EW Fits in Models with Triplet Higgs — Fitting Safely New Models with EW Precision Data

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Introduction

- Most extension of the SM contains extended gauge sector and/or extended Higgs sector

- These new models are severely constrained by precision EW data

- Most analyses utilize SM-like fit with 3 input data points

Models with $\rho = 1$ at tree level:

- MSSM
- models with singlets or doublet Higgses
- models with additional families of fermions

→ SM-like fit with 3 input parameters ok.

Models with $\rho \neq 1$ at tree level:

- SM with triplet Higgs (Blanck & Hollik, 1998)
- LR symmetric model (Czakon et al., 1999)
- * littlest Higgs model (MCC & S. Dawson, 2003)

→ need additional input parameter

SM with an $SU(2)_L$ triplet Higgs (Blank & Hollik, 1998)

SM: $(g, g', v) \to (G_\mu, M_Z, \alpha)$ $\quad \rho = 1 = \dfrac{M_W^2}{M_Z^2 C_\theta^2}$

SM + $SU(2)_L$ triplet Higgs:

$(g, g', v, v') \to (G_\mu, M_Z, \alpha, S_\theta^2)$ $\quad \rho \neq 1$

relation bt M_W & M_Z

Fix M_W^2 using μ-decay:

$$\sqrt{2}\, G_\mu = \frac{\pi \alpha}{M_W^2 S_\theta^2}(1 + \Delta r)$$

$$\Delta r = -\frac{\delta G_\mu}{G_\mu} - \frac{\delta M_W^2}{M_W^2} + \frac{\delta \alpha}{\alpha} - \frac{\delta S_\theta^2}{S_\theta^2}$$

$$= \underbrace{\frac{1}{M_W^2}\left(\Pi^{WW}(0) - \Pi^{WW}(M_W^2)\right)}_{\log.} + \underbrace{\Pi^{\gamma\gamma}(0)'}_{\log.} - \frac{\delta S_\theta^2}{S_\theta^2}$$

top-loop contributions:

$$\frac{1}{M_W^2}\Pi^{WW}(0) \to \frac{\sqrt{2}\, G_\mu}{16\pi^2}(3 m_t^2)\cdot\left(1 + 2 \ln \frac{Q^2}{m_t^2}\right) + \cdots$$

$$\frac{1}{M_W^2}\Pi^{WW}(M_W^2) \to \frac{\sqrt{2}\, G_\mu}{16\pi^2}(3 m_t^2)\cdot\left(1 + 2 \ln \frac{Q^2}{m_t^2}\right) + \cdots$$

$$\Pi^{\gamma\gamma}(0)' \to \ln \frac{m_t^2}{Q^2}$$

renormalization conditions:

① $\mathrm{Re}\,\hat{\Sigma}_T^{W}(M_W^2) = \mathrm{Re}\,\hat{\Sigma}_T^{ZZ}(M_Z^2) = \mathrm{Re}\,\hat{\Sigma}_T^{AZ}(M_Z^2) = 0$

② $\hat{\Sigma}_T^{AZ}(0) = \hat{\Sigma}_T^{AA}(0) = 0$

③ $$\mathrm{Re}\,\frac{\partial \hat{\Sigma}_T^{ZZ}(k^2)}{\partial k^2}\bigg|_{k^2=M_Z^2} = \mathrm{Re}\,\frac{\partial \hat{\Sigma}_T^{AA}(k^2)}{\partial k^2}\bigg|_{k^2=0} = \mathrm{Re}\,\frac{\partial \hat{\Sigma}_T^{W}(k^2)}{\partial k^2}\bigg|_{k^2=M}$$

$$= 0$$

$$\Rightarrow \quad \delta m_Z^2 = \mathrm{Re}\,\{\Pi^{ZZ}(M_Z^2)\}$$

$$\frac{\delta\alpha}{\alpha} = 2\,\frac{\delta e}{e} = \Pi^{\gamma\gamma\prime}(0) + 2\,\frac{g_V^e - g_A^e}{Q_e}\,\frac{\Pi^{\gamma Z}(0)}{M_Z^2}$$

$$\frac{\delta s_\theta^{\ell\,2}}{s_\theta^{\ell\,2}} = \mathrm{Re}\left\{\left(\frac{c_\theta}{s_\theta}\right)\left[\frac{\Pi^{\gamma Z}(M_Z^2)}{M_Z^2}\right]\right\} + \cdots$$

$$\delta G_\mu = -\,\frac{\Pi^{WW}(0)}{M_W^2} + \delta_{VB}$$

$\Pi^{XY} \equiv$ 

$$\rho = 1 = \frac{M_W^2}{M_Z^2 C_\theta^2}$$

SM:

$$\frac{\delta S_\theta^2}{S_\theta^2} = \frac{C_\theta^2}{S_\theta^2}\left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right] = \frac{C_\theta^2}{S_\theta^2}\left[\frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi^{WW}(M_W^2)}{M_W^2}\right]$$

$$\frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2} \rightarrow \frac{2\sqrt{2}G_\mu}{16\pi^2}\frac{1}{C_\theta^2}(3m_t^2)\ln\frac{Q^2}{m_t^2}$$

$$\Rightarrow \frac{\delta S_\theta^2}{S_\theta^2} \sim m_t^2$$

SM + triplet Higgs:

$$\frac{\delta S_\theta^2}{S_\theta^2} = \frac{C_\theta}{S_\theta}\frac{\Sigma^{\gamma Z}(M_Z^2)}{M_Z^2} \sim \ln\frac{m_t^2}{Q^2} \quad !$$

Consistent renormalization scheme important.

