# Two-loop light fermion corrections to Higgs production and decays

Alessandro Vicini

Università degli Studi di Milano and

INFN - Sezione di Milano

Brookhaven - February $4^{th}$, 2005

Work done in collaboration with

U. Aglietti, R. Bonciani, G. Degrassi

Phys.Lett.B595:423, 2004

Phys.Lett.B600:57, 2004

# Plan of the talk

- Motivations
- Computational approach

  The 2-loop master integrals

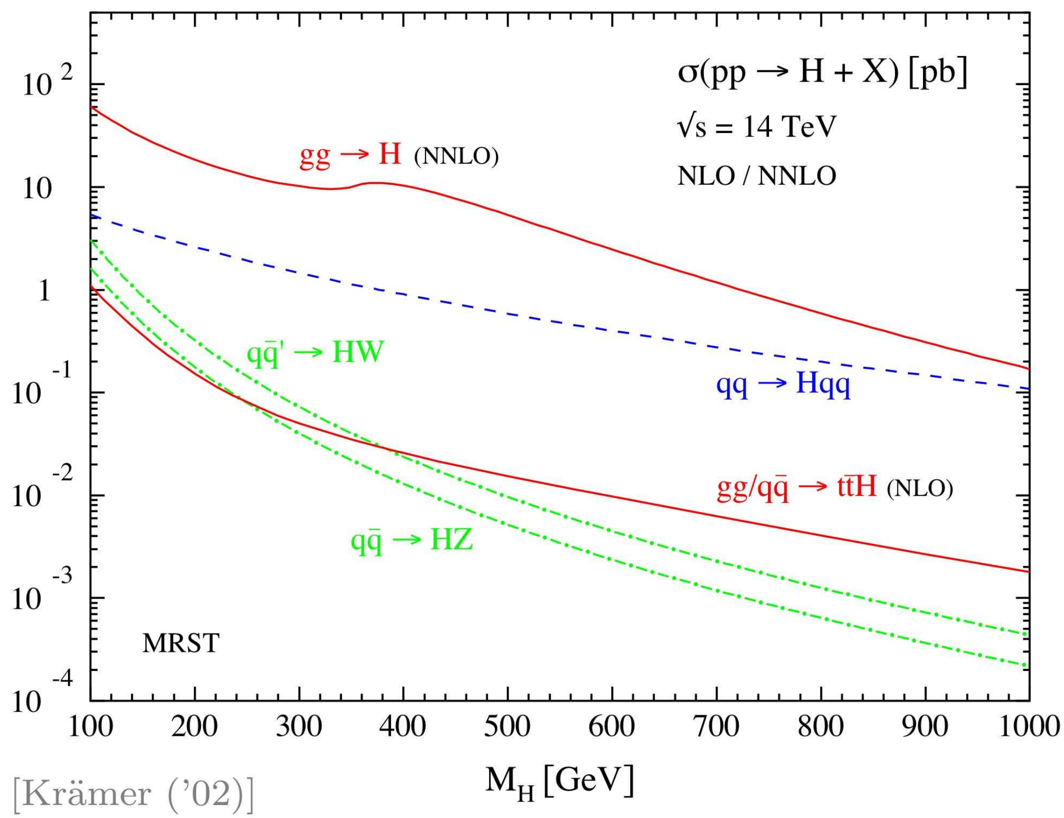
  The Harmonic Polylogarithms
- Numerical results

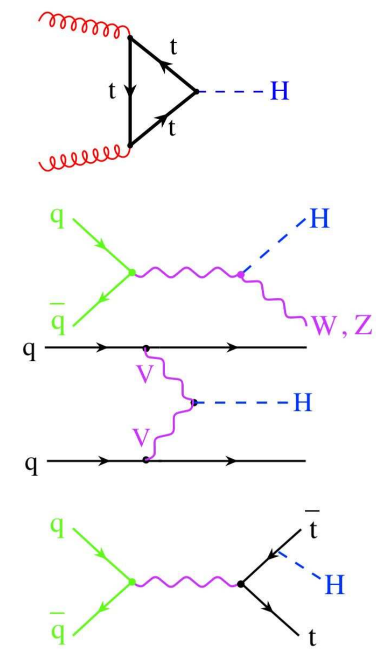
  $g\ g \rightarrow H$ (2-loop light fermions)

  $p\ p \rightarrow H + X$ (NNLO QCD+ 2-loop light fermions)

  $H \rightarrow \gamma\gamma$ (2-loop light fermions + 2-loop QCD)

# SM Higgs production at the LHC




[Krämer ('02)]

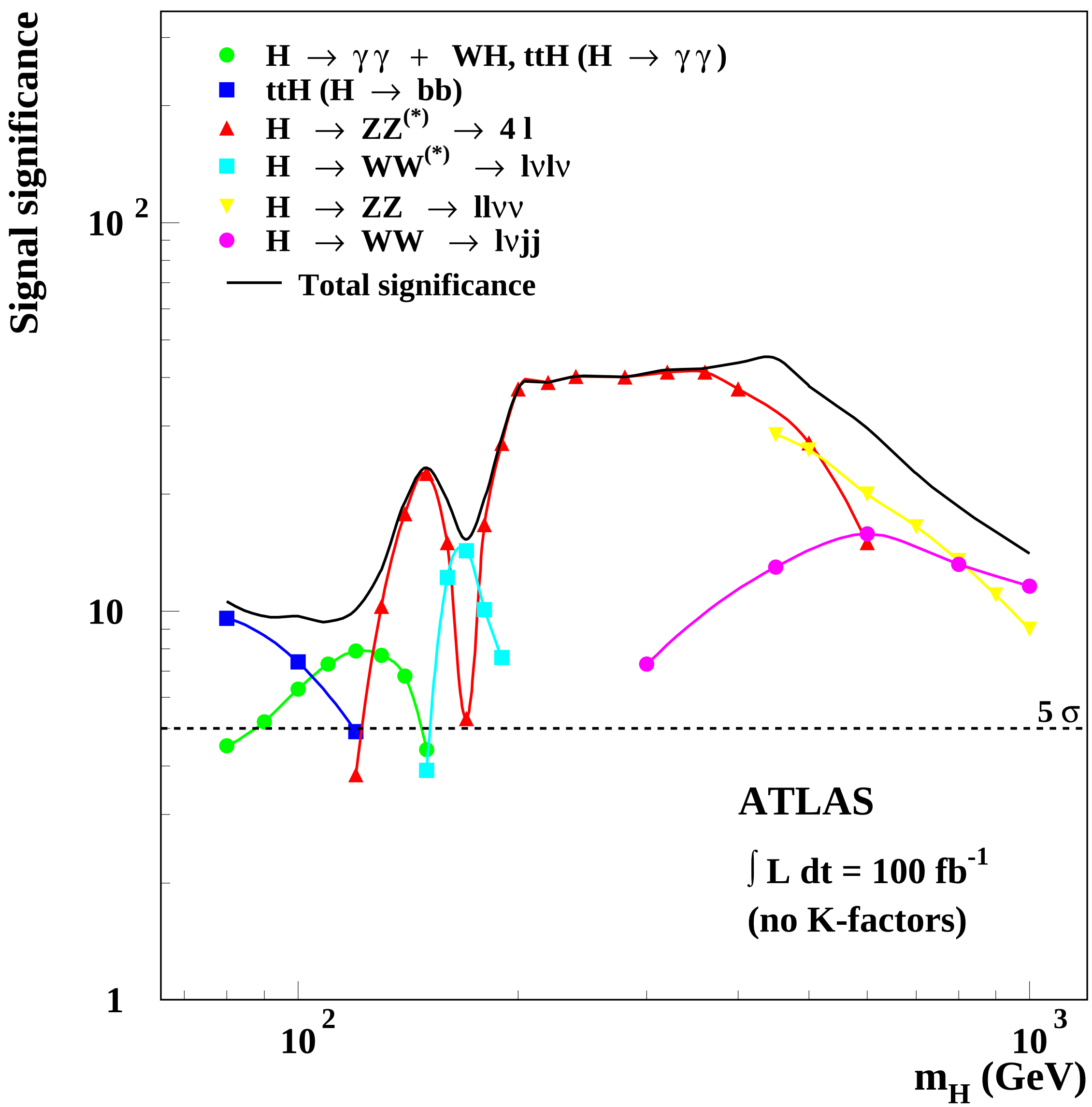
# Higgs production

The gluon fusion production mechanism has the largest rate
even if it starts at 1-loop
$\rightarrow$ need to control, at least, the first quantum corrections (i.e. 2-loop)

