From ep to pp through the Color Glass Condensate

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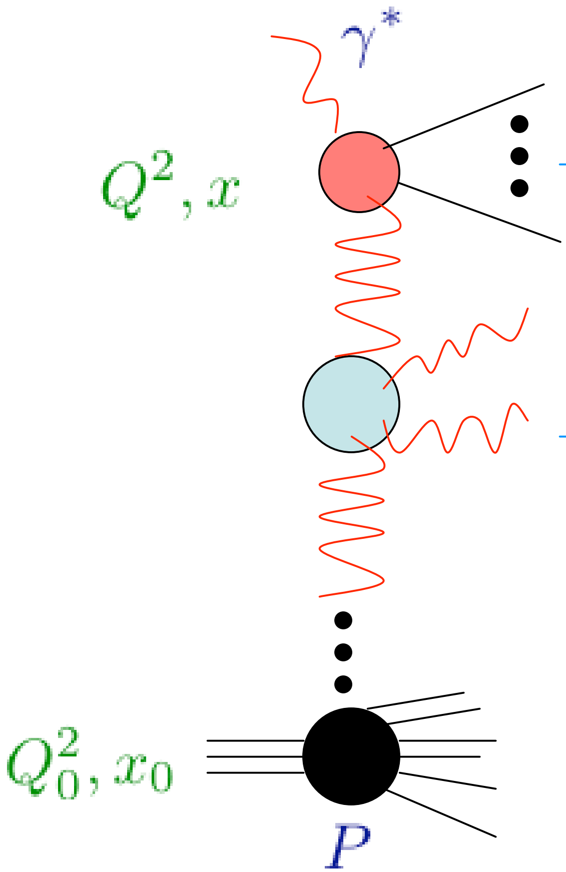
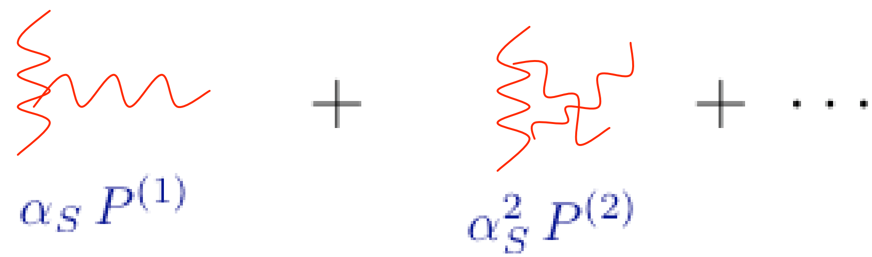
Outline of talk:

- Introduction
- A classical effective theory (and its quantum evolution) for high energy QCD
- Dipoles in the Color Glass Condensate
- Hadronic scattering and k_t factorization in the Color Glass Condensate
- Outlook

STRUCTURE OF HIGHER ORDER CONTRIBUTIONS IN DIS



$$C_2(x, \alpha_S) = \delta(1-x) + \alpha_S C_2^{(1)}(x) + \alpha_S^2 C_2^{(2)} + \cdots$$

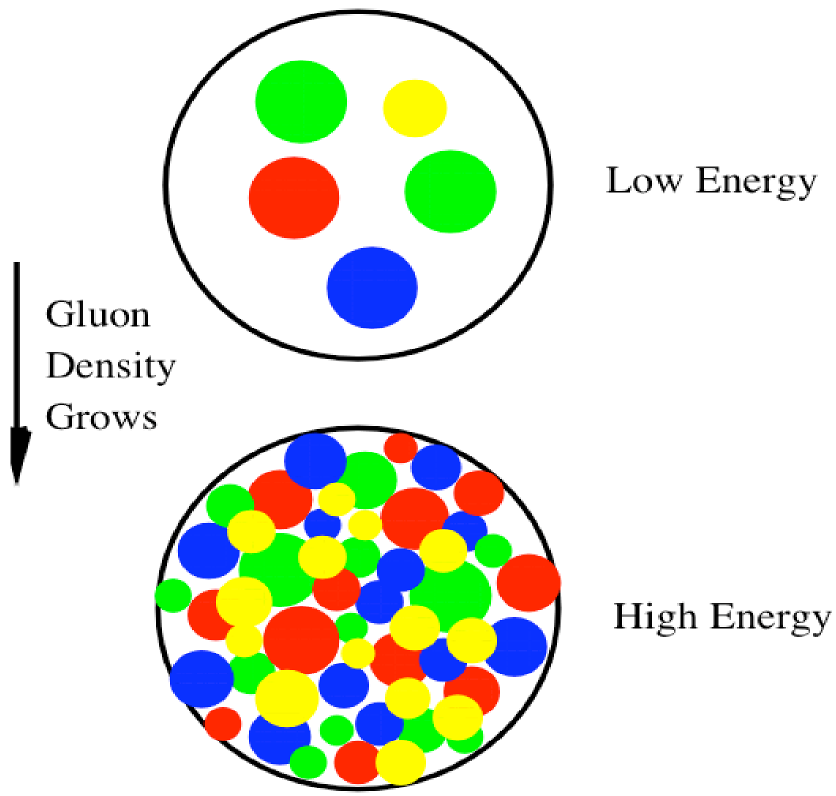


+ *higher twist (power suppressed) contributions...*

- Coefficient functions - C - computed to NNLO for many processes, e.g., gg -> H Harlander, Kilgore; Ravindran, Van Neerven, Smith; …
- Splitting functions -P - computed to 3-loops recently! Moch, Vermaseren, Vogt

