Higgs boson di-photon signal at the LHC: realistic K-factors through NNLO in pQCD

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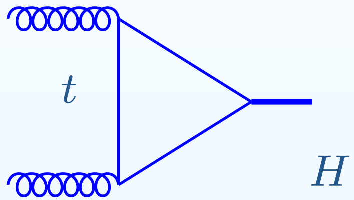
Outline

- Introduction
- Method
- Results
- Conclusions

Introduction

- Observation of the Higgs boson signal at the LHC will be an important milestone for the Standard Model.
- For the light Higgs boson, $gg \to H$ is the dominant production mechanism.



- Large ~ 1.7 NLO K-factor; good for signal-to-background ratio, not so good for the peace of mind. Dawson, Djouadi, Spria, Zerwas
- Fully inclusive Higgs production cross-section in gluon fusion is known through NNLO.

  Harlander and Kilgore
  Anastasiou and Melnikov
  van Neerven, Ravindran, Smith
- $K_{\mathrm{NNLO}} \approx 2$; improved scale stability. NNLO cross-sections match well with "threshold resummed" results

  Catani, Grazziani, de Florian

Introduction

- Inclusive Higgs production cross-section is not a realistic observable.
- For $H \to \gamma\gamma$, the following cuts on the final photons are imposed (ATLAS,CMS):
  - $p_\perp^{(1)} \geq 25 \text{ GeV}$, $p_\perp^{(2)} \geq 40 \text{ GeV}$.
  - $|\eta_{1,2}| \leq 2.5$.
  - Isolation cuts, e.g. $E_{\mathrm{T,hadr}} \leq 15 \text{ GeV}$, $\delta R = \sqrt{\delta\eta^2 + \delta\phi^2} < 0.4$.
- Do the conclusions based on inclusive calculations change when those cuts are imposed?
- Because the Higgs boson is typically produced with $E_h \sim m_h$, and $|p_\perp| \ll m_h$, inclusive calculations should be fairly accurate.
- A more detailed answer requires complete NNLO computation in the presence of a complicated "measurement" function (jet algorithms, cuts on the decay products, jet activity, isolation).

Introduction

- Until very recently, fully differential NNLO calculations were beyond theoretical capabilities.
- At NLO, well-developed, process-independent methods for exclusive computations; simple counter-terms for a single gluon emission (dipoles) that remove the infra-red and collinear singularities from the matrix elements can be constructed and used.

  Catani, Seymour and many many others.
- It is still somewhat of an open problem as to how to generalize those methods to NNLO [Campbell, Glover, Weinzierl, Gehrmann-de Ridder, Gehrmann, Kilgore, Grazzini, Frixione]. The sticky issue is the construction of the NNLO dipoles and their analytic integration over unresolved phase-spaces.
- An alternative approach is to find an algorithm for extracting infra-red and collinear singularities and to rely on numerical evaluation of both singular and non-singular terms.

  Anastasiou, K.M. , Petriello.
- The approach works. $pp \to H \to X \to \gamma\gamma + X$, fully differential w.r.t. all final state particles as an example study.

Method: what do we want

- **Goal**: fully automated, numerical method for extracting and cancelling the infra-red singularities.
  - The NNLO cross-section:

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \mathrm{d}\sigma_{VV} + \mathrm{d}\sigma_{RV} + \mathrm{d}\sigma_{RR}.$$

  - For each component, obtain an expansion:

$$\mathrm{d}\sigma_{AB} = \sum_{j=j_{\min}}^{j=4} \frac{M_j^{\mathrm{AB}}}{\epsilon^j},$$

    where M_j^{AB} are ϵ-independent and integrable throughout the phase-space.
  - M_j^{AB} can be computed numerically. Poles in ϵ cancel, when the $\mathrm{d}\sigma_{AB}$ are combined.
- The method deals with the differential cross-sections $\Rightarrow$ any cuts on the final states are allowed.
- If the fully differential cross-section for $pp \to H + X$ is available, Higgs decays are easy to incorporate.

