Electroweak bosons rapidity distributions at hadron colliders

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Outline

- Introduction
- Method
- Results
    - Dilepton rapidity distribution for E866: sea quark distributions in the proton;
    - W, Z production at the Tevatron and the LHC – precision QCD at hadron colliders.
- Conclusions

Introduction

- LHC is the next big step in particle physics;
- Experiments of unprecedent complexity;
- Typically, bad signal to backrground ratios; cuts on the final states;
- Huge rates for Standard Model processes;
- Theoretical estimates for signals and backgrounds rely on “perturbative” QCD;
- QCD for hadron collider physics includes a variety of things:
  - parton distribution functions (PDFs);
  - jet algorithms;
  - hadronization models;
  - Monte Carlo event generators;
  - perturbative calculations.
- These issues are mutually interconnected.

Introduction

- Processes at hadron colliders can be classified as:
  - Clean, well-studied processes with large cross-sections (calibration, model-independent searches for new physics);
  - Important discovery processes with small cross-sections, submerged into large background (Higgs production);
  - Processes with many particles in the final state; backgrounds for dedicated new physics searches;
  - QCD processes with large cross-sections and large uncertainties (two jet cross-section, heavy flavor production etc.).
- For all of these cases NLO QCD is obligatory; for some NNLO QCD is desirable:
  - at NNLO, one gets for the first time an honest estimate of the theoretical uncertainty: absolutely necessary for calibration processes;
  - For discovery channels, one can get better signal to background ratio (Higgs production);
  - leads to a better understanding of the underlying structure of the theory; may result in important lessons for other processes and methods (resummations).

Introduction

- Production of electroweak bosons is an important process:
- The first application of parton model ideas beyond DIS;
- Discovery of W and Z bosons;
- W mass and width measurements;
- charge assymmetry;
- Excess in dileptons at large invariant masses is a universal new physics signal (Z', extra dimensions, compositness).
- rapidity distribution permits measuring PDFs:

$$\frac{d\sigma}{dY} \sim q\left(\frac{m_\gamma^*}{\sqrt{s}} e^{Y}\right) \bar{q}\left(\frac{m_\gamma^*}{\sqrt{s}} e^{-Y}\right) + \mathcal{O}(\alpha_s)$$

  Recent results from E866 at FNAL on dilepton pair production at $\sqrt{s} = 40\ \mathrm{GeV}$ at hydrogen and deuterium targets;
- Huge rates at the LHC: $W \to l\nu \to 15\ \mathrm{events/sec}$, $Z \to l^+l^- \to 1.5\ \mathrm{events/sec}$.
- if rapidity of Z and W is measured, possible partonic luminosity monitor at the LHC. Dittmar, Pauss, Zürcher

Introduction

- For γ^*, Z, W production a complete control of the final state kinematics is desirable; ideally, partonic level Monte Carlo through $\mathcal{O}(\alpha_s^2)$. It is still in a distant future (see, however, my talk on the Higgs production).
- What is available?
  - Total cross-section to $\mathcal{O}(\alpha_s^2)$. Hamberg, van Neerven, Matsuura (1990)
    Harlander and Kilgore (2002)
  - $p_\perp$ distirbution at $\mathcal{O}(\alpha_s^2)$ and resummed; Collins, Soper, Sterman (1985);
  - rapidity distribution at $\mathcal{O}(\alpha_s)$ Altarelli, Ellis, Martinelli 1979
  - rapidity distribution at $\mathcal{O}(\alpha_s^2)$ Anastasiou, Dixon, K.M. Petriello 2003
- Poor man's solution:

$$\frac{\mathrm{d}^2\sigma_{\mathrm{mod}}}{\mathrm{d}p_\perp \mathrm{d}Y} = \theta(p - p_\perp^{\mathrm{cut}}) \frac{\mathrm{d}^2\sigma}{\mathrm{d}p_\perp \mathrm{d}Y} + \left[\frac{\mathrm{d}\sigma}{\mathrm{d}Y} - \int\limits_{p_\perp^{\mathrm{min}}}^{p_\perp^{\mathrm{max}}} \mathrm{d}p \frac{\mathrm{d}^2\sigma}{\mathrm{d}p_\perp \mathrm{d}Y} \right] \theta(p_\perp^{\mathrm{cut}} - p).$$

- NNLO rapidity distribution is the key to fully control the kinematics.