BFKL evolution: Linear RG in x

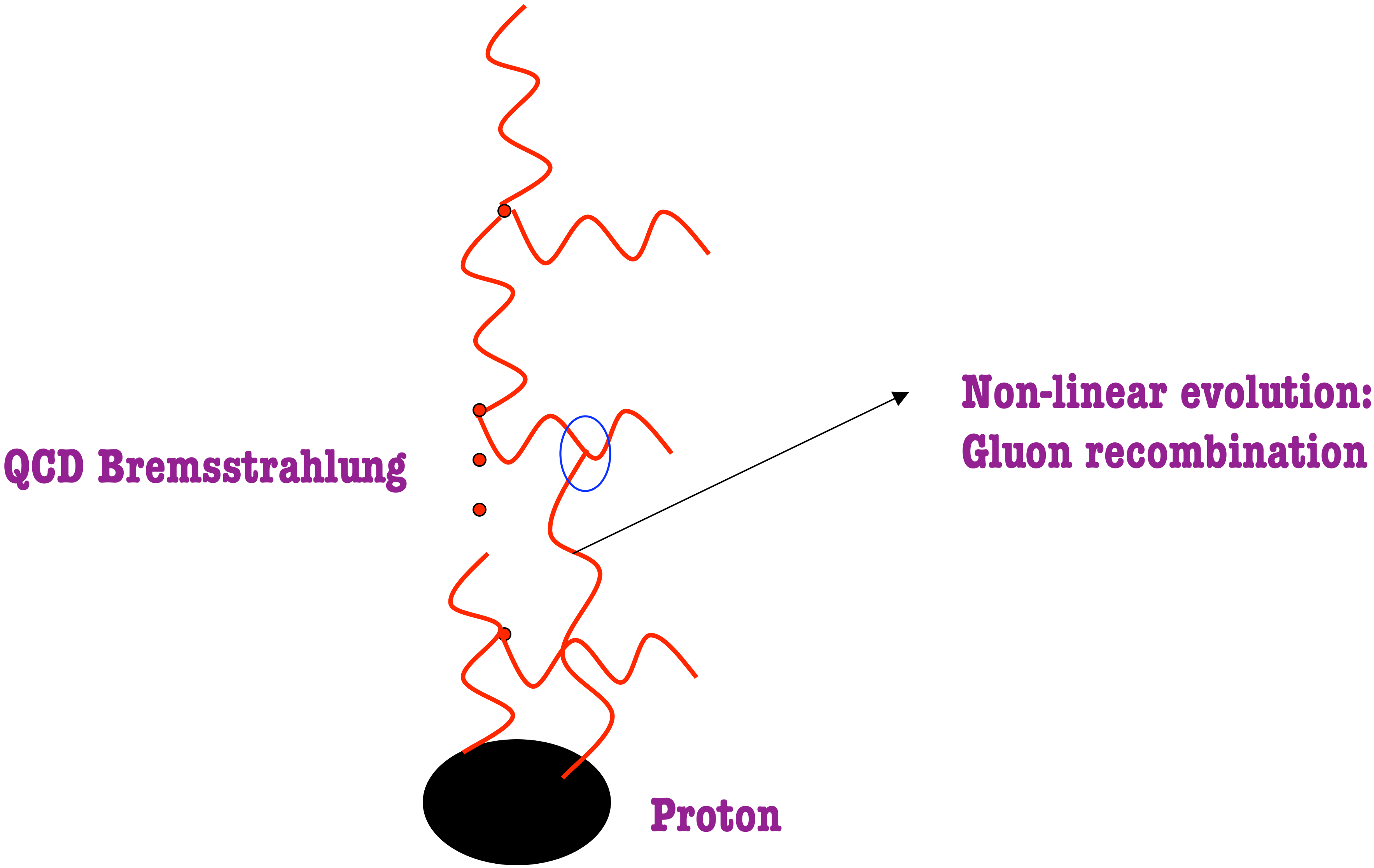
Balitsky-Fadin-Kuraev-Lipatov



- Large x

- Small x

Gluon density saturates at f= $\frac{1}{\alpha_S}$



Proton is a dense many body system at high energies

- Higher twists (power suppressed-in Q^2)

