Model of nonperturbative contributions in q_T resummation

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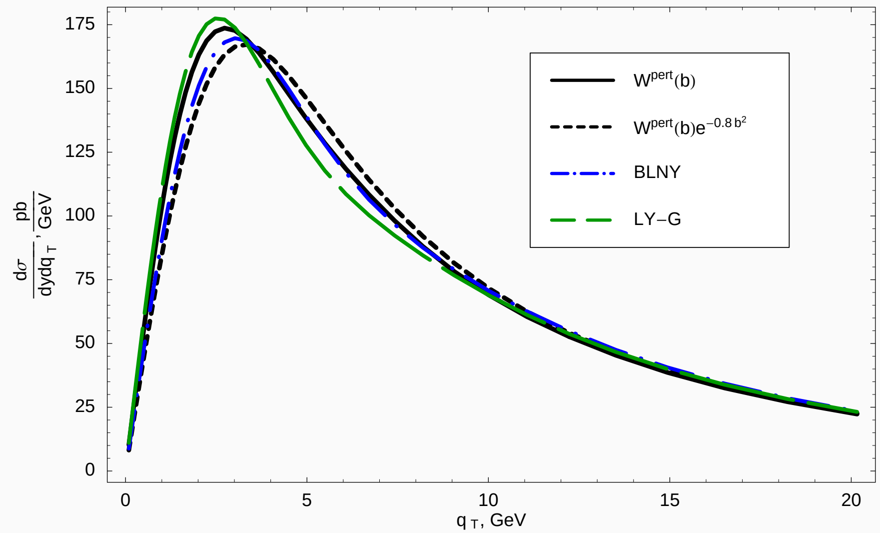
The largest theory uncertainties in the measured M_W arise from

- the model of W boson's recoil in the transverse plane
- parton densities



A W boson acquires $q_T \neq 0$ by recoiling against perturbative or nonperturbative QCD radiation

The peak of $d\sigma/dq_T$ shifts by up to $\sim$ 500 MeV depending on the nonperturbative model (large effect compared to the targeted $\delta M_W \sim$ 30 MeV)

A global analysis of q_T data from production of Drell-Yan pairs and Z bosons reduces this uncertainty to $\sim$ 50 MeV $\Rightarrow$ today's talk

q_T resummation: available methods

- Formalism in impact parameter (b) space *(Collins, Soper, Sterman, 1985)*
  - proved by a factorization theorem for k_T-dependent PDF's *(J. Collins, A. Metz, 2004; X. Ji, J.-P. Ma, F. Yuan, 2004)*
  - theory symmetries preserved automatically
  - conservation of momentum
  - fast and accurate evaluation of Fourier-Bessel transform possible *(ResBos, Balazs, P. N., Yuan)*
- Formalism in q_T space *(Ellis, Veseli)*
- joint resummation *(Li; Kulesza, Sterman, Vogelsang; ...)*
- gauge-invariant k_T-dependent PDFs *(Ji, Ma, Yuan and many others)*
- ...

I will discuss b-space formalism at NLO QCD at $x \gtrsim 10^{-2}$

The resummed cross section in theory

$$\left.\frac{d\sigma_{AB\to VX}}{dQ^2 dy dq_T^2}\right|_{q_T^2\ll Q^2} = \sum_{a,b=g,\overset{(-)}{u},\overset{(-)}{d},\dots} \int_0^\infty \frac{bdb}{2\pi} J_0(q_T b)\widetilde{W}_{ab}(b,Q,x_A,x_B)$$

$$\begin{aligned}\widetilde{W}_{ab}(b,Q,x_A,x_B) &= |\mathcal{H}_{ab}|^2\, e^{-\mathcal{S}(b,Q)}\overline{\mathcal{P}}_a(x_A,b)\overline{\mathcal{P}}_b(x_B,b)\\ &= \widetilde{W}_{LP}(b,Q,x_A,x_B)\otimes \widetilde{W}_{PS}(b,Q,x_A,x_B)\end{aligned}$$

$S(b,Q),\ \overline{\mathcal{P}}_a$ are universal in Drell-Yan-like processes

Leading-power (LP) terms: do not vanish at $b\to 0$

$$\widetilde{W}_{LP}(b,Q) = \sum_{k=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^k \sum_{m=0}^{2k} w_{km}\ln^m(Qb)$$

Power-suppressed (PS) terms are proportional to even powers of b

(Korchemsky, Sterman; Tafat)

$$\widetilde{W}_{PS}(b,Q) \approx \exp\left[-\sum_{p=1}^{\infty} b^{2p} f_p(\ln Q)\right];\quad f_p\sim\Lambda_{QCD}^{2p}$$

The resummed cross section in a global fit

$$\widetilde{W}_{ab}(b,Q) \equiv \widetilde{W}_{pert}(b,Q)e^{-S_{NP}(b,Q)},$$

where

- at $b \lesssim 1$ GeV^{-1},

$$\widetilde{W}_{pert}(b,Q) = \sum_{k=0}^{N}\left(\frac{\alpha_s}{\pi}\right)^k \sum_{m=0}^{2k} w_{km} \ln^m (Qb)$$