Method

- It took more than 20 years to go from $\mathcal{O}(\alpha_s)$ to $\mathcal{O}(\alpha_s^2)$ for $\mathrm{d}\sigma/\mathrm{d}Y$. Why?
- Part of the reason is a misconception:
    - Common belief: complexity in higher orders originates from virtual loops;
    - In reality: complexity in higher orders originates from singular integration of tree level graphs.
- In a way, loops are simple, real emission is not.
- Loops are simple, because the structure is well-understood:
    - Integration-by-parts identities [Chetyrkin, Tkachov];
    - Automatic solution of recurrence relations [Laporta];
    - Methods to compute master integrals [Smirnov, Tausk, Gehrmann, Remiddi].
- Can a similar understanding of the mathematical structure of real emissions be reached?

Method

- Key idea:
  - Map phase-space → loop integrals using the optical theorem:

$$\sum |T_{in}|^2 \sim \mathrm{Im}(T_{ii}).$$

  - Use multi-loop methods
- Total cross-section: the on-shell conditions

$$2\pi i \delta(P_\gamma^2 - m_\gamma^2) \to \frac{1}{P_\gamma^2 - m_\gamma^2 - i\delta} - \mathrm{c.c.}$$

- Rapidity distribution: create a ‘fake’ particle

$$2\pi i \delta\left(\frac{P_\gamma \cdot [p_1 - u p_2]}{P_\gamma \cdot p_2}\right) \to \frac{P_\gamma \cdot p_2}{P_\gamma \cdot [p_1 - u p_2] - i\delta} - \mathrm{c.c.}$$

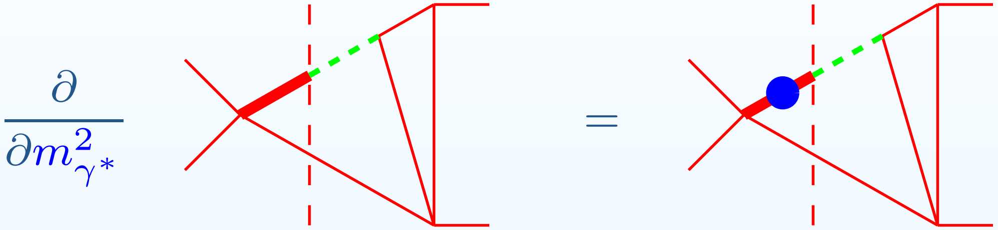
- Since both, the on-shell and the rapidity constraints are polynomial in momenta, multiploop methods are applicable without any modification.

Method: rapidity master integrals

- IBP and recurrence relations express any relevant integral (out of 10^6 that are needed) through few master integrals:
    - 5 V-V master integrals,
    - 5 R-V master integrals,
    - 21 R-R master integrals.
- The process is characterized by a topology, not the particle content; as a consequence master integrals are the same for γ^*, W, Z, H etc. production;
- V-V and R-V master integrals are known two-loop or one-loop integrals “multiplied” by phase-space factors.
- R-R master integrals were unknown and hard to evaluate by a brute force

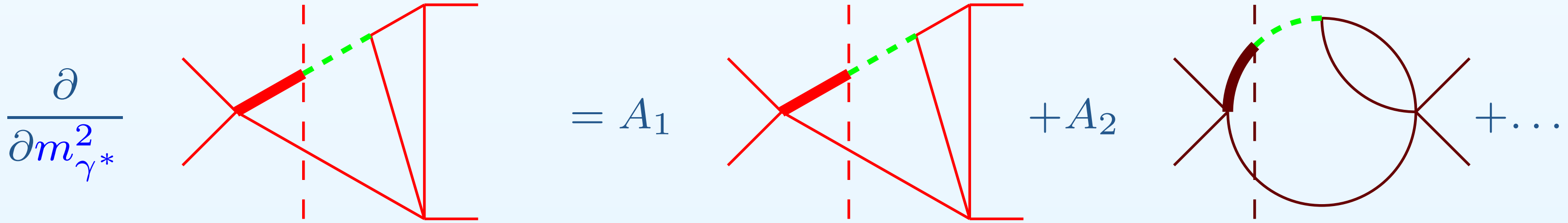
Method: differential equations

- Two kinematic variables: $m_{\gamma^*}^2, u \sim \exp(2Y)$. Form diff. eqs. (Kotikov, Gehrmann, Remiddi) for the “cut-integrals”



$$\frac{\partial}{\partial m_{\gamma^*}^2} \qquad = $$

- Apply IBP reduction to the r.h.s



$$\frac{\partial}{\partial m_{\gamma^*}^2} \qquad = A_1 \qquad + A_2 \qquad + \ldots$$