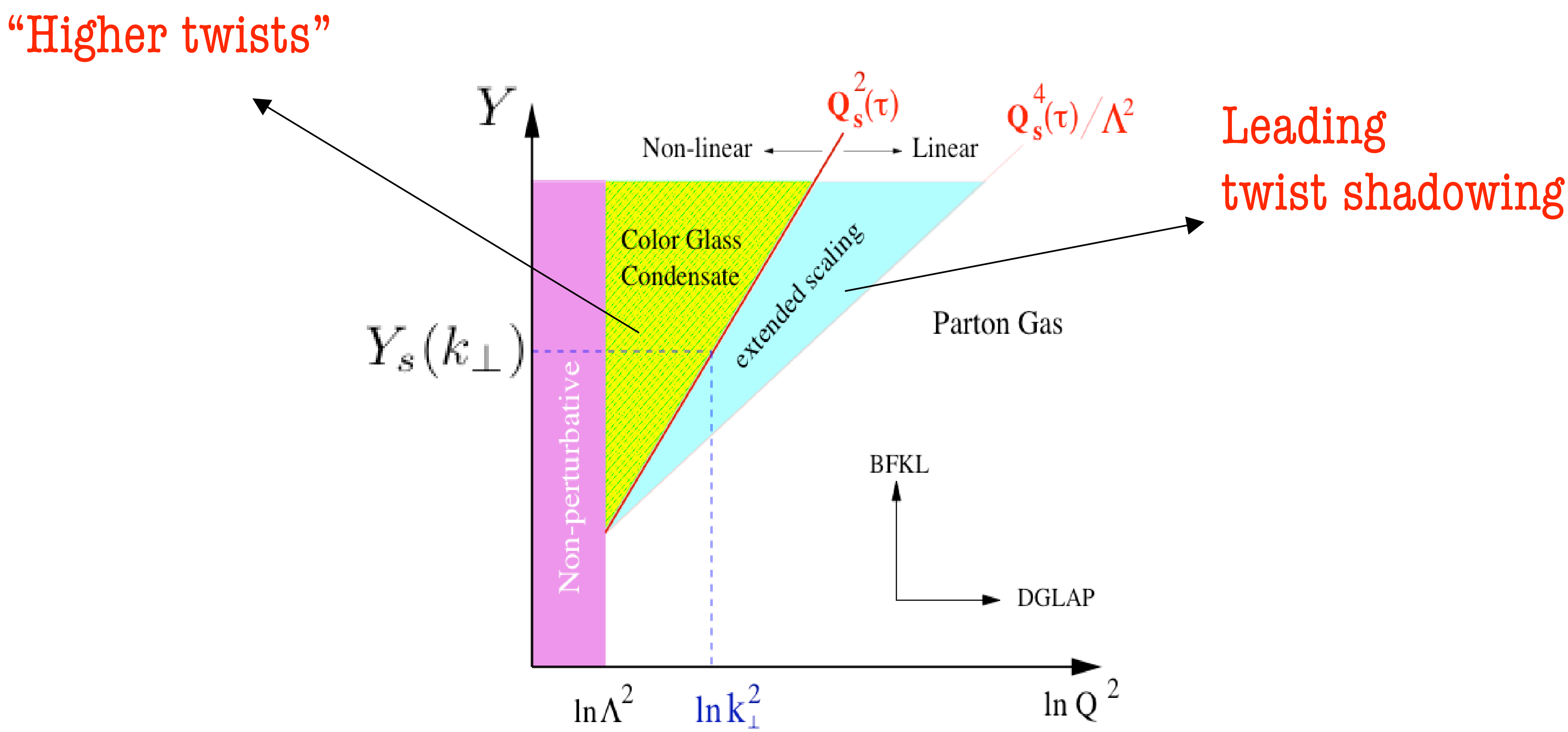
  are important when: $Q^2 \approx Q_s^2(x) >> \Lambda^2_{\mathrm{QCD}}$

- Leading twist "shadowing" of these contributions can

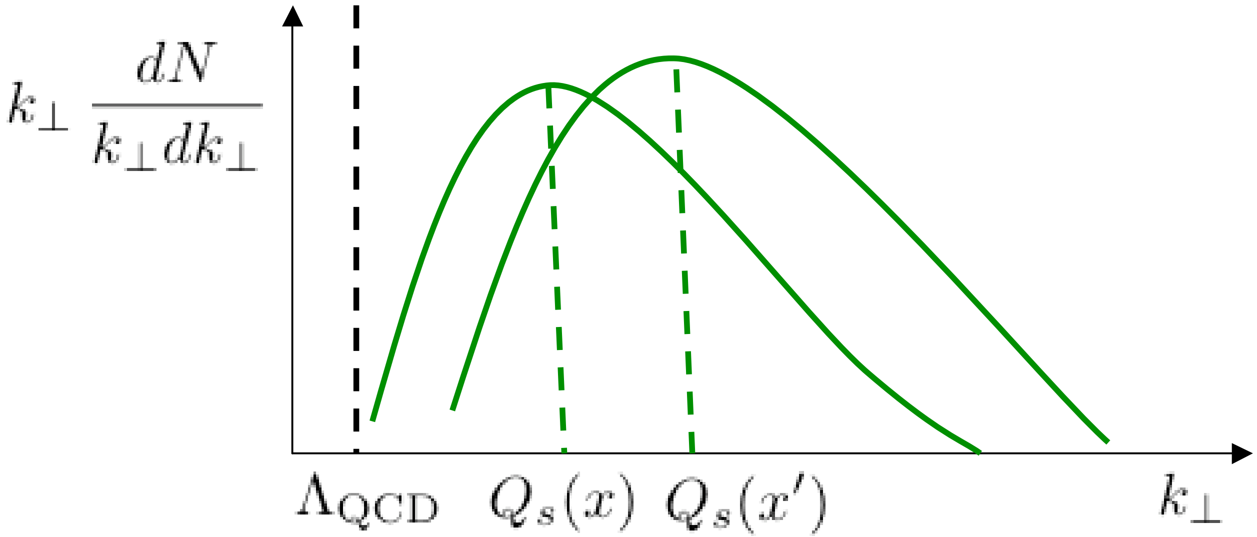
  extend up to $Q^2 >> Q_s^2(x)$ at small x.

Need a new organizing principle-
beyond the OPE- at small x.

Novel regime of QCD evolution at high energies



Hadron at high energies is a Color Glass Condensate



- ✓ *Dynamical degrees of freedom are colored gluons*
- ✓ *These are coupled to random light cone sources-time scales much larger than natural time scales-very similar to spin glasses*
- ✓ *Bosons with large occupation # ~* $\frac{1}{\alpha_S}$ *- form a condensate*
- ✓ *Typical momentum of gluons is* Q_s

Correlation Functions

JIMWLK => An infinite hierarchy of ordinary differential equations for gluon correlators $< A_1 A_2 \cdots A_n >_y$

$$< O[\alpha] >_Y = \int [d\alpha] O[\alpha] W_Y[\alpha]$$

Weight functional for random sources -Gaussian in MV

$$=> \frac{\partial}{\partial Y} < O[\alpha] >_Y = < \frac{1}{2} \int_{x,y} \frac{\delta}{\delta \alpha_Y^a(x)} \chi_{x,y}^{ab} \frac{\delta}{\delta \alpha_Y^b(y)} O[\alpha] >_Y$$