- $\sigma(gg \rightarrow H)$
  QCD: available at NNLO
  enhance the lowest order cross-section by 60-70 %
  residual theoretical uncertainty: below 10%
  A. Djouadi, D. Graudenz, M. Spira, P. Zerwas S.Dawson
  R. Harlander, W. Kilgore S. Catani, D. de Florian, M. Grazzini
  C. Anastasiou, K. Melnikov V. Ravindran, J. Smith, W. van Nerveen

  EW: large $m_t$ expansion: below 1%; caveat: not a well convergent expansion
  A. Djouadi, P. Gambino

# Higgs decay modes



## Decay $H \to \gamma\gamma$

The decay $H \to \gamma\gamma$ is a rare process ($BR \sim 10^{-3}$), but,
despite of its small branching ratio, it has a clear signature
$\to$ important channel if the Higgs is light
$\to$ mandatory to have an accurate and stable theoretical prediction

- $H \to \gamma\gamma$
  QCD: available at NLO, small positive correction: $\mathcal{O}(2\%)$
  H. Zheng, D. Wu   A. Djouadi, M. Spira, J. van der Bij, P. Zerwas   S. Dawson, R. Kaufmann
  K. Melnikov, O. Yakovlev   M. Inoue, R. Najima, T. Oka, J. Saito   M. Steinhauser
  J. Fleischer, O. Tarasov, V. Tarasov

  EW: $\mathcal{O}(G_\mu m_t^2)$ and $\mathcal{O}(G_\mu m_H^2)$: corrections below 1%
  Y. Liao, X. Li   J. Korner, K. Melnikov, O. Yakovlev

# Motivations

2-loop EW light fermion corrections to $\sigma(gg \to H)$ and to $\Gamma(H \to \gamma\gamma)$
gauge-invariant subset, which could be numerically not negligible
massless fermions, sum over the generations

- are EW effects relevant? may be

in view of the high accuracy reached in the QCD sector

New techniques to evaluate 2-loop Master Integrals
important check of the validity of this approach

# Evaluation of the probability amplitude

Projection of the amplitude to extract the form-factors

standard projectors

Simplification of each scalar expression to master integrals

express scalar products as propagators

exploit IBP

if no more relations among the integrals can be found,

then we consider the remaining integrals MI

Calculation of the master integrals in terms of GHPL

each MI satisfies a set of differential equations

w.r.t. its kinematical invariants

# Structure of the amplitude and projectors

process: $g(q_1,\mu)\,g(q_2,\nu)\,\rightarrow\,H$

$$T^{\mu\nu} = q_1^\mu\, q_1^\nu\, T_1 + q_2^\mu\, q_2^\nu\, T_2 + q_1^\mu\, q_2^\nu\, T_3 + q_1^\nu\, q_2^\mu\, T_4 + (q_1 \cdot q_2)\, g^{\mu\nu}\, T_5 + \epsilon^{\mu\nu\rho\sigma}\, q_{1\rho}\, q_{2\sigma}\, T_6$$

$T_6 = 0$ $gg \rightarrow H$: absence of triangle subamplitudes
$H \rightarrow \gamma\gamma$: purely imaginary, it does not contribute at this order

on-shell gluons, photons $\rightarrow T_{1,2,3}$ do not contribute
gauge invariance $\rightarrow T_4 = -T_5$

Projector to extract the contribution to $T_5$ from each Feynman diagram

$$P^{\mu\nu} = \frac{1}{(D-2)\,(q_1 \cdot q_2)} \left( g^{\mu\nu} - (q_1^\mu q_2^\nu + q_1^\nu q_2^\mu)/q_1 \cdot q_2 \right)$$

# Integration by parts identities (IBP)

$$\int d^n k \frac{\partial}{\partial k_\mu} \left( \frac{\left(k^\mu, q_i^\mu, \ldots\right) \, (k \cdot q_1)^{\alpha_1} \cdots (k \cdot q_n)^{\alpha_n}}{[k^2 - m_0^2]^{\beta_0} [(k+q_1)^2 - m_1^2]^{\beta_1} \cdots [(k+q_n)^2 - m_n^2]^{\beta_n}} \right) = 0$$

Topology = assignement of $q_i$, $m_i$, all $\beta_i \neq 0$

The IBPs relate integrals of a given topology with different sets of indices $\alpha_i, \beta_j$ among themselves and to simpler subtopologies (i.e. some $\beta_i = 0$)

e.g. $(2,1) = c_1 \ (1,1) + c_2 \ (1,0) + c_3 \ (0,1)$

$(1,2) = c_4 \ (1,1) + c_5 \ (1,0) + c_6 \ (0,1)$

Useful when writing the differential equations for one MI
to express the differential equation in terms of the MI+known functions

In some cases (e.g. specific choices of masses and momenta)
the system of the IBPs expresses an integral as a combination only of simpler subtopologies (i.e. it is not a MI)

# Differential equations for the Master Integrals

Any amplitude F satisfies a (set of) differential equation(s) $\boxed{\dfrac{\partial F}{\partial s_k} = rhs}$

w.r.t. its kinematical invariants: e.g. $s_1 = q_1^2,\, s_2 = q_2^2,\, s_3 = (q_1 - q_2)^2$

We obtain the eqs. inverting the following system

$$q_i^\mu \frac{\partial F}{\partial q_j^\mu} \;=\; q_i^\mu \sum_k \left( \frac{\partial s_k}{\partial q_j^\mu} \right) \frac{\partial F}{\partial s_k}$$

We explicitly evaluate the derivatives $\frac{\partial F}{\partial q_j^\mu}$

and use the IBPs to reduce all integrals with higher exponents $\alpha_i, \beta_i$

to a combination either of the MIs, or of simpler topologies.

An amplitude may have in general more than one MI

$\rightarrow$ system of (one or more) (coupled) first order linear differential equations

$$\begin{cases} \frac{\partial}{\partial s_k} I_1 \;=\; a_{11}\, I_1 + \cdots + a_{1N}\, I_N + (simpler\, topologies)_1 \\ \vdots \\ \frac{\partial}{\partial s_k} I_N \;=\; a_{N1}\, I_N + \cdots + a_{NN}\, I_N + (simpler\, topologies)_N \end{cases}$$

This system can be solved by means of the Euler's variation of the constant method.

The inhomogeneous term is always exactly known.

The method to solve the differential equations is constructive:
starting from the tadpole, we derive eqs. for the self-energy, vertex, box,...

