Uncertainties in Sudakov Form Factors

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- Introduction, spacelike Sudakov FF in Herwig++
- Numerical study
- Conclusion

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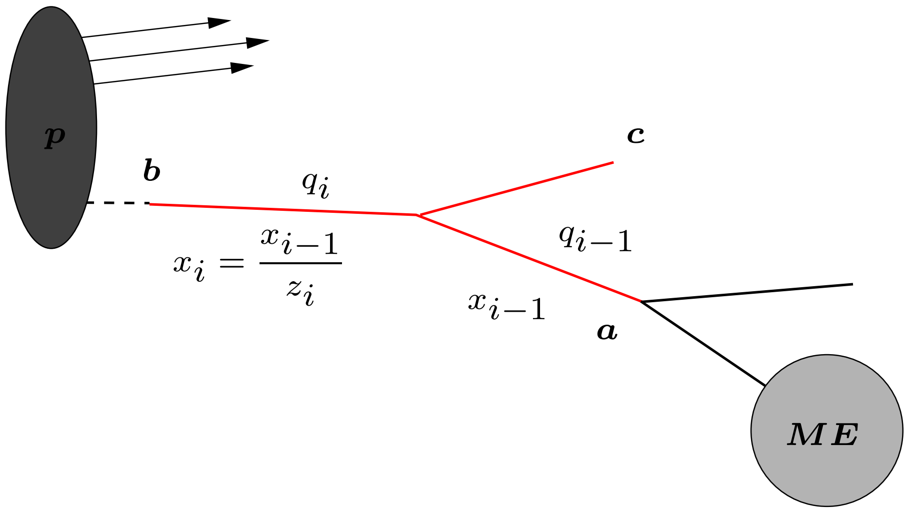
Backward branching kinematics in Herwig++

Consider only single branching $b \rightarrow ac$:



Sudakov decomposition $q_i = \alpha_i p + \beta_i n + q_{\perp i}$. Basis $(p, n)\|$ proton direction. Kinematics of shower reconstructed from

$$\alpha_i = \frac{\alpha_{i-1}}{z}, \qquad \boldsymbol{q}_{\perp i} = \frac{\boldsymbol{q}_{\perp i-1} - \boldsymbol{p}_{\perp i}}{z_i} \; .$$

$$\boldsymbol{p}^2_{\perp i} = (1 - z_i)^2 \tilde{q}^2_i - z_i Q^2_g \; .$$

Q_g closely related to parton shower cutoff.

Sudakov form factor for space–like branchings

The Sudakov form factor for spacelike backward evolution of a parton a from the hard scale $\tilde{q}_{\max}$ down to some scale $\tilde{q}$,

$$S_a(\tilde{q}, \tilde{q}_{\max}; x, \tilde{q}_0) = \exp\left[-\sum_b \mathcal{I}_{ba}(\tilde{q}, \tilde{q}_{\max}; x, \tilde{q}_0)\right] \; . \qquad (1)$$

The sum on the right hand side (rhs) is over all possible splittings into partons of type b and

$$\mathcal{I}_{ba}(\tilde{q}, \tilde{q}_{\max}; x, \tilde{q}_0) = \int_{\tilde{q}^2}^{\tilde{q}^2_{\max}} \frac{d\tilde{q}^2}{\tilde{q}^2} \int_{z_0}^{z_1} dz \frac{\alpha_S(z, \tilde{q}^2)}{2\pi} \frac{x' f_b(x', \tilde{q}^2)}{x f_a(x, \tilde{q}^2)} P_{ba}(z, \tilde{q}^2) \; . \qquad (2)$$

Choosing the argument of $\alpha_S(Q)$ as $Q = (1 - z_i)\tilde{q}_i$ we may now rewrite the integral (2) as

$$\mathcal{I}_{ba}(\tilde{q}, \tilde{q}_{\max}; x, Q_g) = \int_{\tilde{q}^2}^{\tilde{q}^2_{\max}} \frac{d\tilde{q}^2}{\tilde{q}^2} \int_0^1 dz \frac{\alpha_S[(1-z)\tilde{q}]}{2\pi} \frac{x' f_b(x', \tilde{q}^2)}{x f_a(x, \tilde{q}^2)} P_{ba}(z, \tilde{q}^2) \Theta(\mathrm{P.S.}) \; . \qquad (3)$$

Single branching type Sudakov

We only consider specific branchings $b \to ac$, i.e. formally

$$S_a(\tilde{q}, \tilde{q}_{\rm max}; x, Q_g) = \prod_b S_{ba}(\tilde{q}, \tilde{q}_{\rm max}; x, Q_g) \ .$$

And the branching probability density ("At which $\tilde{q}$ is my next branching?")

$$\mathcal{P}_{ba}(\tilde{q}, \tilde{q}_{\rm max}; x, Q_g) = \mathcal{I}'_{ba}(\tilde{q}; x, Q_g) S_{ba}(\tilde{q}, \tilde{q}_{\rm max}; x, Q_g) \ .$$

Note that this is properly normalized as

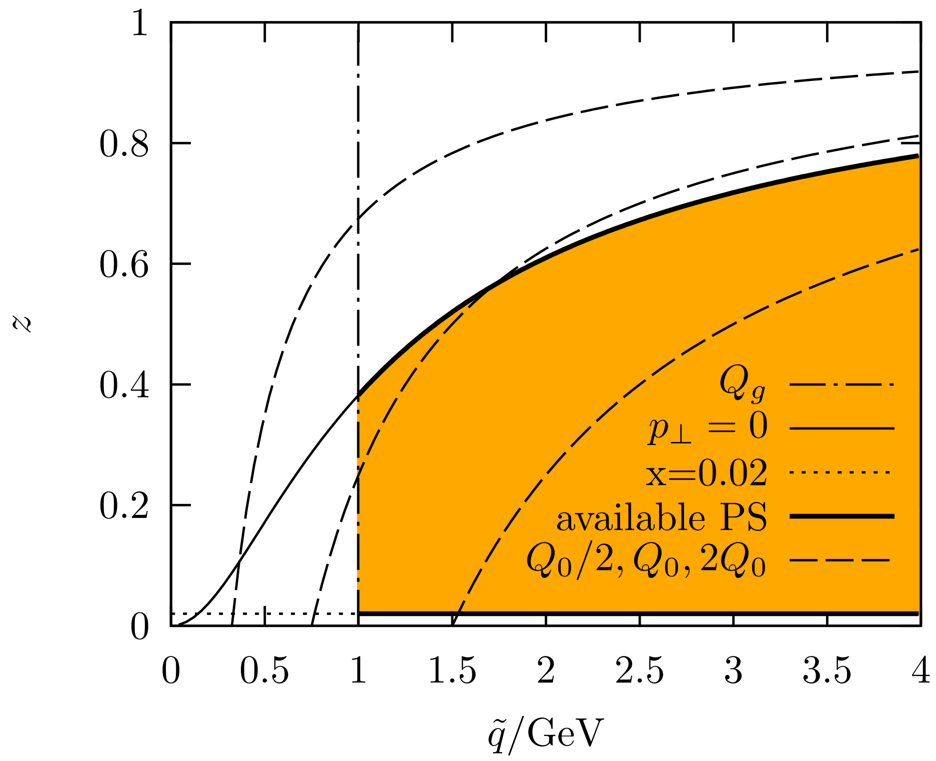
$$\int_{\tilde{q}_{\rm max}}^{\tilde{q}_0} \mathcal{P}_{ba}(\tilde{q}, \tilde{q}_{\rm max}; x, Q_g)\, d\tilde{q} = 1 - S_{ba}(\tilde{q}_0, \tilde{q}_{\rm max}; x, Q_g) \ .$$

$$= P(\text{“any branching”})$$

where, of course,

$$S_{ba}(\tilde{q}_0, \tilde{q}_{\rm max}; x, Q_g) = P(\text{“no branching”})$$

Available phase space



Limited by several factors:

- Real transverse momentum,

$$x < z < 1 + \frac{Q_g}{2\tilde{q}} - \sqrt{\left(1 + \frac{Q_g}{2\tilde{q}}\right)^2 - 1}$$

- $\tilde{q} > Q_g$
- Maybe $\alpha_S(Q) = 0$ at low $\tilde{q}$. Interesting when we vary $Q \to Q/2, 2Q$.

