{31}{3}H_{0,0,0}+4H_{0,0,0,0}+8H_2\zeta_2
\end{aligned}$$

$$\begin{aligned}
&+\frac{22}{3}H_3+8H_4\Big]+(1-x)\Big[\frac{367}{18}+\frac{1}{2}\zeta_2{}^2+2H_{-3,0}-2H_{-2}\zeta_2-4H_{-2,-1,0}-10H_{-2,0}-2H_{0,0}\\
&+2H_{-2,0,0}+2H_0\zeta_3+H_{0,0}\zeta_2-H_{0,0,0,0}+8H_1\zeta_2+\frac{140}{3}H_1\Big]+(1+x)\Big[32H_{-1}\zeta_2-18\zeta_2\\
&-23\zeta_3+\frac{26}{3}H_{-1,0}-16H_{-1,0,0}-32H_{-1,2}-\frac{481}{18}H_0-29H_0\zeta_2+5H_{0,0,0}+24H_3+\frac{70}{3}H_2\Big]\\
&-2\zeta_2-2\zeta_3+32H_0+14H_0\zeta_2+2H_{0,0,0}-16H_3\Big)+16C_Fn_f\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[2\zeta_3\\
&-\frac{20}{9}\zeta_2-\frac{4}{3}H_{-2,0}-\frac{8}{3}H_{-1}\zeta_2-\frac{40}{9}H_{-1,0}-\frac{4}{3}H_{-1,0,0}+\frac{8}{3}H_{-1,2}+\frac{2}{3}H_0\zeta_2+\frac{20}{9}H_{0,0}+\frac{4}{3}H_{0,0,0}\\
&-\frac{4}{3}H_3\Big]+(1-x)\Big[\frac{61}{9}-\frac{8}{3}H_1\Big]+(1+x)\Big[2H_{0,0}-\frac{8}{3}H_{-1,0}+\frac{41}{9}H_0-\frac{4}{3}H_2\Big]\Big)\\
&+16C_F{}^2\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[9\zeta_3-7\zeta_2{}^2+12H_{-3,0}-64H_{-2}\zeta_2-16H_{-2,-1,0}-6H_{-2,0}\\
&+52H_{-2,0,0}+56H_{-2,2}-12H_{-1}\zeta_2-72H_{-1}\zeta_3-16H_{-1,-2,0}+96H_{-1,-1}\zeta_2-80H_{-1,-1,0,0}\\
&-96H_{-1,-1,2}-80H_{-1,0}\zeta_2-6H_{-1,0,0}+44H_{-1,0,0,0}+12H_{-1,2}+8H_{-1,2,0}+64H_{-1,3}+3H_0\\
&+3H_0\zeta_2+26H_0\zeta_3+28H_{0,0}\zeta_2+9H_{0,0,0}-12H_{0,0,0,0}-12H_2\zeta_2-6H_3-4H_{3,0}-24H_4\Big]\\
&-(1-x)\Big[15+8H_{-3,0}+8H_{-2,0,0}+61H_0+6H_0\zeta_3+2H_{0,0}\zeta_2-6H_{0,0,0,0}+12H_1\zeta_2+60H_1\\
&+8H_{1,0}\Big]+(1+x)\Big[24\zeta_2+57\zeta_3+10H_{-2,0}-48H_{-1}\zeta_2-4H_{-1,0}+40H_{-1,0,0}+48H_{-1,2}\\
&+59H_0\zeta_2-22H_{0,0}-35H_{0,0,0}-22H_2-4H_{2,0}-44H_3\Big]+8\zeta_2-42\zeta_3-4H_{-2,0}+42H_0\\
&-38H_0\zeta_2+14H_{0,0}-16H_2+26H_{0,0,0}+24H_3\Big)\,.
\end{aligned} \tag{4.10}$$

Finally the Mellin inversion of $\gamma^{(2)s}_{\rm ns}(N)$ in Eq. (3.9) leads to the following result for the leading (third-order) difference $P^{(2)s}_{\rm ns}(x)$ of the 'valence' and 'minus' splitting functions:

$$\begin{aligned}
P^{(2)s}_{\rm ns}(x) \;=&\; 16n_f\frac{d^{abc}d_{abc}}{n_c}\Big(\frac{1}{2}(1-x)\Big[\frac{50}{3}+\frac{41}{12}\zeta_2-\frac{5}{4}\zeta_2{}^2-H_{-3,0}+H_{-2}\zeta_2-H_{-2,0,0}+\frac{9}{4}H_3\\
&+2H_{-2,-1,0}+\frac{3}{2}H_{0,0}\zeta_2-\frac{1}{2}H_1\zeta_2-\frac{3}{4}H_{1,0,0}+\frac{91}{12}H_1\Big]+\frac{1}{2}(1+x)\Big[H_{-1,-1,0}-\frac{3}{2}H_{-1}\zeta_2+\frac{3}{4}H_0\\
&-\frac{13}{6}H_{-1,0}+\frac{1}{2}H_{-1,0,0}+2H_{-1,2}-\frac{3}{2}H_{-2,0}+\frac{9}{4}H_0\zeta_2+\frac{29}{12}H_{0,0}+\frac{41}{12}H_2-H_2\zeta_2-\frac{1}{2}H_{2,0,0}\\
&+\frac{3}{2}H_4\Big]-\frac{1}{3}\Big(\frac{1}{x}+x^2\Big)\Big[3H_{-1}\zeta_2+2H_{-1,-1,0}-2H_{-1,0,0}-2H_{-1,2}+H_1\zeta_2\Big]+\frac{1}{3}x^2\Big[5\zeta_3-2H_3\\
&+2H_{-2,0}+4H_0\zeta_2-2H_{0,0,0}+2H_1\zeta_2\Big]+\frac{91}{24}H_0+\zeta_3-\frac{9}{2}\zeta_2+\zeta_2{}^2-H_0\zeta_3-H_0\zeta_2-2H_{0,0}\zeta_2\\
&+\frac{3}{8}H_{0,0}-\frac{1}{4}H_{0,0,0}+\frac{1}{2}H_{0,0,0,0}+H_{-2,0}-H_3\Big)\,.
\end{aligned} \tag{4.11}$$

Of particular interest is the end-point behaviour of the harmonic polylogarithms at x – $x\to 1$, where logarithmic singularities occur. In the limit $x\to 0$, the factors $\ln x$ are relat trailing zeroes in the index field, whereas in the limit $x\to 1$ factors of $\ln(1-x)$ emerge

S. MOCH

Motivation

NNLO F_L

Some facts about F_L

- Theory
  - Longitudinal structure function $F_L = F_2 - 2xF_1$ vanishes at tree level
  - Complete NNLO analysis of F_L (or $R = \frac{\sigma_T}{\sigma_L}$) requires three-loop coefficient functions
- Experiment
  - Various methods for measurement of F_L
    - R-ratio
    - derivative $\frac{\partial F_L}{\partial \ln y}$ (inelasticity y)
    - fits of the shape of F_L in cross section
  - HERA results for F_L

    H1 collaboration '01; H1 collaboration '03; Klein '03; Lobodzinska '03

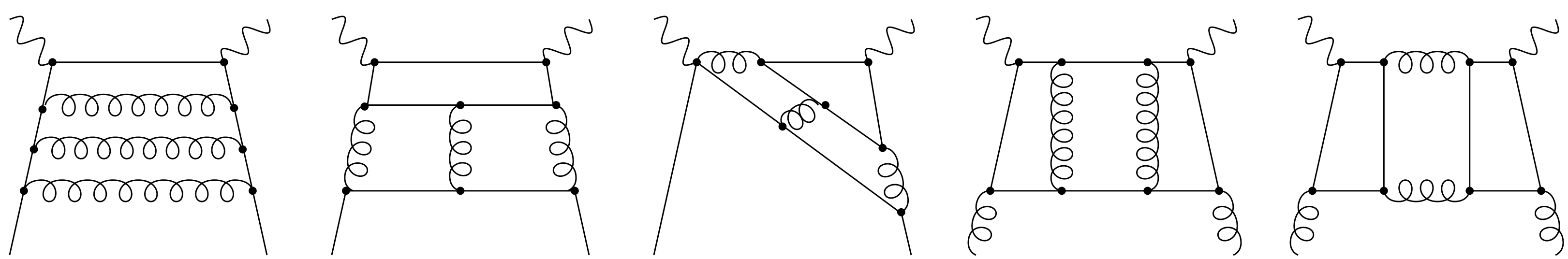
S. Moch

Flavour classes (a subtlety)