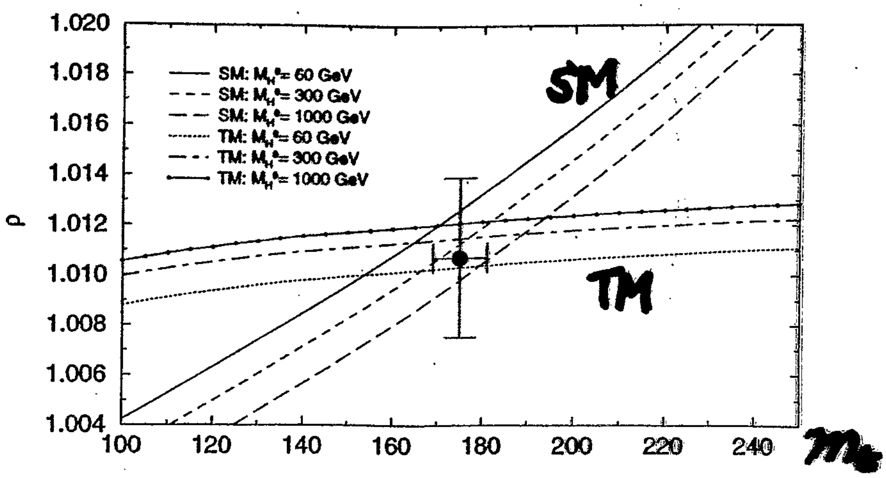
SM + one SU(2) triplet Higgs (Blank & Hollik 1998):

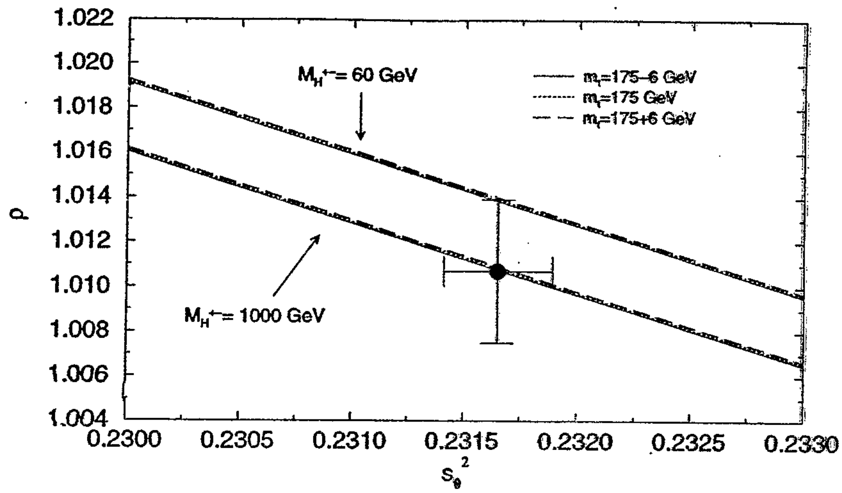
SM:

$\Delta\rho_t \sim m_t^2$

TM:

$\Delta\rho_t \sim \ln m_t^2$





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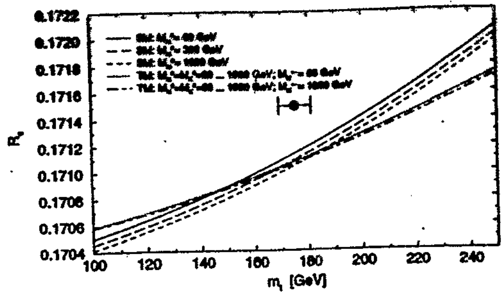
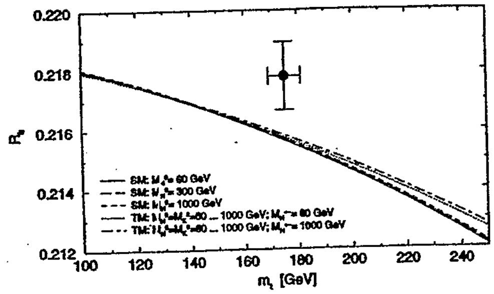


Figure 5.8: Top mass dependence of R_c in the SM and the TM for various Higgs masses. The error bar of R_c covers the full vertical axis.

Rb



mt

Figure 5.9: Top mass dependence of R_b in the SM and the TM for various Higgs masses.

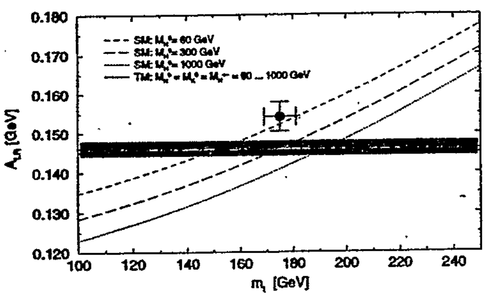


Figure 5.10: Left/Right asymmetry in the SM and the TM. The shaded area corresponds to a variation of $s_\theta{}^2 = 0.23165 \pm 0.00024$.