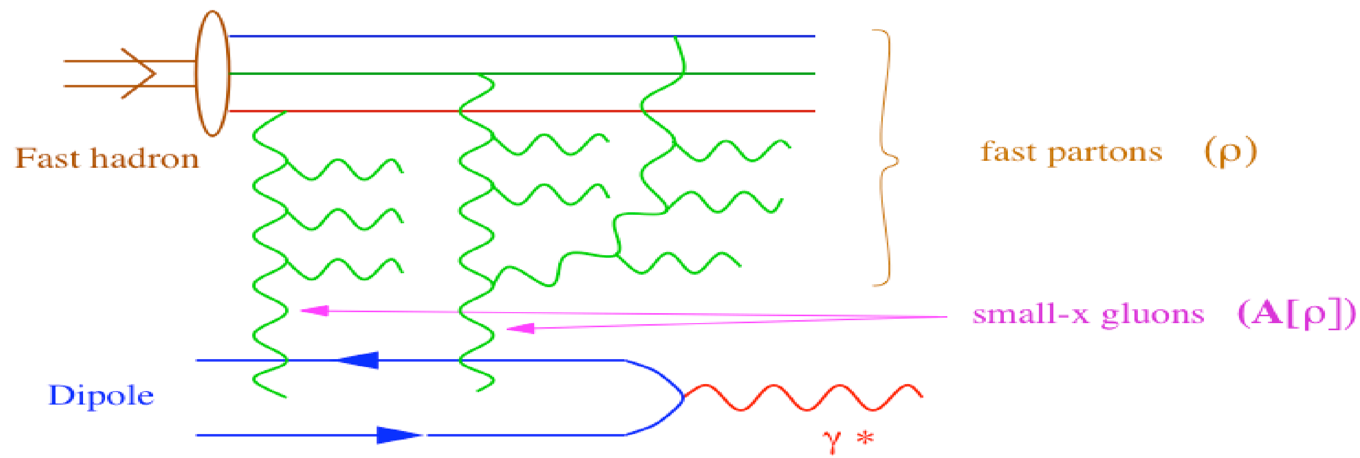
"time"

"diffusion coefficient"

For the gluon density $< \alpha(x_\perp) \alpha(y_\perp) >_Y$ for $g\,\alpha << 1$

Recover the BFKL equation in low density limit

DIS:



$$\sigma^{\gamma^* p}(x, Q^2) = \int_0^1 dz \int d^2 r |\psi(z, r; Q^2)|^2 \sigma_{\rm dipole}(x, r)$$

$$\text{where } \sigma_{\rm dipole}(x, r) = 2 \int d^2 b \, (1 - S(x, r, b))$$

IN CGC: $S(x, r, b) = \frac{1}{N_c} < {\rm Tr} V^\dagger(x) V(y) >_Y \equiv 1 - \mathcal{N}_Y(r, b)$

$$V^\dagger(x) = \mathcal{P} \exp\left(ig \int dx^- \alpha_a(x^-, x) T^a \right)$$

Models for dipole cross-section

Golec-Biernat-Wusthoff:

$$\sigma_{\mathrm{dipole}}(x, r_\perp) = \sigma_0 \left[1 - \exp\left(-r_\perp^2/4R_0^2(x)\right)\right]$$

Bartels-Golec-Biernat-Wusthoff:

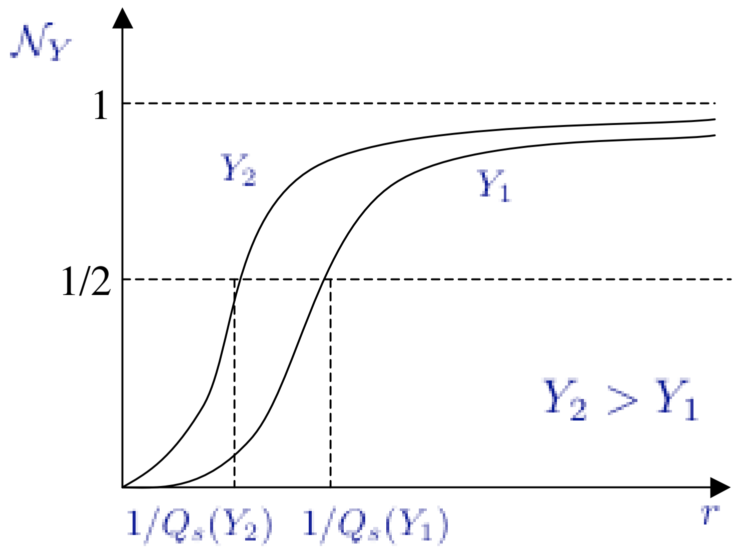
$$\sigma_{\mathrm{dipole}}(x, r_\perp) = \sigma_0 \left[1 - \exp\left(-\frac{\pi^2 r_\perp^2 \alpha_s(\mu^2)\, xG(x, \mu^2)}{3\,\sigma_0}\right)\right]$$

MV-Gaussian sources:

$$\sigma_{\mathrm{dipole}}(x, r_\perp) = \sigma_0 \left[1 - \exp\left(-Q_s^2\, r_\perp^2 \ln(1/r_\perp \Lambda)\right)\right]$$

The BK equation= Mean Field JIMWLK

$$\frac{\partial \mathcal{N}_Y(x,y)}{\partial Y} = \bar{\alpha}_s \int_z \frac{(x-y)^2}{(x-z)^2(y-z)^2} \{\underbrace{\mathcal{N}_Y(x,z) + \mathcal{N}_Y(z,y) - \mathcal{N}_Y(x,y)}_{\text{BFKL}} - \underbrace{\mathcal{N}_Y(x,z)\mathcal{N}_Y(z,y)}_{\text{Non-linear}}\}$$



- From saturation condition,

  $\mathcal{N} = 1/2$ when $r \sim 1/Q_s(Y)$ =>

  $$Q_s^2(Y) \approx Q_0^2\, e^{\lambda Y} \text{ with } \lambda \sim 4.8\,\alpha_s$$