- At three loops new contributions to coefficient functions emerge
- Charge factor distinguishes flavour classes $\langle e^k \rangle = \frac{1}{n_f} \sum_{i=1}^{n_f} e_i^k$

flavor factor	fl_2	fl_{11}	fl_{02}	fl_2^g	fl_{11}^g
non-singlet	1	$3\langle e \rangle$	0	–	–
singlet	1	$\frac{\langle e \rangle^2}{\langle e^2 \rangle}$	1	1	$\frac{\langle e \rangle^2}{\langle e^2 \rangle}$

- Feynman diagrams (examples)



fl_2 fl_{02} fl_{11} fl_2^g fl_{11}^g

S. MOCH

NNLO SCHEME CHANGES...

$$
\begin{aligned}
K_{22}^{N(2)} = \; & \gamma_{qq}^{N(2)} + C_{2,g}^{N(1)}\gamma_{gq}^{N(1)} + \gamma_{gq}^{N(0)}\left(C_{2,g}^{N(2)} - C_{2,g}^{N(1)}C_{2,q}^{N(1)}\right) + 2\beta_0\left[2C_{2,q}^{N(2)} - \left(C_{2,q}^{N(1)}\right)^2 - \frac{C_{L,q}^{N(2)}C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}}\right] \\
& + \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - \gamma_{qg}^{N(1)}\right] - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left[\gamma_{qg}^{N(2)} - C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}\right)\right. \\
& \left. + \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 4\beta_0\right)\left(C_{2,g}^{N(1)}C_{2,q}^{N(1)} - C_{2,g}^{N(2)}\right) + \gamma_{qg}^{N(0)}\left(\frac{C_{L,g}^{N(2)} - C_{L,q}^{N(1)}C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}}\right)\right] \\
& + \frac{\left(C_{L,q}^{N(1)}\right)^2}{\left(C_{L,g}^{N(1)}\right)^2}\left[\left(C_{2,q}^{N(1)}\right)^2\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - C_{2,g}^{N(1)}\gamma_{qg}^{N(1)} + \gamma_{qg}^{N(0)}\left(C_{2,g}^{N(1)}C_{2,q}^{N(1)} - C_{2,g}^{N(2)}\right)\right] \\
& + \frac{C_{L,g}^{N(2)}}{\left(C_{L,g}^{N(1)}\right)^2}\left[C_{L,q}^{N(1)}\gamma_{qg}^{N(0)} + C_{L,q}^{N(2)}\gamma_{qg}^{N(0)} - C_{2,g}^{N(1)}C_{L,q}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right)\right] - 2\frac{C_{2,g}^{N(1)}C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)}\right)^2}\gamma_{qg}^{N(0)} \\
& - \frac{\beta_1}{\beta_0}\left\{\gamma_{qg}^{N(1)} + \gamma_{qg}^{N(0)}\left(C_{2,g}^{N(1)} - \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}}\right) + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - \gamma_{qg}^{N(1)}\right]\right. \\
& \left. + \frac{C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)}\right)^2}\gamma_{qg}^{N(0)}\left(C_{L,g}^{N(2)} - C_{L,q}^{N(1)}C_{2,g}^{N(1)}\right)\right\} + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right)\left(\gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}\right)
\end{aligned}
$$

A. GUFFANTI

...NNLO Mellin Transforms

$$
\begin{aligned}
& +\zeta(2)S_{1,-1}(N) + \left[\zeta(2)\log(2) - \frac{5}{8}\zeta(3)\right][S_1(N) - S_{-1}(N)] \\
& -\frac{3}{40}\zeta(2)^2 + \frac{5}{8}\zeta(3)\log(2) - \frac{1}{2}\zeta(2)\log^2(2) \\
= & -2S_{-2,1,1}(N) + S_1(N)S_{-2,1}(N) + S_{-2,2}(N) + S_{-3,1}(N) \qquad (122)
\end{aligned}
$$

$$
\begin{aligned}
S_{1,2,-1}(N) = & (-1)^N \mathbf{M}\left\{\frac{1}{1+x}[\mathrm{Li}_2(-x)\log(1+x) + 2\mathrm{S}_{1,2}(-x)]\right\}(N) \\
& -\log(2)[\mathrm{S}_{1,2}(N) - S_{1,-2}(N)] - \frac{1}{2}\zeta(2)S_{1,-1}(N) \\
& + \left[\frac{1}{4}\zeta(3) - \frac{1}{2}\zeta(2)\log(2)\right][S_1(N) - S_{-1}(N)] \\
& + \frac{6}{5}\zeta(2)^2 - 3\mathrm{Li}_4\left(\frac{1}{2}\right) - \frac{23}{8}\zeta(3)\log(2) + \zeta(2)\log^2(2) - \frac{1}{8}\log^4(2) \qquad (123)
\end{aligned}
$$

$$
\begin{aligned}
S_{1,2,1}(N) = & \mathbf{M}\left\{\left[\frac{1}{x-1}(\mathrm{Li}_2(x)\log(1-x) + 2\mathrm{S}_{1,2}(x))\right]_+\right\}(N) + \zeta(2)S_{1,1}(N) \\
= & -\mathbf{M}\left[\frac{\mathrm{Li}_3(1-x)}{x-1}\right](N) + \mathbf{M}\left[\left(\frac{1}{x-1}\right)_+ \mathrm{S}_{1,2}(x)\right](N) \\
& + S_1(N)S_3(N) + \frac{1}{2}S_1^2(N)S_2(N) + \frac{1}{2}S_2^2(N) - \frac{1}{2}\zeta(2)S_1^2(N) \\
& + S_4(N) - \frac{1}{2}\zeta(2)S_2(N) - \frac{8}{5}\zeta^2(2) \qquad (124)
\end{aligned}
$$