quite different from available phase space in standard DGLAP or Pythia

Numerical study

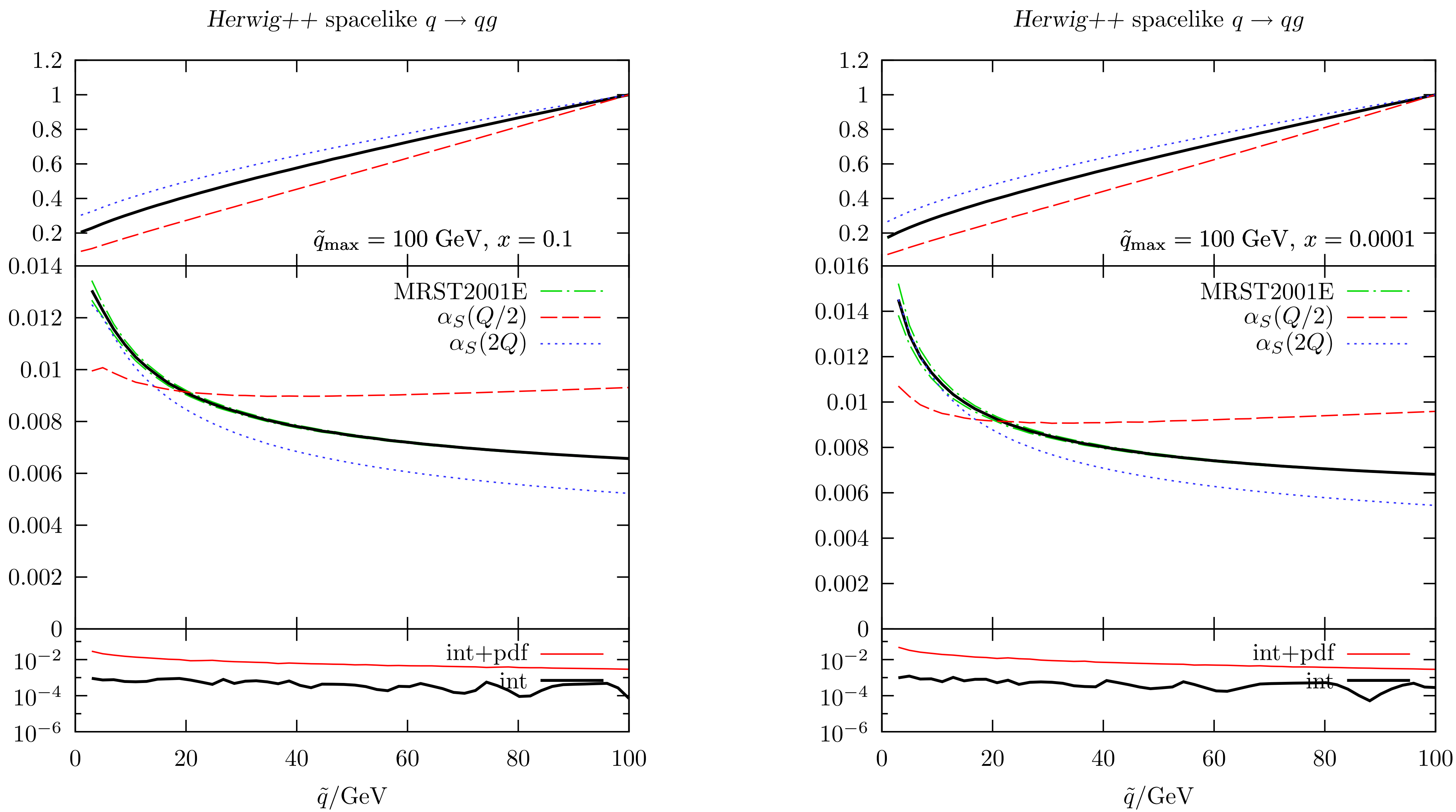
We consider

- different types of splittings.
- low and high x and $\tilde{q}_{\max}$.
- pdf errors from MRST/CTEQ,
- $\alpha_S(Q)$ errors from scale variation in comparison.
- NP treatment of $\alpha_S(Q)$.
- no study of effects beyond NLL.
- no kinematics from other generators
- strictly only first emission (vetos. . .)

We always show

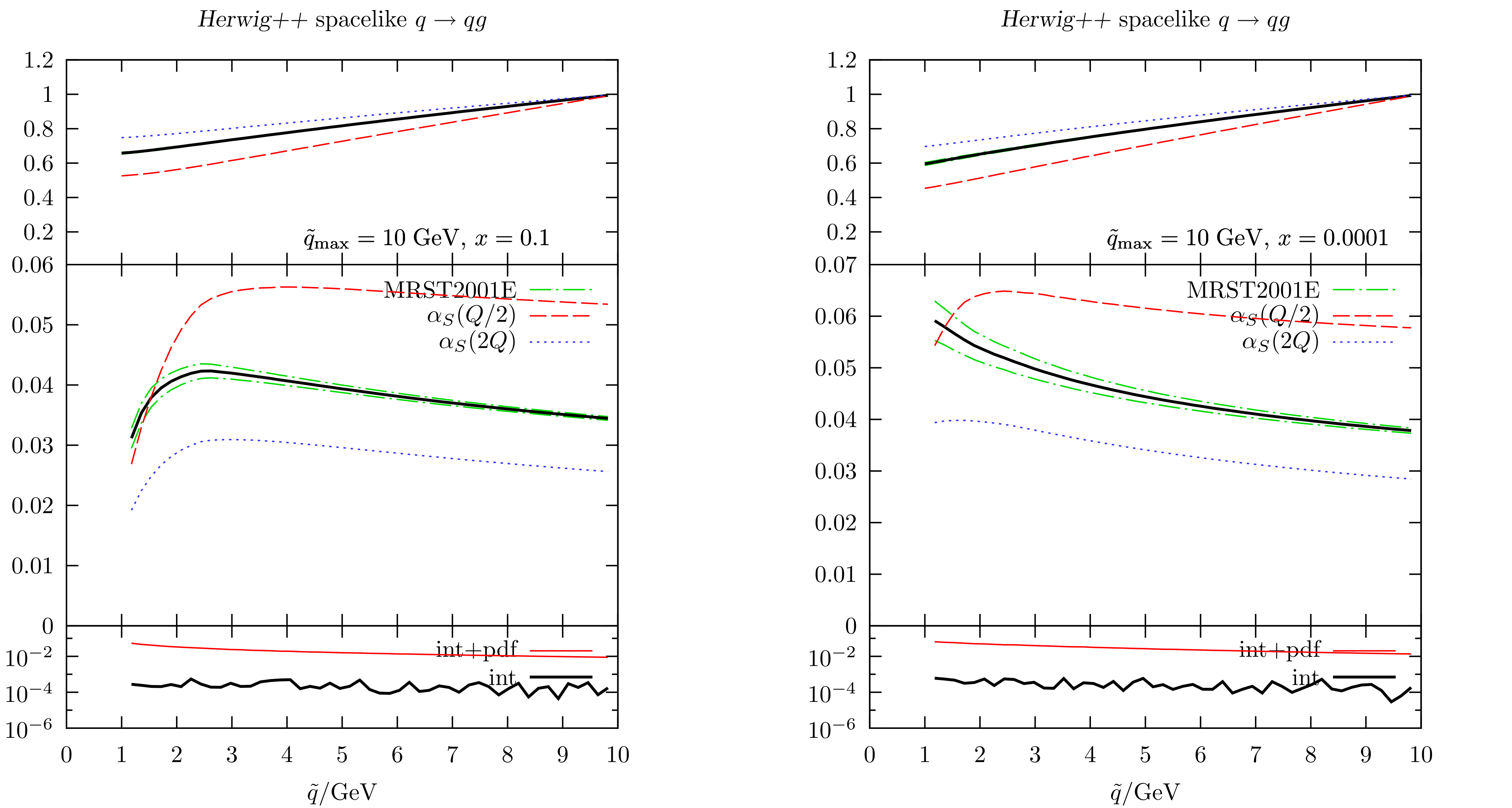
- Sudakov form factor (top panel)
- Branching probability density (middle panel)
- Error information (bottom panel)

$q \to qg$, high $\tilde{q}_{\max}$



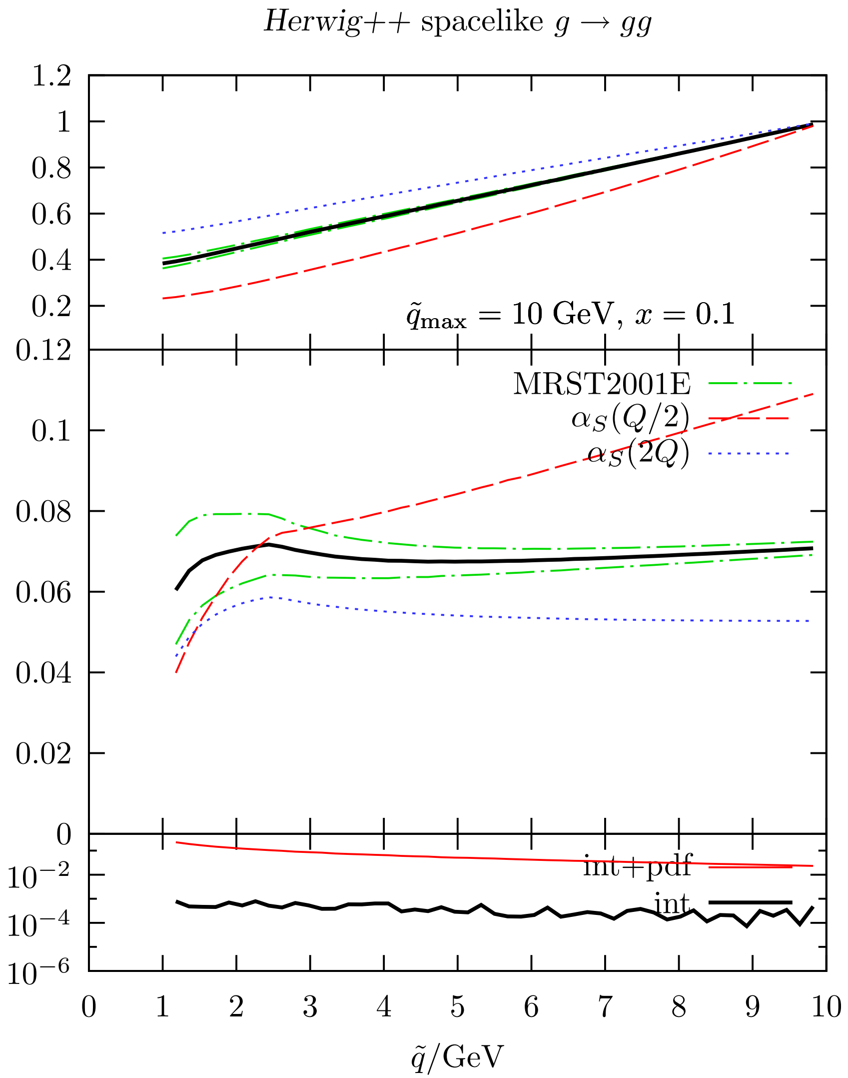
α_S uncertainty clearly dominates the error.