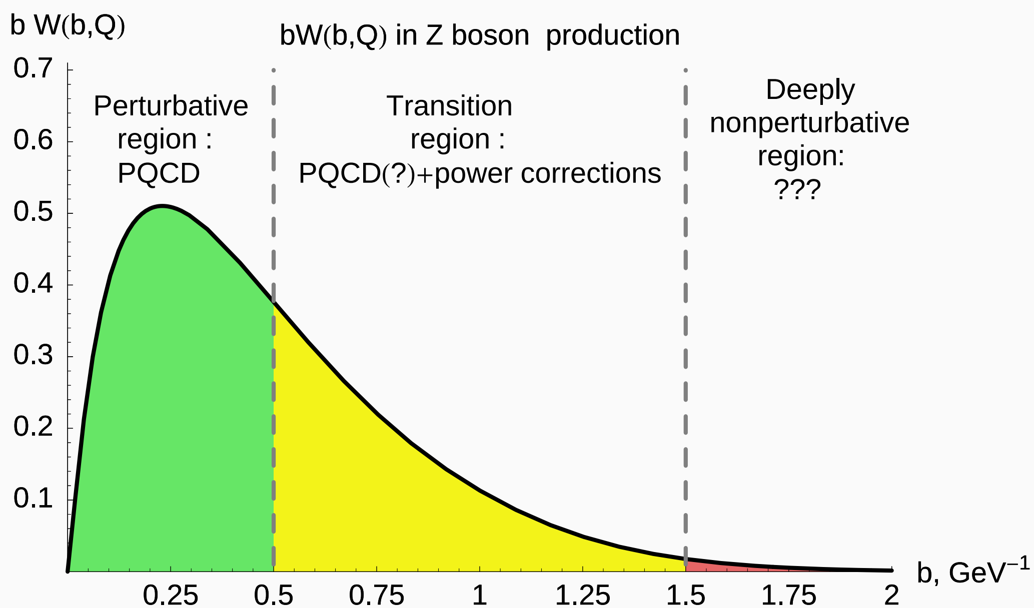
- $\widetilde{W}_{pert}(b,Q)$ is continued in some fashion to $b > 1$ GeV^{-1};
- $e^{-S_{NP}}$ is the universal effective nonperturbative exponent to be found from the fit:

$$e^{-S_{NP}(b,Q)} \equiv \frac{\widetilde{W}}{\widetilde{W}_{pert}} = \frac{\widetilde{W}_{LP} \otimes \widetilde{W}_{PS}}{\widetilde{W}_{pert}}$$

- if $\widetilde{W}_{pert} \approx \widetilde{W}_{LP}$ at all b, the fit should prefer

$$S_{NP}(b,Q) \approx -\ln\left[\widetilde{W}_{PS}(b,Q)\right] \approx b^2 f(\ln Q) \oplus \text{ small corrections}$$

$bW(b,Q)$ in Z boson production



- ❒ $b \lesssim 0.5$ GeV^{-1} : $\widetilde{W}(b,Q) \approx \widetilde{W}_{pert}(b,Q)$

  contributes most of the rate at the Tevatron

- ❒ $0.5 \lesssim b \lesssim 1.5-2$ GeV^{-1} : higher-order terms in α_s and b^p important; contributes some variations in $d\sigma/dq_T$ at $q_T \lesssim 10$ GeV
- ❒ $b \gtrsim 1.5 - 2$ GeV^{-1} : terra incognita; tiny contributions

$\widetilde{W}_{pert}(b,Q)$ at large b: the b_* prescription *(Collins, Soper, 1982; CSS, 1985)*

$$\widetilde{W}(b,Q) = \widetilde{W}_{pert}(b_*,Q)e^{-S_{NP}(b,Q;b_{max})}$$

$$b_*(b,b_{max}) \equiv \frac{b}{\sqrt{1+\left(\frac{b}{b_{max}}\right)^2}} = \begin{cases} b & \text{at } b \ll b_{max} \\ b_{max} & \text{at } b \gg b_{max} \end{cases}$$

$$\widetilde{W}_{pert}(b_*,Q) = \sum_j \sigma_0 e^{-S_{pert}(b_*,Q)}$$
$$\times \left[\mathcal{C}_{j/a}\otimes f_{a/A}\right](x_A,b_*,\mu_F(b_*))\left[\mathcal{C}_{\bar{j}/b}\otimes f_{b/B}\right](x_B,b_*,\mu_F(b_*))$$

$$\left[\mathcal{C}_{j/a}\otimes f_{a/A}\right](x,b_*,\mu_F(b_*)) = \int_x^1 \frac{d\xi}{\xi}\mathcal{C}_{j/a}\left(\frac{x}{\xi},\frac{b_*\mu_F(b_*)}{b_0}\right) f_{a/A}(\xi,\mu_F(b_*))$$

with $b_0 \equiv 2e^{-\gamma_E} \approx 1.123$; $\mu(b_*) \approx b_0/b_*$.