Method: the sketch of the algorithm

- The method applies to VV, RV and RR, with minimal modifications. I focus on RR since it is where the bottleneck is.
- The algorithm:
  - map the differential phase-space onto the unit hypercube:

$$\int \prod_i \frac{\mathrm{d}p_i^{-1}}{2p_i^0} \delta^d \left(P_{\mathrm{in}} - \sum p_i \right) .... \Rightarrow \int_0^1 \prod_j \mathrm{d}x_j x_j^{-a_j \epsilon} (1 - x_j)^{-b_j \epsilon} ...$$

  - use the “sector decomposition” to disentangle overlapping singularities; Binoth, Heinrich , Denner, Roth.
  - use “plus”-distribution expansion for book-keeping:

$$\frac{1}{x^{1+a\epsilon}} = -\frac{1}{a\epsilon}\delta(x) + \left[\frac{1}{x}\right]_+ - a\epsilon \left[\frac{\ln x}{x}\right]_+ + ...$$

- The outcome: all the singularities from RR diagrams are extracted without a single integration.

Method: the phase space parameterization

- Convenient phase-space parameterization is crucial for the efficiency;
- Different parameterizations for different diagrams;
- The "energy" parameterization $(z = m_h^2/s_{\rm part})$:

$$N \int_0^1 \{{\rm d}\lambda_i\}[\lambda_1(1-\lambda_1)]^{1-2\epsilon}[\lambda_2(1-\lambda_2)]^{-\epsilon}[\lambda_3(1-\lambda_3)]^{-\epsilon}\times[\lambda_4(1-\lambda_4)]^{-\epsilon-1/2}D^{2-d};$$

$$\begin{aligned} N &= \Omega_{d-2}\Omega_{d-3}(1-z)^{3-4\epsilon}/2^{4+2\epsilon}, \\ D &= 1-(1-z)\lambda_1\left(1-\vec{n}_1\cdot\vec{n}_2\right)/2 \;> 0, \\ 1-\vec{n}_1\cdot\vec{n}_2 &= 2\left[\lambda_2+\lambda_3-2\lambda_2\lambda_3+2(1-2\lambda_4)\sqrt{\lambda_2(1-\lambda_2)\lambda_3(1-\lambda_3)}\right]. \end{aligned}$$

- Expressions for invariant masses may look complicated; the guiding principle is the simplicity of the singularity structure.

$$s_{13} = -(1-z)\lambda_1(1-\lambda_2), \quad s_{23} = -(1-z)\lambda_1\lambda_2,$$
$$s_{34} = (1-z)^2\lambda_1(1-\lambda_1)\left(1-\vec{n}_1\cdot\vec{n}_2\right)/2/D,$$

Method: the structure of singularities

- The usual way to talk about singularities is in terms of their physical origin (infra-red, collinear, UV). For our purposes this is not very relevant.
- Mathematical structure of singularities of the matrix elements is important and can be very different:
  - factorized: $\frac{1}{\lambda_1 \lambda_2}$;
  - line singularities: $\frac{1}{|\lambda_1 - f(\lambda_i)|}$;
  - entangled : $\frac{1}{(\lambda_1+\lambda_3)(\lambda_1+\lambda_2)}$;
- Factorized singularities are treated using the expansion in plus-distributions; a good phase-space parameterization puts as many as possible singular limits into a factorized form.
- "Bad" line singularities can be avoided by a variable transformation:

$$\frac{1}{|\lambda_1 - f(\lambda_i)|} \rightarrow \frac{1}{|\lambda_1 - \lambda_2'|}.$$

- Entangled singularities are disentangled by using the sector decomposition.

Method: the sector decomposition

- Consider

$$I = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^\epsilon y^\epsilon}{(x+y)^2} F_J[s_{ab}(x,y)].$$

- Split the integration region into two sub-regions $x > y$ and $y > x$. The integral I is written accordingly as the sum of two terms, $I = I_1 + I_2$. We have

$$I_1 = \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \frac{x^\epsilon y^\epsilon}{(x+y)^2} F_J[s_{ab}(x,y)], \quad I_2 = \int_0^1 \mathrm{d}y \int_0^y \mathrm{d}x \frac{x^\epsilon y^\epsilon}{(x+y)^2} F_J[s_{ab}(x,y)].$$

- Remap each integration to the unit hypercube. In I_1, make the change $y^{'} = y/x$; in I_2, set $x^{'} = x/y$. Performing these changes of variables (and rewriting $x^{'} \rightarrow x, y^{'} \rightarrow y$ for notational ease), we obtain

$$I_1 = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^{-1+2\epsilon} y^\epsilon}{(1+y)^2} F_J[s_{ab}(x,xy)], \quad I_2 = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{y^{-1+2\epsilon} x^\epsilon}{(1+x)^2} F_J[s_{ab}(xy,y)].$$

- The singularities in I_1 and I_2 are now in a factorized form, and can be extracted with the expansion in plus distributions.