The inhomogeneous terms contain simpler topologies
If we keep them in their integral representation,
$\rightarrow$ the solution of the diff.eq. is a repeated integral with one extra-integration:

$$F = \int ds f(s) \int ds_1 g_1(s_1) \cdots \int ds_n g_n(s_n)$$

## Solving the differential equations

• in $D$ dimensions: in some cases a solution for arbitrary $D$ has been found in terms of Hypergeometric functions

• expanding in $D-4$: conceptual and practical advantages

practical: renormalization $\leftrightarrow$ subtraction of the poles in $D-4$

conceptual: all the coefficients of the various terms of the expansion are functions of a class called Harmonic PolyLogarithms (HPL) or their generalization (GHPL)

order by order, all terms in the diff. eqs. are written as a repeated integral
this structure suggests the rules of an Hopf algebra

# The Harmonic Polylogarithms (HPL) Remiddi, Vermaseren, Gehrmann

Basis of functions:

$$f(-1;x) = 1/(1+x), \quad f(0;x) = 1/x, \quad f(1;x) = 1/(1-x),$$

Any HPL satisfies the relation

$$H(\vec{a};x) = \int_0^x dt f(a,t)\, H(\vec{b};t)$$

When $\vec{a}$ has only one component, we define

$$H(-1;x) = \log(1+x), \quad H(0;x) = \log(x), \quad H(1;x) = -\log(1-x),$$

When $\vec{a}$ has more than one component, we have a repeated integral, like e.g.

$$H(-1,0;x) = \int_0^x dt \frac{1}{t+1} \int_0^t ds \frac{1}{s}$$

The analyticity properties can be read from the functions $f(a;x)$ in each integral.

$\longrightarrow$ transparent: analytic continuation prescriptions
convergence of the power expansions (cfr.numerical evaluation)

The HPL form an Hopf algebra, i.e.

$$H(a,\vec{b};x) = H(a,x)H(\vec{b};x) - H(b_1,a,\vec{b}_{n-2};x) - \cdots - H(\vec{b},a;x)$$

# The Generalized Harmonic Polylogarithms (GHPL)

The HPLs allow to study a wide class of single-threshold physical problems,
but are not in general sufficient, when a diagram involves more than one threshold.

In our problem: two massive variables, $s$ and $m^2$ $\to$ one adimensional variable $x = s/m^2$
two thresholds, $m^2$ and $4\,m^2$, in the same diagram

$\longrightarrow$ Enlargement of the basis of elementary functions
same algebraic structure

$$f_i(a;x) = \Bigg\{ \frac{1}{x}, \frac{1}{1+x}, \frac{1}{1-x}, \frac{1}{\sqrt{x(4+x)}}, \frac{1}{\sqrt{x(4-x)}}, \frac{1}{4+x}, \frac{1}{4-x}$$

$$\frac{1}{x-\exp(i\pi/3)}, \frac{1}{x-\exp(-i\pi/3)}, \frac{1}{(1-x)\sqrt{x(4+x)}}, \frac{1}{(1-x)\sqrt{x(4-x)}}, \Bigg\}$$

This set of functions closes the algebra.

- it is not the only possible extension of the HPL
  e.g. HPL with two variables have been studied

# Numerical evaluation of the GHPL

the GHPLs can be represented as repeated integrals

$$\text{e.g. } H(-r,-4,-1;x) = \int_0^x dt \frac{1}{\sqrt{t(4+t)}} \int_0^t ds \frac{1}{4+s} \int_0^s dr \frac{1}{1+r}$$

the singularity and branch cut structure can be read from the basis functions $f(a;x)$

consistently with the Feynman prescription for the propagators,
we perform the analytic continuation as

$$H(\vec{a};x) \rightarrow H(\vec{a};x-i\varepsilon)$$

in our example, when $x > 0$ all integrands are well defined
the intervals $-1 < x \leq 0$, $-4 < x < -1$ and $x \leq -4$ have to be discussed separately
the starting integral breaks down into the sum of several terms
each with a well defined prescription

The Hopf algebra can be exploited to put in evidence the most singular term of a given GHPL, leaving the finite remnant for the numerical integration

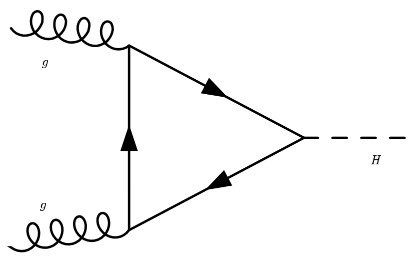
$$H(-4,-r,-1;x) = H(-4;x)H(-r,-1;x) - \int_0^x dt \frac{1}{\sqrt{t(4+t)}} H(-4;t)H(-1;t)$$

package in Mathematica, in `fortran77`, (C++ in progress)

## The gluon fusion process:

$$\sigma\,(g\,g \to H) \;=\; \frac{G_\mu \alpha_s^2}{512\,\sqrt{2}\,\pi}\left|\mathcal{G}^{1l} + \mathcal{G}^{2l}\right|^2 \,,$$

lowest order

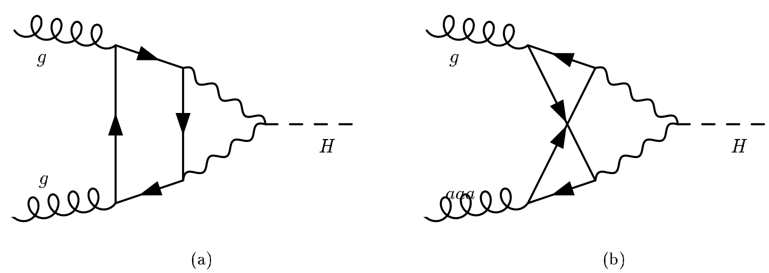



(a)

$$\mathcal{G}_t^{1l} \;=\; -4\,t_{_H}\left[2-(1-4\,t_{_H})\,H\left(-r,-r;-\frac{1}{t_{_H}}\right)\right], \qquad t_{_H} = m_t^2/m_{_H}^2$$

$$H(-r,-r;x) = \frac{1}{2}\log^2\left(\frac{\sqrt{x+4}-\sqrt{x}}{\sqrt{x+4}+\sqrt{x}}\right) \;.$$

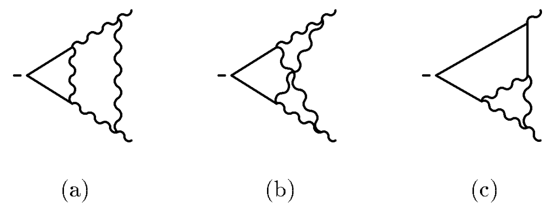
# The gluon fusion process: 2-loop corrections



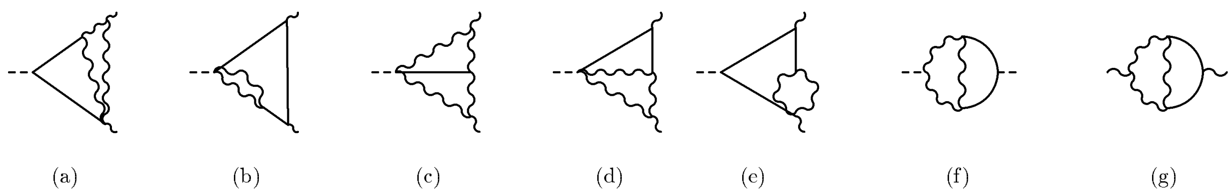

- light fermions= $u, d, c, s$ + $b$ (diags with Z-exchange)
- gauge invariant subset of Feynman diagrams
- all fermion masses set to zero
- the $WWH$ and $ZZH$ vertices avoid the Yukawa coupling suppression
- all diagrams are UV- and IR-finite

# 2-loop topologies relevant for gluon fusion and $H \to \gamma\gamma$

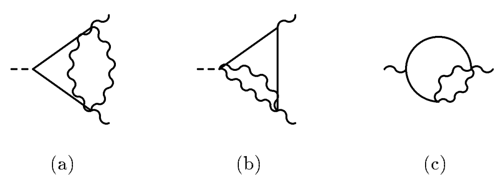
6 denominators




5 denominators

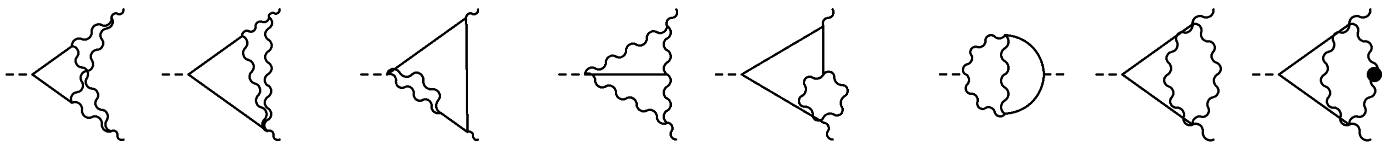



4 denominators




solid lines: massive
wavy lines: massless

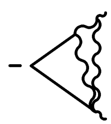
# 2-loop MI relevant for gluon fusion and $H \to \gamma\gamma$