- Solve differential equations order by order in ϵ;
- Boundary conditions can obtained from simple kinematic limits;
- Hierarchical solution: simpler master integrals are non-homogeneous terms in differential equations for more complicated master integrals;

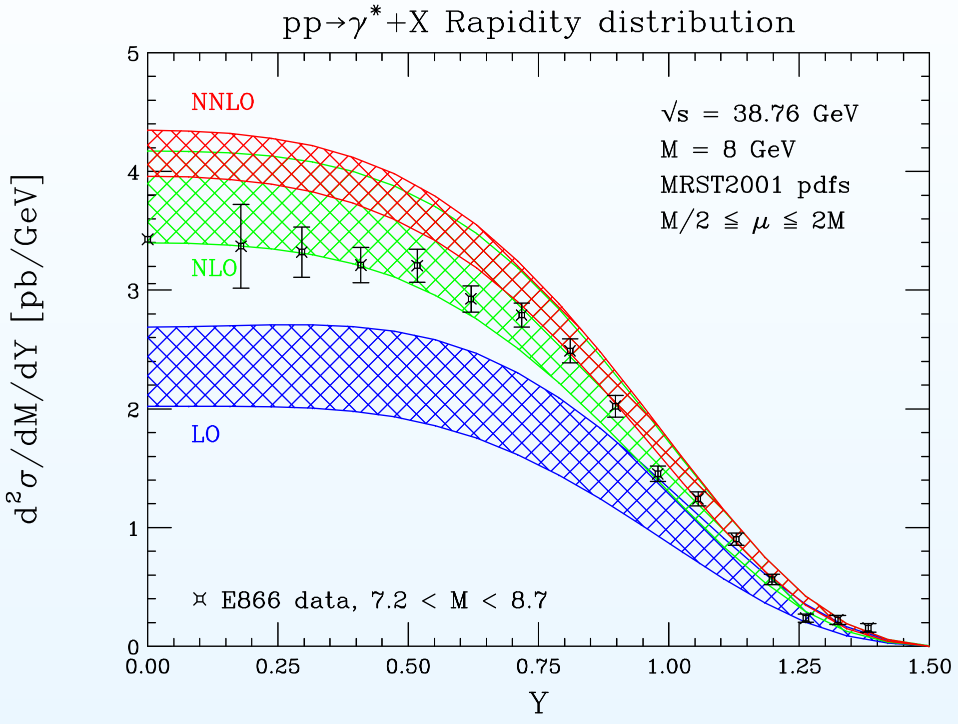
Results: E866

- Measures rapidity distribution of the μ pairs in pp and pd collisions at $\sqrt{s} \approx 40\ \mathrm{GeV}$ (fixed target)
- Sensitive to sea anti-quark distributions in the proton

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y} \sim e_u^2 u(x_1)\bar{u}(x_2) + e_d^2 d(x_1)\bar{d}(x_2) + (x_1 \leftrightarrow x_2),$$