- Many numerical/analytical studies for fixed and running coupling

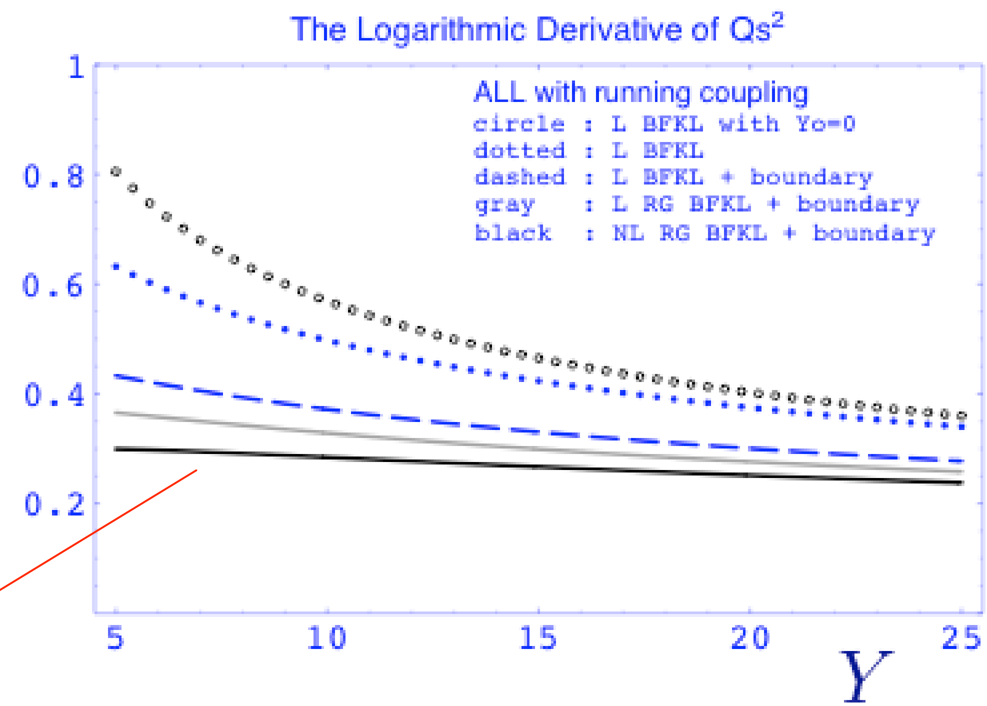
How does Q_s behave as function of Y?

Fixed coupling LO BFKL: $Q_s^2 = Q_0^2 e^{c\,\bar{\alpha}_s Y}$

LO BFKL+ running coupling: $Q_s^2 = \Lambda_{\rm QCD}^2 e^{\sqrt{2b_0 c(Y+Y_0)}}$

Re-summed NLO BFKL + CGC:

$$\lambda \equiv \frac{d \ln Q_s^2}{dY}$$

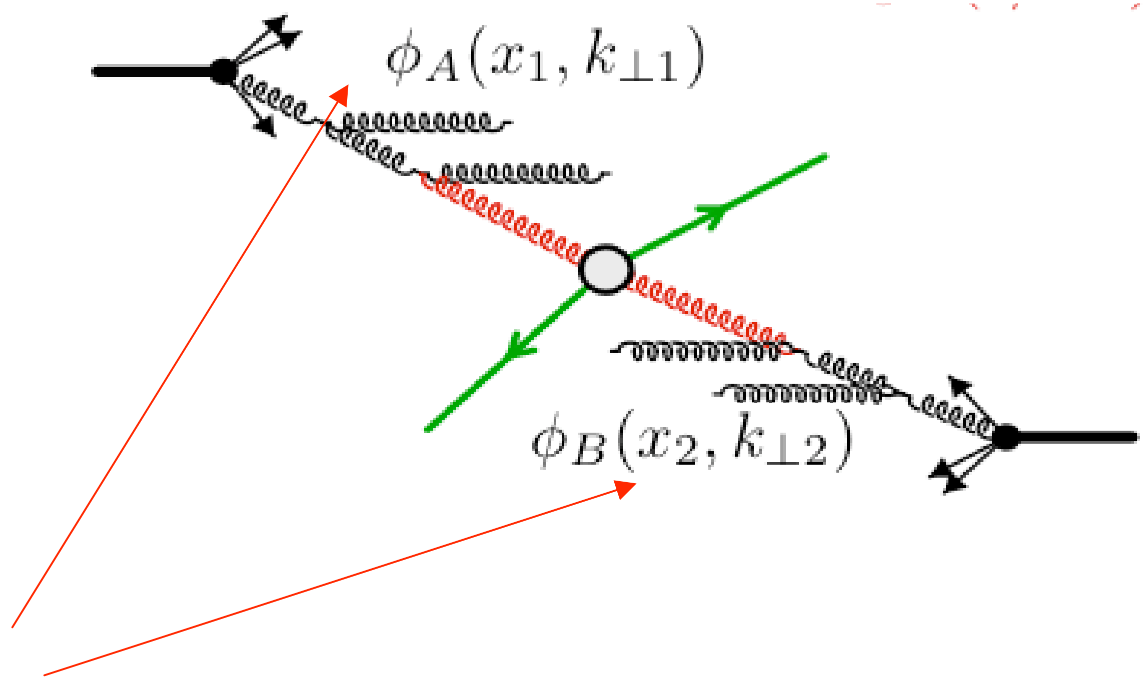


Triantafyllopoulos

Very close to G.B-W fit!

k_t factorization:

$\Lambda_{QCD} << M << \sqrt{s}$



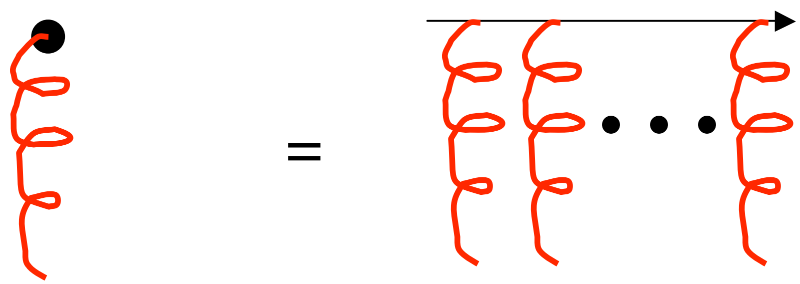
Are these "un-integrated gluon distributions" universal?