$$
\begin{aligned}
\rightarrow S_{-1,-1,-2}(N) = & (-1)^{N+1}\mathbf{M}\left\{\frac{1}{1+x}[F_1(x) + \log(1-x)\mathrm{Li}_2(-x)]\right\}(N) \\
& + (-1)^{N+1}\mathbf{M}\left\{\frac{1}{1+x}\left[\frac{1}{2}\mathrm{S}_{1,2}(x^2) - \mathrm{S}_{1,2}(x) - \mathrm{S}_{1,2}(-x)\right]\right\}(N) \\
& + \frac{1}{2}\zeta(2)[S_{-1,1}(N) - S_{-1,-1}(N)] + \left[\frac{9}{8}\zeta(3) - \frac{3}{2}\zeta(2)\log(2) - \frac{1}{6}\log^3(2)\right]S_{-1}(N) \\
& - \frac{1}{10}\zeta(2)^2 + \frac{17}{8}\zeta(3)\log(2) - \frac{7}{4}\zeta(2)\log^2(2) - \frac{1}{6}\log^4(2) \\
= & (-1)^{N+1}\mathbf{M}\left\{\frac{1}{1+x}[\mathrm{S}_{1,2}(-x) + \mathrm{Li}_2(-x)\log(1+x) + \mathrm{Li}_2(-x)\log(1-x)]\right\}(N) \\
& + S_1(N)S_{2,-1}(N) + S_{2,-2}(N) + S_{3,-1}(N) + S_{-1}(N)S_3(N) \\
& + \frac{1}{2}S_2(N)S_{-2}(N) + \frac{1}{2}S_{-1}^2(N)S_{-2}(N) \\
& + [S_1(N) - S_{-1}(N)][S_2(N) - S_{-2}(N)]\log 2 + \frac{1}{2}\zeta(2)S_1(N)S_{-1}(N) \\
& + S_{-4}(N) + 2\log(2)[S_3(N) - S_{-3}(N)] + \left[\frac{1}{2}\zeta(2) - \log^2(2)\right]S_2(N) \\
& + S_{-2}(N)\log^2(2) - \left[\frac{1}{4}\zeta(3) - \frac{1}{2}\zeta(2)\log(2)\right]S_1(N) \\
& + \left[\frac{3}{4}\zeta(3) - \frac{1}{2}\zeta(2)\log(2)\right]S_{-1}(N) - 4\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{3}{2}\zeta^2(2) \\
& - \frac{5}{2}\zeta(3)\log(2) + \frac{1}{2}\zeta(2)\log^2(2) - \frac{1}{6}\log^4(2) \qquad (125)
\end{aligned}
$$

$(O(\alpha^2))$

J. Blümlein

Physics information of $d\sigma/dY$

- High precision measurements at fixed-target collisions (E866), the Tevetron, and the LHC.
- Standard caddle: extract pdf's and partonic luminosities

$$\frac{d\sigma}{dY} = [\text{quark density}]_1 \left(\frac{M}{E_{cm}} e^{Y}\right) \times [\text{quark density}]_2 \left(\frac{M}{E_{cm}} e^{-Y}\right) + \mathcal{O}(\alpha_s)$$

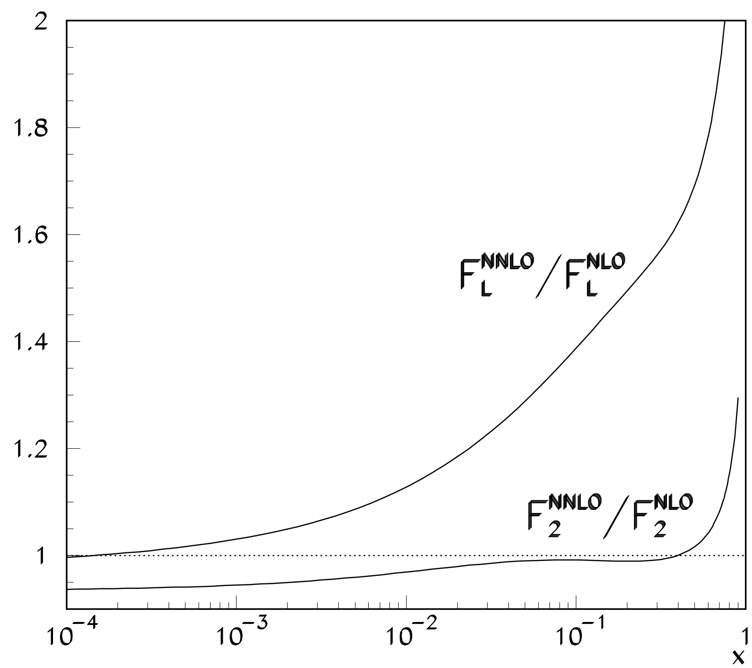
- Precision electroweak measurements at hadron colliders
  - weak mixing angle from forward-backward asymmetry
  - W-mass measurements (sensitive to pdf's)
- Determination of new gauge boson couplings to quarks, . . .

C. Anastasiou

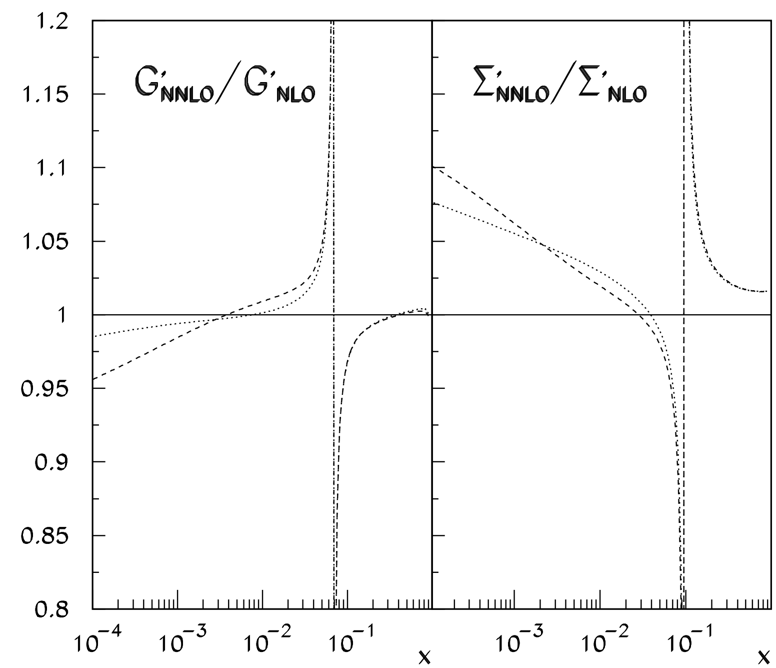
GOOD NEWS: NNLO FITS POSSIBLE

HOW BIG IS THE IMPACT OF HIGHER ORDER PERTURBATIVE CORRECTIONS?