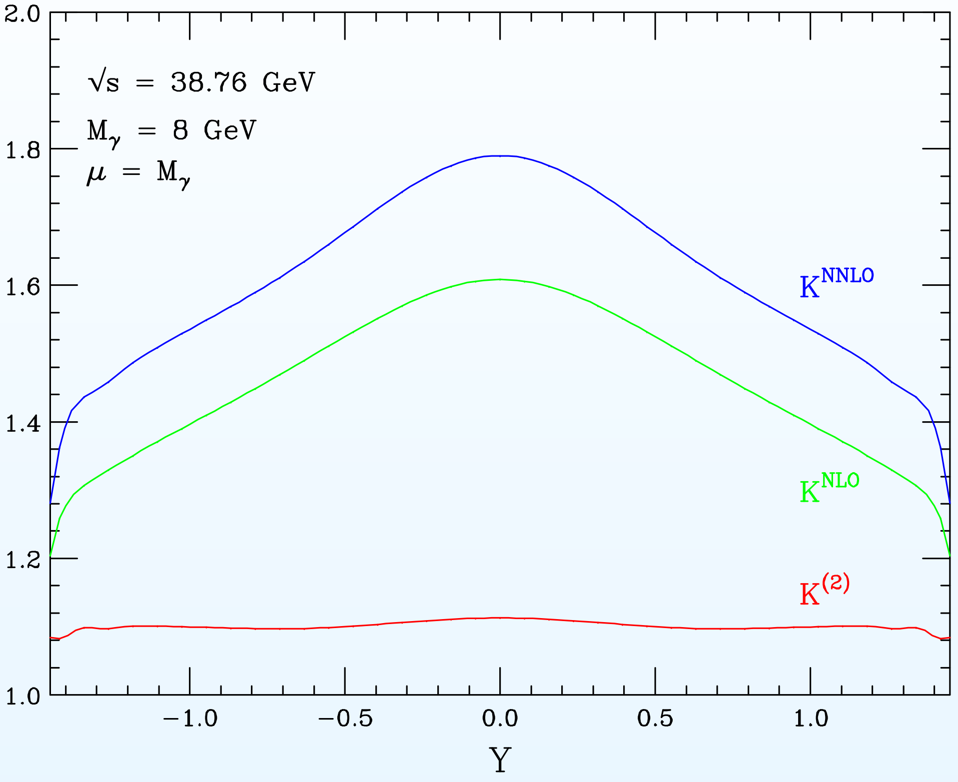
- $\bar{u}$ and $\bar{d}$ are not well-known for $x \sim 1$; E866 is sufficiently sensitive to study this issue.
- The NLO corrections are $\sim 40\%$ in the central Y region, the scale dependence is $\sim 20\%$.
- What about the NNLO corrections?
    - ○ Do they stabilize the theory prediction and by how much?
    - ○ Do they improve or make worse the quality of the theory/data comparison?
    - ○ Is it justified to use constant K-factor for all rapidities?

E866: DY rapidity distribution



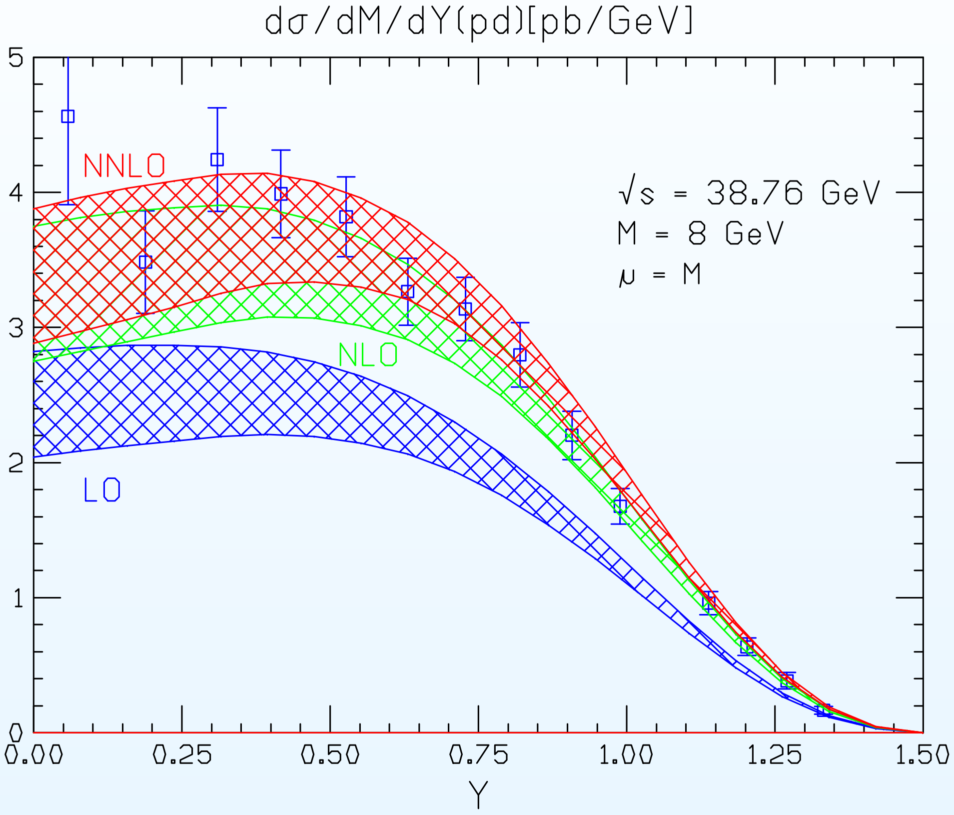
- Substantial improvement in scale stability;
- NNLO distribution sharper in central rapidity regions (smaller factorization scales are appropriate);
- Too many anitquarks in the proton.