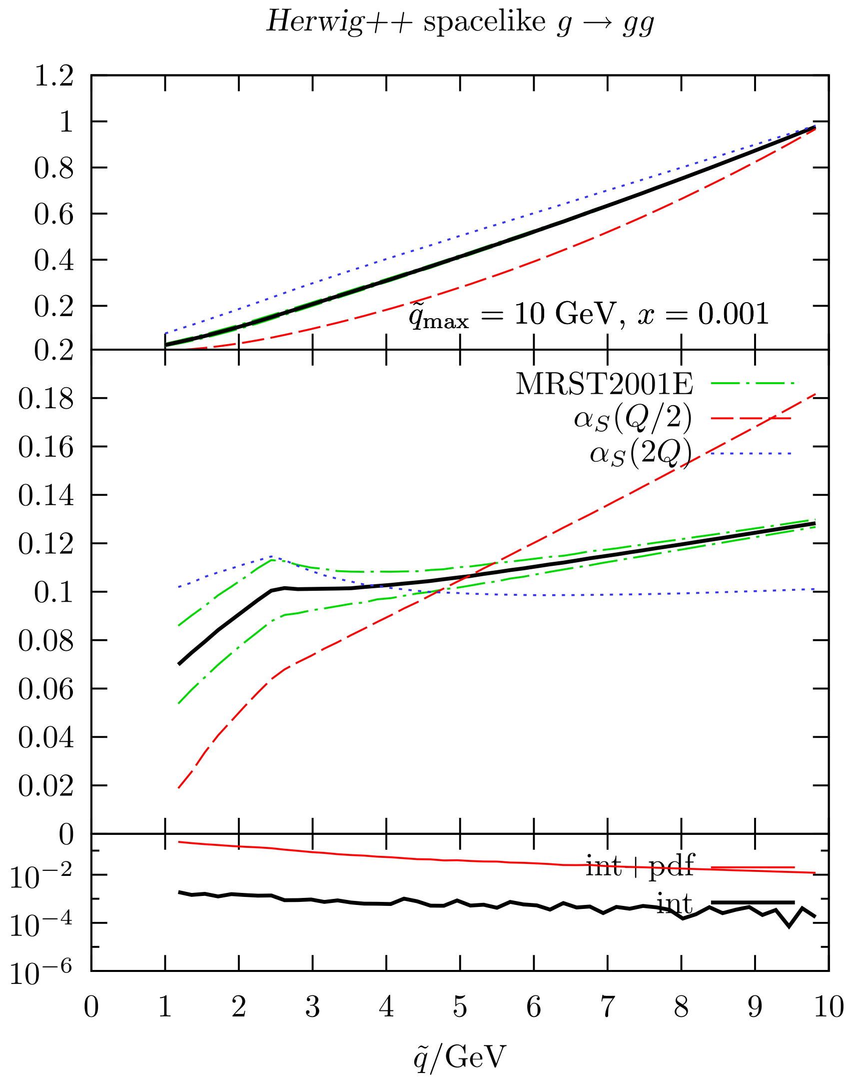
$q \to qg$, low $\tilde{q}_{\max}$



pdf error more significant for small $\tilde{q}_{\max}$ and small x.

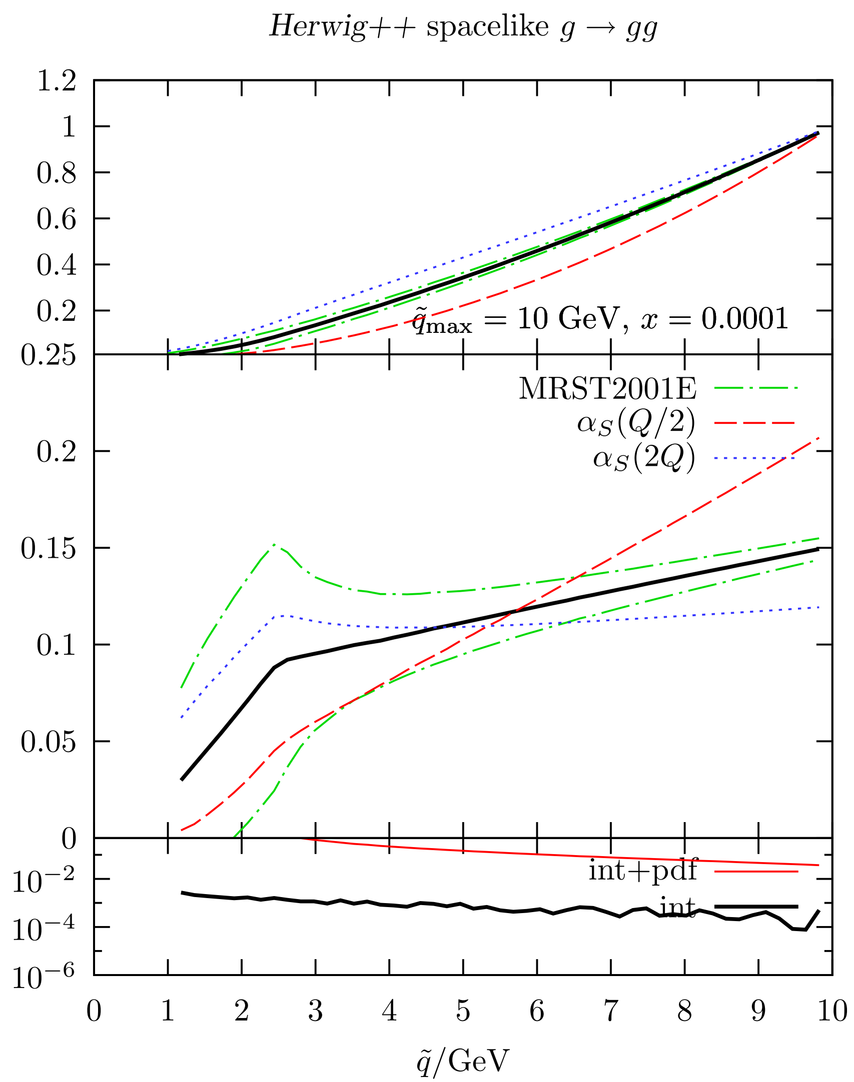
$g \to gg$, low $\tilde{q}_{\max}$

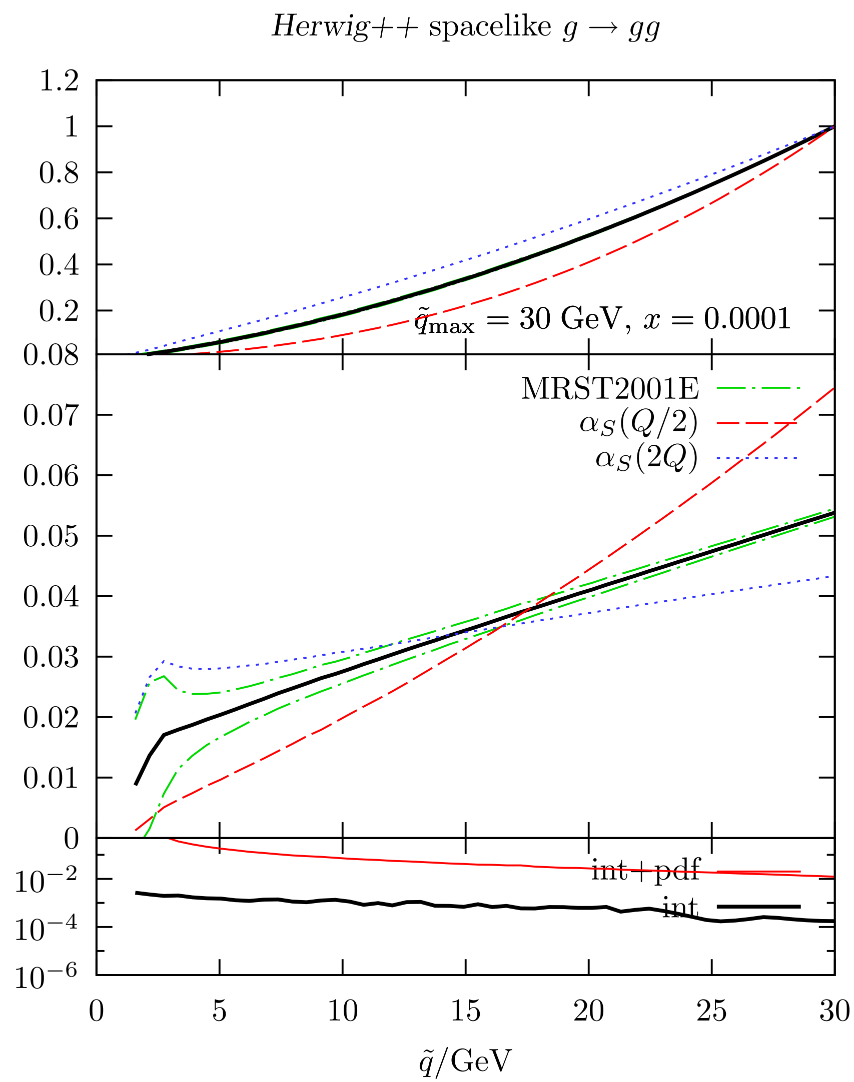




Gluon also uncertain at large x.