"Dipoles"-with evolution a la JIMWLK / BK

Virtual photon production in forward p-p

Kopeliovich, Raufeisen, Tarasov
Gelis, Jalilian-Marian

Scattering off classical field:

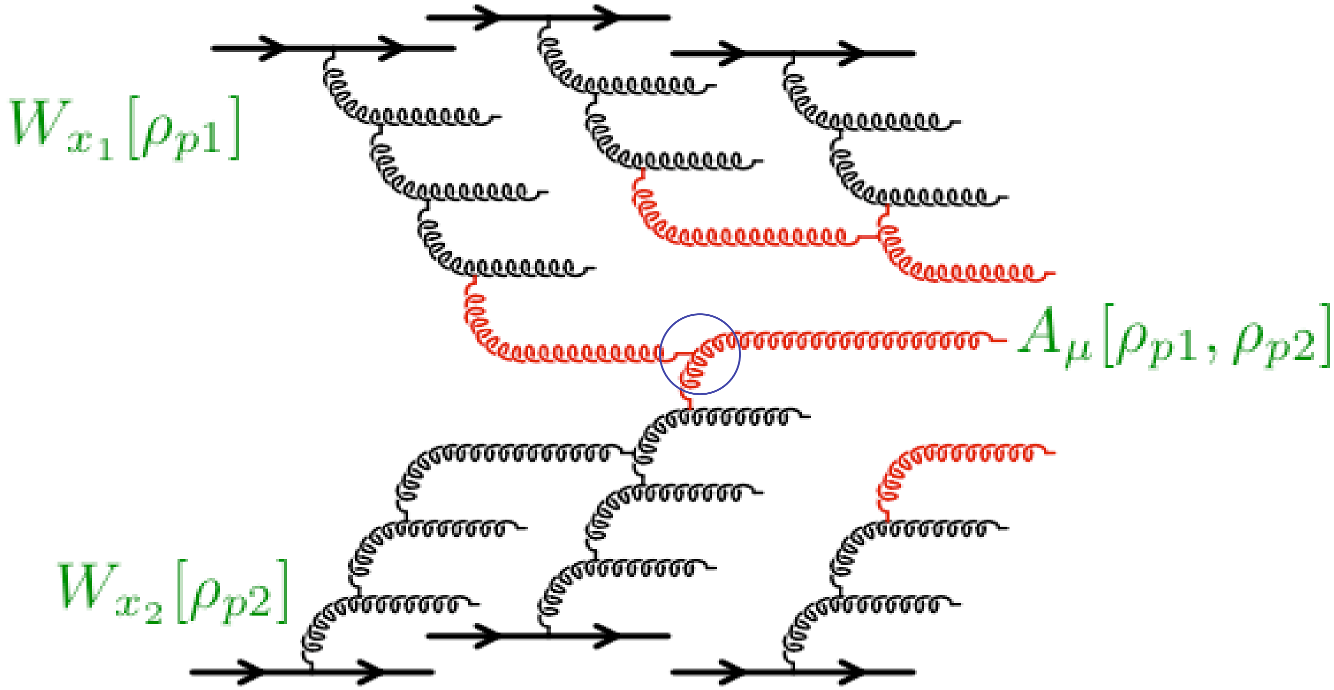


$= O(1)$

$$\sigma_{\gamma^*} = \int_0^1 dz \int d^2 r_\perp \, |\phi|^2 \, \sigma_{\text{dipole}}(r_\perp)$$

Same dipole correlator as in e-p

HADRONIC COLLISIONS IN THE CGC FRAMEWORK



Solve Yang-Mills equations for two light cone sources: ρ_{p1} & ρ_{p2}

For observables $O(A_\mu(\rho_{p1}, \rho_{p2}))$ average over $W_{x1}[\rho_{p1}]$ & $W[\rho_{p2}]$

Systematic power counting for scattering in the CGC

- Gluon & quark production to lowest order in sources (the dilute/pp case).

$$(\rho_{p1}/k_\perp^2,\ \rho_{p2}/k_\perp^2 << 1)$$

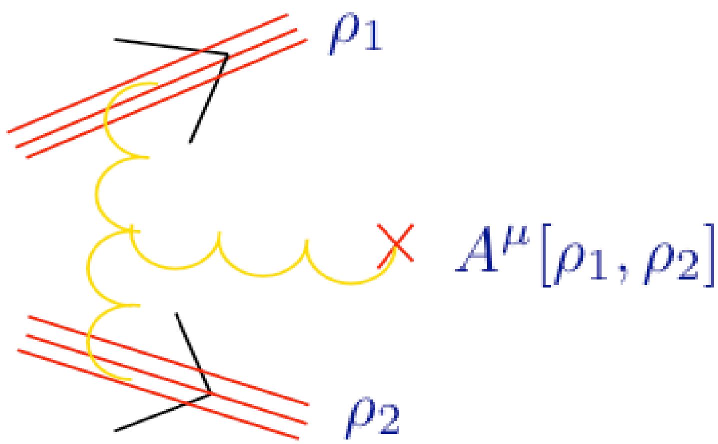
- Gluon & quark production to lowest order in one source & all orders in the other (the semi-dense/forward p-p case).