(a) (b) (c) (d) (e) (f) (g) (h)

For each MI, analytical expression in terms of GHPLs and HPLs

Example:



$$= \left(\frac{\mu^2}{m^2}\right)^{2\epsilon} \sum_{i=-1}^{0} \epsilon^i F_i^{(7)} + \mathcal{O}(\epsilon)\,,$$

$$aF_{-1}^{(7)} = \frac{1}{2x}[3H(-r,-r,-1;x) - 2H(0,-r,-r;x) - H(0,0,-1;x)]\,,$$

$$\begin{aligned} aF_{0}^{(7)} = \frac{1}{2x}[&6H(-r,-r,-r,-r;x) - 12H(-r,-r,-1,-1;x) \\ &+6H(-r,-r,0,-1;x) - 3H(-r,-4,-r,-1;x) + 2H(0,-r,-4,-r;x) \\ &-2H(0,0,-r,-r;x) + 4H(0,0,-1,-1;x) - H(0,0,0,-1;x)]\,. \end{aligned}$$

# Analytical results for $gg \to H$

units $\alpha/(2\pi s^2)(m_{_W}^2/m_{_H}^2)$:

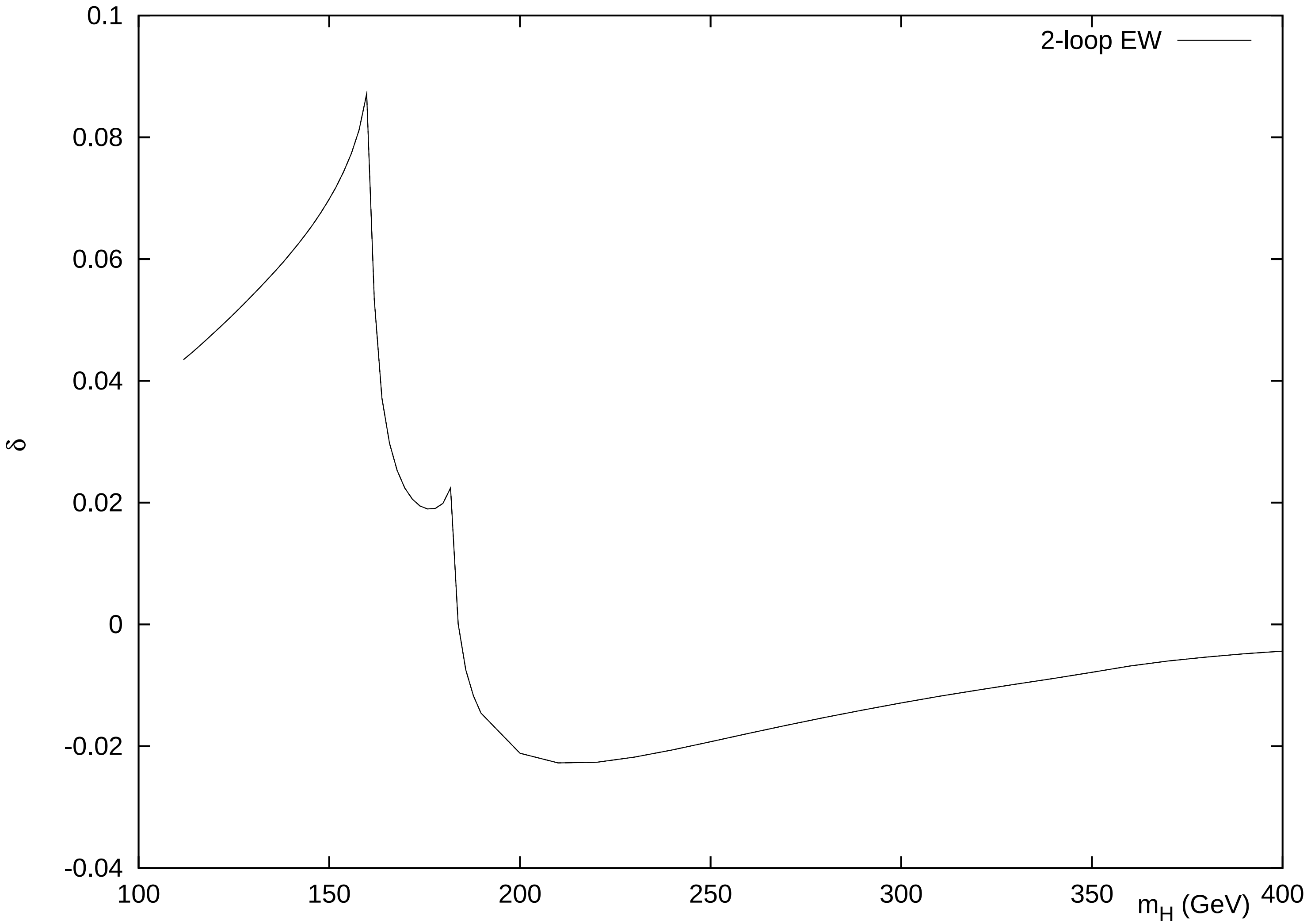
$$\mathcal{G}_{lf}^{2l} = \frac{2}{c^4}\left(\frac{5}{4} - \frac{7}{3}s^2 + \frac{22}{9}s^4\right) A_1\left[z_{_H}\right] + 4\, A_1\left[w_{_H}\right]\ ,$$

$w_{_H} \equiv m_{_W}^2/m_{_H}^2, \quad z_{_H} \equiv m_{_Z}^2/m_{_H}^2, \quad s^2 \equiv \sin^2\theta_W, \quad c^2 = 1 - s^2$

$$\begin{aligned} A_1[x] \;=\; & -4 + 2\,(1-x)\,H\left(-1;-\frac{1}{x}\right) - 2\,x\,H\left(0,-1;-\frac{1}{x}\right) + 2\,(1-3\,x)\,H\left(0,0,-1;-\frac{1}{x}\right) \\ & + 2\,(1-2\,x)\,H\left(0,-r,-r;-\frac{1}{x}\right) - 3\,(1-2\,x)\,H\left(-r,-r,-1;-\frac{1}{x}\right) \\ & - \sqrt{1-4\,x}\,\left[2\,H\left(-r;-\frac{1}{x}\right) - 3\,(1-2\,x)\,H\left(-4,-r,-1;-\frac{1}{x}\right)\right. \\ & \left. + 2\,(1-2\,x)\,H\left(-r,0,-1;-\frac{1}{x}\right) + 2\,(1-2\,x)\,H\left(-r,-r,-r;-\frac{1}{x}\right)\right]\ . \end{aligned}$$

$\sigma(gg \to H) = \sigma_0\,(1+\delta)\,, \qquad \Gamma\,(H \to gg) = 8m_H^3/\pi^2\;\sigma(gg \to H)$