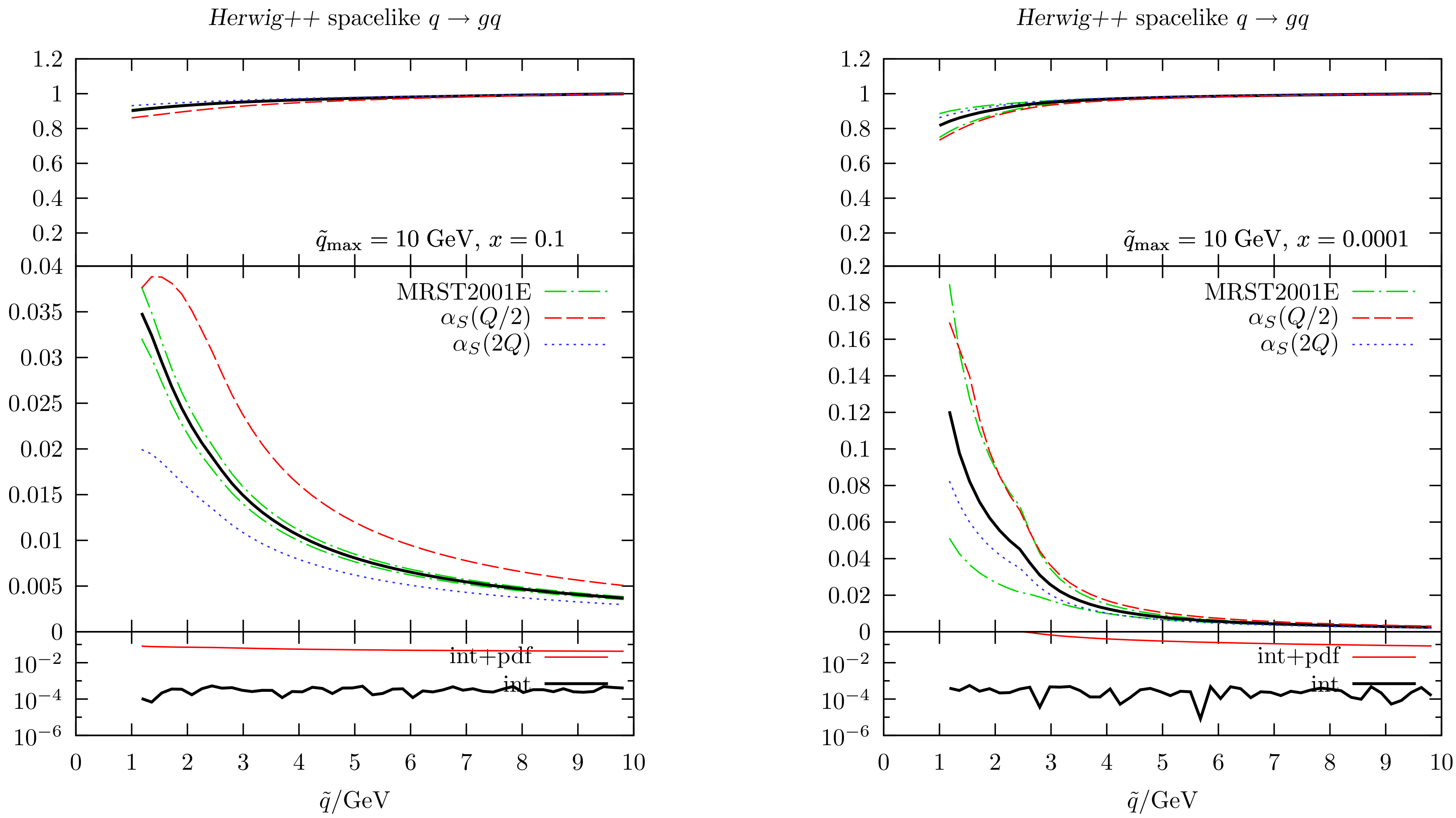
$g \to gg$, lower x, higher $\tilde{q}_{\max}$





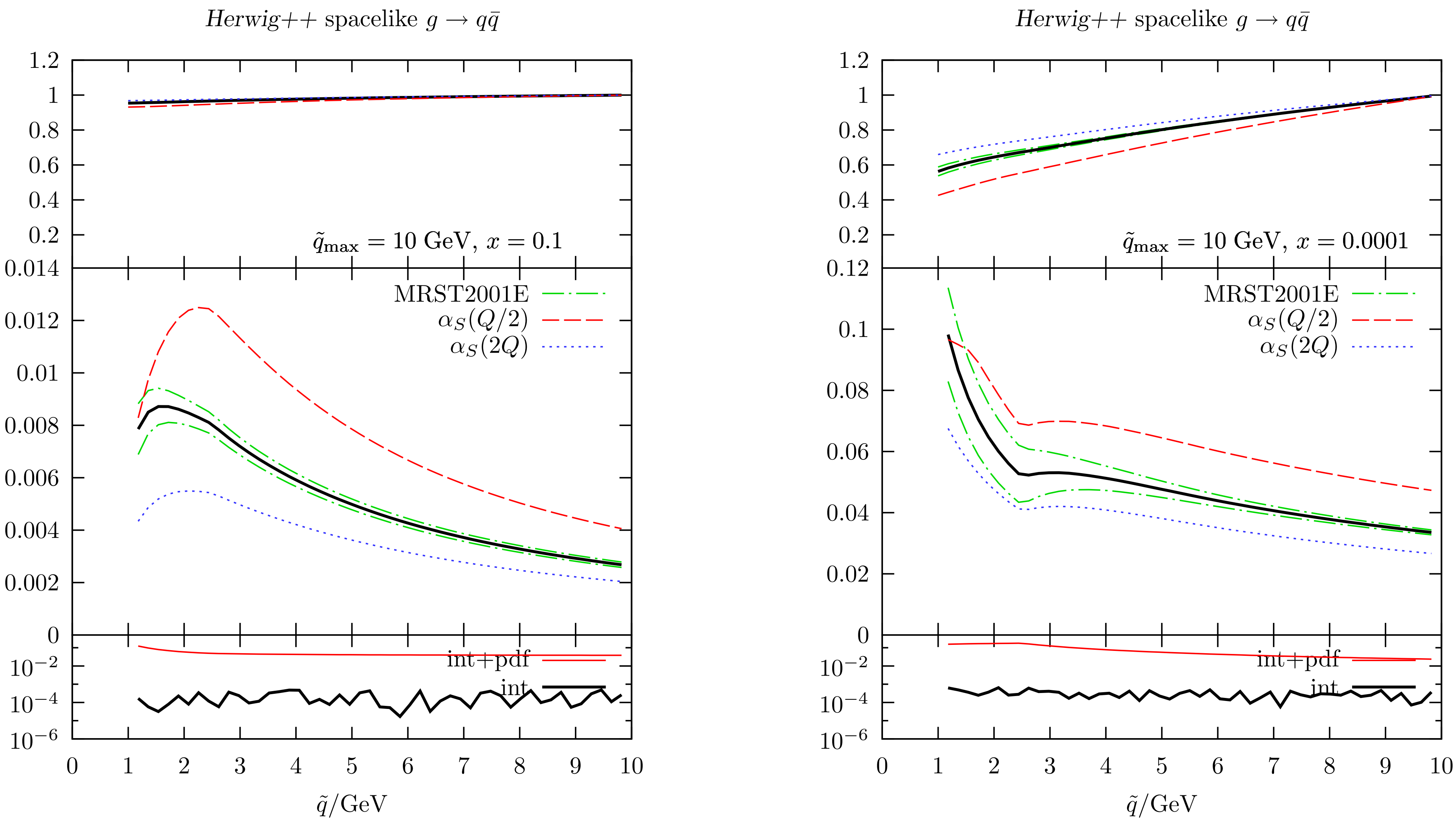
Quite sizable at very small x. Shrinks again for larger $\tilde{q}_{\max}$.