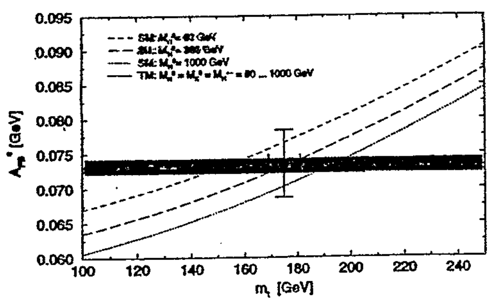
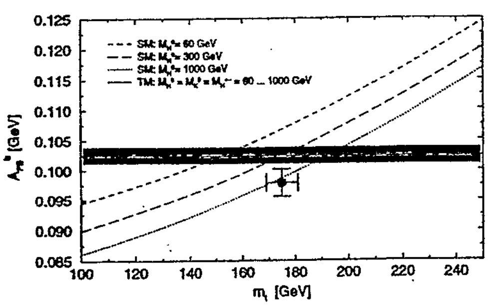


Figure 5.11: Forward/backward asymmetry for charm quarks in the SM and the TM. The shaded area corresponds to a variation of $s_\theta{}^2 = 0.23165 \pm 0.00024$.

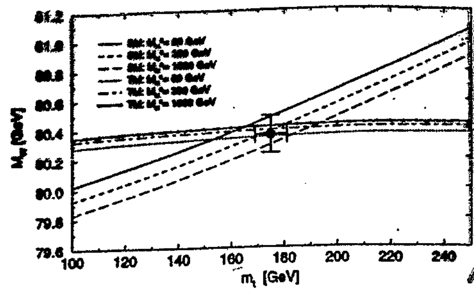
A_{FB}^b



mt

Figure 5.12: Forward/backward asymmetry for bottom quarks in the SM and the TM. The shaded area corresponds to a variation of $s_\theta{}^2 = 0.23165 \pm 0.00024$.

Mw



SM

TH

Mt

Figure 5.1: Top mass dependence of M_W in the SM and the TM for various doublet Higgs masses M_{H^0}. The input values for the TM Higgs masses M_{K^0} and $M_{H^\pm}$ are 300 GeV.

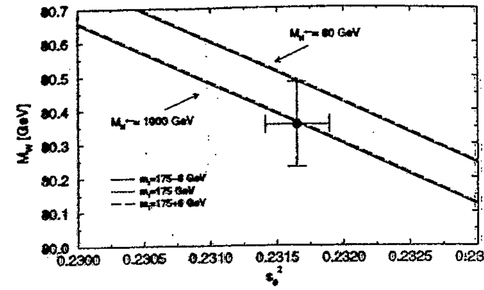


Figure 5.2: Dependence of M_W on the input parameter $s_\theta{}^2$ for various values of m_t and $M_{H^\pm}$ in the TM. The masses for the neutral Higgs bosons are fixed at 300 GeV.

Γz

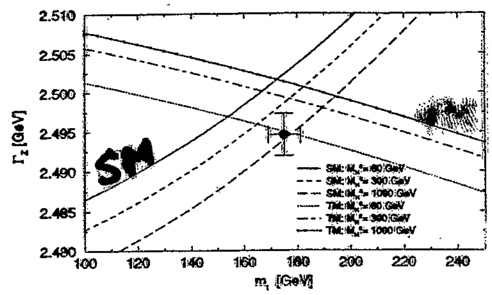


Figure 5.5: Top mass dependence of the total Z width in the SM and the TM for various doublet Higgs masses M_{H^0}. The input values for the TM Higgs masses M_{K^0} and $M_{H^\pm}$ are 300 GeV.

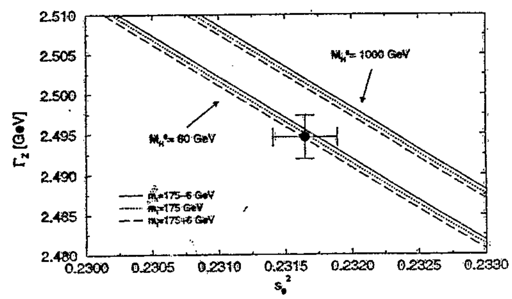


Figure 5.6: Dependence of the total Z width on the input parameter $s_\theta{}^2$ for various values of m_t and M_{H^0}. The masses of the triplet Higgs bosons are fixed at 300 GeV.

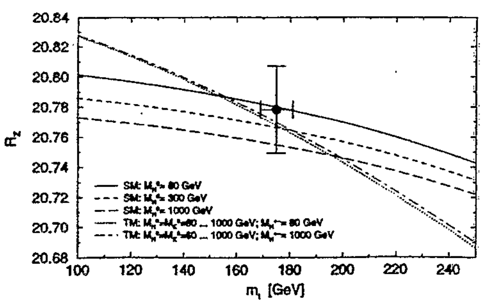


Figure 5.7: Top mass dependence of R_Z in the SM and the TM for various Higgs masses.