Upper constraint on b_{max}

In $\left[\mathcal{C}_{j/a} \otimes f_{a/A}\right](x, b_*, \mu_F(b_*))$, we choose

$$\mu_F(b_*) \approx \frac{b_0}{b_*}$$

to prevent appearance of large collinear logarithms in

$$C_{j/a}\left(x, \frac{b_* \mu_F(b_*)}{b_0}\right) = \sum_{k,m} \left(\frac{\alpha_s}{\pi}\right)^k \left[P_{j/a}(x) \ln^m\left(\frac{b_* \mu_F(b_*)}{b_0}\right) + \dots\right]$$

The collinear logs are resummed by DGLAP equations in $f_{a/A}(x, \mu_F(b_*))$

The PDF parametrizations are only available at $\mu_F > Q_{ini} = 1 - 1.3$ GeV
$\Rightarrow b_{max}$ cannot exceed $b_0/Q_{ini} \sim 0.86 - 1.123$ GeV^{-1}

$\Rightarrow W^{pert}(b_*, Q)$ deviates from the exact PQCD result at $b \sim 1$ GeV^{-1}!

$\Rightarrow$ Compensated in part by the phenomenological $S_{NP}(b, Q; b_{max})$

$\Rightarrow$ Can affect validity of the calculation???

NLO global analysis of q_T distributions

(R. Brock, F. Landry, P.N., C.-P. Yuan, 2002)

- simultaneous fit to low-Q Drell-Yan (E288, E605, and R209) and Tevatron Run-0 and Run-1 Z data

- realized in b_* prescription with $b_{max} = 0.5 \text{ GeV}^{-1}$

- the best-fit $S_{NP}(b)$ is quadratic in b (Gaussian)

$$S_{NP}(b) = b^2 \left[g_1 + g_2 \ln\left(\frac{Q}{3.2 \text{ GeV}}\right) + g_1 g_3 \ln\left(100 x_A x_B\right)\right],$$

  with $g_1 = 0.21 \text{ GeV}^2$, $g_2 = 0.68 \text{ GeV}^2$, $g_3 = -0.6$

- parametrizations with 2 parameters or linear terms in b fail spectacularly ($\chi^2/d.o.f. > 3$)

Unanswered questions

- ❐ Why $\chi^2/d.o.f. = 176/119 \sim 1.48$?

- ❐ Is $S_{NP}(b, Q = M_Z) \approx (2.7\ \text{GeV}^2) b^2$ indeed mostly nonperturbative?

- ❐ Why 3 large parameters g_1, g_2, and g_3 are required to get a good fit?

Qiu & Zhang: $g_3 \neq 0$ is an artifact of b_* prescription with small b_{max}?

If so, good fits would prefer $|g_3| \to 0$

- ❐ Can the calculation of $\widetilde{W}_{pert}(b, Q)$ in the transition region $b \sim 1$ GeV^{-1} be improved?

The "$2b_*$ prescription"

1. Take the original b_* prescription

$$\widetilde{W}(b,Q) = \widetilde{W}_{pert}(b_*,Q)e^{-S_{NP}(b,Q;b_{max})}$$

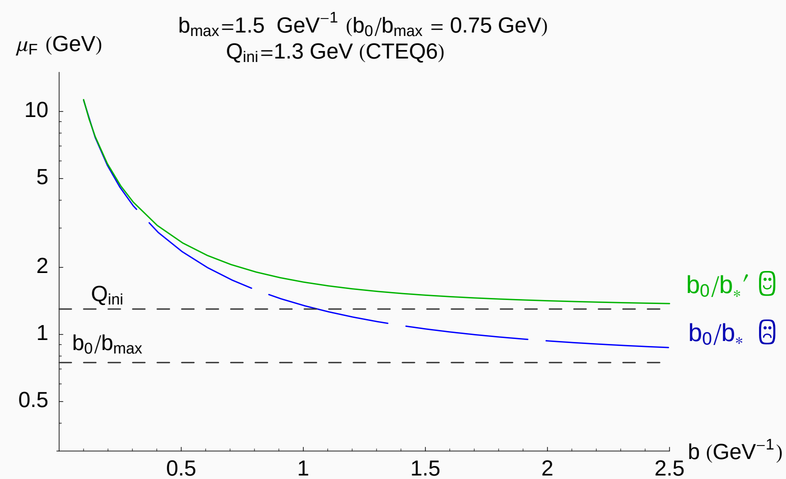
2. Choose $\mu_F = b_0/b'_*$ in $\left[\mathcal{C}_{j/a} \otimes f_{a/A}\right](x, b_*, \mu_F)$, with

$$b'_* \equiv b_*(b, b'_{max}),$$

and

$$b'_{max} = \min\left(b_{max}, b_0/Q_{ini}\right)$$

$$\mu_F = \begin{cases} \sim 1/b & \text{for} \quad b \ll b_0/Q_{ini} \\ Q_{ini} & \text{for} \quad b \gtrsim b_0/Q_{ini} \end{cases}$$



b_{max} can be safely increased at least up to $2-3$ GeV^{-1},
but the scale μ_F in $f_{a/A}(x,\mu_F)$ never goes below Q_{ini}