$q \to gq$



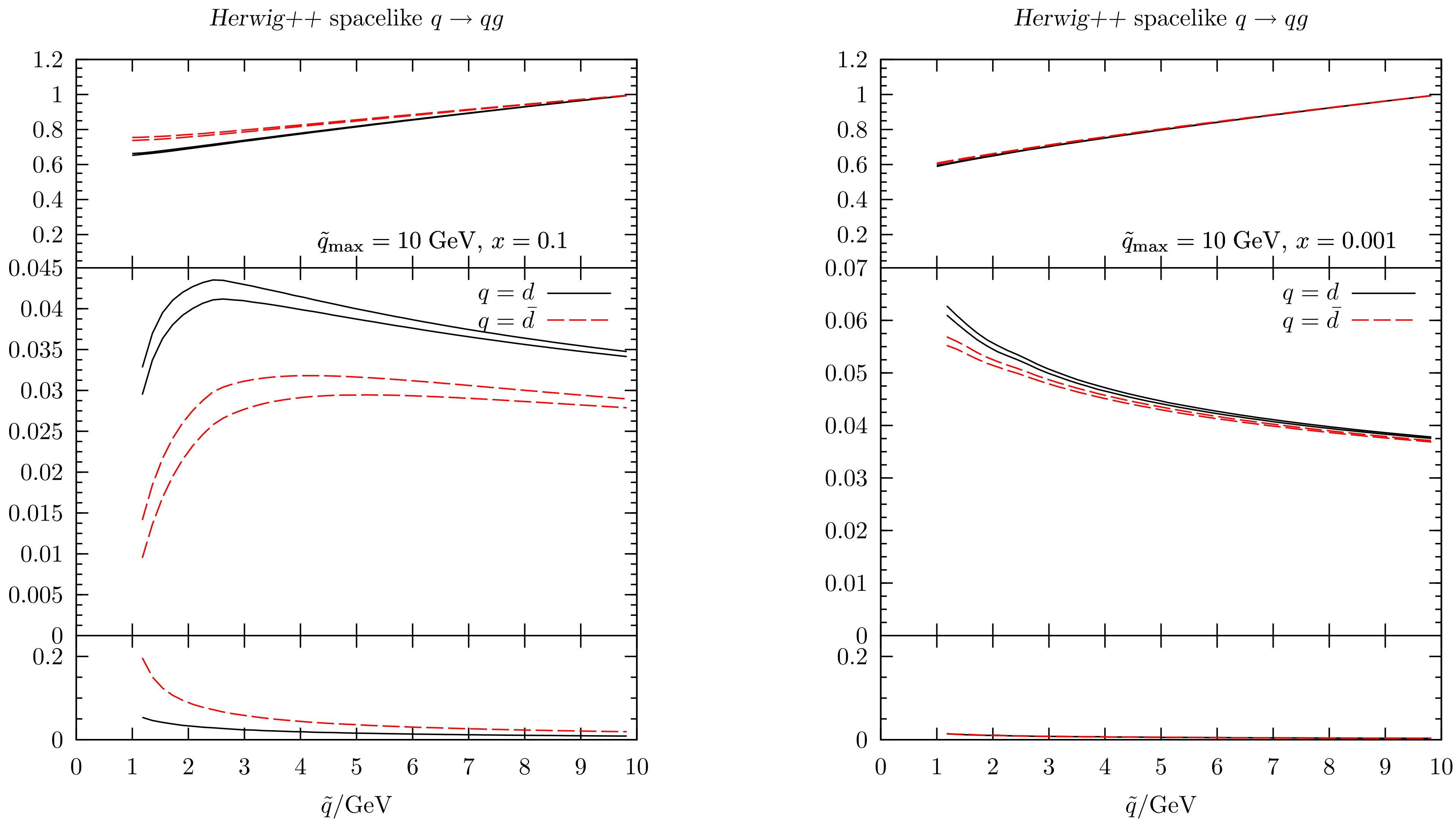
Unlikely type of branching. Small x more uncertain.

$g \longrightarrow q\bar{q}$



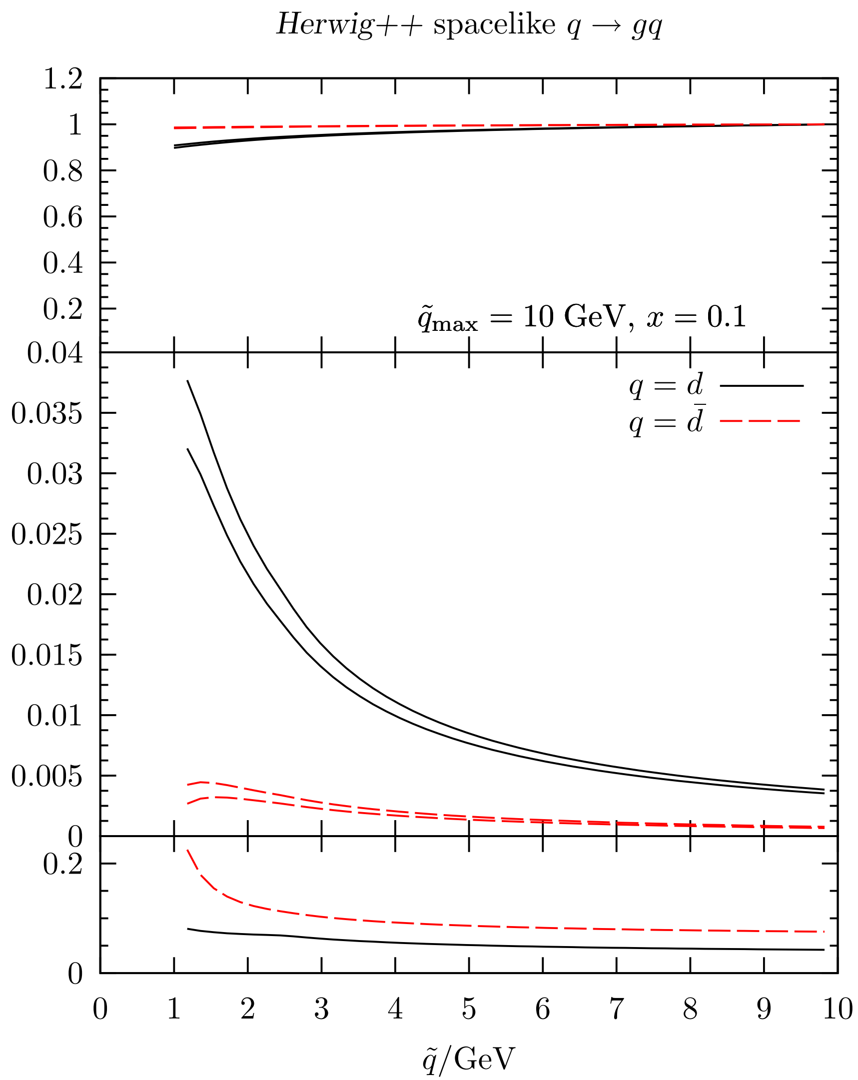
Branching *and* uncertainty become more important at smaller x.

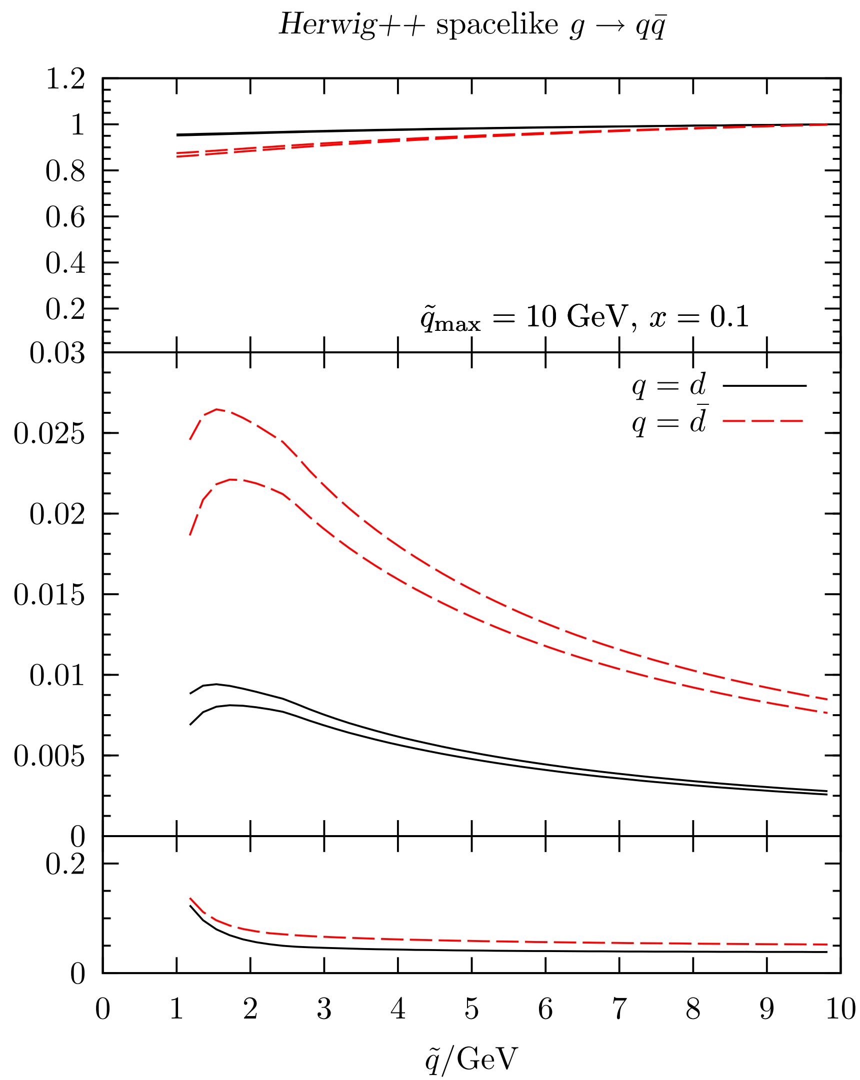
$q \longleftrightarrow \bar{q}$ (1)



Valence $\longleftrightarrow$ sea. Valence wants to remain intact.