E866: DY rapidity distribution



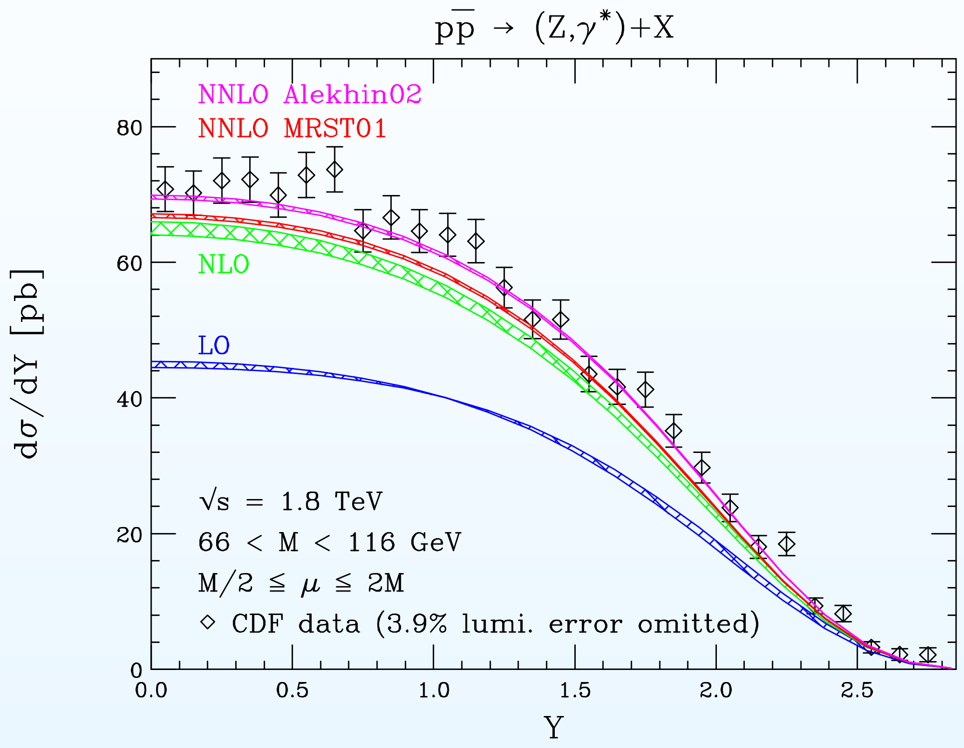
- $d\sigma_{\mathrm{LO}} \times K$ fails.
- $d\sigma_{\mathrm{NLO}} \times K$ is accurate to $1 - 3\%$.

E866: Alekhin PDFs



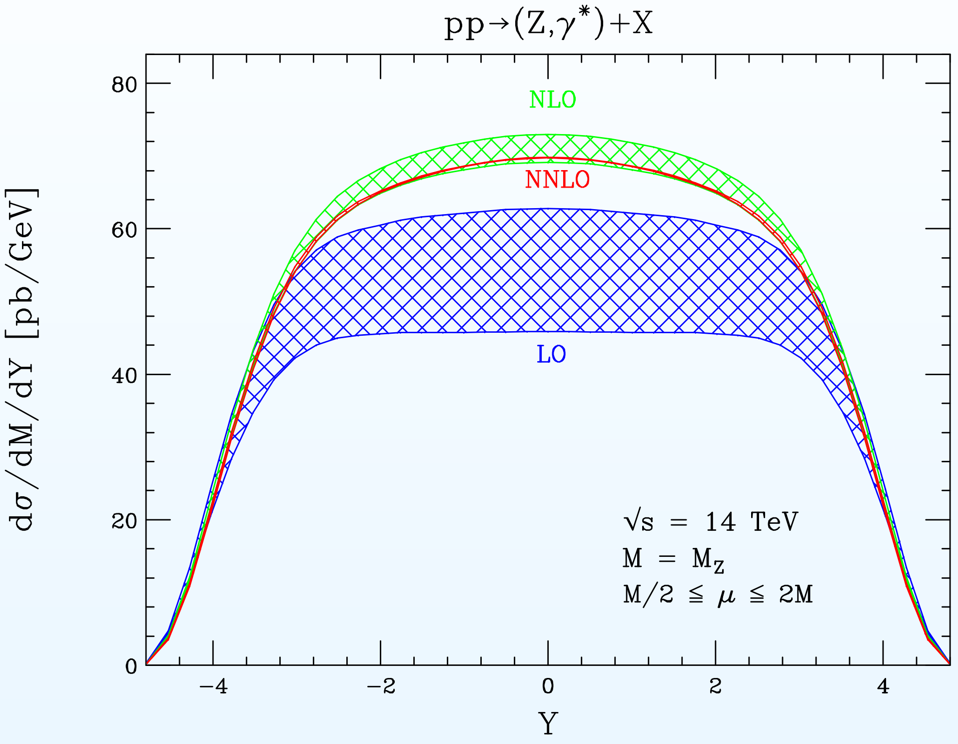
- Alekhin fits to DIS; DY rapidity distribution is the prediction.
- PDF uncertainties are large.