$\sigma_0$ is the lowest order (1-loop) cross-section



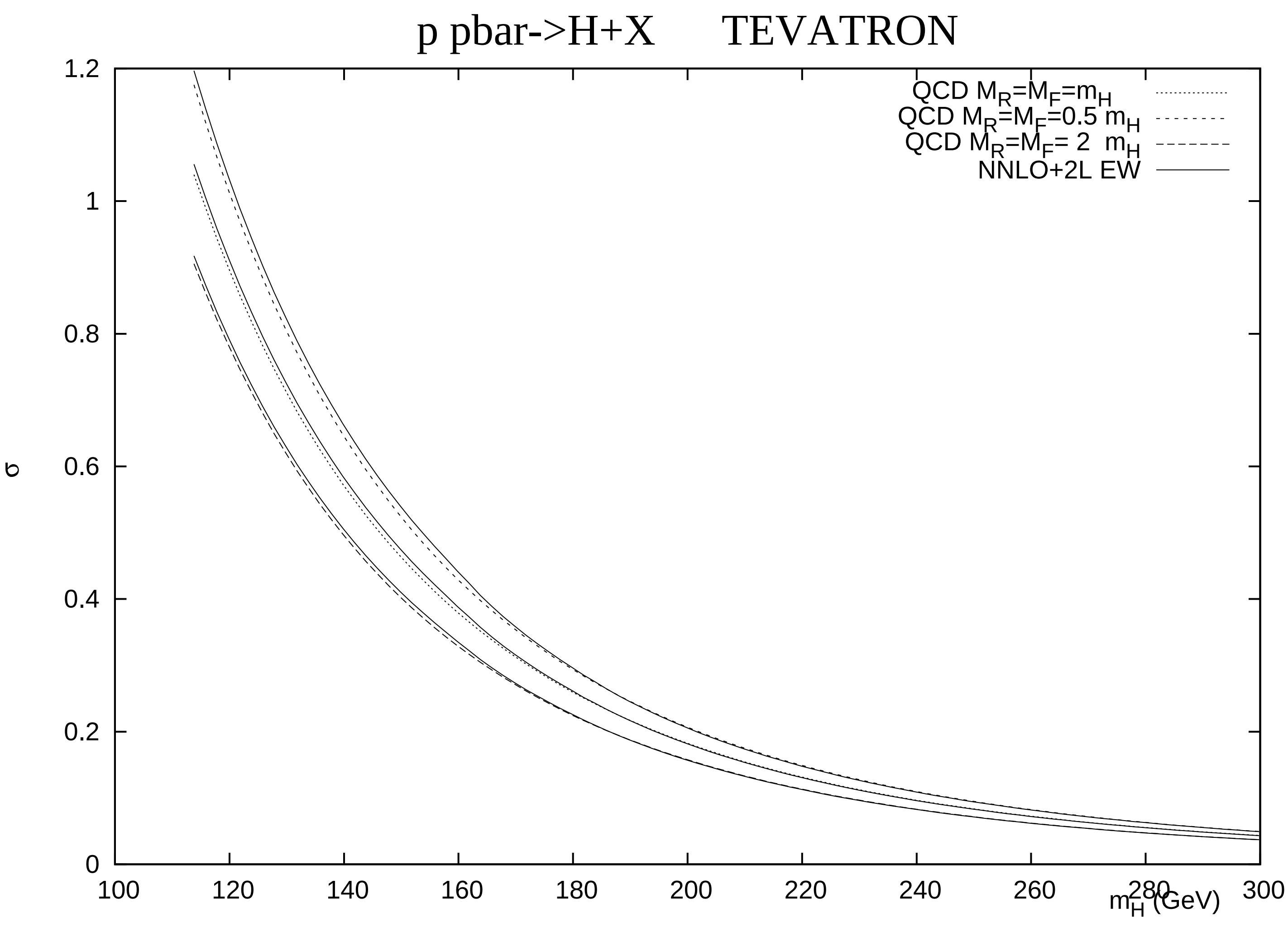

Relative corrections to the production cross section $\sigma(gg \to H)$ and to the decay width $\Gamma\,(H \to gg)$ given by 2-loop light-fermion corrections

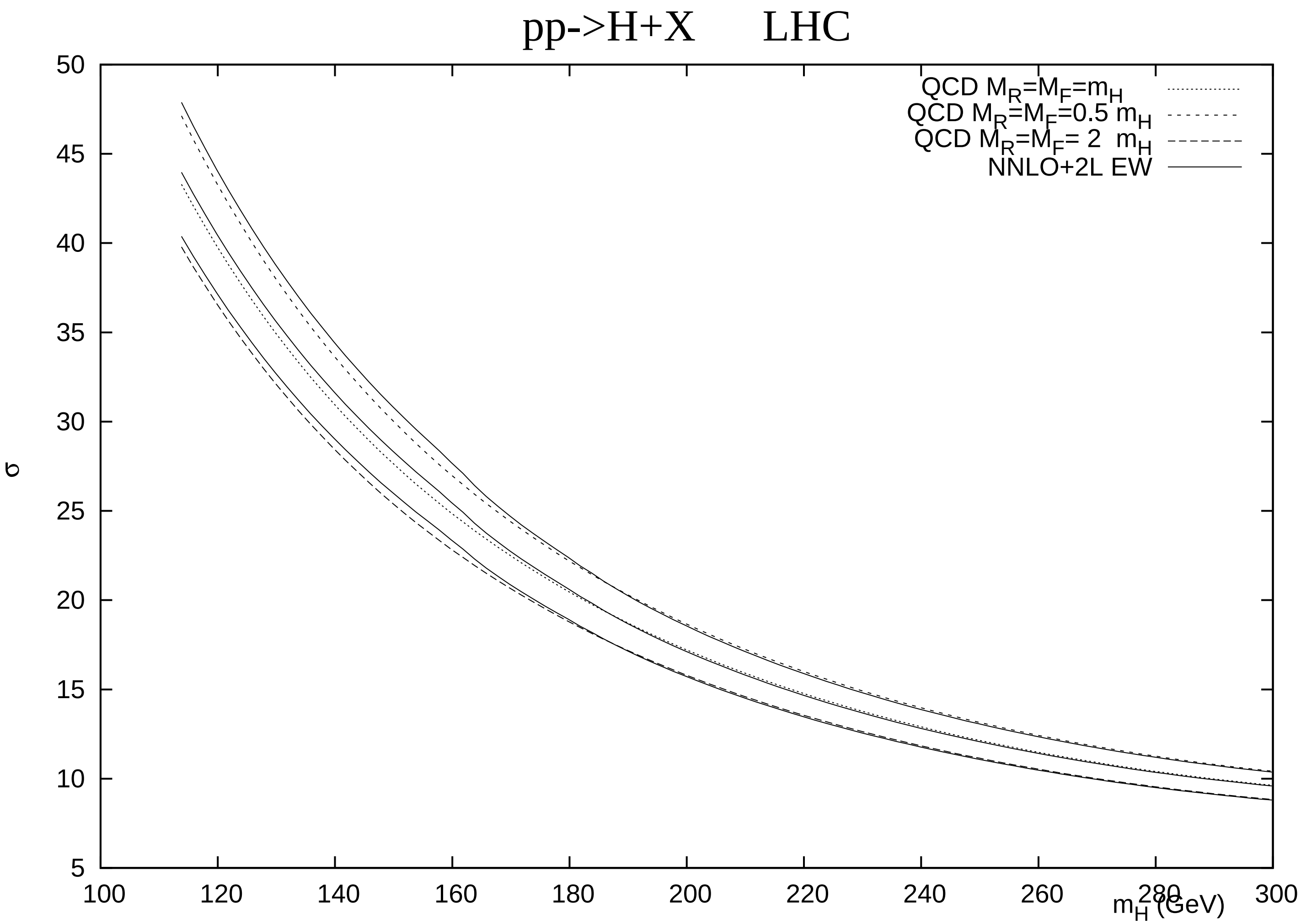
# Cross-section $\sigma(pp \to H)$ NNLO QCD+2-loop EW

Thanks to: M. Grazzini, for the fortran code for Higgs-production at NNLO QCD based on S. Catani, D. de Florian, M. Grazzini, JHEP 0105:025,2001, JHEP 0201:015,2002