Method: general comments

- The method works; you are about to see an existence proof in a few minutes.
- The method is general. The major problem is efficiency. Careful organization of the calculation becomes an important issue.
- For example, each application of sector decomposition increases the size of the expression; for Higgs production we have to consider approximately a hundred sectors.
- The method is "topological":

$$\mathcal{M}^2 \sim \frac{\mathrm{Num}(s_{ij}, F_J(s_{ij}))}{\prod s_{ab}}.$$

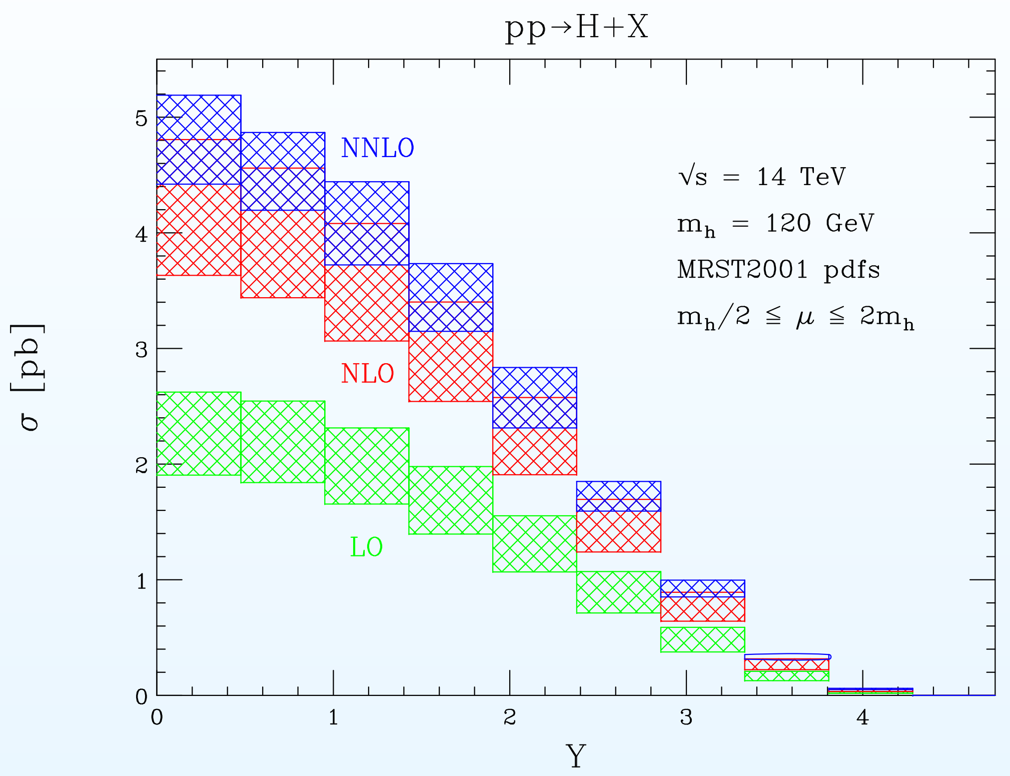
  Denominators are sector decomposed, while numerators are kept written through the invariant masses $\Rightarrow$ solution valid for all $2 \to 1$ processes.
- Not an event generator, but can be turned into one.

Numerical implementation: FEHiP

- The calculation is implemented into a FORTRAN code, FEHiP:
  http://www.phys.hawaii.edu/ kirill/FEHiP.htm
- Currently includes $H \to \gamma\gamma$ decay mode only.
- Uses VEGAS as implemented in the CUBA library Han (2004)
- For a given value of μ_f and the range of x, pdfs are interpolated by a set of quadratic polynomials when the program is initialized. Speeds up the computation enormously.
- Run times are strongly observable-dependent (3 GHz PC $\Leftrightarrow$ $\approx 10^5$ Vegas evaluations per hour). For 1% precision:

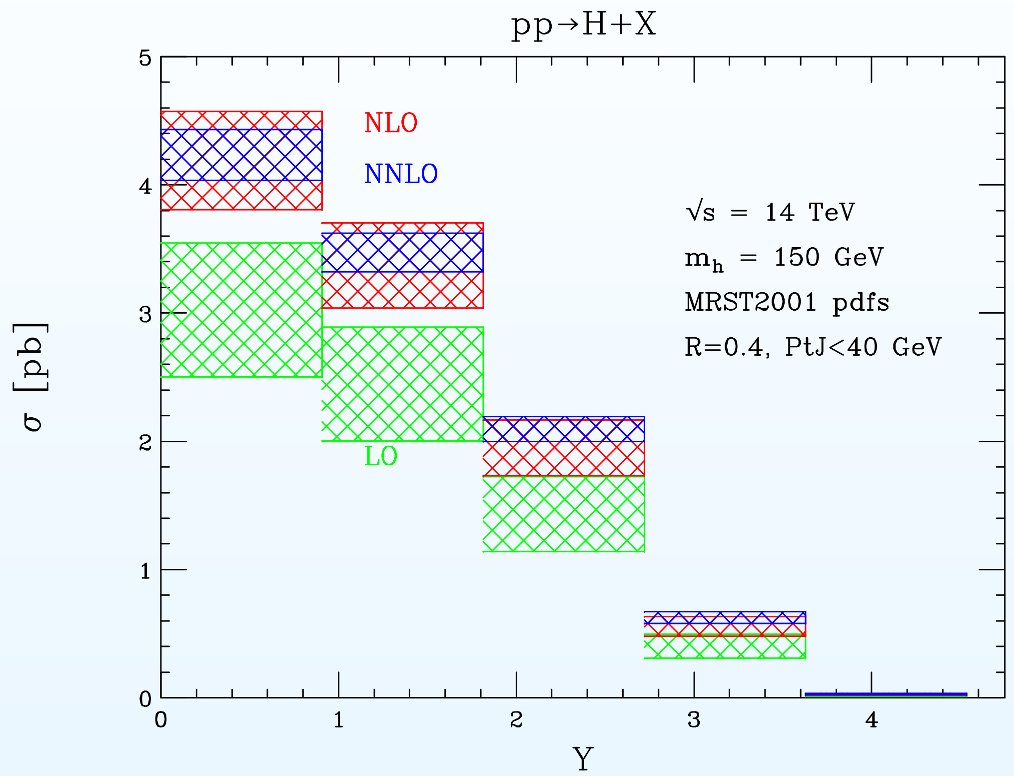
Observable	Number of evaluations	Time
$\sigma_{pp\to H}$	$1.5 - 2 \times 10^5$	1 hour
$\sigma_{pp\to H}^{\rm veto}$	8×10^5	8 hours
$\sigma_{pp\to H\to\gamma\gamma}^{\rm basic\ cuts}$	7×10^6	3 days
$\frac{{\rm d}\sigma_{pp\to H\to\gamma\gamma}^{\rm basic\ cuts}}{{\rm d}Y_\gamma}$	$\geq \times 10^7$	5 days

Results: Higgs rapidity distribution



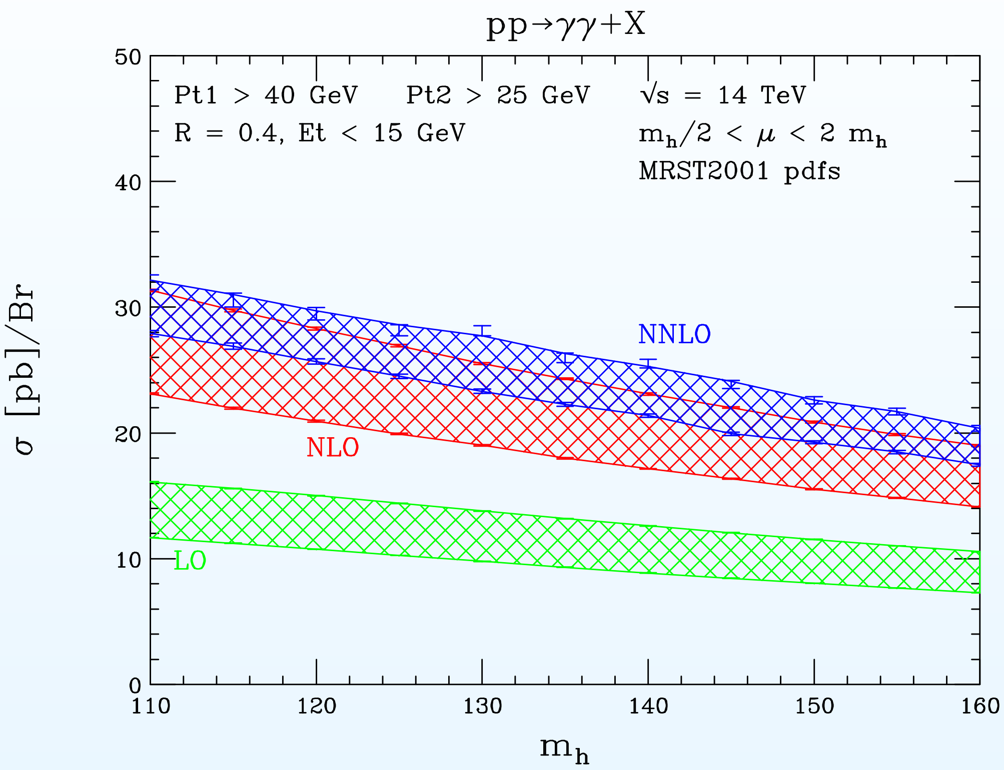
- NNLO corrections important, but things do look convergent;
- Improved stability w.r.t. scale variations;
- Insignificant rapidity dependence of the K-factor.