Minimal Left-Right Symmetric Model $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

(Czakon, Gluza, Zralek, Jegerlehner, 1999)

Minimal Model:

$$\Phi \sim (\tfrac{1}{2}, \tfrac{1}{2}, 0) \sim \begin{pmatrix} \kappa & \\ & \kappa' \end{pmatrix} \quad : \quad \text{EWSB}$$

$$\Delta_L \sim (1, 0, 2) \sim \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} \quad \equiv \quad 0$$

$$\Delta_R \sim (0, 1, 2) \sim \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \quad : \quad SU(2)_R \times U(1)_{B-L} \to U(1)_Y$$

$$(g, g', \kappa, \kappa', v_R) \to (e, M_{W_1}, M_{W_2}, M_{Z_1}, M_{Z_2})$$

$$\frac{\delta s_\theta^2}{s_\theta^2} \sim 2\frac{c_\theta^2}{s_\theta^2} \frac{(\delta M_{Z_1}^2 + \delta M_{Z_2}^2) - (\delta M_{W_1}^2 + \delta M_{W_2}^2)}{(M_{Z_1}^2 + M_{Z_2}^2) - (M_{W_1}^2 + M_{W_2}^2)} + \cdots$$

$$\sim -2\frac{c_\theta^2}{s_\theta^2}(c_\theta^2 - s_\theta^2)\frac{\delta M_{W_1}^2}{M_{W_2}^2 - M_{W_1}^2} + \cdots$$

$$\longrightarrow \frac{\sqrt{2}\, G_\mu}{8\pi^2} c_\theta^2 \left(\frac{c_\theta^2}{s_\theta^2} - 1\right)\left(\frac{M_{W_1}^2}{M_{W_2}^2 - M_{W_1}^2}\right)(3m_t^2)$$

For $M_{W_2} \gtrsim 400$ GeV, smaller than SM log. term.

$M_{W_2}^2 \to \infty$ limit: 4 inputs v.s. 3 inputs

Introduction: Littlest Higgs Model

- Electroweak precision constraints ⇒ SM Higgs has to be light
- To stabilize m_H needs new states ~ few TeV to cancel the quadratic divergences
- Little Higgs models — an alternative to SUSY as a solution to the gauge hierarchy problem
- consider minimal realization of this idea: the Littlest Higgs model - a non-linear σ model based on $SU(5)/SO(5)$

THE MODEL

- Global symmetry : $SU(5) \xrightarrow{\langle\Sigma\rangle} SO(5)$

$$\langle\Sigma\rangle \equiv \Sigma_0 = \begin{pmatrix} & & \mathbb{1}_{2\times2} \\ & 1 & \\ \mathbb{1}_{2\times2} & & \end{pmatrix}$$

$$\Sigma = e^{2i\frac{\Pi}{f}}\,\Sigma_0$$

- Reduce the global symmetry by gauging some part of it $\Rightarrow$ pseudo-Goldstone

gauged subgroup:

$$[SU(2)\times U(1)]_1 \times [SU(2)\times U(1)]_2$$

$$\xrightarrow{\quad\Sigma_0\quad} [SU(2)\times U(1)]_{SM}$$

- turning off either copy of $[SU(2)\times U(1)]_{1,2}$ restores some global symmetry $SU(3)_{1,2}$ which forbids m_H^2

- Collective symm. breaking $\Rightarrow$ m_H^2 at 2-loop

- Goldstone bosons:

$$24 - 10 = 14 = 4 \oplus 10$$

$$= \underbrace{1_0 \oplus 3_0}_{\substack{\text{long. comp.}\\ \text{of } Z_H,\, W_H,\, A_H}} \oplus \underbrace{2_{\pm 1/2}}_{\substack{\text{SM}\\ \text{doublet}\\ h}} \oplus \underbrace{3_{\pm 1}}_{\substack{\text{triplet}\\ \phi}}$$

- Quadratic divergences cancelled at one-loop by new states:

$W, Z, B \leftrightarrow W_H, Z_H, B_H$

$t \leftrightarrow T$

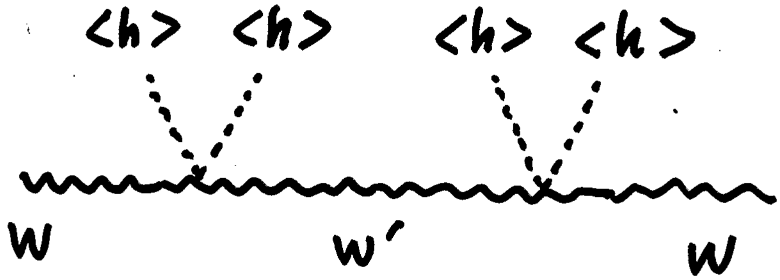
$H \leftrightarrow \phi$

cancellation between states with same spin statistics

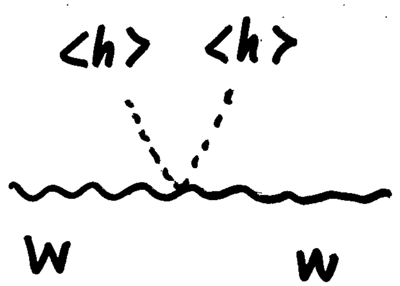
collective symm. breaking ensures relations between coupling consts.

- opposite signs
- equality bt coupling consts.