$2b_*$ prescription: factorization scale dependence

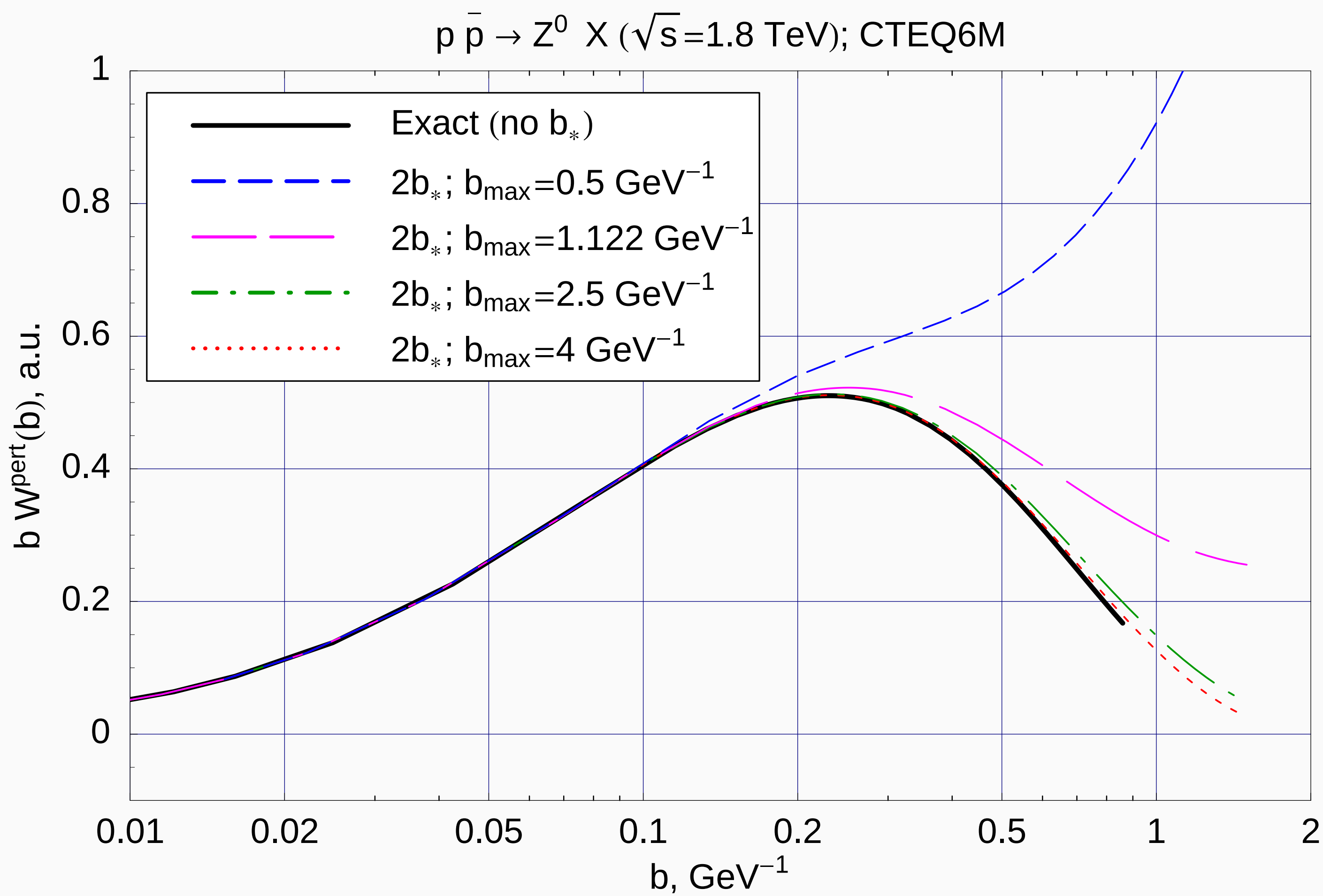
- If $\mu_F \sim Q_{ini}$, large non-resummed logarithms appear at $b_* \gg b_0/Q_{ini}$

$$C_{j/a}\left(x, \frac{b_* \mu_F}{b_0}\right) = \sum_{k,m} \left(\frac{\alpha_s}{\pi}\right)^k \left[P_{j/a}(x) \ln^m\left(\frac{b_* \mu_F}{b_0}\right) + ...\right]$$

- should not create problems, because the region $b_* \gg b_0/Q_{ini}$ is exponentially suppressed by $e^{-S_{pert}(b_*,Q) - S_{NP}(b,Q)}$
  - confirmed by a numerical calculation

- Properties of $2b_*$ prescription
- at $b_{max} \leq b_0/Q_{ini}$, reduces to the original b_* prescription
- no new parameters (utilizes freedom in the choice of μ_F)
- preserves continuity of $\widetilde{W}(b,Q)$ and its derivatives
- the balance of pert. and nonpert. contributions in $\widetilde{W}(b,Q)$ is smoothly changed by varying b_{max}
- at $b_{max} \gg b_0/Q_{ini}$, is structurally and numerically close to the leading-log extrapolation of $\widetilde{W}_{pert}(b,Q)$, such as that in the principal value resummation *(Sterman; Kulesza, Sterman, Vogelsang...)*

Perturbative form factors $b\widetilde{W}^{pert}(b,Q)$ and $b\widetilde{W}^{pert}(b_*,Q)$ in the $2b_*$ prescription for the Tevatron Run-1 Z boson production



Global fits in $2b_*$ prescription New!