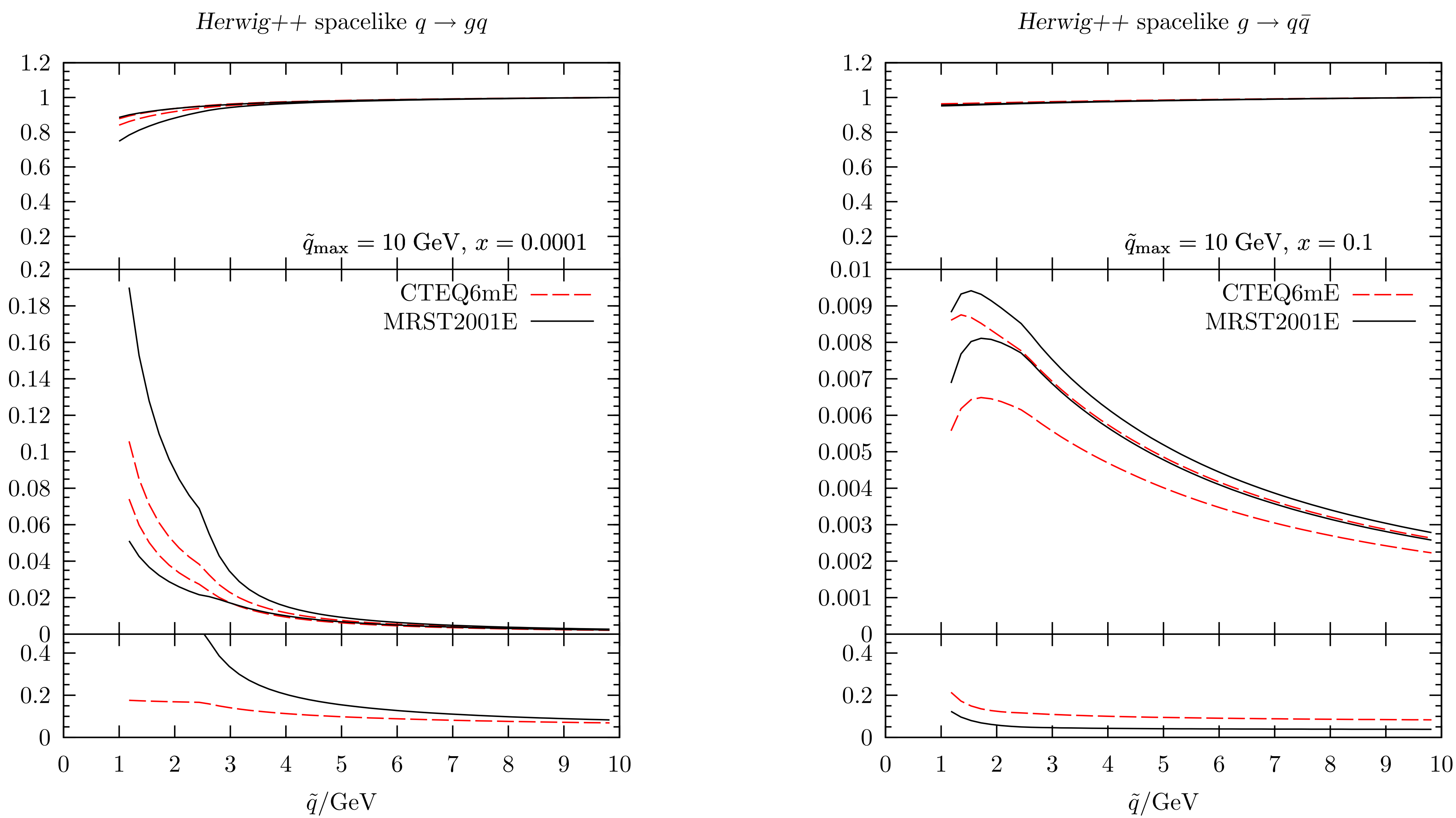
$q \longleftrightarrow \bar{q}$ (2)





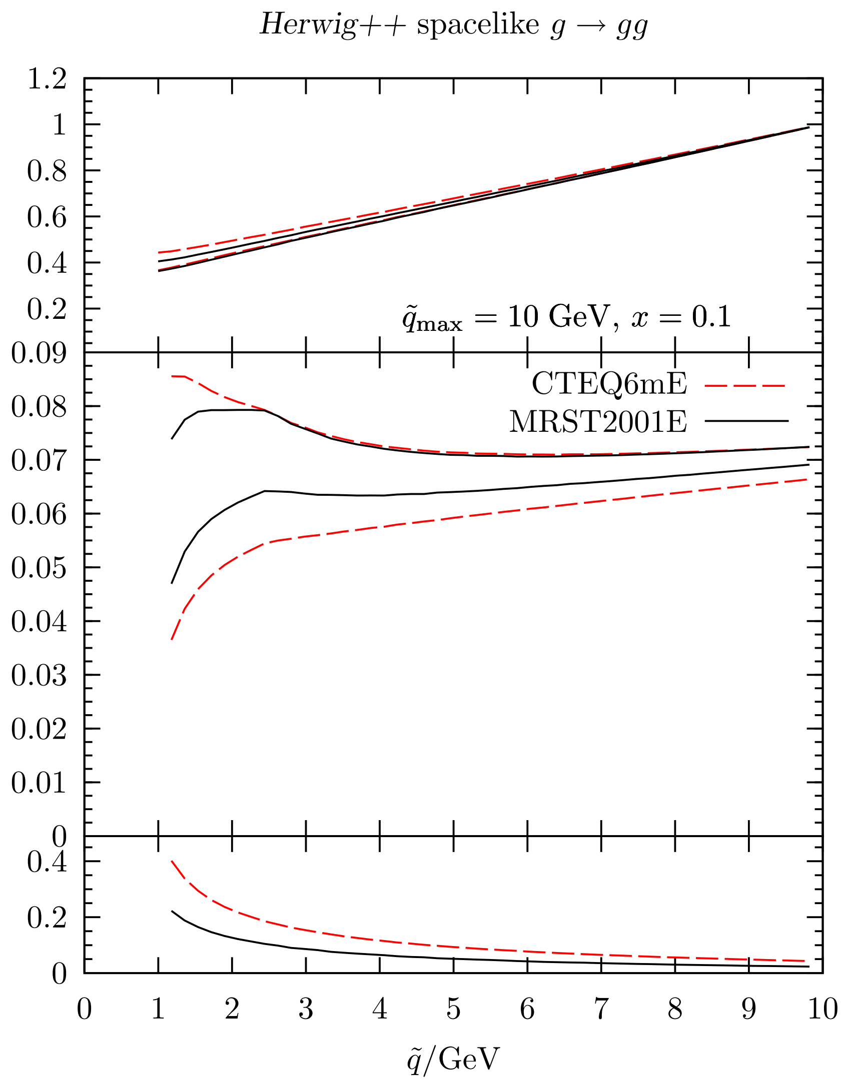
In both cases: pdf error more sizable for sea, esp at large large x.

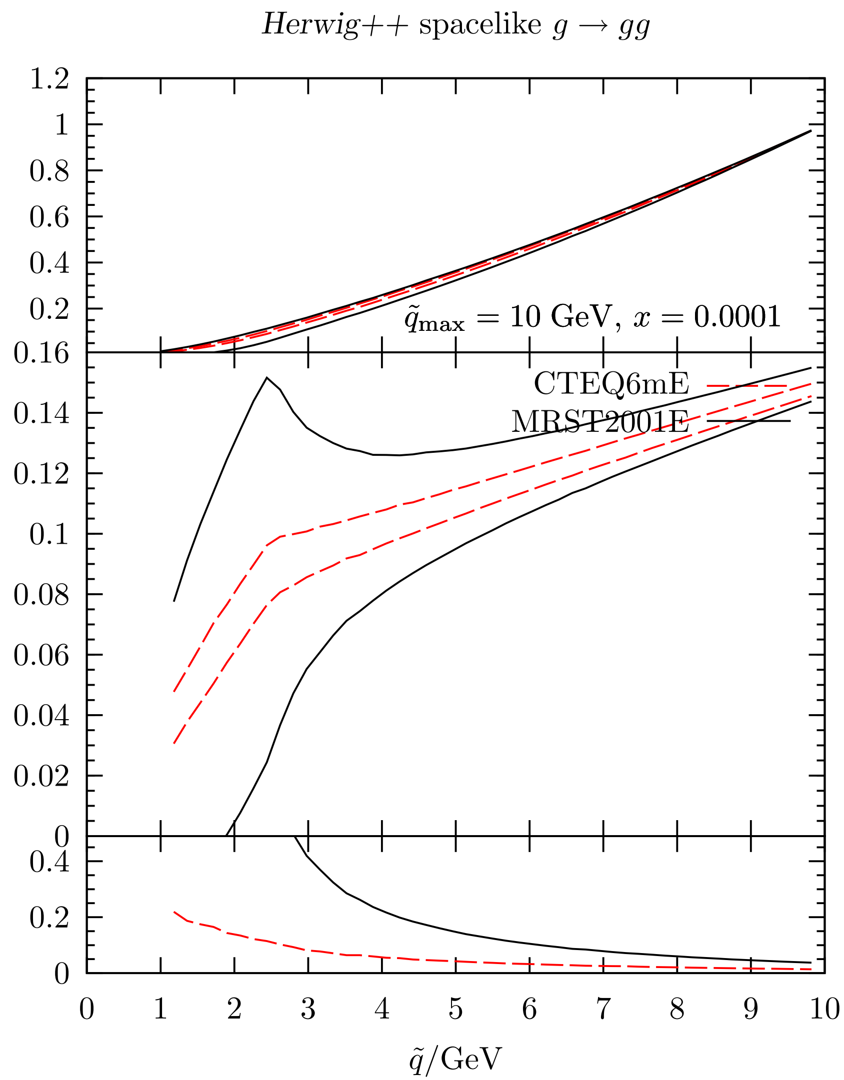
MRST vs CTEQ errors (1)



Some differences in error estimate. No contratictions.