Due to coupling to heavy particles W', Z', Φ, T :



$$M_W^2 \sim \underset{\substack{\uparrow \\ \text{SM}}}{m_W^2} \left(1 + \frac{v^2}{f^2} \cdots\cdots\right)$$



m_W^2

SM

Electroweak Precision Constraints in Littlest Higgs Model

(M.-C.C. & S. Dawson, Phys. Rev. D70, 015003 (2004))

- naturalness requires $f \simeq (1-2)$ TeV ($\Lambda \sim 4\pi f$)
- existing tree level analyses (3 inputs even for $v' \neq 0$!!)

  $\Rightarrow$ $f > (3-4)$ TeV

- One-loop contributions important

  ① tree-level corrections (higher order terms in Chiral perturbation theory) $\sim \frac{v^2}{f^2}$

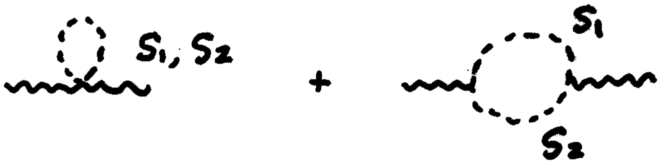
  one-loop radiative corrections $\sim \frac{1}{16\pi^2}(1)$

  For $f \sim$ few TeV $\Rightarrow \frac{v^2}{f^2} \sim \frac{1}{16\pi^2}$

  $\Rightarrow$ we found cancellations between tree-level and one-loop corrections

  $\Rightarrow$ a low cutoff with $f \sim 2$ TeV is still allowed

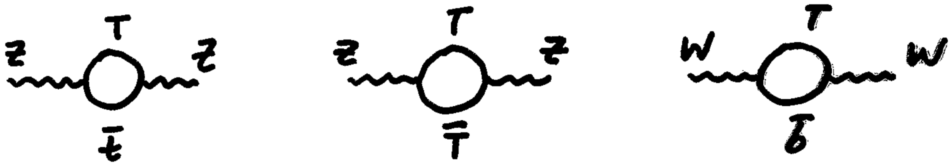
② Heavy scalar fields do NOT decouple !!



$\sim (m_1^2 - m_2^2)\left(1 + \ln \frac{Q^2}{m^2}\right)$

unless both scalar fields have degenerate masses, the scalar contributions grow with $\Delta m_{21}^2 \sim f^2$

③ Heavy top T contributes ~ log.



Renormalization

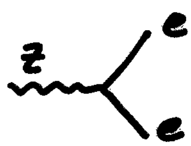
- a valid renormalization scheme requires 4 input data points : $\rho = M_W^2 / M_Z^2 C_\theta^2$

$$\begin{array}{lll} SM: & (g, g', v) & \rho = 1 \\ LH: & (g, g', v, v') & \rho \neq 1, \text{ relations bt } M_W^2 \text{ \& } M_Z^2 \end{array}$$

Trade $(G_\mu, \alpha(M_Z), M_Z, S_\theta^2)$ for (g, g', v, v')

LEP definition :

$$4 S_\theta^2 - 1 \equiv \frac{\mathrm{Re}(g_V^e)}{\mathrm{Re}(g_A^e)} \quad ,$$



$: (g_V + \gamma_5 g_A)$

$$= 4 S_W^2 + \frac{v^2}{f^2}(\cdots\cdots)$$

$$LH : \qquad S_W^2 = \frac{g'^2}{g'^2 + g^2}$$

$$S_\theta^2 + \Delta S_\theta^2 = S_W^2$$

$$\Delta S_\theta^2 = - \frac{1}{2\sqrt{2}\, G_\mu f^2} \Big[S_\theta^2 c^2 (c^2 - s^2) - C_\theta^2 (c'^2 - s'^2)(-2 + 5c'^2) \Big]$$

Fix M_W^2 using μ-decay:

$$G_\mu = \frac{\pi \alpha}{\sqrt{2}\, M_W^2\, S_\theta^2} \left[1 + \Delta r_{tree} + \Delta r' \right]$$

where

$$\Delta r_{tree} = - \frac{\Delta S_\theta^2}{S_\theta^2} + \frac{c^2 s^2}{\sqrt{2}\, G_\mu f^2}$$

$$\Delta r' = - \frac{\delta G_\mu}{G_\mu} - \frac{\delta M_W^2}{M_W^2} + \frac{\delta \alpha}{\alpha} - \frac{\delta S_\theta^2}{S_\theta^2}$$

$$= \frac{1}{M_W^2} \left(\Pi^{WW}(0) - \Pi^{WW}(M_W^2) \right) + \Pi^{\gamma\gamma\prime}(0)$$

additional contributions from new heavy particles

$$- \frac{C_\theta}{S_\theta} \frac{\Pi^{\gamma Z}(M_Z^2)}{M_Z^2}$$

$\leftarrow v' \neq 0$

fermion contributions log.