98 data points

- Tevatron Run-1 Z boson production (CDF, D0)
  - $Q \approx M_Z$, $\sqrt{s} = 1.8$TeV, $p_T < 10$ GeV
  - sizable errors
- Fixed-target Drell-Yan pair production (E288, E605, R209)
  - $Q = 5 - 18$ GeV, $p_T < 1.4$ GeV
  - small statistical errors, incomplete systematical errors; 2 outlier points in E605 sample contribute $\delta\chi^2 \approx 25$

Nonperturbative function:

$$S_{NP}(b) = b^{2-\beta}\left[g_1 + g_2 \ln\left(\frac{Q}{3.2\text{ GeV}}\right) + g_1 g_3 \ln\left(100 x_A x_B\right)\right],$$

where $\beta = 0$ (Gaussian form) or free

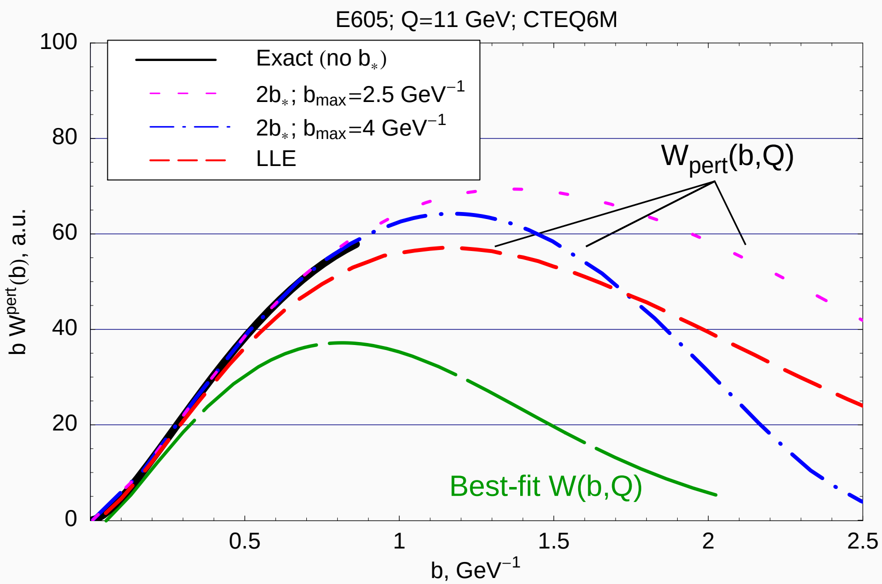
Scan over $b_{max} = 0.5 - 2.5$ GeV^{-1}

Summary of results (PRELIMINARY)

- Increasing b_{max} up to $1 - 1.5$ GeV^{-1} improves the quality of the fit
  - χ^2 and $|S_{NP}(b,Q)|$ decrease
  - Best-fit $|g_3| \approx 0$
  - Best-fit $\beta = -0.2\ (+0.3)$ in Drell-Yan (Z) experiments; correlated with normalizations of DY data; $\beta = 0$ in the next slides
- The preferred $S_{NP}(b,Q)$ is close to a two-parameter Gaussian form, $S_{NP}(b,Q) \approx [g_1 + g_2 \ln(Q/3.2)]\, b^2$
- Small, but non-zero, g_3 and β are needed because of high accuracy of E288 and E605 data