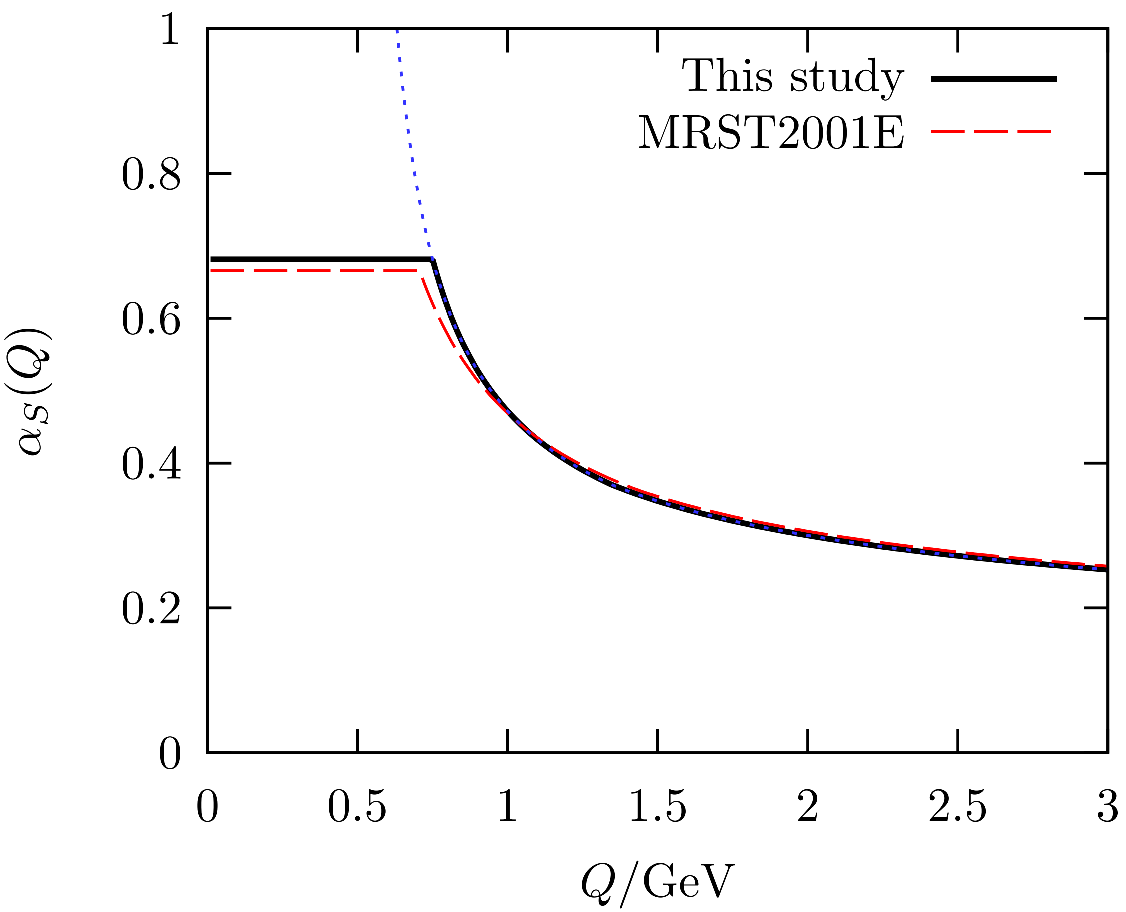
MRST vs CTEQ errors (2)





MRST seem to be slightly more conservative.

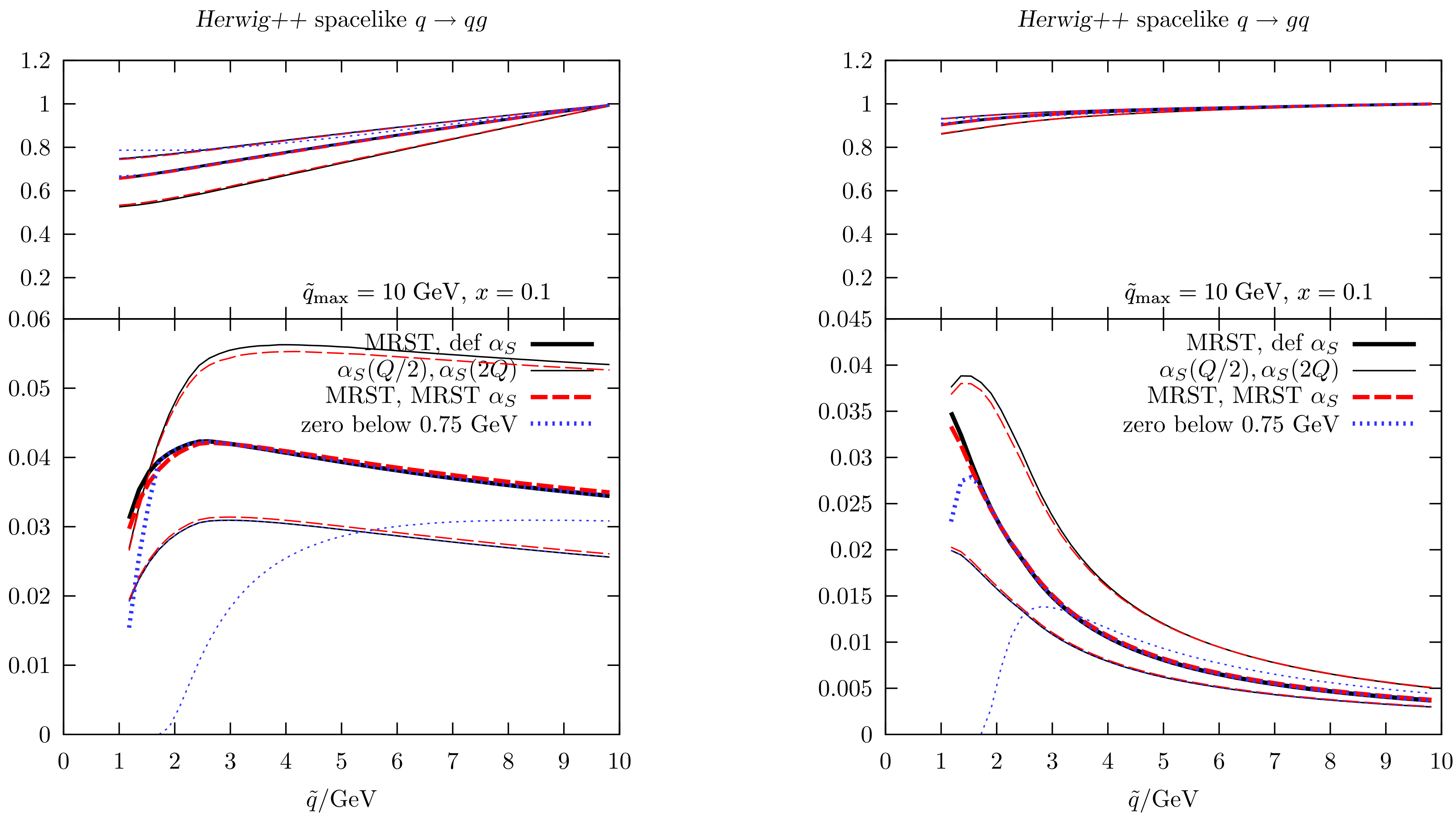
Non–perturbative α_S



Consider

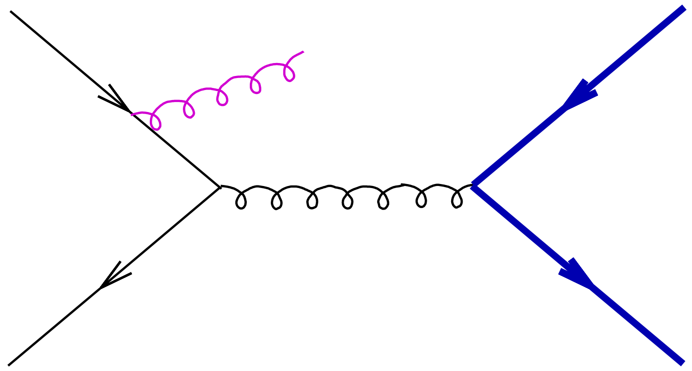
- Different parametrizations of α_S.
- Freeze below $Q_0 = 0.75$ GeV
- Zero below $Q_0 = 0.75$ GeV

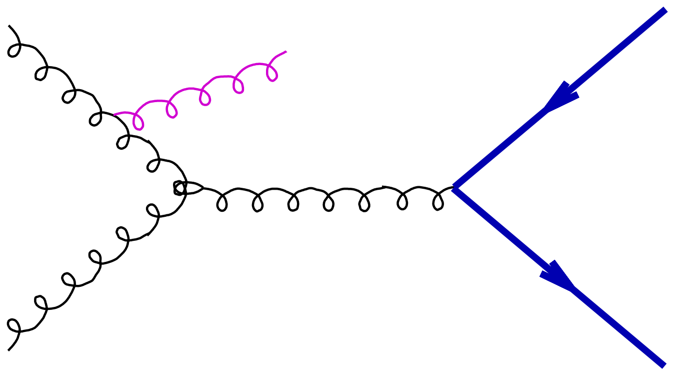
NP α_S effect



Modelling NP region may be important!