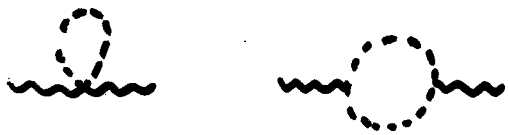
SM:

$$G_\mu = \frac{\pi \alpha}{\sqrt{2}\, M_W^2\, S_\theta^2} \left[1 + \Delta r \right]$$

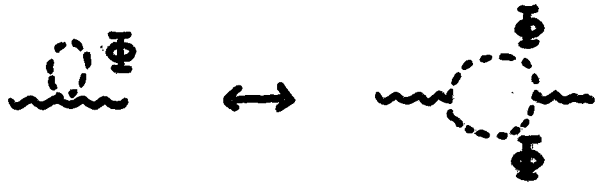
$$\Delta r = \frac{1}{M_W^2} \left(\Pi^{WW}(0) - \Pi^{WW}(M_W^2) \right) + \Pi^{\gamma\gamma\prime}(0)$$

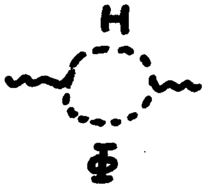
$$- \frac{C_\theta^2}{S_\theta^2} \left(\frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi^{WW}(M_W^2)}{M_W^2} \right)$$

Non-decoupling of scalar fields



$m^2 \left(1 + \ln \frac{Q^2}{m^2}\right)$ cancels :



However, 

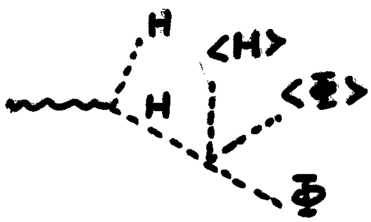
$\sim M_\Phi^2 \left(1 + \ln \frac{Q^2}{M_\Phi^2}\right)$ remains

$\to \Pi^{XX'}$ diverges as $M_\Phi^2 \sim f^2 \to \infty$

- More precisely,

$$\Delta r_Z^S \sim \frac{1}{16\pi^2} \frac{1}{v^2} \underbrace{\left(\frac{v'}{v}\right)^2}_{\text{coupling constant}} M_\Phi^2$$

$\langle \Phi \rangle \equiv v' = \frac{\lambda_{h\Phi h}}{2\lambda_\phi^2} \frac{v^2}{f}$



(i) if $v' = 0$ (for fixed f) $\Leftrightarrow$ setting $\lambda_{h\Phi h} = 0$ by hand

then $\Delta r_Z^S \equiv 0$ (Custodial symm.)