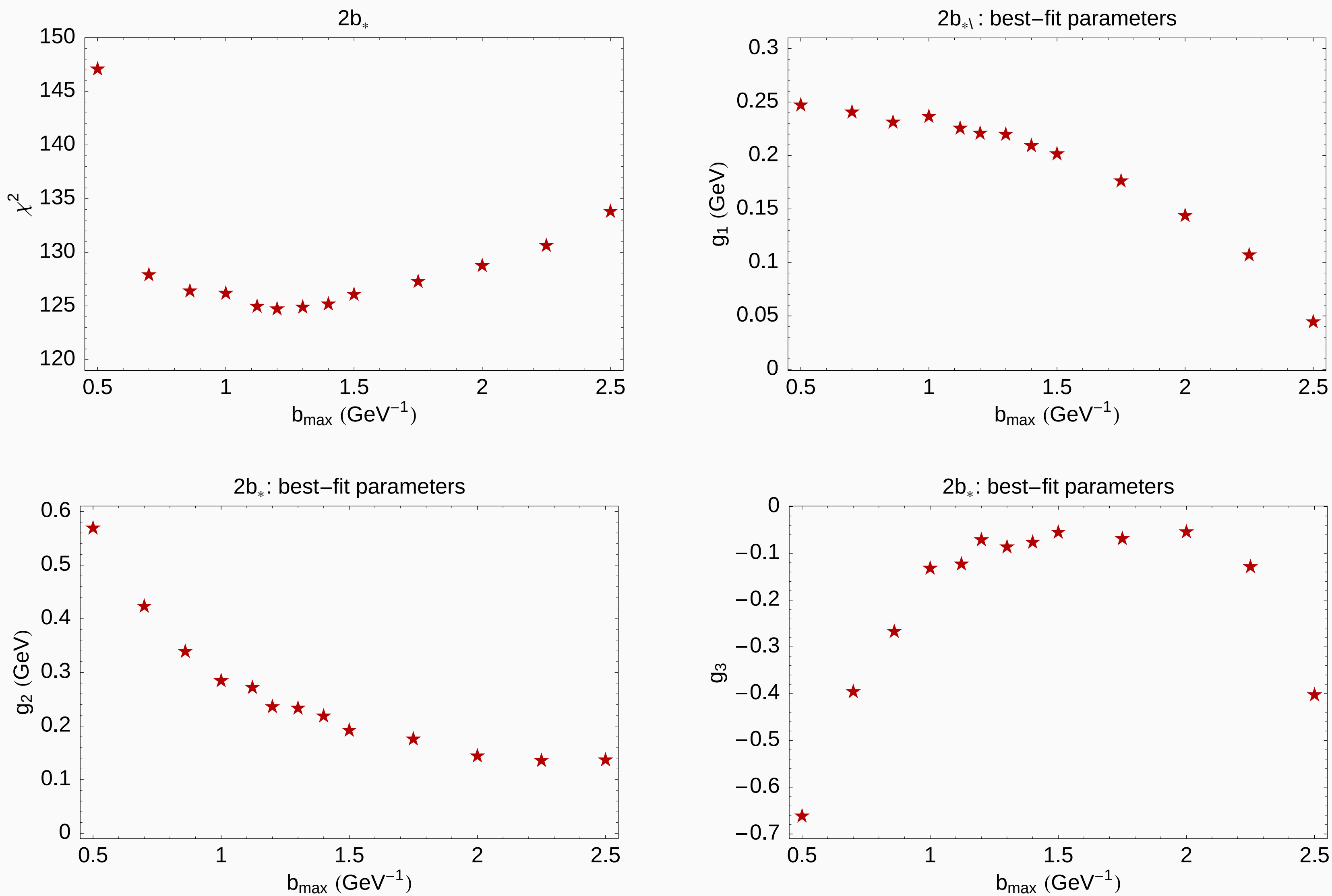
Choosing $b_{max} > 1.5$ GeV^{-1}

- ❒ Z production is described well for b_{max} up to $3-4$ GeV^{-1}
- ❒ Description of low-Q Drell-Yan data worsens for $b_{max} > 1.5$ GeV^{-1} because of rapid variations in $\widetilde{W}_{pert}(b,Q)$ at $b = 1.5-3$ GeV^{-1}



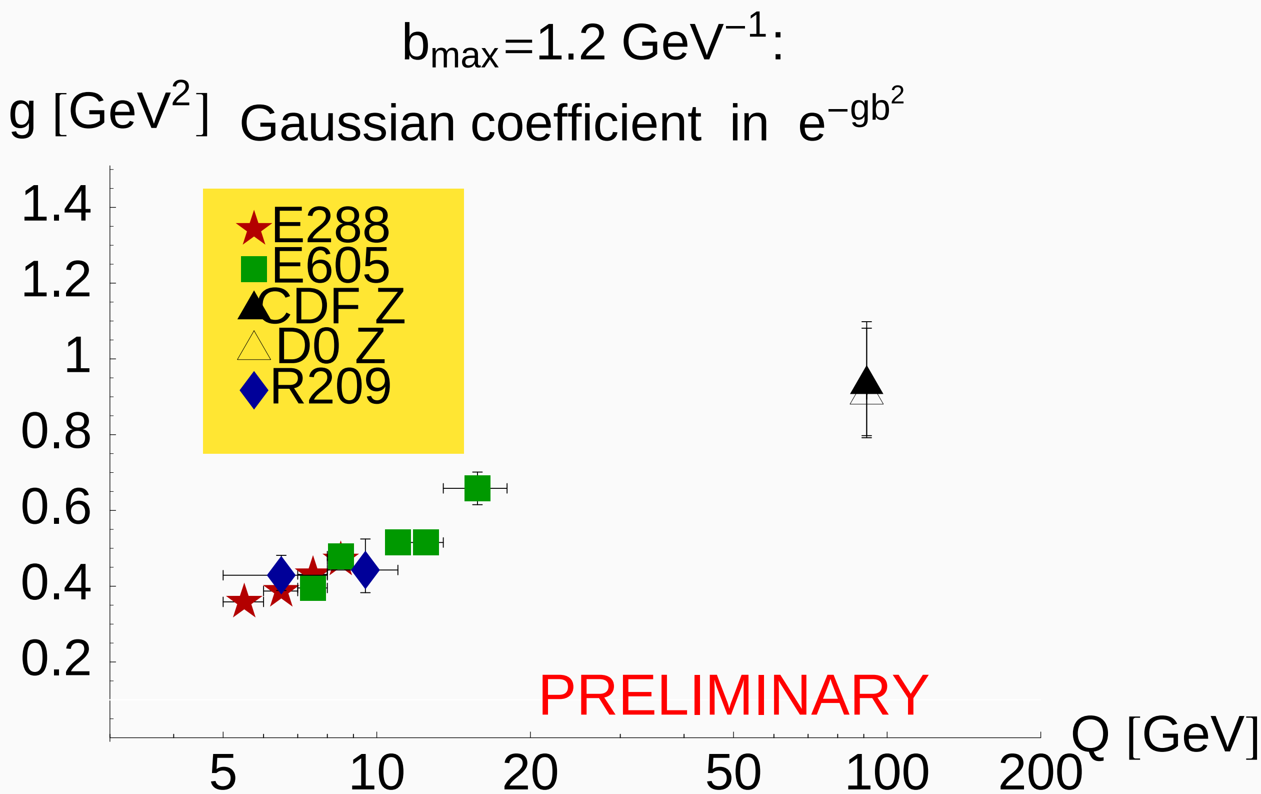
- ❒ The variations reflect absence of important higher-order logs $\sum_{k=N+1}^{\infty} \alpha_s^k \sum_m w_{km} \ln^m(Qb_{(*)})$
- ❒ are not easily compensated by adjustments in $S_{NP}(b,Q)$
- ❒ Similar features are present in the leading-log extrapolation
- ❒ $b_{max} \sim 1-1.5$ GeV^{-1} is the optimal range