Z at the Tevatron



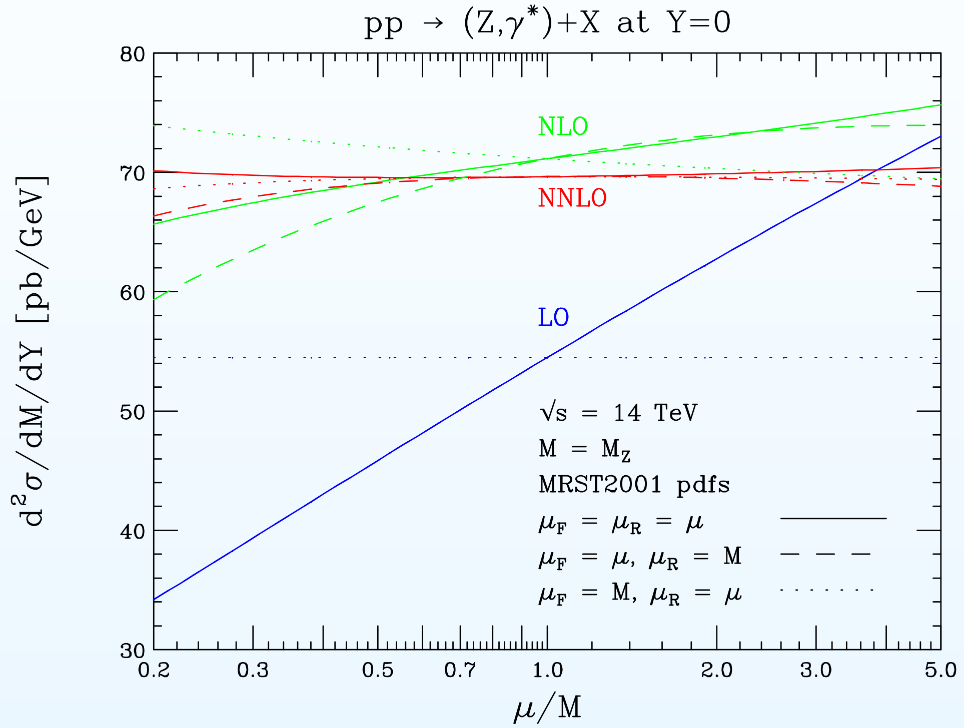
- Unnaturally small scale dependence at LO (c.f. large shift from LO to NLO).
- The width of the NNLO band is 1%.
- Both Alekhin and MRST are consistent with the data (given the error bars).