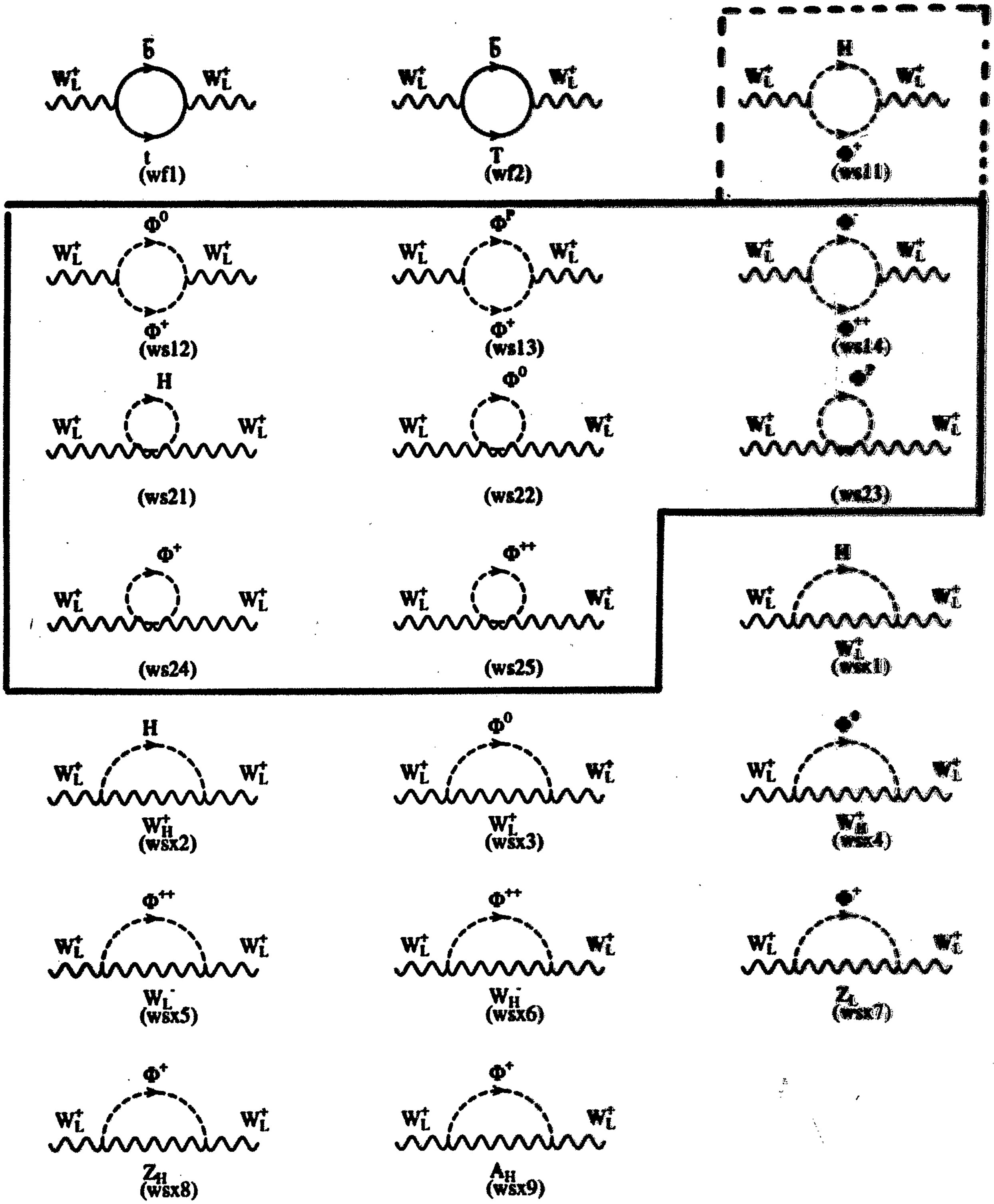


FIG. 11: Complete list of diagrams due to fermions and scalar fields to the self-energy of the Standard Model W gauge boson.

(ii) Take $f \to \infty$: decouple or not depending on how the limit is taken

(a) $\mu^2 \sim f^2/16\pi^2 = a_{uv} \frac{f^2}{16\pi^2}$

a_{uv}: unknown $\mathcal{O}(1)$ coeff. parameterized by UV completion

keep μ^2 (thus v^2) fixed by adjusting a_{uv}

$$v^2 = \frac{\mu^2}{\lambda_{h^4} - \frac{\lambda^2_{h\Phi h}}{\lambda_{\Phi^2}}} \quad \text{fixed} \quad (m_H^2 \text{ fixed} \sim v^2)$$

Recall:

$$v' = \frac{\lambda_{h\Phi h}}{2\lambda_{\Phi^2}} \frac{v^2}{f}$$

$$v' \sim \frac{v^2}{f}, \quad \Delta r_Z^S \sim \frac{1}{16\pi^2} \frac{1}{v^2} \frac{v^2}{f^2} f^2 \sim \frac{1}{16\pi^2}$$

Non-decoupling!

(Note: $\Delta_{tree} \to 0$ as $f \to \infty$)

(b) without tuning a_{uv}, $f \to \infty \Rightarrow \mu^2 \to \infty$

v^2 fixed by taking $\lambda_{h^4} \to \infty$ ($m_H \to \infty$!)

$$\Delta r_S^Z \sim \frac{1}{v^2} \left(\frac{\lambda_{h\Phi h}}{\lambda_{\Phi^2}}\right)^2 \left(\frac{v}{f}\right)^2 \lambda_{\Phi^2} f^2 \to \frac{\lambda^2_{h\Phi h}}{\lambda_{\Phi^2}} \to 0$$

decouple!

$\frac{1}{4}\lambda_{h^4} = \lambda_{\Phi^2} \to \infty$ Perturbation Theory breaks down!

Cancellation bt tree and 1-loop corrections to M_W

(19)