$$(\rho_p/k_\perp^2 << 1,\ \rho_A/k_\perp^2 \sim 1)$$

- Gluon & quark production to all orders in both sources (the dense/high energy pp case)

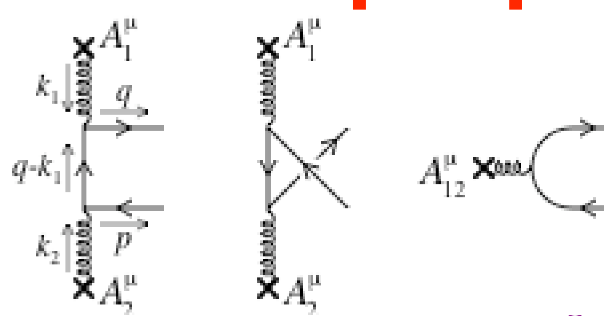
$$(\rho_{A1}/k_\perp^2 \sim 1, \rho_{A2}/k_\perp^2 \sim 1)$$

Inclusive gluon production in hadronic collisions to lowest order in ρ_1 & ρ_2 and α_S expressed in k_t factorized form.



This diagram in $A^\tau = 0$ gauge is equivalent to sum of all Bremsstrahlung diagrams in covariant gauge.

Inclusive pair production in CGC framework



$A^\mu_{1.2}[\rho_1, \rho_2] \propto O(\rho_1 \rho_2)$

Abelian

Non-Abelian vertex here is the Lipatov vertex

$$\frac{d\sigma}{dy_p dy_q d^2p_\perp d^2q_\perp} = \frac{1}{(2\pi)^6}\frac{1}{(N_c^2-1)^2}\int \frac{d^2k_{1\perp}}{(2\pi)^2}\frac{d^2k_{2\perp}^2}{(2\pi)^2}\delta(\vec{k}_{1\perp}+\vec{k}_{2\perp}-\vec{p}_\perp-\vec{q}_\perp)$$
$$\times\phi_1(k_{1\perp})\phi_2(k_{2\perp})\frac{\mathrm{Tr}\left(|m_{ab}^{-+}(k_1,k_2;q,p)|^2\right)}{k_{1\perp}^2 k_{2\perp}^2}$$

$|m_{ab}^{-+}(k_1,k_2;q,p)|^2$ *is identical to Collins & Ellis' k_t factorization result*

$$\frac{d\phi_1(k_{1\perp},x_\perp)}{d^2x_\perp} = \frac{\pi g^2}{k_\perp^2}\int d^2r_\perp e^{-i\vec{k}_\perp\cdot\vec{r}_\perp} < \rho_a(x_\perp+\frac{r_\perp}{2})\rho_a(x_\perp-\frac{r_\perp}{2}) >_\rho$$

is the un-integrated gluon distribution in the Gaussian MV-model

$$\frac{\mathrm{Tr}\left(|m_{ab}^{-+}(k_1,k_2;q,p)|^2\right)}{k_{1\perp}^2 k_{2\perp}^2}$$