Z at the LHC



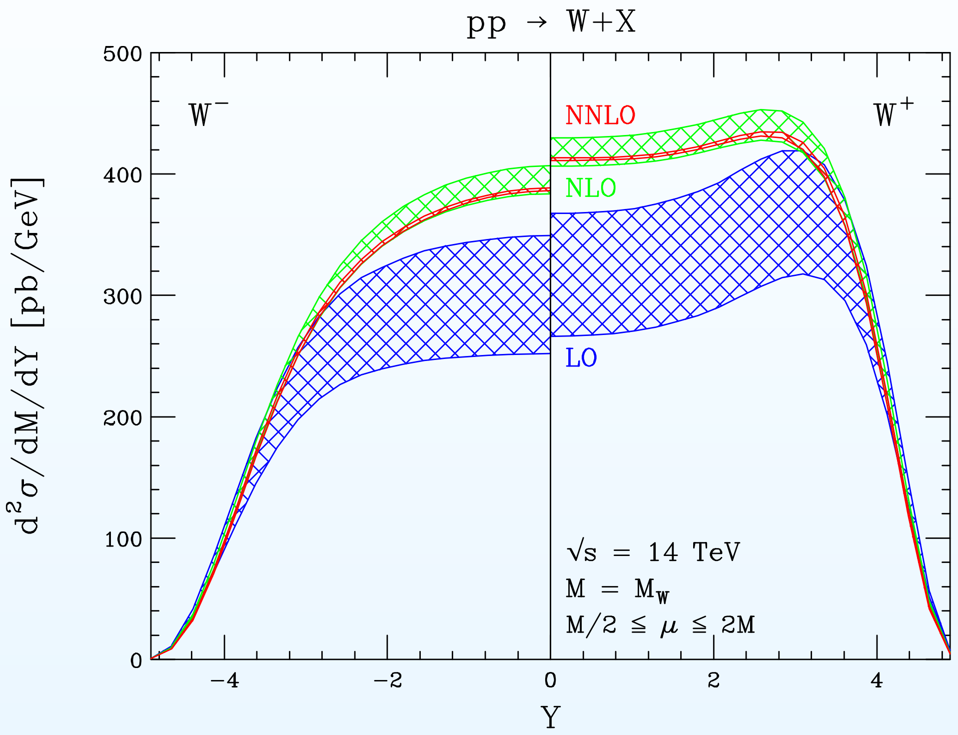
- Notice remarkable scale stability at NNLO (the width of the NNLO band is 0.2%.
- No uncertainty from perturbative QCD is left.

Z at the LHC



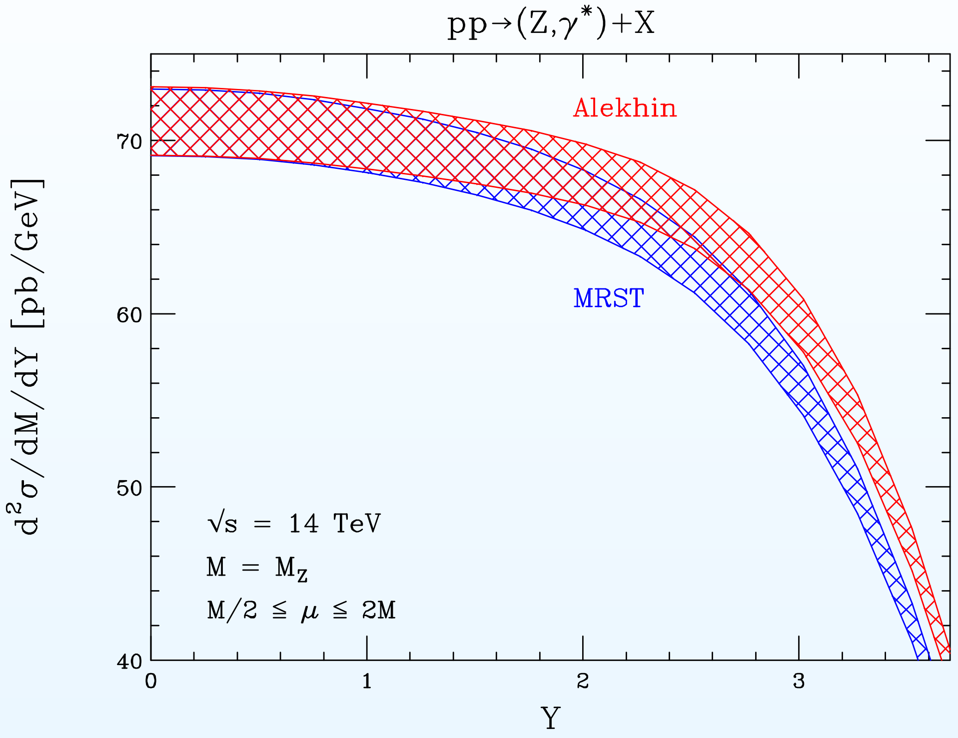
- A more detailed investigation of the scale variation.