f= 2 TeV

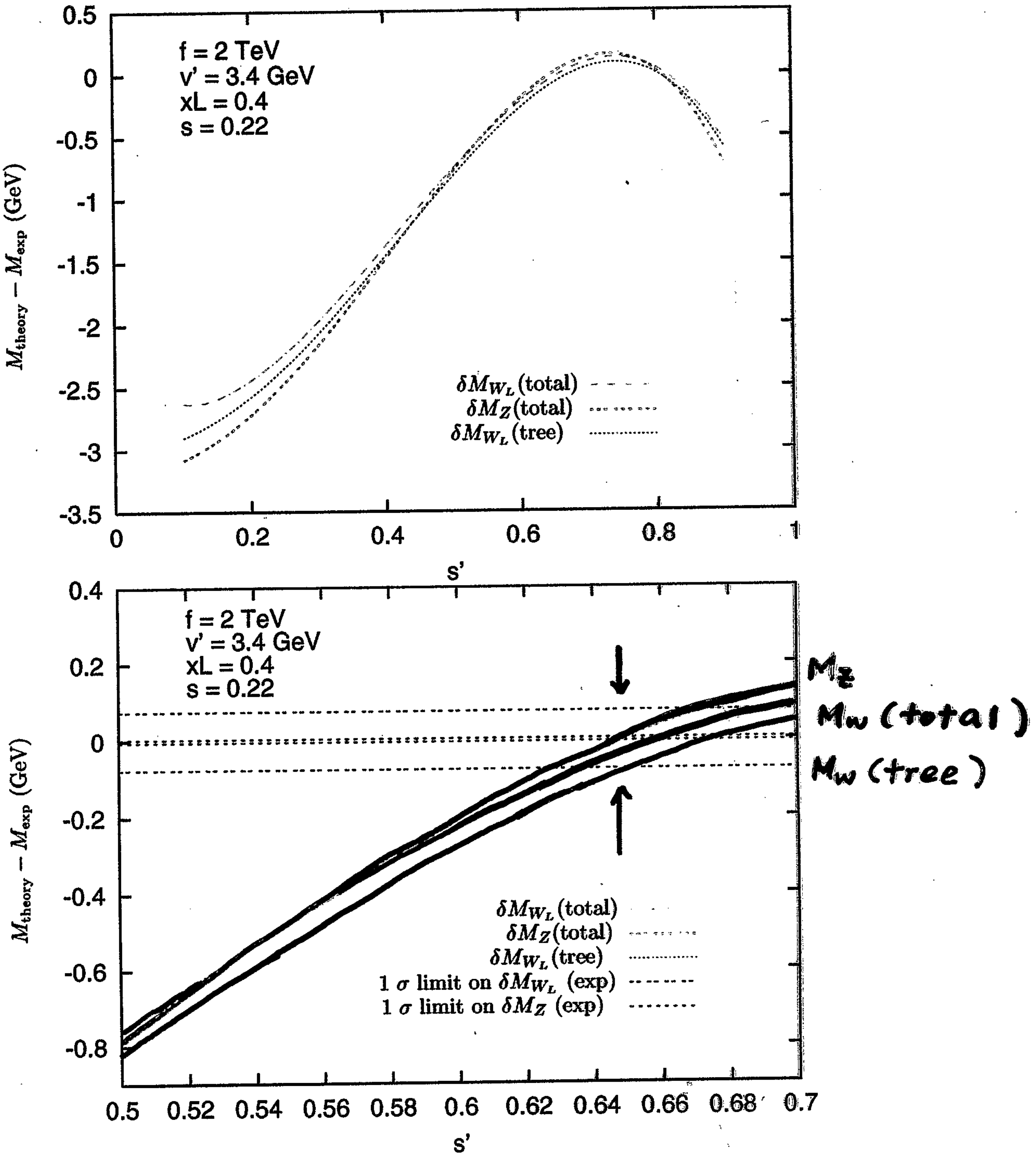
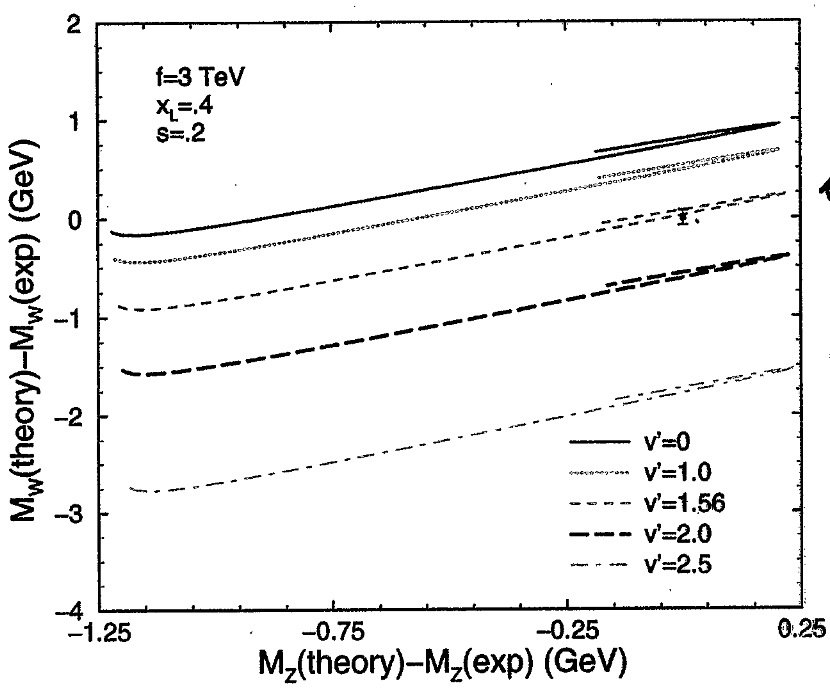


FIG. 1: Prediction for M_{W_L} as a function of the mixing angle s' at the tree level and the one-loop level. Also plotted is the correlation between M_Z and s' for fixed s, v' and f. The cutoff scale f in this plot is 2 TeV, the $SU(2)$ triplet VEV $v' = 3.4\, GeV$, the mixing angle $s = 0.22$, and $x_L = 0.4$.

s': $U(1)_1 \times U(1)_2 \to U(1)_Y$

$v' \neq 0$ required to agree with data at low f



0
1.0
$v' = 1.56$ GeV
2.0
2.5

FIG. 4: Parametric plot of $M_{W_L} - M_Z$ in terms of s' for different values of the $SU(2)$ triplet VEV, $v' =$ 0, 1.0, 1.56, 2.0 and 2.5. The cutoff scale f is 3 TeV, the mixing angle $s = 0.2$, and $x_L = 0.4$. The data point with error bars on M_{W_L} and M_Z is also shown.

Parametric in terms of s'

$f = 3$ TeV

Conclusions

- No Model-independent Precision EW fit

- $\rho \neq 1$: valid renormalization scheme requires

  4 input parameters

  → soften the constraints ($m_t^2 \to \ln m_t^2$)
  → many new models which have been ruled out by SM-like fit may still be alive if the renormalization scheme is properly defined according to the EW structure of the model

- heavy particles (esp. triplet Higgs) do NOT decouple

  → Precision EW Constraints ⇒ <u>upper</u> bound on Λ
  *→ Perturbation theory breaks down, EFT NOT valid

- In littlest Higgs model, consistent analysis shows $f = 2$ TeV is still allowed.