$2b_*$ prescription: scan over b_{max} (PRELIMINARY)



Improvement in the properties of the fits at $b_{max} = 1 - 2\ \mathrm{GeV}^{-1}$

Nonperturbative smearing g preferred by individual mass bins
($b_{max} = 1.2$ GeV^{-1})



$g_3 \approx 0$, $\beta \approx 0$

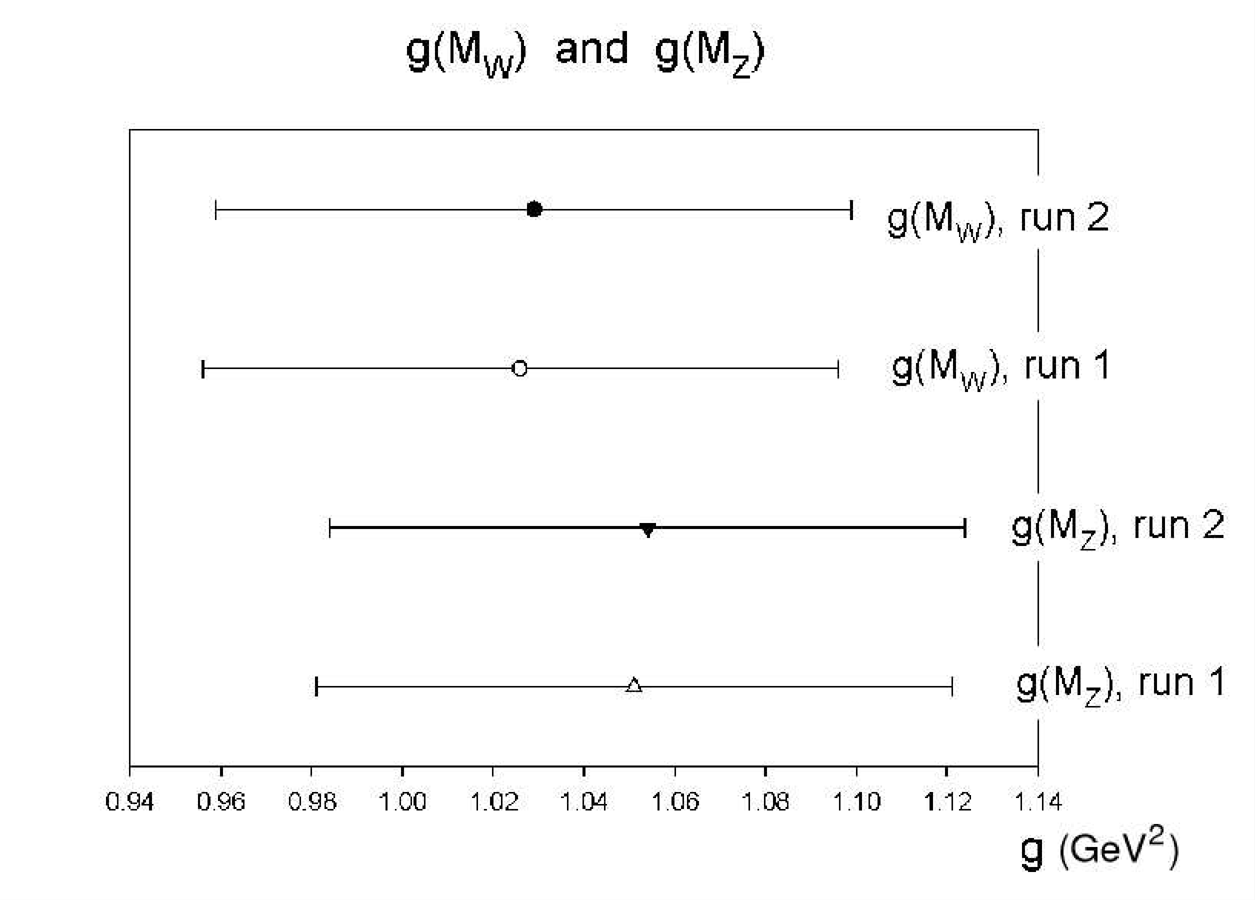
$S_{NP}(b) \approx g b^2$,

with
$g \sim \langle k_T^2 \rangle / 2$

Dependence of best-fit $g(Q)$ on $\ln Q$ is approximately linear

$g(Q)$ at $Q = M_W$ and $Q = M_Z$ in the best fit ($b_{max} = 1.2$ GeV^{-1})