is well defined in the collinear limit of $|k_{1\perp}|,|k_{2\perp}|\to 0$

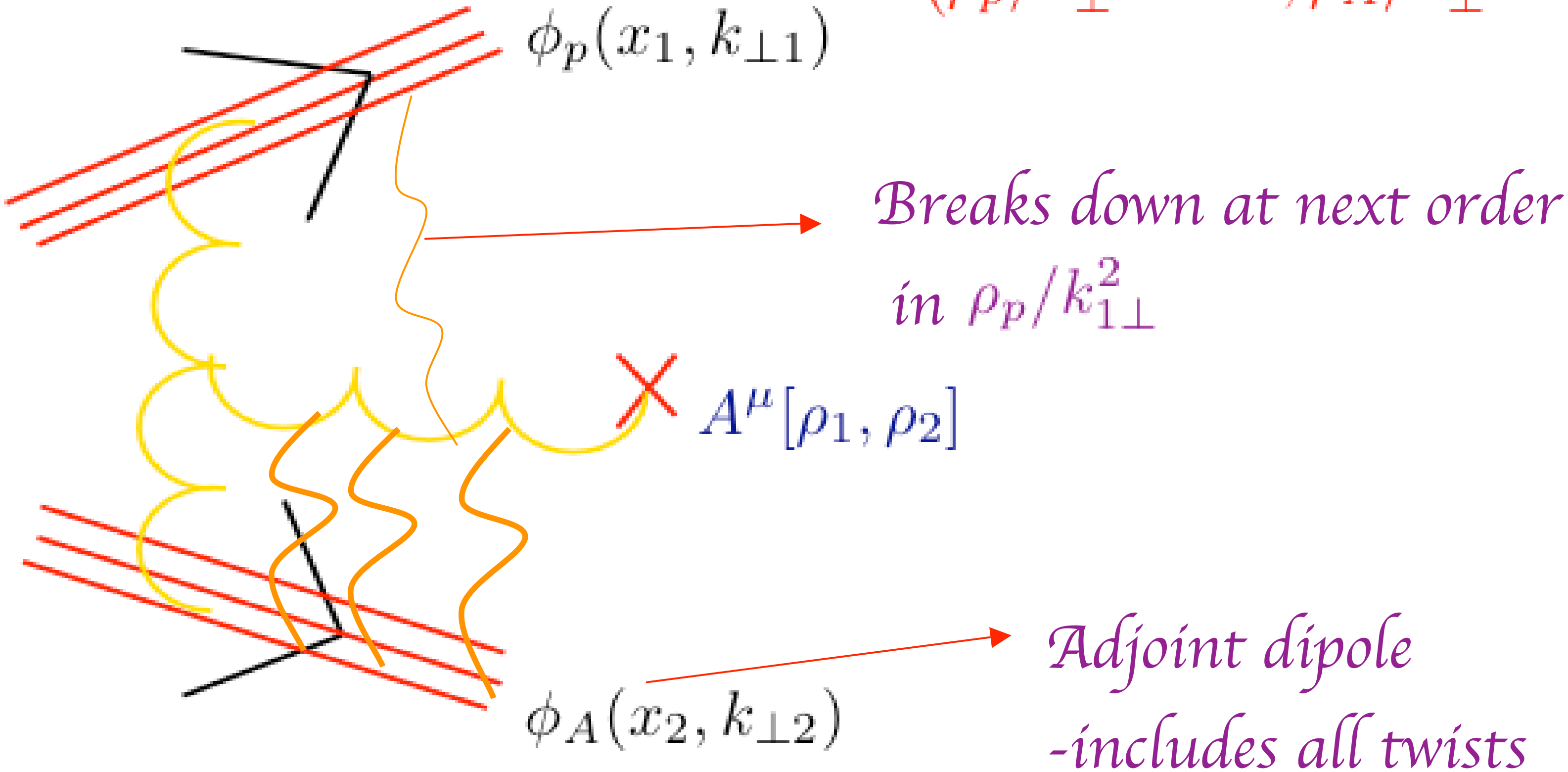
↓

$|M|^2_{gg\to q\bar{q}}$ *after integration over azimuthal angles*

Recover lowest order collinear factorization result

Systematic power counting-inclusive gluon production

$(\rho_p/k_\perp^2 << 1, \rho_A/k_\perp^2 \sim 1)$



- K_t holds for inclusive gluon production lowest order in $\rho_p/k_{\perp 1}^2$ but all orders in $\rho_A/k_{\perp 2}^2$

☆ Result for gluon multiplicity in forward pp

$$N_g = \frac{4g^2 N_c}{\pi^2 (N_c^2 - 1) q_\perp^2} \int \frac{d^3q}{(2\pi)^3 2E_q} \frac{d^2k_\perp}{(2\pi)^2} \int d^2x_\perp \frac{d\phi_p(k_\perp, x_\perp)}{d^2x_\perp} \frac{d\phi_A(q_\perp - k_\perp, x_\perp - b)}{d^2x_\perp}$$

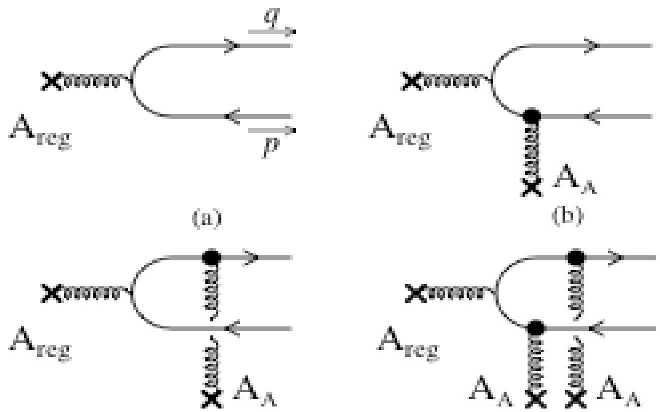
Result is k_t factorized into product of "collinear" and "all-twist" un-integrated distributions.

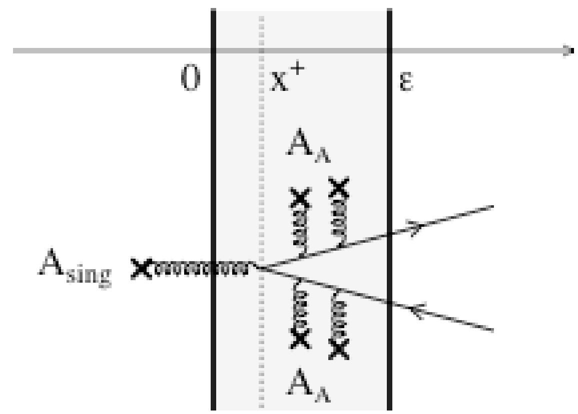
$$\phi_A(k_\perp, x_\perp) \propto < U^\dagger_{ab} U_{bc} >_{\rho_A}$$

-Is non-linear, contains gluon density to all orders- proportional to un-integrated gluon density at large k_t

Note: for forward hadron production–from valence partons, "fundamental" dipole - opposed to "adjoint" dipole here

Quark production to all orders in forward pp





$$\frac{d\sigma^{pA\to q\bar{q}X}}{dy_p dy_A d^2p_\perp q_\perp} \propto \phi_p \times [A\phi_{g,g} + (B\phi_{g;q\bar{q}} + c.c) + C\phi_{q\bar{q};q\bar{q}}]$$

Two point-dipole operator in target

3- & 4- point "multipole" operators

More non-trivial evolution with rapidity...

$$\frac{d\sigma^{pA\to q\bar{q}X}}{dy_p dy_A d^2p_\perp q_\perp} \propto \phi_p \times [A\phi_{g,g} + (B\phi_{g;q\bar{q}} + c.c) + C\phi_{q\bar{q};q\bar{q}}]$$

$$< U_A(x_\perp)U_A^\dagger(y_\perp) >$$

$$< U_F(x_\perp)\tau^a U_F^\dagger(y_\perp)U_F(y'_\perp)\tau^b U_F(x'_\perp) >$$

$$< U_F(x_\perp)\tau^a U_F^\dagger(y_\perp)\tau^{b'}(U_A^{a'b'})^\dagger(y'_\perp) >$$

- *RG evolution given by JIMWLK equations-can be tested at LHC*

- *Numerical methods exist to compute n-point correlators for more exclusive final states.*

Outlook: The demise of the Structure function ?

- Dipoles (and multipole) operators may be more relevant observables at high energies-depend on k_t & impact parameter

- Are universal-process independent.

- RG running of these operators - detailed tests of high energy QCD.