Results: Higgs rapidity distribution and the jet veto



- Relevant for heavier Higgs discovery ($H \to WW \to l\nu l\nu$);
- Only events with $p_{\perp}^{J} < 40 \text{ GeV}$ are accepted ⇔ cuts away $pp \to t\bar{t}$ background;
- Smaller K-factors, compared to inclusive case;
- $p_{\perp}^{H}$ increases from NLO to NNLO ($p_{\perp}^{H,\text{NLO}} = 37.6 \text{ GeV}, \;\; p_{\perp}^{H,\text{NNLO}} = 44.6 \text{ GeV}$; hence, larger fraction of the NNLO cross-section is removed.

Results: realistic di-photon cross-sections



- $p_\perp^{\gamma,1} > 40 \text{ GeV}$ and $p_\perp^{\gamma,2} > 25 \text{ GeV}$; $|\eta^{\gamma,1(2)} < 2.5$.
- Isolation cut: $E_\perp^{\text{hadr}} < 15 \text{ GeV}$ for $R < 0.4$.

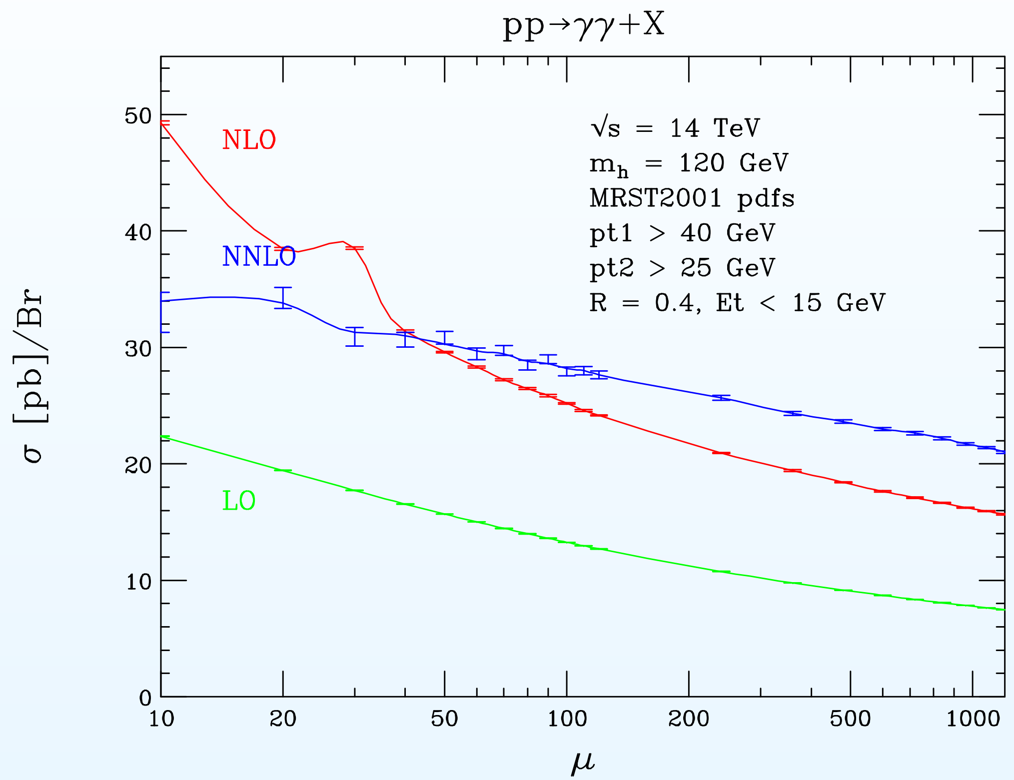
Results: realistic di-photon cross-sections