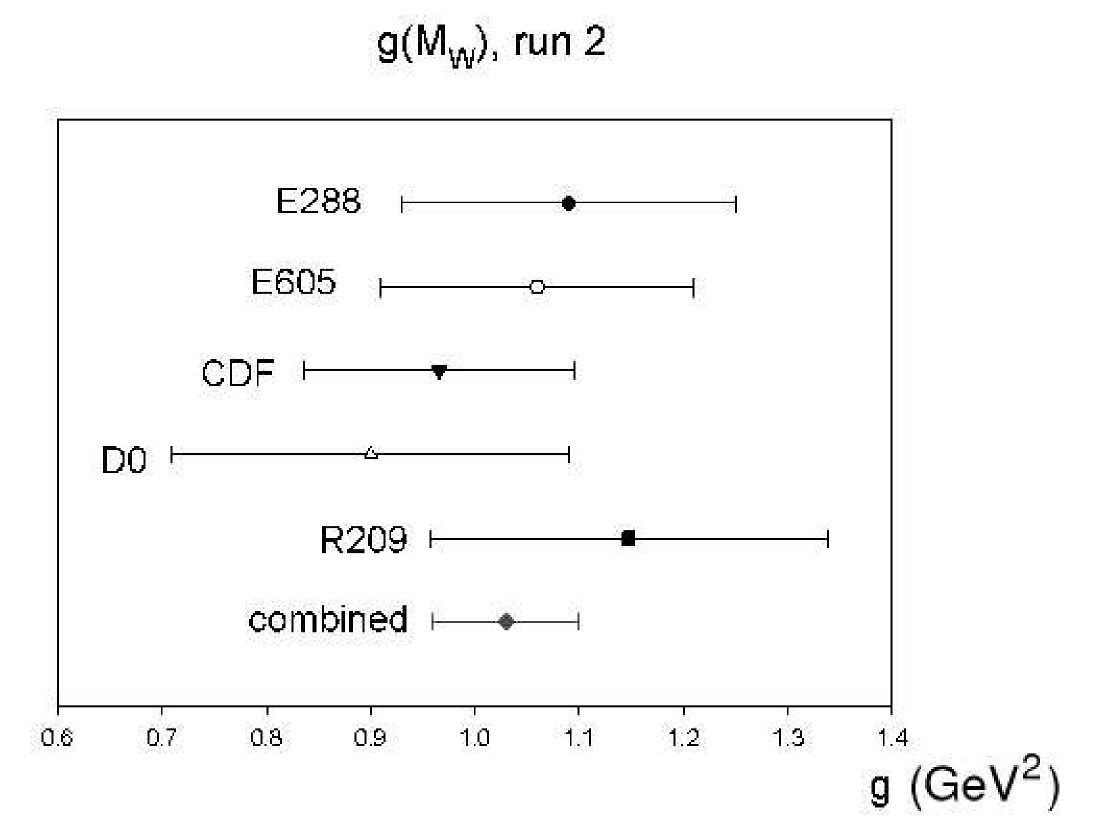
- Obtained using a Lagrange multiplier method



- Errors are for $\delta\chi^2_{tot} = 1$

- Translates into a variation $\approx \pm 50$ MeV in the peak of $d\sigma(W)/dq_T$

$g(M_W)$: constraints from individual experiments



All data sets agree within errors; constraints from low-Q DY and Z Run-1 data are comparable

Conclusions

- ❐ Modifications in b_* prescription improve description of perturbative contributions at $b \sim 1$ GeV^{-1} and lead to better agreement with the data

- ❐ High quality of the obtained global fits supports universality of k_T-dependent factorization in Drell-Yan-like processes

- ❐ For $b_{max} \sim 1-1.5$ GeV^{-1}, the data prefer a nearly Gaussian $S_{NP}(b, Q)$ with approximately linear universal dependence on $\ln Q$ ($g_3 \approx 0$)

- ❐ Our preliminary estimate is $S_{NP}(b, Q = M_W) \approx (1.03 \pm 0.07) b^2$ for $b_{max} = 1.2$ GeV^{-1}

- Much more work is needed to investigate
  - agreement between the different experiments;
  - correlations between $S_{NP}(b, Q)$ and normalizations of low-Q DY data;
  - correlations between $S_{NP}(b, Q)$ and PDF's;
  - simultaneous fit of $S_{NP}(b, Q)$ and PDF's $\Rightarrow$ tools developed within CTEQ
  - effect of the NNLO corrections
  - rapidity dependence
- CTEQ W & Z working group systematically explores these topics