$\sigma(pp \to H)$ evaluated with MRST 2002 NLO and NNLO

$$\begin{aligned}
\sigma(pp \to H + X) &= \sum_{a,b} \int_0^1 dx_1 dx_2 \; f_{a,p}(x_1, M^2)\, f_{b,p}(x_2, M^2) \; \int_0^1 dz \delta \left( z - \frac{\tau_H}{x_1 x_2} \right) \hat{\sigma}_{ab}(z) \\
\hat{\sigma}_{gg}(z) &= \hat{\sigma}_0 \left( 1 + K_{gg}(\alpha_s(\mu^2), \mu^2, M^2, \alpha_{em}) \right) \\
K_{gg} &= \left( c_1 \frac{\alpha_s(\mu^2)}{\pi} + c_2 \left( \frac{\alpha_s(\mu^2)}{\pi} \right)^2 + d_1 \frac{\alpha_{em}}{2\pi} \right) \delta(1-z) + \\
&\quad + \left( k_1 \frac{\alpha_s(\mu^2)}{\pi} + k_2 \left( \frac{\alpha_s(\mu^2)}{\pi} \right)^2 \right)
\end{aligned}$$


p pbar->H+X TEVATRON
QCD M_R=M_F=m_H
QCD M_R=M_F=0.5 m_H
QCD M_R=M_F= 2 m_H
NNLO+2L EW
σ
m_H (GeV)
0
0.2
0.4
0.6
0.8
1
1.2
100
120
140
160
180
200
220
240
260
280
300



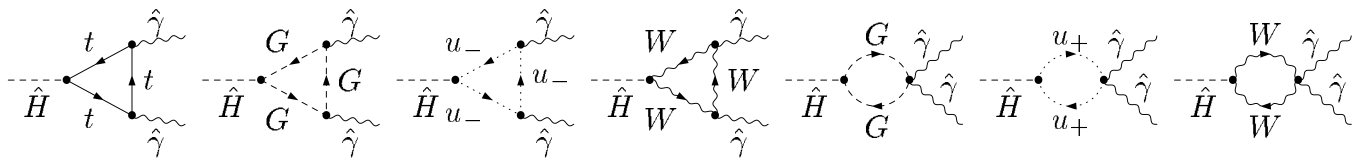
pp->H+X LHC
QCD M_R=M_F=m_H
QCD M_R=M_F=0.5 m_H
QCD M_R=M_F= 2 m_H
NNLO+2L EW
σ
m_H (GeV)
50
45
40
35
30
25
20
15
10
5
100
120
140
160
180
200
220
240
260
280
300

## Top corrections to $g\ g \to H$ G. Degrassi, F. Maltoni, Phys. Lett.B600:255

The relevant diagrams, including the top-bottom doublet (W-diagrams) or the top only (Z-diagrams) have been evaluated by means of a Taylor expansion valid up to the first, W-W, threshold.

Small negative correction: they reduce the 1-loop partonic cross-section of approximately 1 per cent.

# The decay $H \to \gamma\gamma$: lowest order



$$\Gamma\,(H \to \gamma\,\gamma) \;=\; \frac{G_\mu \alpha^2\, m_{_H}^3}{128\,\sqrt{2}\,\pi^3}\,|\mathcal{F}|^2\;,$$

$$\mathcal{F}^{1l} \;=\; \mathcal{F}^{1l}_{_W} + \frac{4}{9}\,N_c\,\mathcal{F}^{1l}_t\;,$$

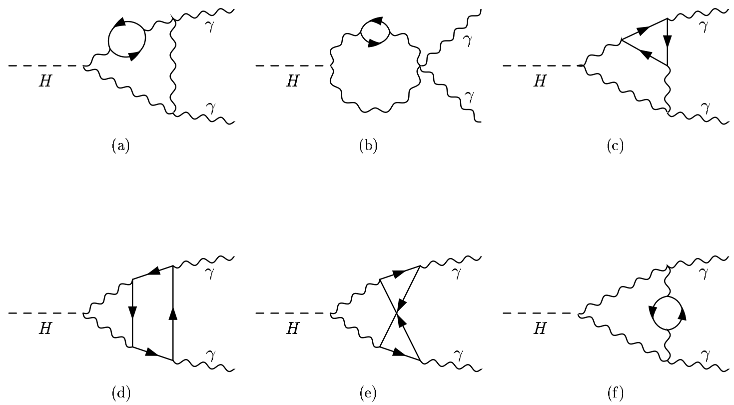
$$\mathcal{F}^{1l}_{_W} \;=\; 2\,(1+6\,w_{_H}) - 12\,w_{_H}\,(1-2\,w_{_H})\,H\left(-r,-r;-\frac{1}{w_{_H}}\right)\;,$$

$$\mathcal{F}^{1l}_t \;=\; -4\,t_{_H}\left[2-(1-4\,t_{_H})\,H\left(-r,-r;-\frac{1}{t_{_H}}\right)\right],$$

dominance of the $W$–loop (W and $t$ comparable for $m_{_H} \sim 600$ GeV)

destructive interference of $W$ and $t$

# $H \to \gamma\gamma$: 2-loop EW light fermion corrections




- light fermions= leptons, $u, d, c, s$ and $b$ (diags with Z-exchange)
- gauge invariant subset of Feynman diagrams
- all fermion masses set to zero
- the $WWH$ and $ZZH$ vertices avoid the Yukawa coupling suppression
- all diagrams are IR-finite

# The background field gauge (BFG)

Cornwall, Abbott, Denner Dittmaier Weiglein

Splitting of the fields in a classical and a quantum components

The gauge fixing breaks only the gauge invariance of the quantum part

Green's functions with external classical fields satisfy simple Ward Identities

Larger number of Feynman rules, but, cleaner rearrangement of the amplitude (usually the WI lead to useful cancelations)

The gauge fixing modifies the Feynman rules involving one or two quantum fields

In the BFG $\xi_Q = 1$ EW SM the vertex $\hat{\gamma} W^{\pm} \phi^{\mp}$ is absent.
Massless fermions have vanishing coupling with the scalars

$\rightarrow$ the bosonic lines in the loop are only the physical vectors $W$ and $Z$.

# Renormalization

The calculation of the 2-loop light fermion EW corrections to $H \to \gamma\gamma$ requires mass and coupling constants renormalization in the 1-loop amplitude

only the $W$ mass is renormalized, in the on-shell scheme

all contributions given by the external photons vacuum polarizations
$e_0 \to e$ in the couplings $\widehat{\gamma}WW$
the $HWW$ vertex is expressed in terms of $G_\mu$

The renormalization of the 1-loop amplitude requires the evaluation of counterterms and diagrams including $\mathcal{O}(D-4)$
but, all these terms exactly cancel in the physical amplitude