PERTURBATIVE COEFFICIENTS

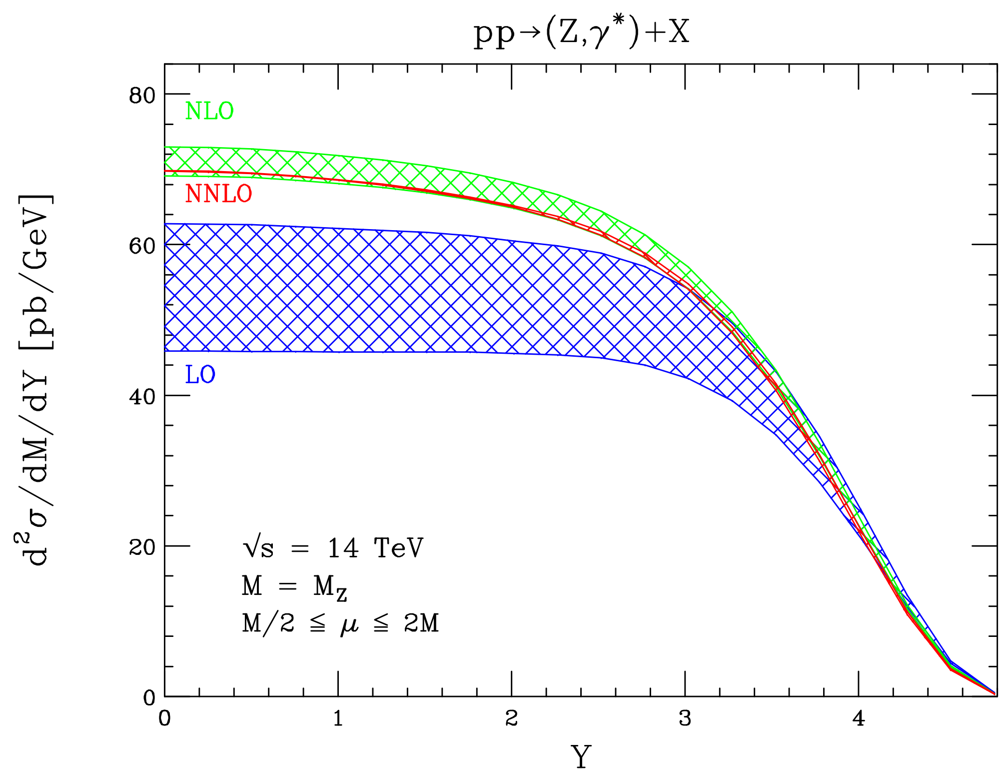


EVOLUTION



ALEKHIN

On-shell Z boson at the LHC



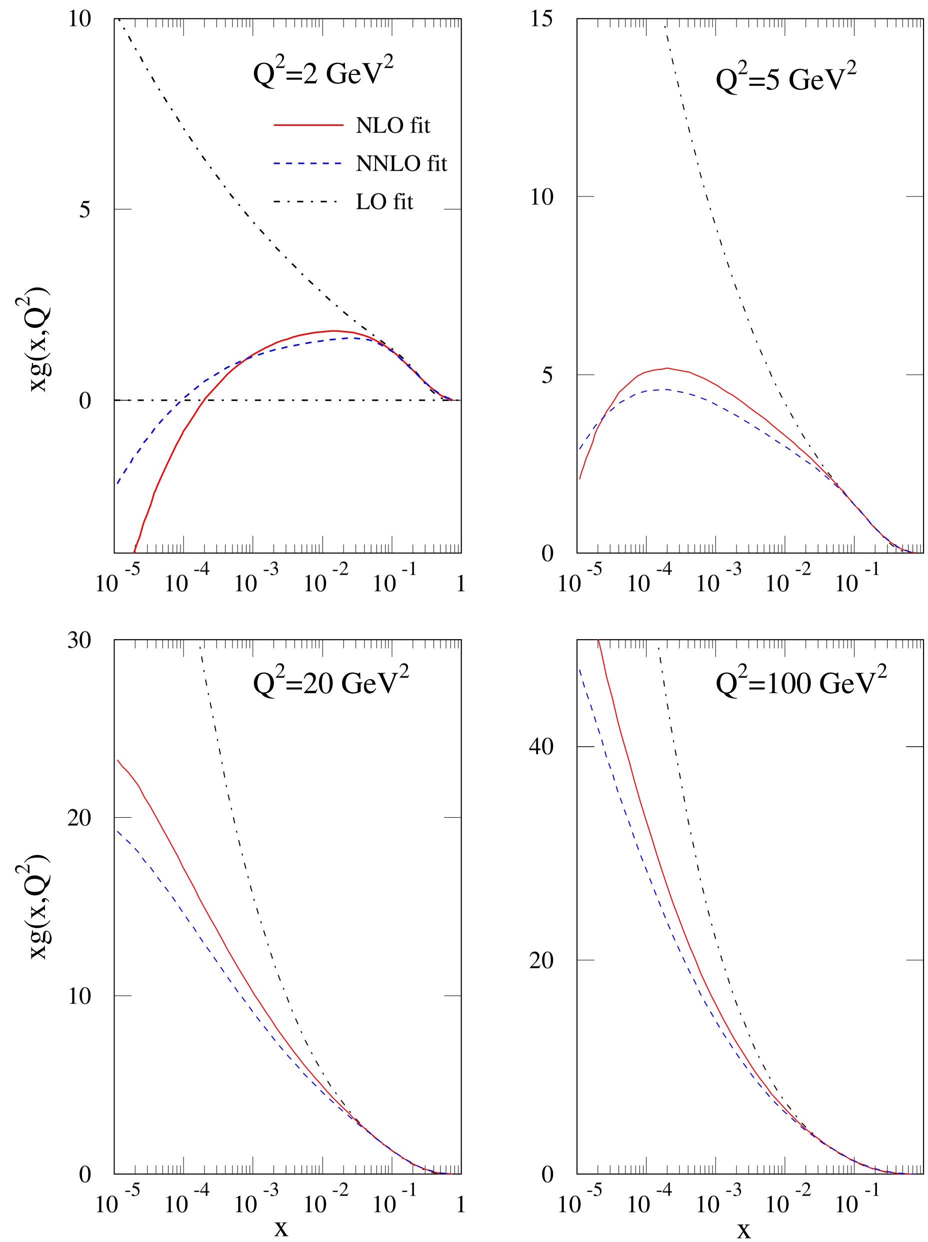
- small NNLO scale uncertainty: $(30\% - 25\%)(LO) \to (6\%)(NLO) \to 0.1\%(Y = 0) - 1\%(Y \leq 3) - 3\%(Y \simeq 4)(NNLO)$
- shape stabilizes at NNLO

C. Anastasiou

$xg(x, Q^2)$ going from LO $\rightarrow$ NLO $\rightarrow$ NNLO.

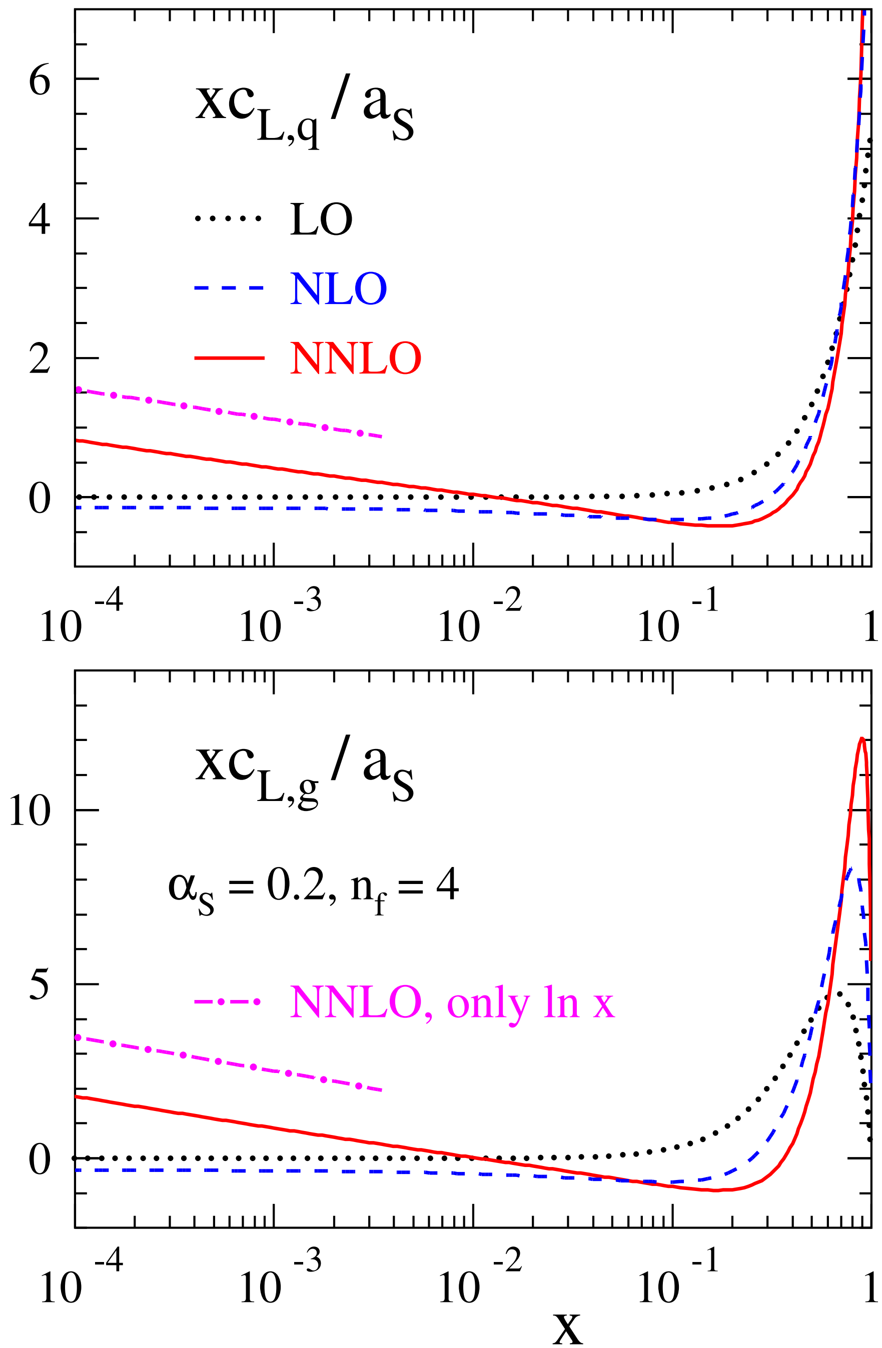
GOOD NEWS:
NNLO GLUON LESS NEGATIVE

Gluon LO , NLO and NNLO



R. THORNE

NNLO F_L IMPORTANT!