$W^{\pm}$ at the LHC



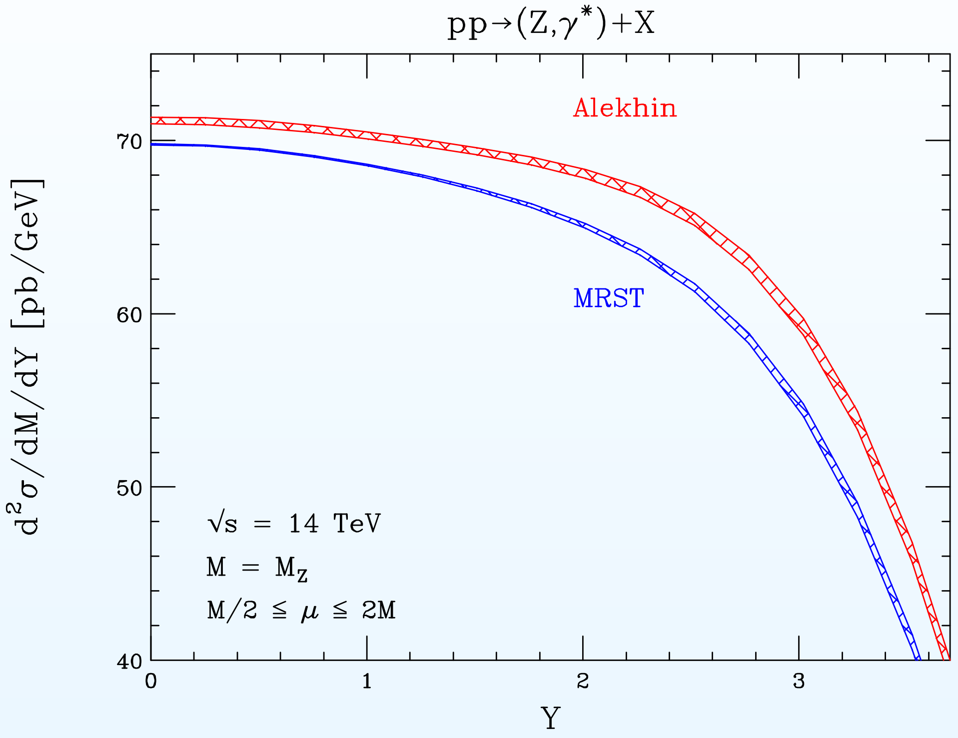
- Very good stability; no QCD uncertainty.
- Different distribution shapes for $W^{\pm}$.
- $W^{\pm}$ charge asymmetry is very stable agains higher order QCD effects and PDF uncertainties.

PDF uncertainties and the LHC



- Let us treat different PDFs (MRST, CTEQ, Alekhin) as different models; can we distinquish between them at the LHC given projected error bars?
- No, if the NLO QCD theory is used; the scale uncertainty is too large.

PDF uncertainties and the LHC



- With the NNLO QCD theory, the scale dependence is gone;
- This makes the PDF uncertainty the largest theory uncertainty.

Features of the result

- Huge ($\sim 90\%$) cancellation of $q\bar{q}$ and qg at NNLO; the gg becomes relevant.
- Large cancellation between “hard” and “soft” (universal) parts of the result; “soft” contributions do not dominate.
- No difference between slow and fast PDFs evolution; $\mathrm{N}^3\mathrm{LO}$ evolution kernels for DGLAP [Moch, Vermasseren, Vogt] do not have a large impact on the prediction.
- Small corrections at the LHC is the result of the cancellation of PDFs changes and the NNLO coefficient fucntion; both are relevant.
- The major theory uncertainty is due to PDFs.
- Numerical program VRAP http://www.slac.stanford.edu/lance/Vrap

Conclusions

- New method for NNLO calculations (real radiation) in QCD; applicable to many phenomenologicaly relevant applications;
- Rapidity distributions for γ^*, W, Z:
  first NNLO calculation of any distribution in QCD for collider physics.
- E866 data/theory: too many antiquarks in the proton at moderate x in existing PDFs; the $\bar{d}/\bar{u}$ ratio at $x \geq 0.2$ is not correctly described; requires PDF re-fitting.
- QCD predictions for Z, W are possible with sub-percent precision; major uncertainty from PDFs;
- For the 1% precision, other effects like EW corrections have to be incorporated.
- Z and W production should become “standard candels” for the LHC and the Tevatron partonic luminosity monitoring.

PS. Fully differential calculations for W, Z production may be getting within reach. See my talk on the Higgs production.