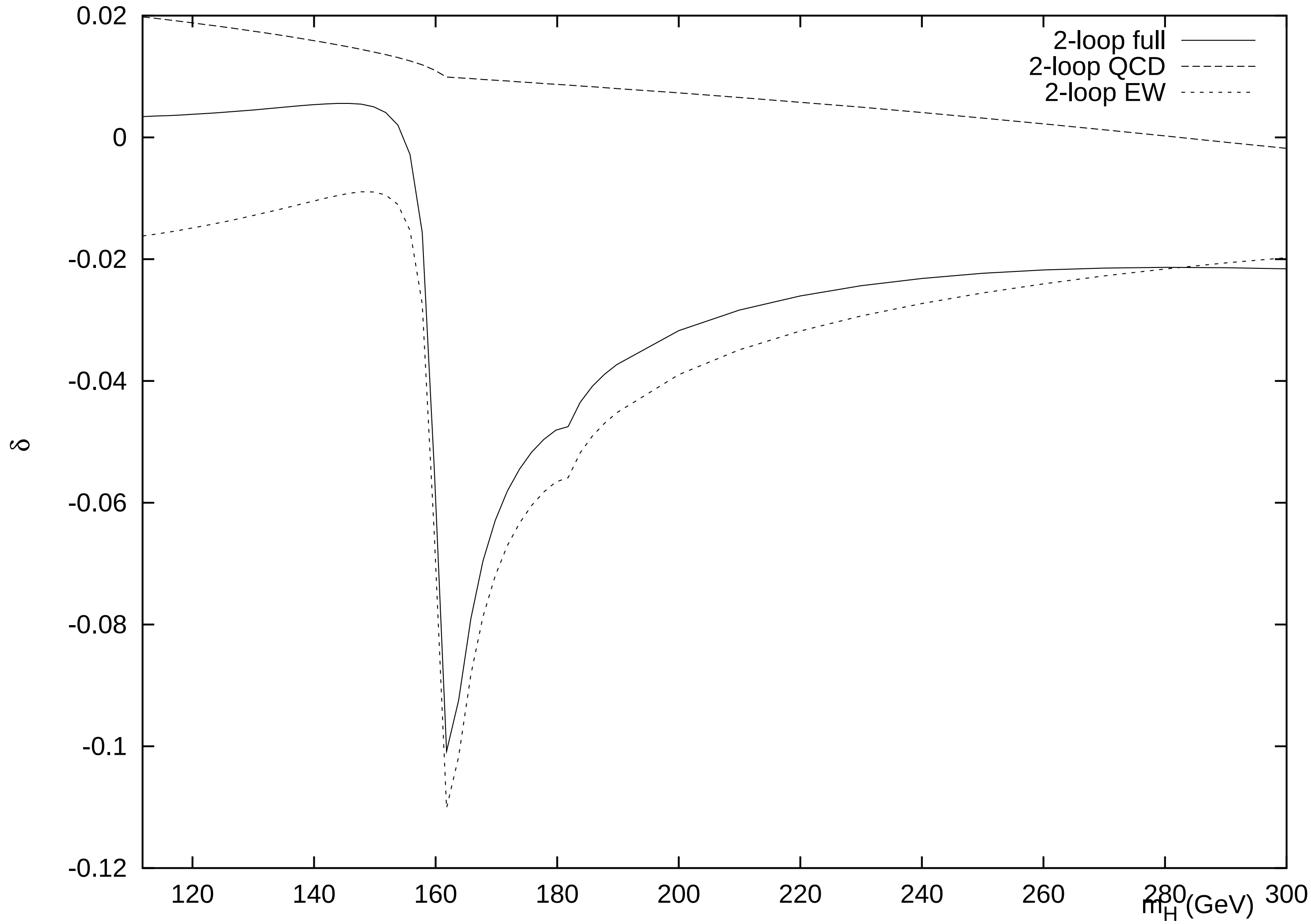
# Analytical results for $H \to \gamma\gamma$

units $\alpha/(2\pi s^2)(m_{_W}^2/m_{_H}^2)$:

$$\mathcal{F}_{lf}^{2l} = 2\,N_c\,A_2\,[-2/9, w_{_H}] + 3\,A_2\,[0, w_{_H}] + \frac{2\,N_c}{c^4}\left(\frac{11}{36} - \frac{19}{27}\,s^2 + \frac{70}{81}s^4\right)A_1\,[z_{_H}]$$
$$+\frac{3}{c^4}\left(\frac{1}{2} - 2\,s^2 + 4\,s^4\right)A_1\,[z_{_H}]\ ,$$

where

$$A_2[q,x] = -8\,(1+q) + 4\,(1+q)\,(1-x)\,H\left(-1;-\frac{1}{x}\right) - 2\,(1+2\,q\,x\ )\,H\left(0,-1;-\frac{1}{x}\right)$$
$$-\frac{2}{3}\,(5-12\,x)\,H\left(-r,-r;-\frac{1}{x}\right) - 6\,(1+q-3x-2\,q\,x)\,H\left(-r,-r,-1;-\frac{1}{x}\right)$$
$$+2\ (1+2\,q)\ \left[(1-2\,x)\ H\left(0,-r,-r;-\frac{1}{x}\right) + (1-3\,x)\ H\left(0,0,-1;-\frac{1}{x}\right)\right]$$
$$-\sqrt{1-4\,x}\,\left\{2\ (1+2\,q)\ H\left(-r;-\frac{1}{x}\right) - 6\,q\ (1-2\,x)\ H\left(-4,-r,-1;-\frac{1}{x}\right)\right.$$
$$\left.+4\,q\ (1-2\,x)\ \left[H\left(-r,0,-1;-\frac{1}{x}\right) + H\left(-r,-r,-r;-\frac{1}{x}\right)\right]\right\}$$
$$+\frac{6\,(1-2\,x)^2}{\sqrt{1-4\,x}}\,H\left(-r,-1;-\frac{1}{x}\right)\ ,$$




Relative corrections to the decay width $\Gamma(H \to \gamma\gamma)$.

# Comments

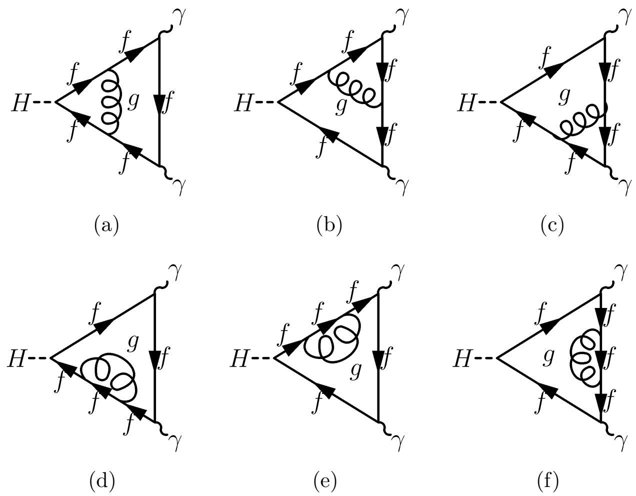
- As in the gluon fusion case,
  the corrections have a peaked structure in correspondence of the $WW$ and $ZZ$ thresholds

- The strong peak at the $WW$ threshold is due to the presence in $A_2[x]$ of a factor $1/\sqrt{1-4x}$
  which clearly diverges at the threshold (i.e. at $x = 1/4$)

- The numerical evaluation has been done introducing the decay width $\Gamma_{W}$ in the $W$ boson mass
  which regularizes the threshold divergence

- The 2-loop EW light fermion corrections almost cancel the QCD ones
  having the same size ($< 2\%$) but opposite sign

# $H \to \gamma\gamma$: 2-loop QCD corrections




- Exact analytical results
- The corrections can be evaluated for any value of $m_H$