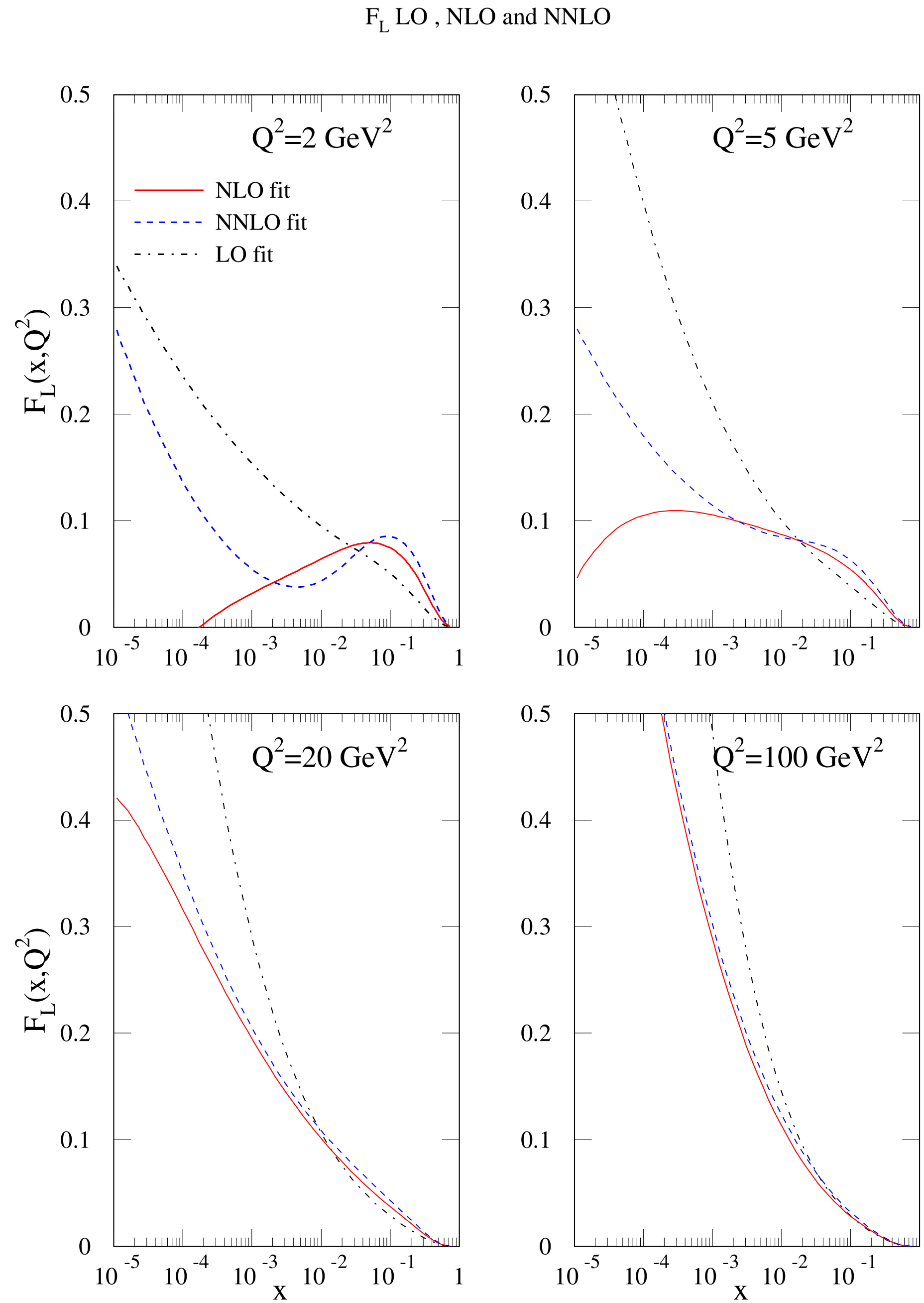
- Perturbative expansion of singlet-quark and gluon coefficient functions $c_{L,\mathrm{q}}$ and $c_{L,\mathrm{g}}$ for F_L with $n_f = 4$ and $\alpha_s(\mu^2) = 0.2$ (results divided by $a_\mathrm{s} = \alpha_\mathrm{s}/(4\pi)$)
- LO and NLO contributions remarkably small
- At small-x $c_{L,\mathrm{q}}$ and $c_{L,\mathrm{g}}$ dominated by NNLO term
- Leading small-x term at NNLO results $xc^{(3)}_{L,a} \sim \ln x$

S. Moch

Also instability in physical, gluon dominated, quantity $F_L(x,Q^2)$ going from LO $\rightarrow$ NLO $\rightarrow$ NNLO.

Note very large effect of exact NNLO coefficient function.