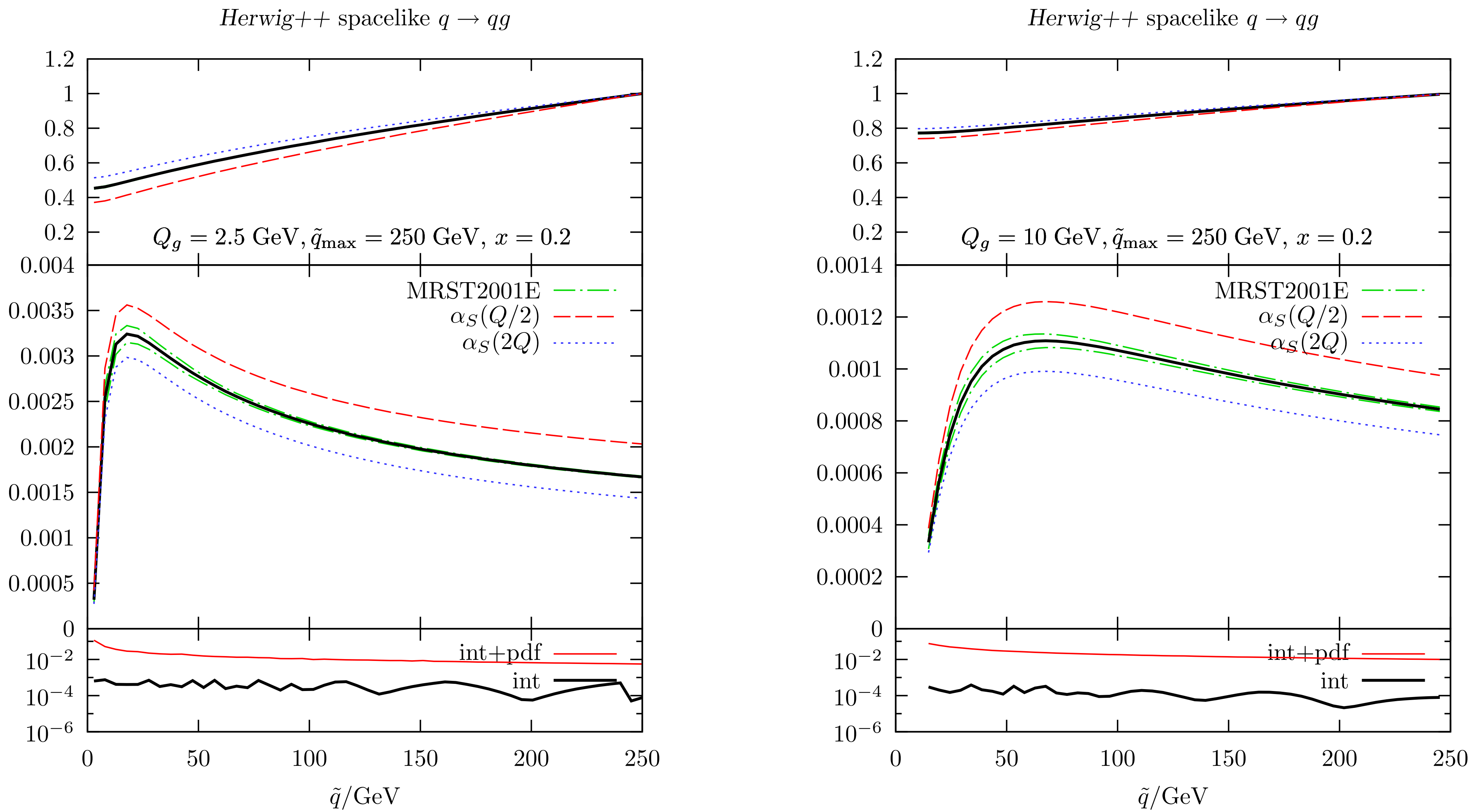
$t\bar{t}$ production





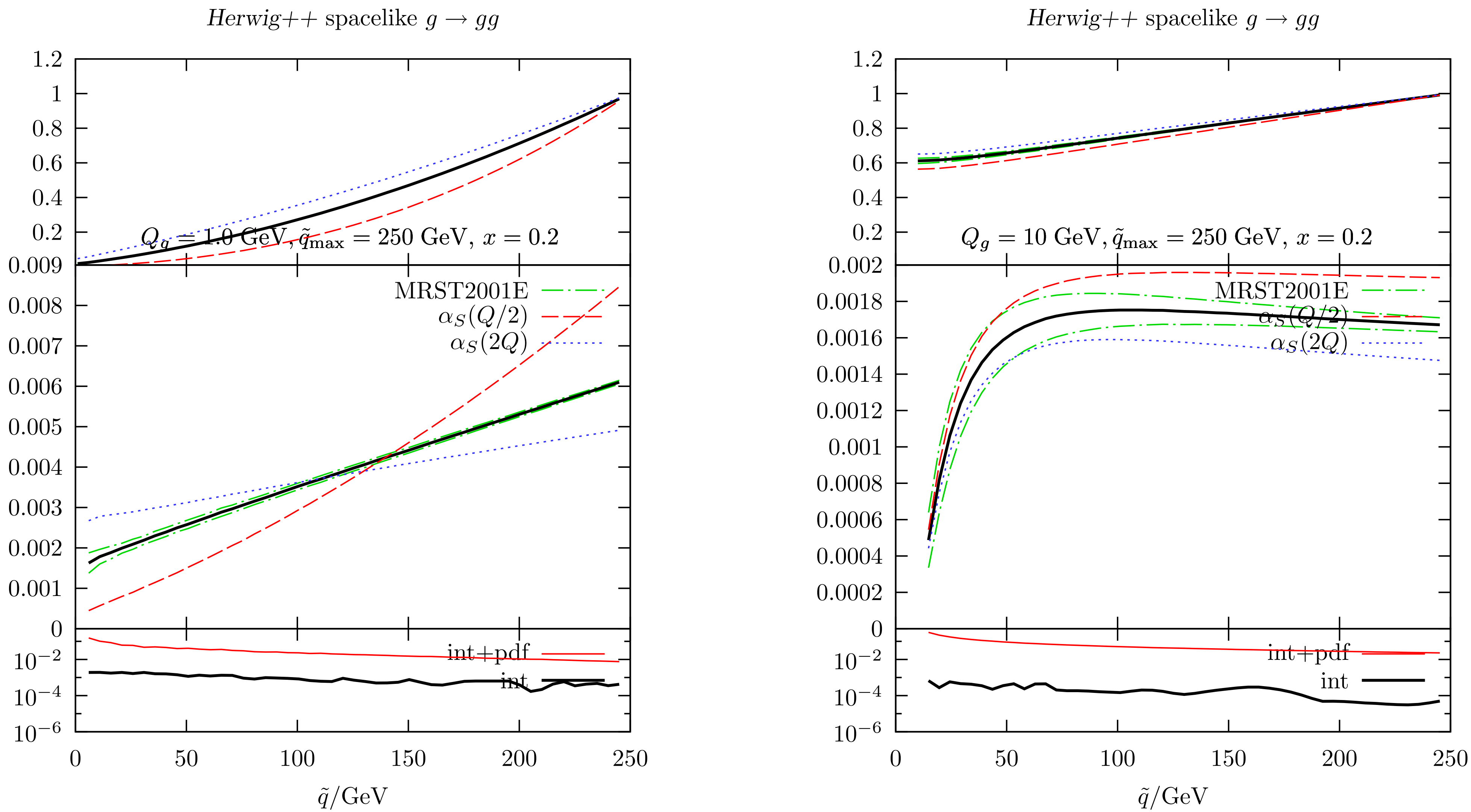
- pdf uncertainties in initial state radiation.
- Sudakovs with cutoff Q_g should give estimate of probability for extra jets with $E_T \sim Q_g$.
- $\rightarrow$ look at high $\tilde{q}_{\max}$, $x \sim 0.2, 0.3$.

$t\bar{t}$ kinematics, $q \to qg$



pdf uncertainties small at large x (considered valence quarks).

$t\bar{t}$ kinematics, $g \to gg$



Sizable for gluons.

Some remarks/conclusions

- Sudakov FF can be a useful tool for understanding certain effects.
- effect of pdf errors mostly small.
- *BUT* can be large in places ☛ $t\bar{t}$?
- all compared to α_S scale uncertainties.
- very sensitive to non–perturbative α_S and scale variations in general (used for tuning. . .).
- NLL effects may be interesting to look at in greater detail?!