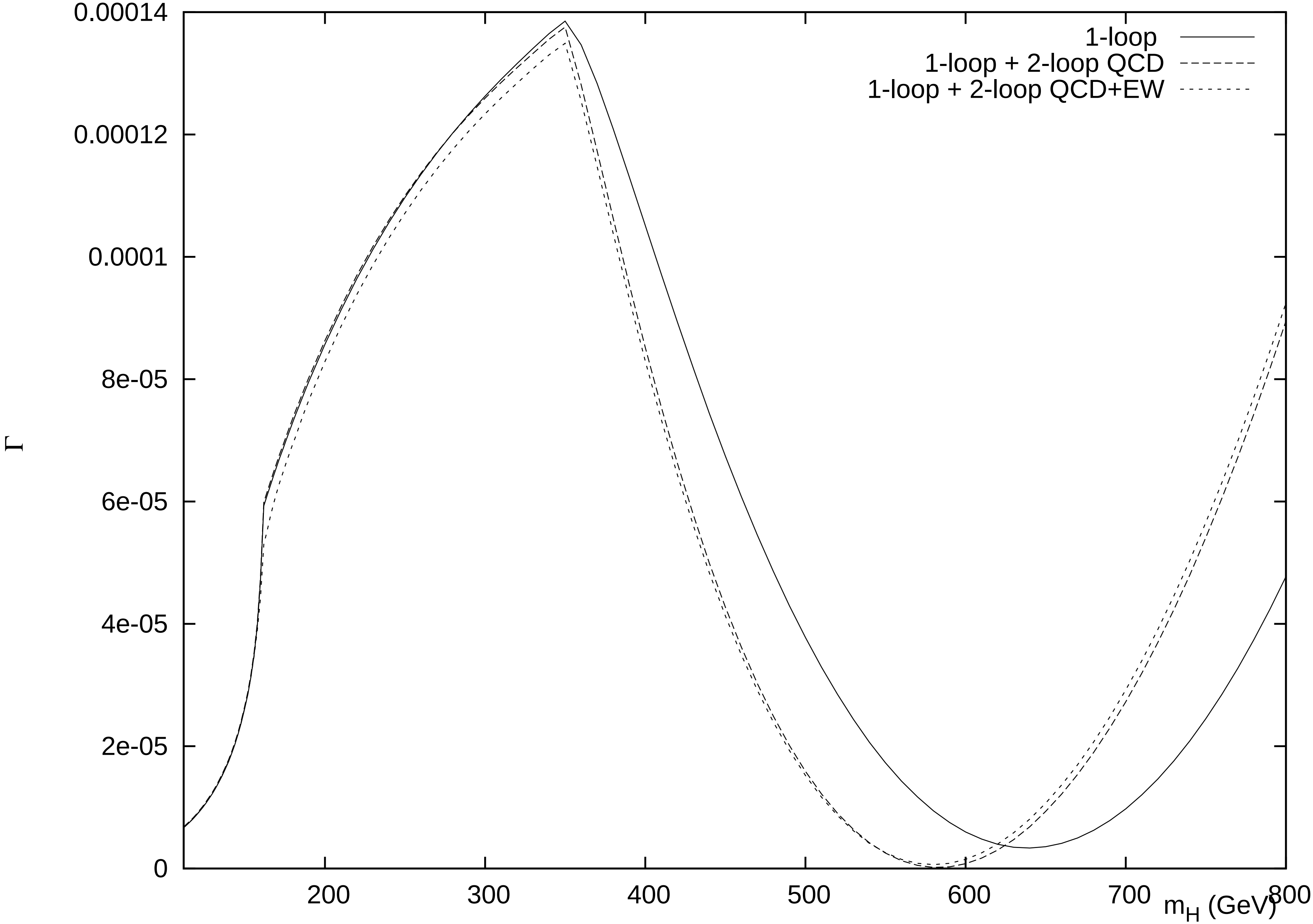
and are not restricted to the region below the $t - \bar{t}$ threshold.

- Perfect agreement with the existing literature.

Defining the variable $x$ as:

$$x = \frac{\sqrt{-m_H^2 + 4m_t^2} - \sqrt{-m_H^2}}{\sqrt{-m_H^2 + 4m_t^2} + \sqrt{-m_H^2}}. \quad (1)$$

$$
\begin{aligned}
\mathcal{F}_t^{(2l)} \quad = \quad & \frac{144\zeta^2(2)}{5(1-x)^5} - \frac{72\zeta^2(2)}{(1-x)^4} + \frac{48\zeta(3)}{(1-x)^4} + \frac{72\zeta^2(2)}{(1-x)^3} - \frac{96\zeta(3)}{(1-x)^3} - \frac{36\zeta^2(2)}{(1-x)^2} + \frac{44\zeta(3)}{(1-x)^2} + \frac{20}{(1-x)^2} \\
. \quad & + \frac{36\zeta^2(2)}{5(1-x)} + \frac{4\zeta(3)}{(1-x)} - \frac{20}{(1-x)} + \Big[\frac{64\zeta(3)}{(1-x)^5} + \frac{16\zeta(2)}{(1-x)^4} - \frac{160\zeta(3)}{(1-x)^4} - \frac{32\zeta(2)}{(1-x)^3} + \frac{160\zeta(3)}{(1-x)^3} \\
& + \frac{24}{(1-x)^3} + \frac{20\zeta(2)}{(1-x)^2} - \frac{80\zeta(3)}{(1-x)^2} - \frac{36}{(1-x)^2} - \frac{4\zeta(2)}{(1-x)} + \frac{16\zeta(3)}{(1-x)} + \frac{12}{(1-x)}\Big] H(0,x) \\
& + \Big[\frac{64}{(1-x)^4} - \frac{128}{(1-x)^3} + \frac{80}{(1-x)^2} - \frac{16}{(1-x)}\Big] H(0,-1,0,x) \\
& - \Big[\frac{64}{(1-x)^5} - \frac{160}{(1-x)^4} + \frac{160}{(1-x)^3} - \frac{80}{(1-x)^2} + \frac{16}{(1-x)}\Big] H(0,-1,0,0,x) \\
& + \Big[\frac{32\zeta(2)}{(1-x)^5} - \frac{80\zeta(2)}{(1-x)^4} - \frac{24}{(1-x)^4} + \frac{80\zeta(2)}{(1-x)^3} + \frac{48}{(1-x)^3} - \frac{40\zeta(2)}{(1-x)^2} - \frac{24}{(1-x)^2} + \frac{8\zeta(2)}{(1-x)}\Big] H(0,0,x) \\
& + \Big[\frac{128}{(1-x)^5} - \frac{320}{(1-x)^4} + \frac{320}{(1-x)^3} - \frac{160}{(1-x)^2} + \frac{32}{(1-x)}\Big] H(0,0,-1,0,x) \\
& - \Big[\frac{48}{(1-x)^5} - \frac{88}{(1-x)^4} + \frac{44}{(1-x)^3} - \frac{10}{(1-x)^2} + \frac{6}{(1-x)}\Big] H(0,0,0,x) \\
& + \Big[\frac{8}{(1-x)^5} - \frac{20}{(1-x)^4} + \frac{20}{(1-x)^3} - \frac{10}{(1-x)^2} + \frac{2}{(1-x)}\Big] H(0,0,0,0,x) \\
& - \Big[\frac{32}{(1-x)^5} - \frac{80}{(1-x)^4} + \frac{80}{(1-x)^3} - \frac{40}{(1-x)^2} + \frac{8}{(1-x)}\Big] H(0,0,1,0,x) \\
& - \Big[\frac{16}{(1-x)^4} - \frac{32}{(1-x)^3} + \frac{20}{(1-x)^2} - \frac{4}{(1-x)}\Big] H(0,1,0,x) \\
& + \Big[\frac{112}{(1-x)^5} - \frac{280}{(1-x)^4} + \frac{280}{(1-x)^3} - \frac{140}{(1-x)^2} + \frac{28}{(1-x)}\Big] H(0,1,0,0,x) \\
& + \Big[\frac{16}{(1-x)^4} - \frac{32}{(1-x)^3} + \frac{36}{(1-x)^2} - \frac{20}{(1-x)}\Big] H(1,0,0,x)
\end{aligned}
$$



1-loop
1-loop + 2-loop QCD
1-loop + 2-loop QCD+EW
0.00014
0.00012
0.0001
8e-05
6e-05
4e-05
2e-05
0
200
300
400
500
600
700
800
$m_H$ (GeV)
Γ

## Summary

2L EW light fermion corrections to $\sigma(gg \to H)$ not negligible
enhancement of the partonic cross-section up to 9%
enhancement at the hadronic level, up to 4%

2L EW light fermion corrections to $H \to \gamma\gamma$ smaller
$m_H < 161$ GeV, partially cancel the QCD terms

2L QCD corrections to $H \to \gamma\gamma$
available in analytical form for any value of $m_H$.

Analytical approach: MIs expressed in terms of HPLs and GHPLs
several cancelations in the physical amplitude
exact cancelation of all terms $\mathcal{O}(D-4)$
simple numerical implementation