- For standard ATLAS cuts, the $pp \to H + X \to \gamma\gamma + X$ cross-section is:

m_h, GeV	$\sigma_{\mathrm{LO}}/Br_{\gamma\gamma}$, pb		$\sigma_{\mathrm{NLO}}/Br_{\gamma\gamma}$, pb		$\sigma_{\mathrm{NNLO}}/Br_{\gamma\gamma}$, pb	
	$\mu = m_h/2$	$\mu = 2m_h$	$\mu = m_h/2$	$\mu = 2m_h$	$\mu = m_h/2$	$\mu = 2m_h$
110	16.10	11.67	31.33	23.12	31.76	27.68
115	15.59	11.23	29.75	21.97	30.99	26.80
120	15.02	10.76	28.30	20.93	29.41	25.67
125	14.40	10.27	26.94	19.92	28.79	24.56
130	13.79	9.79	25.52	19.03	28.46	23.29
135	13.19	9.32	24.31	17.99	26.64	22.05

- Note, that $\mu = m_h/2$ seems to be an appropriate scale..

Results: realistic di-photon cross-sections



- Scale dependence of the di-photon signal with standard cuts suggests $\mu \sim m_h/2$.
- Choosing smaller scale makes the cross-section larger; this leads to a better agreement with the threshold resummed prediction.

Results: realistic di-photon signal

- Is it sufficient to know only inclusive K-factors?

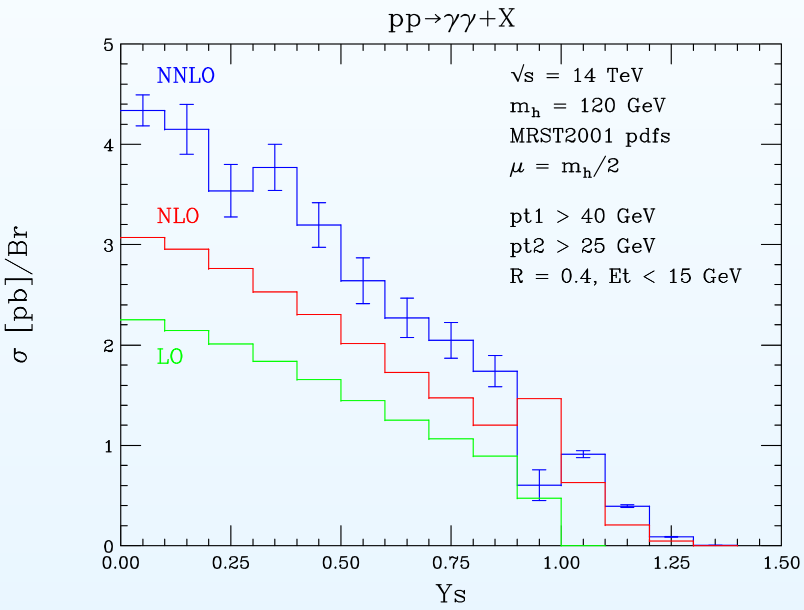
m_h, GeV	$\sigma^{\rm cut}_{\rm NNLO}/\sigma^{\rm inc}_{\rm NNLO}$	$K^{(2)}_{\rm cut}/K^{(2)}_{\rm inc}$
110	0.559	0.93
115	0.589	0.95
120	0.601	0.95
125	0.632	0.98
130	0.669	1.00
135	0.668	1.00