NOT SO GOOD NEWS: SMALL x F_L UNSTABLE



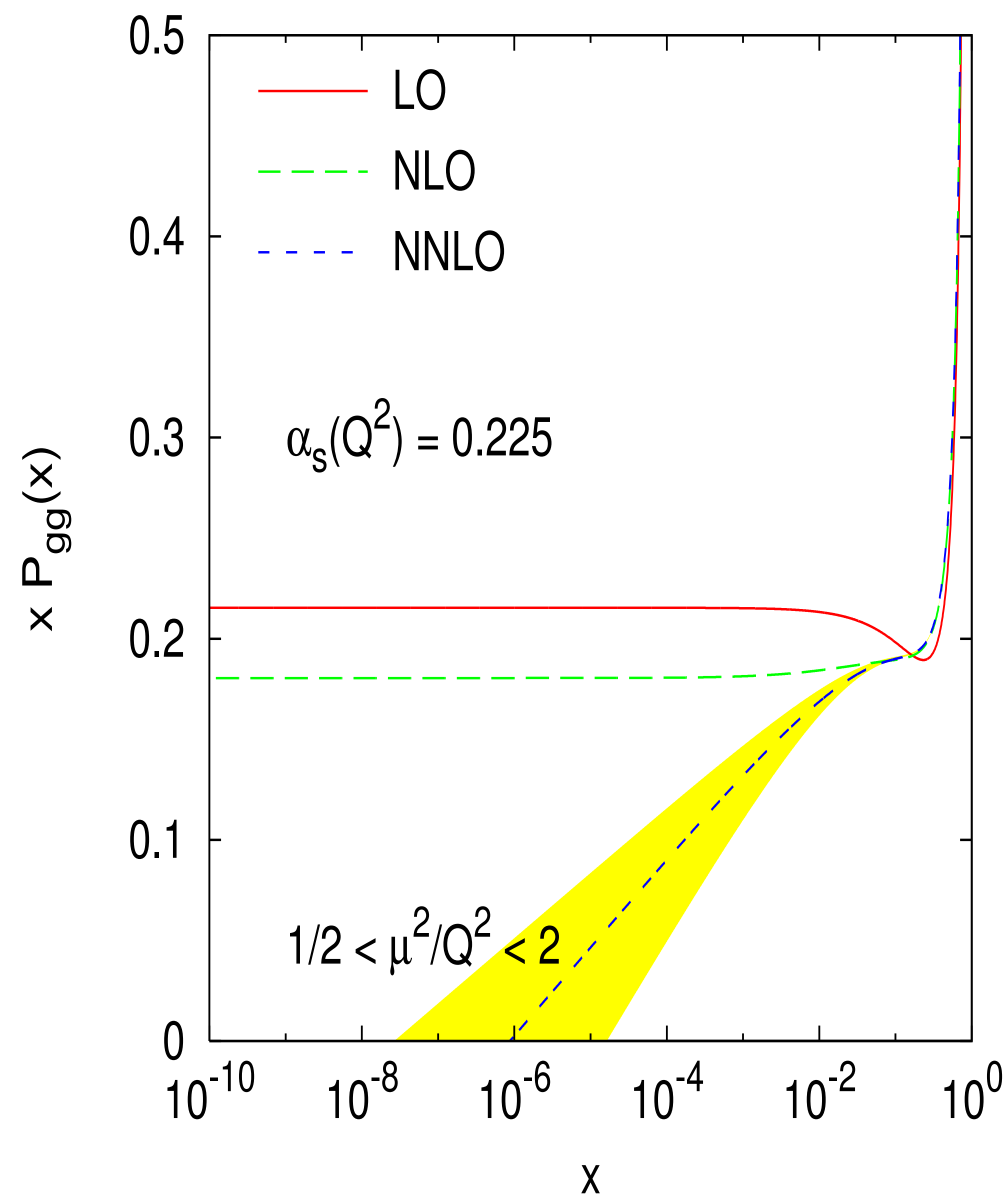
R. THORNE

Perturbative structure of P_{gg}

- Small-x gluon splitting function has logarithmic enhancements:

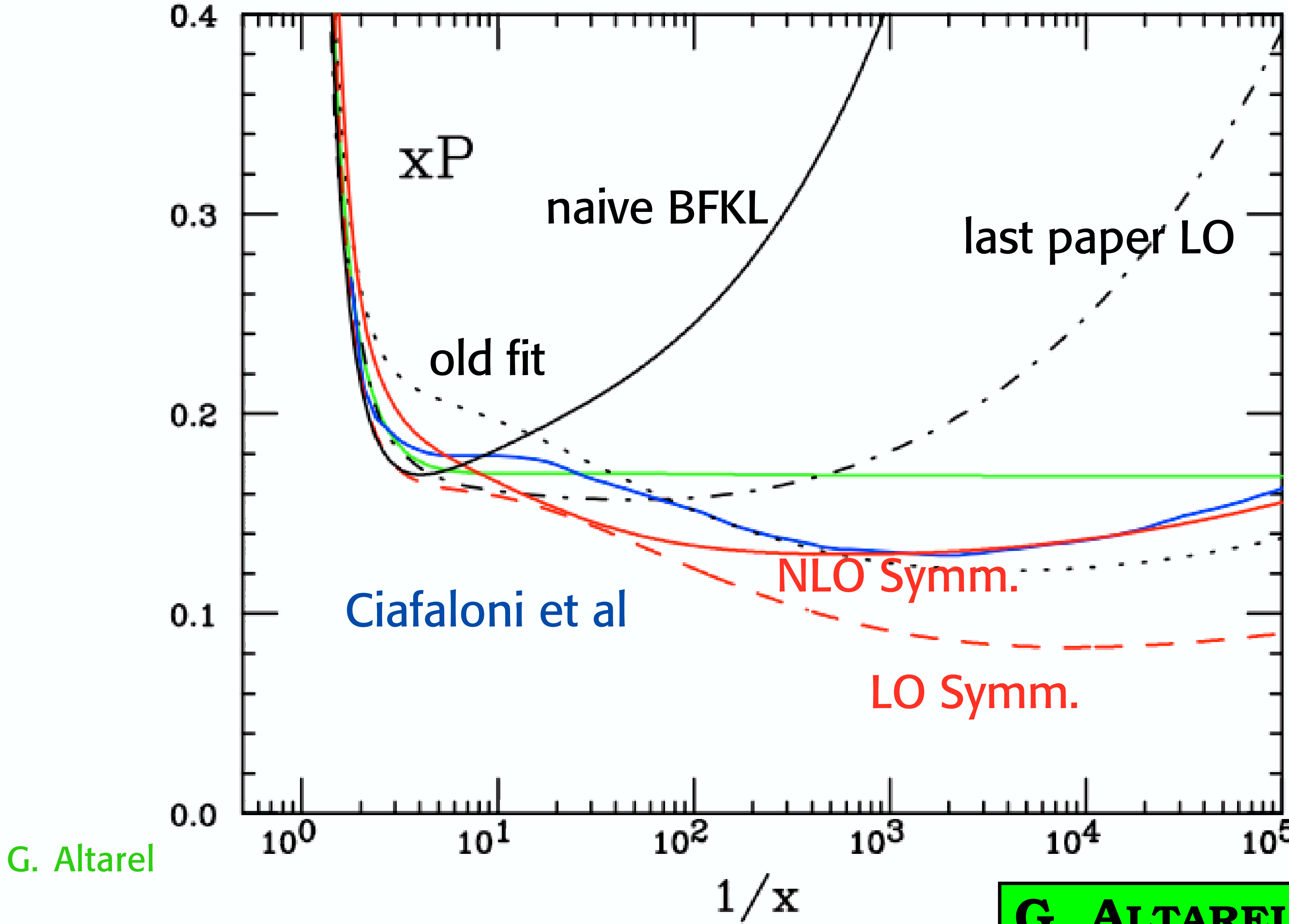
$$xP_{gg}(x) = \sum_{n=1} \alpha_s^n \ln^{n-1} \frac{1}{x} + \sum_{n=2} \alpha_s^n \ln^{n-2} \frac{1}{x} + \dots$$

- NNLO (α_s^3): first small-x enhancement in gluon splitting function.



