Axion Academic Training
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Coherence with applications to Axions.

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- **Definition**
- **Coherence in lasers**
- **Axion photon coherence/oscillations**
- **Applications**

Coherence

1. The quality or state of cohering, especially a logical, orderly, and aesthetically consistent relationship of parts!
2. The property of being coherent, as of waves!

3. The distance over which interference will occur. The coherence length of an optical source is affected by the size of the source, spatial coherence, the phase purity of the source, temporal coherence, and the spectral bandwidth of the light.

Temporal/Spatial Coherence

- A characteristic of laser output, calculated by dividing the speed of light by the linewidth of the laser beam. The temporal coherence length of different lasers thus varies from a few centimeters to many meters and longer.

- The maintenance of a fixed-phase relationship across the full diameter of a cross section of a laser beam.

Degree of coherence

For lightwaves, the magnitude of the degree of coherence is equal to the visibility, V, of the fringes of a two-beam interference test, as given by

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

where $I_{\max}$ is the intensity at a maximum of the interference pattern, and $I_{\min}$ is the intensity at a minimum.
Light is considered to be highly coherent when the degree of coherence exceeds 0.9, partially coherent for values less than 0.9 but more than nearly zero values, and incoherent for nearly zero values.

Axion/photon oscillations in a magnetic field *B* with length *L*

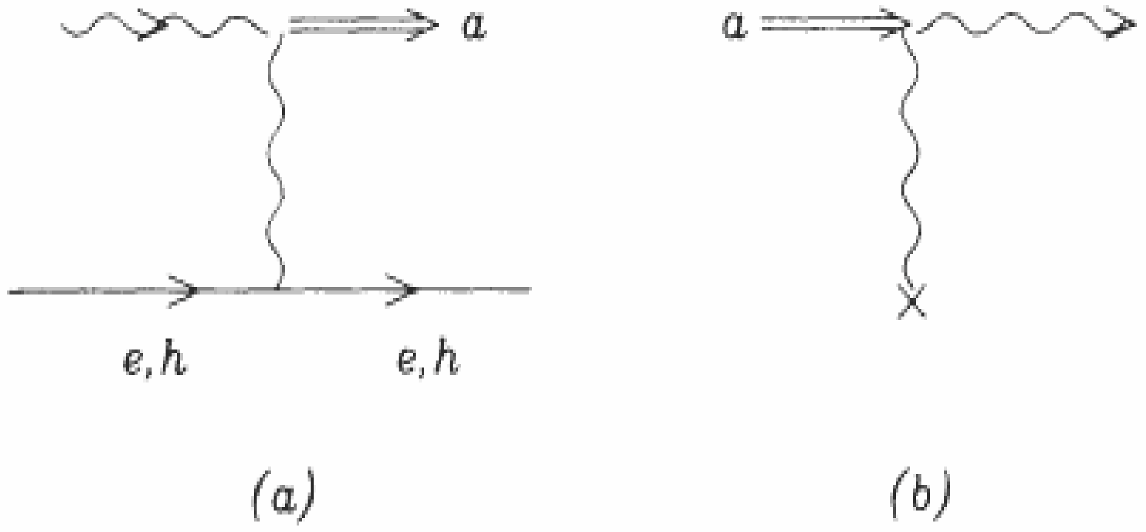


FIG. 1. (a) Axion production by the Primakoff effect. (b) Axion conversion to a photon by the same process.

Axion to photon conversion probability

$$P_{a\to\gamma} = \left(\frac{Bg_{a\gamma\gamma}}{2}\right)^2 L^2 \frac{\sin^2(qL/2)}{(qL/2)^2}$$

With $q = m_a^2/2\omega$, is the momentum transfer and m_a, ω the axion mass and energy respectively.