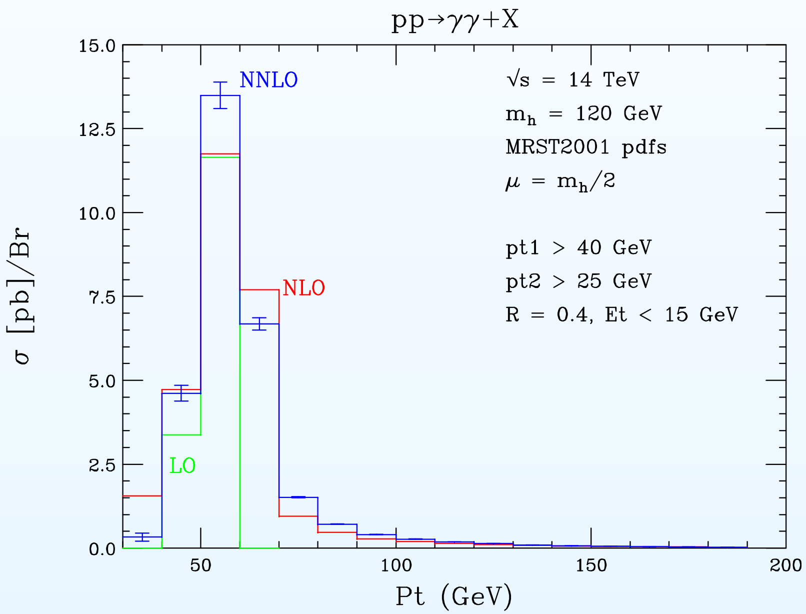
$$K^{(2)} = \frac{\sigma_{\rm NNLO}}{\sigma_{\rm NLO}}.$$
$$\mu = \frac{m_h}{2}.$$

- The ratio of inclusive to "cut" K-factors is the best guess for the NNLO differential cross-section if only inclusive NNLO K-factor and the differential NLO cross-section were available.
- The heavier the Higgs, the smaller the impact of the cuts on the K-factor is.

Results: di-photon distributions

- Rapidity and $p_\perp$ distributions of the photon can be used as additional discriminators between the signal and the background.





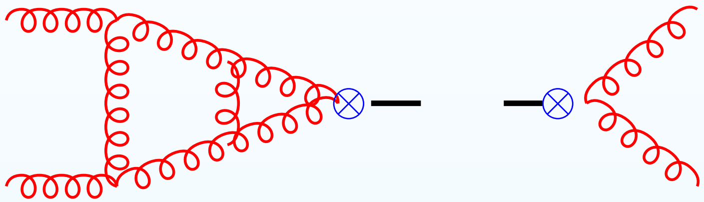
- $Y_s = |\eta^{1,\gamma} - \eta^{2,\gamma}|/2; \qquad p_t = (p_\perp^{1,\gamma} + p_\perp^{2,\gamma})/2.$
- Y_s distribution of the background is almost flat Bern, Dixon, Schmidt.
- p_t distribution of the two photons is saturated around $m_h/2 \pm 10\ \mathrm{GeV}$. The shape is stable against NNLO corrections. Can this be used to enhance signal/background ratio?

Conclusions

- New method for NNLO calculations (real radiation) in QCD; applicable to many phenomenologicaly relevant processes;
- Existence proof: first NNLO calculation of the fully differential cross-section for any process at hadron colliders;
- Complete control over the kinematics of the final states allows arbitrary cuts to be imposed;
- State-of-the-art calculation of the Higgs signal in the di-photon channel for the LHC;
- We plan to extend FEHiP to include $H \to W^+W^-$ and $H \to ZZ$ decays in the future.
- We hope FEHiP will be useful in devising strategies to enhance the signal to background ratio and facilitate the Higgs boson discovery.

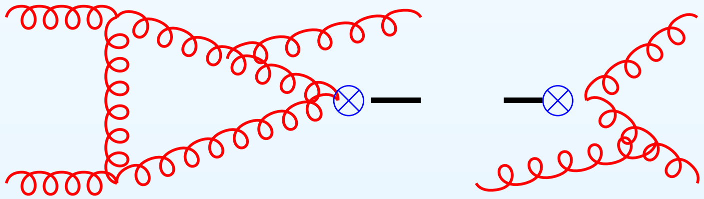
Method: NNLO contributions

- Double Virtual (V-V) (Harlander, 2000)



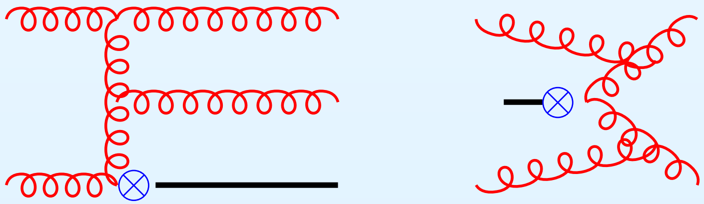
+ 148 diagrams

- Real - Virtual (V-R)



+ 559 diagrams

- Double Real (R-R)



+ 675 diagrams