G. SALAM

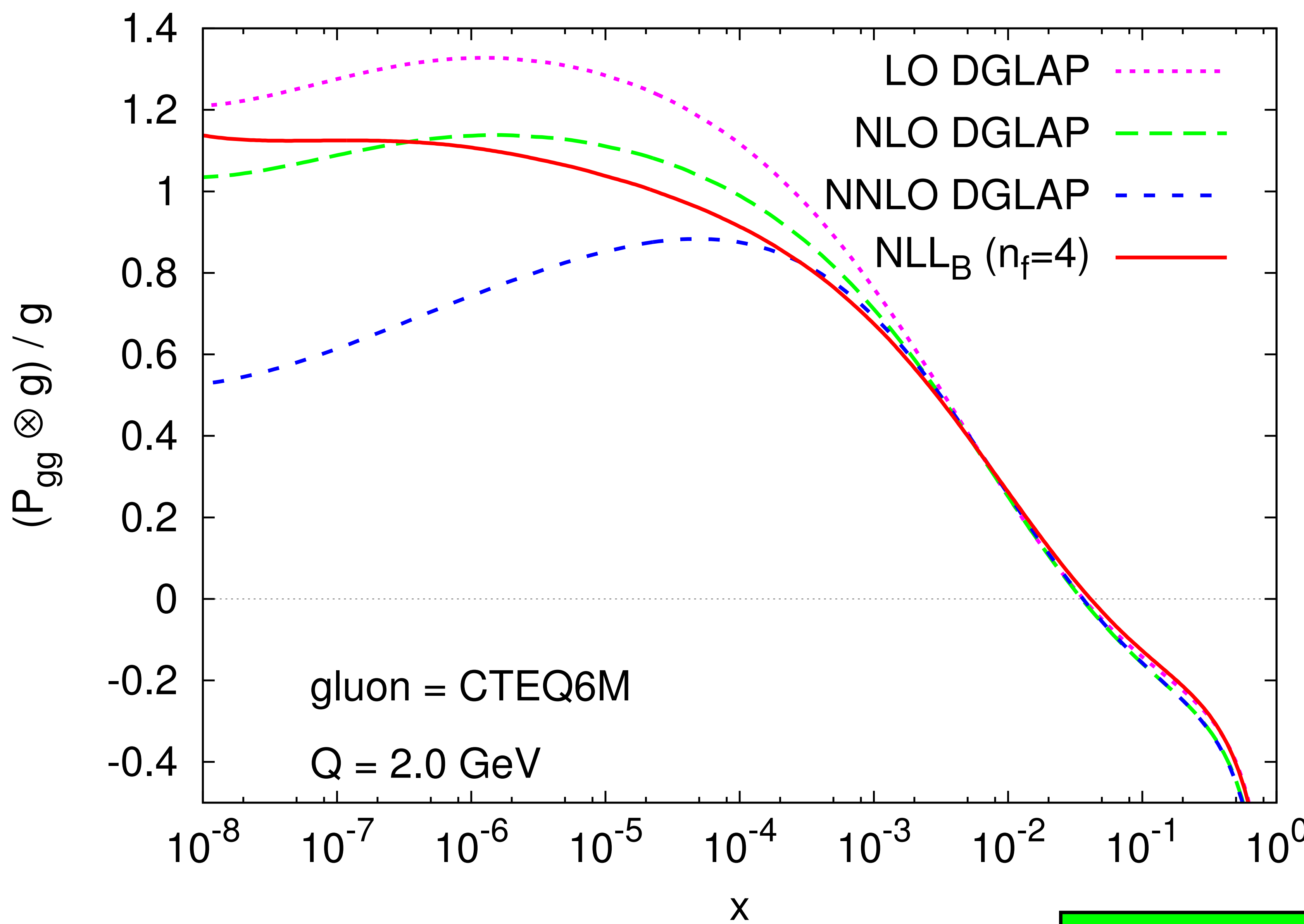
RESUMMED PREDICTIONS CONVERGE!



G. ALTARELLI

Evolution Stabilized

$P_{gg} \otimes g(x)$



G. Salam

THEORETICAL ISSUES: *Soft gluon resummation*

Extending the range of perturbative QCD

- Soft and collinear gluons generate *large logarithms* in QCD cross sections near kinematic thresholds.

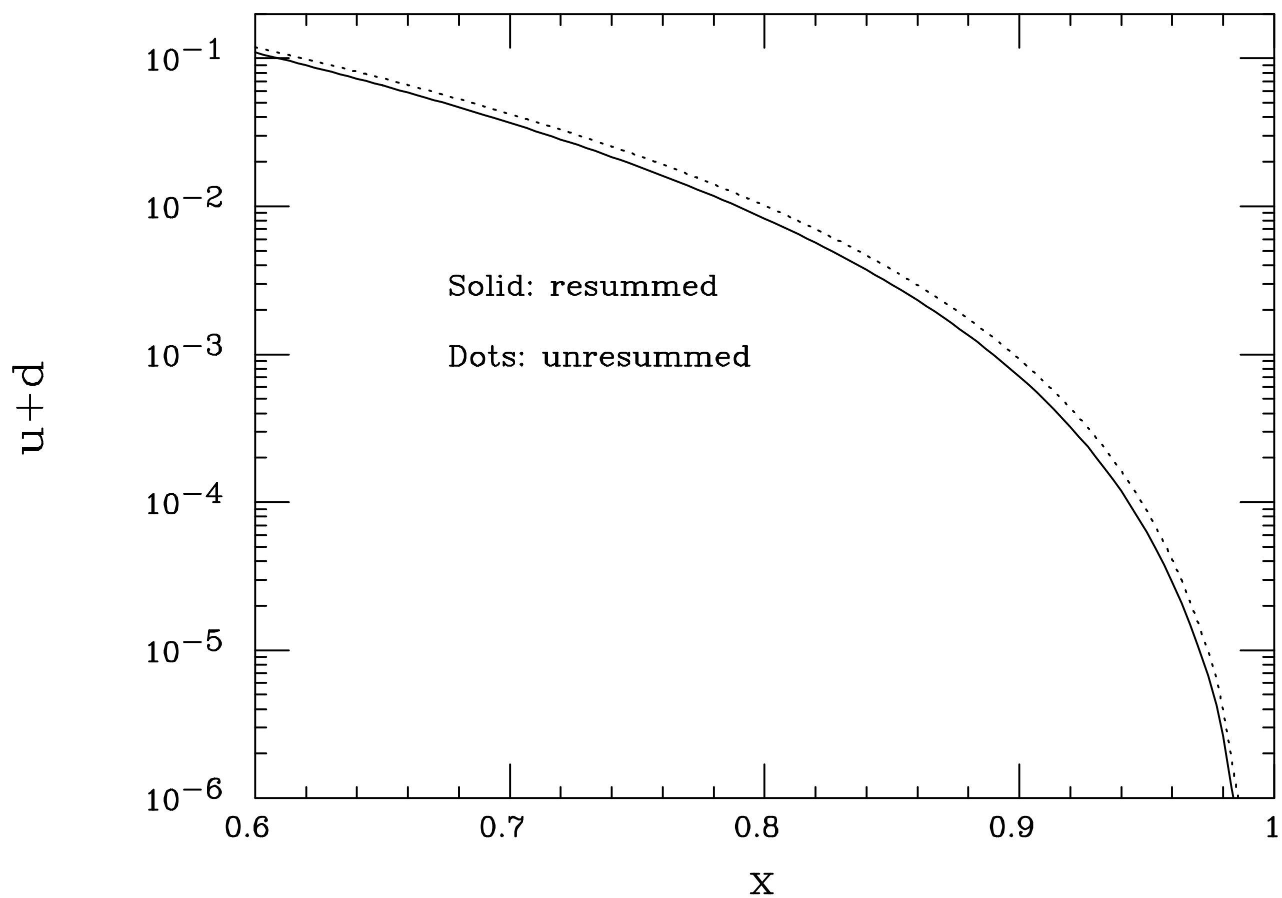
$$\text{DIS} \quad \longrightarrow \alpha_s^n \log^{2n-1}(1-x)/(1-x)$$

- Soft and collinear logarithms can be computed to all orders and they *exponentiate* in moment space.
$\sum_k \alpha_s^k \sum_p^{2k} c_{kp} L^p \to \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right]$
- Resummation *extends the range* of perturbation theory
$\alpha_s L^2 \ll 1 \longleftrightarrow \alpha_s \ll 1$
- Resummation reaches beyond perturbation theory
finite order $\longrightarrow$ resummation $\longrightarrow$ power corrections