The coherence length is defined by

$$qL < \pi \Rightarrow L < \frac{2\pi\omega}{m_a^2}$$

Question:

- How is the momentum transfer q balanced?
- By the magnetic field, over the De Broglie length

$$d\lambda = h/q$$

All the participating virtual photons along this wavelength take part in the process coherently, hence the L^2 effect, similar to the coherent Compton scattering…

- Also the effect is proportional to the energy density in the magnet, hence the B^2 dependence…

Axion-photon conversion in gas

- For wave-numbers k_ω and k_a, we have coherence when (attention, different units!)

$$L\left|k_\omega - k_a\right| < \pi, \text{ with}$$

$$k_\omega = \frac{\omega}{c} n = \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{2\omega^2}\right] \text{ and}$$

$$k_a = \frac{pc}{\hbar c} = \frac{\sqrt{(\hbar\omega)^2 - m_a^2}}{\hbar c} \approx \frac{\omega}{c}\left[1 - \frac{m_a^2}{2(\hbar\omega)^2}\right]$$

$$\left(\frac{\omega_p^2}{2\omega} - \frac{m_a^2}{2\omega\hbar^2}\right)\frac{L}{c} < \pi \;\Rightarrow L < \frac{2\pi\omega}{m_a^2}, \text{ when } \omega_p = 0, \text{ and } \hbar = c = 1$$

Hence, (after some algebra) for coherence requirements we get

$$\omega_p = \omega\left(\frac{m_a}{\hbar\omega}\right)\left[1 \pm \frac{\pi}{kL}\left(\frac{\hbar\omega}{m_a}\right)^2\right]$$

$$k = \frac{\omega}{c} = \frac{4.5\times10^{18}\,\mathrm{s}^{-1}}{3\times10^{10}\,\mathrm{cm/s}} = 1.5\times10^{8}\,\mathrm{cm}^{-1}$$

for $\hbar\omega = 3\mathrm{keV}$

Example with ^{4}He gas

for $m_a = 1\text{eV},\ \omega_p = 1.5\times 10^{15}\,\text{s}^{-1}\cdot(1\pm 0.0002)$

$\omega_p = \omega_p^{STP}\sqrt{p},\ \ldots\ p = 13.4\text{Atm, at 293K}$

Then at 1.8K, $p_{1.8K} = \dfrac{1.8}{293}p = 0.082\cdot(1\pm 0.0004)\ \text{Atm}$

Generalized for any m_a

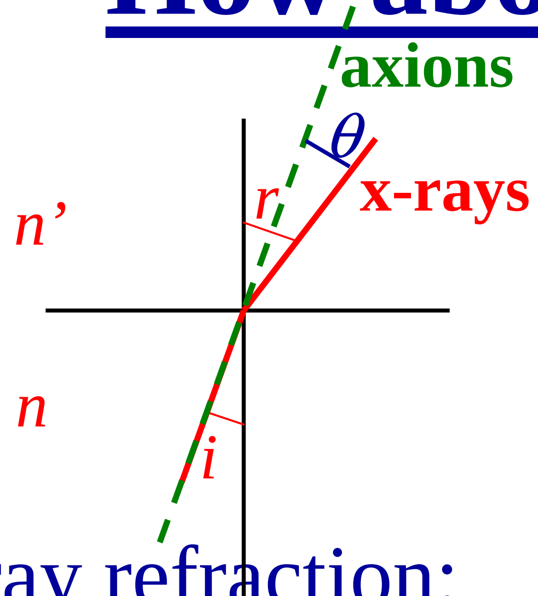
$$p_{1.8K} \approx 80\text{mb}\left(\frac{m_a}{1\text{eV}}\right)\left[1\pm 0.0004\left(\frac{1\text{eV}}{m_a}\right)^2\right]$$

for $m_a = 1\text{eV},\quad \delta T = 0.7\text{mK},\quad \text{or } 0.04\%$

$m_a = 0.25\text{eV},\quad \delta T = 12\text{mK},\quad \text{or } 0.6\%$

$m_a = 0.1\text{eV},\quad \delta T = 70\text{mK},\quad \text{or } 4\%$

How about spatial coherence?



$$\frac{\sin i}{\sin r} = \frac{n'}{n}$$

x-ray refraction:

1. The x-rays/axions do not overlap
2. The x-rays travel longer paths than axions

The coherence is lost…

The important parameter here is pressure uniformity in cross section…

A numerical example

For a 10m long magnet and axion/photon missing each other by 1 wavelength

$$\theta \le 25\times 10^{-12}\, rad$$

$$\frac{\sin i - \sin r}{\sin r} = \frac{n' - n}{n} \Rightarrow \frac{\Delta n}{n} = 27\times 10^{-12},\ \text{for } i = \frac{\pi}{4}$$

$$n = 1 - \frac{\omega_p^2}{2\omega^2} = 1 - \frac{1}{2}\left(\frac{1.5\times 10^{15}\,\text{s}^{-1}}{2\pi\times 7.2\times 10^{18}\,\text{s}^{-1}}\right)^2 \Rightarrow n = 1 - 5.5\times 10^{-8}$$

$$\Delta n = 5.5\times 10^{-8}\left(\frac{\Delta T}{1.8\text{K}}\right)$$

$$\ldots \Delta T \le 1\text{mK}\left(\frac{1\text{eV}}{m_a}\right)$$

Two important consequences

- Pressure (temperature) uniformity requirements are strict with gas buffer. Can use a laser light to probe the uniformity, e.g. 5mK temperature gradient in the cross section, along the whole length of the magnet can produce up to 4mm shift on a red laser light…(effect goes as $(\lambda_1/\lambda_2)^2 = 2000^2$).
- Assuming multiple refractions axion to photon conversion in stars (e.g. sun) does not suffer completely from short coherent lengths but lengths could add linearly over long distances…

Consider the picture:

- Axions produce the photon field in phase, up to coherence length
- Beyond that length axions lag behind by π due to their mass. When the phase of the "new" photon field is 180^{o} out of phase it adds destructively and the total photon field is zero!
- …what happened to the energy conservation?

Actually photons and axions mix…

- A transverse magnetic field is necessary
- The mixing angle is

G. Raffelt and L. Stodolsky PRD 37 (1988) 1237.

$$\phi \approx \frac{B g_{a\gamma\gamma}/2}{-\left(\omega_p^2 - m_a^2\right)/2\omega}$$

$$P_{a\to\gamma} = \left(\frac{B g_{a\gamma\gamma}}{2}\right)^2 L^2 \frac{\sin^2\left(qL/2\right)}{\left(qL/2\right)^2}$$

- The oscillation length L is $qL = 2\pi \Rightarrow L = \frac{4\pi\omega}{m_a^2}$

For axion and photon planewaves

$a = ae^{i\left(\omega t - \vec{k}\cdot\vec{r}\right)}, A = Ae^{i\left(\omega t - \vec{k}\cdot\vec{r}\right)}$ and small mixing angle ϕ

$$\begin{bmatrix} A_p(L) \\ a(L) \end{bmatrix} = R(L) \begin{bmatrix} A_p(0) \\ a(0) \end{bmatrix},$$

with

$$R(L) = \begin{pmatrix} 1 + \left[iqL - \left(1 - e^{-iqL}\right)\right]\phi^2 & \phi\left(1 - e^{-iqL}\right) \\ \phi\left(1 - e^{-iqL}\right) & e^{-iqL} + \phi^2\left[\left(1 - e^{-iqL}\right) - iqL\right] \end{pmatrix}$$

For solar axions

$a(0)=1,\ A(0)=0$ (normalized to 1), then

$$A_p(L)=\phi\left(1-e^{-iqL}\right)=\left[\phi-\phi\cos qL\right]+i\phi\sin qL$$

The number of photons are

$$\left|A(L)\right|^2=2\phi^2\left(1-\cos qL\right)=\left(2\phi\right)^2\sin^2\frac{qL}{2}$$

The number of axions are

$$1-\Re\left\{R_{22}\right\}=1-\left(2\phi\right)^2\sin^2\frac{qL}{2}$$

For axion Production in a magnetic field with a laser light (e.g. PVLAS or E840 @BNL)

$a(0) = 0, A(0) = 1$ (normalized to 1),
then the attenuation of the component
parallel to the magnetic field is

$$\delta = 1 - \Re\left\{R_{11}(L)\right\} = \phi^2\left[1 - \cos qL\right] = 2\phi^2 \sin^2\frac{qL}{2}$$

<u>Summary</u>

- Coherence is a delicate matter and special attention needs to be taken

- Axion/photon coherent oscillations happen in magnetic fields as long as they travel together in time and in space