Impact of large x resummation

Largex $u + d$ *quark distribution*

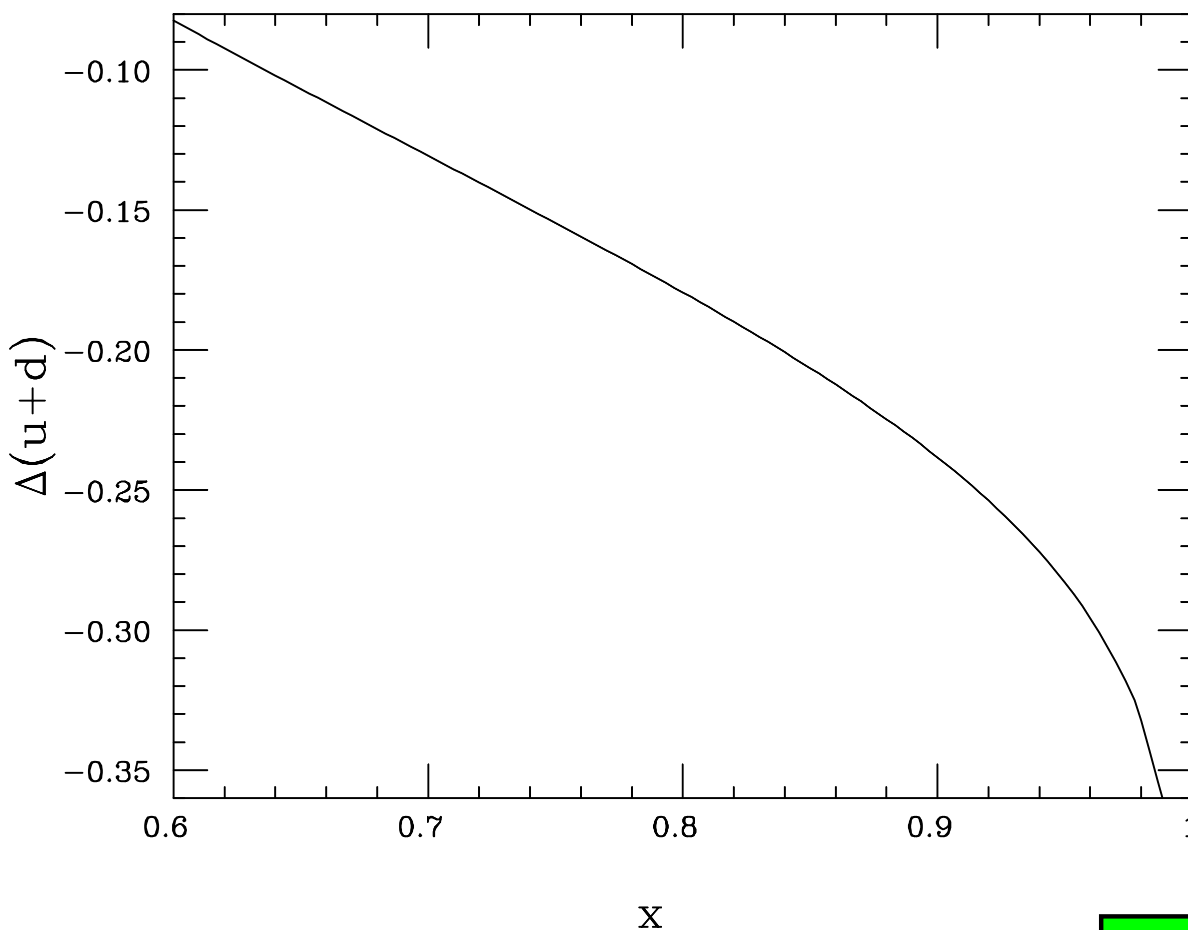
preliminary



IMPACT OF LARGE x RESUMMATION

Relative variation of $u + d$ at large-x

preliminary

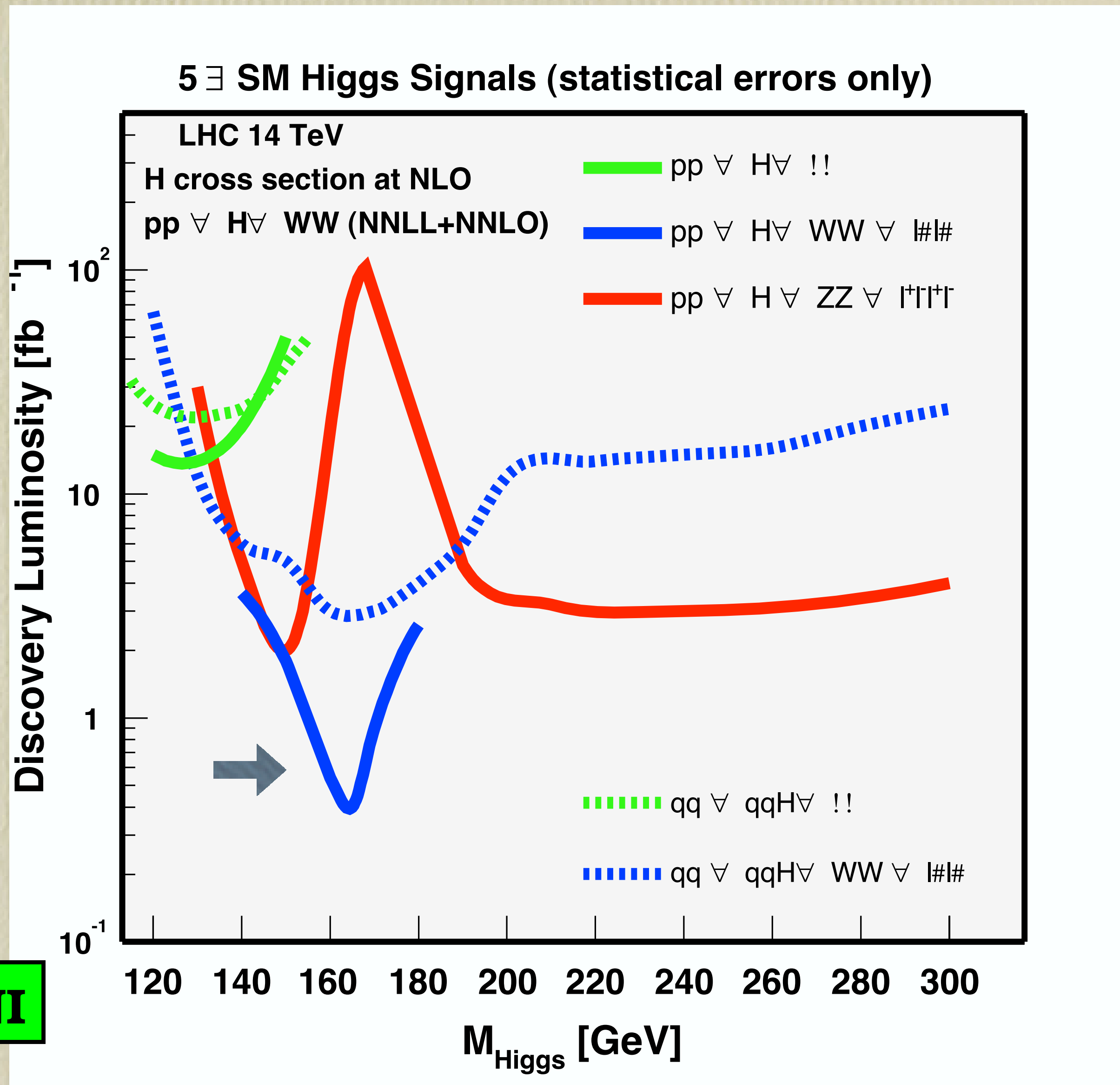


A recent application in gg H W W l l

G. Davatz, G. Dissertori, M. Dittmar, F. Pauss, MG ffi2004ffi

Use results for gg H spectrum at NNLL+NLO to correct ffireweightffievents generated with PYTHIA

Apply the resummation formalism to WW pair production → NLL+LO results used to correct PYTHIA main background



M. GRAZZINI

II: MINI–SUMMARY

- RAPID PROGRESS IN THE INCLUSION OF NEW DATASETS & SUBTLE THEOR. EFFECTS IN GLOBAL PARTON FITS
- PROBLEMS: INCOMPATIBLE DATA? INSUFFICIENT THEORY?
- NNLO COMPUTATIONS FOR INCLUSIVE & LESS INCLUSIVE QUANTITIES
- LARGE x RESUMMATION AVAILABLE, SMALL x RESUMMATION CONVERGING TO RESULT
- ACCURATE ESTIMATE OF PDF ERRORS BEHIND THE CORNER